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Empirical relation between the energy of the topmost particle and the mean binding energy of nuclei*

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On the basis of experimental knowledge, it is shown that separation energy thresholds cannot be interpreted as experimental single-particle energies, and that the energy of the topmost particle in conventional nuclear single-particle potentials is smaller in absolute magnitude than the mean binding energy. Connections with Hugenholtz and Van Hove's theorem are briefly discussed.

Usually, hole spectra exhibit a peak at separation thresholds.[1] The corresponding separation energies (with their signs reversed) are the energies needed to remove a particle from a nucleus, leaving the residual nucleus in its lowest state. Despite its intuitive appeal, the identification of separation energies and binding (or single-particle) energies has been criticized by Meldner[2] as an unjustified procedure. His argument, however, is model-dependent. The purpose of this note is to verify his assertion on the basis of experimental knowledge, and thereby to derive an important inequality between the conventionally defined single-particle energy of the last particle and the mean binding energy of heavy nuclei.

The important empirical fact relevant to our argument is the systematic trend[3] observed in nuclei that the conventionally defined single-particle potential for individual nucleons becomes stronger as the mass number A increases. This is typically reflected in the well-known empirical relation $\hbar\omega \simeq 40A^{-1/3}$ MeV, where ω is the harmonic oscillator frequency. This trend indicates that when a particle is removed the individual binding field shifts *upwards*. This upward shift requires energy. Therefore, to remove a particle it is not sufficient to supply an energy equal to the binding energy of this particle; we must supply an extra energy, which is needed to shift the remaining particles upwards. This extra energy is what is called the "rearrangement energy."[4] As a result, the total energy required to remove a particle in the single-particle state i, which energy is the separation energy $S(i)$, corresponds to the binding energy of some other particle in a lower state:

$$-S(i) < E(i), \tag{1}$$

where $-E(i)$ is the binding energy of the particle in i. Hence, as claimed by Meldner, it is wrong to interpret $-S(i)$ as "experimental" single-particle energies for hole states.

It is also found[5] experimentally that for heavy nuclei $S(F)$, where F denotes the highest occupied single-particle state, is systematically smaller than the mean binding energy $-E_{av}$; for $A > 200$, $S(F)$ is of the order of 5.5 to 6 MeV, whereas E_{av} is of the order of -7.5 MeV.[5,6] Combining this fact with (1), we obtain

$$E(F) > E_{av} \quad \text{for heavy nuclei}. \tag{2}$$

It is noted that this inequality is expected to hold even for infinite nuclear matter. For, as $A \to \infty$, $-S(F)$ and E_{av} will become equal so as to satisfy the saturation condition, $-S(F) = (A+1)E_{av} - AE_{av}$. The upward shift of each binding field also becomes vanishingly small, as the above-mentioned trend[3] indicates. However, the rearrangement energy is governed by $A \times$ (upward shift). Therefore, the rearrangement energy will remain finite as $A \to \infty$[7]; $E(F)$ will not approach $-S(F)$. It should be pointed out that our result (2) does *not* contradict Hugenholtz and Van Hove's theorem.[8] The energy of the topmost particle in their argument is given by $(\partial E_0/\partial A)_\Omega$, where E_0 and Ω are the total energy and the volume of the system, respectively. By definition, this quantity is essentially equal to $-S(F)$. Therefore, the theorem should be stated as "$-S(F) = E_{av}$," instead of $E(F) = E_{av}$ as stated in Ref. 8, which is just the saturation condition. Note that the distinction between the conventionally defined single-particle energy and the separation energy is one of the points made in the present paper. Recently, in the framework of modern Brueckner theory,[9] different definitions have been introduced, equating real parts of (generally) complex single-particle energies and separation thresholds.

Since our inequality (2) is an empirical relation, it can be used as a test on theoretical relationships between the conventional single-particle energy and the total binding energy of heavy nuclei; contrary to their intention in Ref. 8, Hugenholtz and

Van Hove's theorem does not serve this purpose because of their special way of defining the single-particle energy, as discussed above. Particularly noteworthy is that existing nuclear-matter calculations[10] with realistic nucleon-nucleon interactions fail to satisfy (2). The case of heavy (but finite) nuclei involves further complications.[9] These problems will be treated in a separate paper.

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[3]See, for example, A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, p. 240.

[4]There exist numerous papers on this subject. See, in particular, K. A. Brueckner, Phys. Rev. 97, 1353 (1955), Sec. III, and Ref. 1 above.

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