

## Study of the lowest-lying $4^-$ states in $^{90}\text{Zr}$ and $^{92}\text{Zr}^\dagger$

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Using particle  $\gamma$ -coincident techniques, an upper limit of about  $8 \mu\text{b}/\text{sr}$  is placed on the forward angle cross section for the excitation in  $(p, p')$  at  $E_p = 17.2 \text{ MeV}$  of the  $4^-$  state in  $^{90}\text{Zr}$  at  $2.738 \text{ MeV}$ . This upper limit is consistent with distorted-wave Born-approximation calculations using realistic interactions and reasonable wave functions. A  $4^-$  state in  $^{92}\text{Zr}$  is identified at  $2.740 \pm 0.010 \text{ MeV}$  through the reaction  $^{93}\text{Nb}(d, ^3\text{He})^{92}\text{Zr}$  at  $E_d = 17.2 \text{ MeV}$ .

NUCLEAR REACTIONS  $^{90}\text{Zr}(p, p'\gamma)$ ,  $^{93}\text{Nb}(d, ^3\text{He})$ ,  $E_p = 17.2 \text{ MeV}$ ,  $E_d = 17.2 \text{ MeV}$ ; measured  $\sigma(E_{p'}, \theta)$ ,  $\sigma(E_{^3\text{He}}, \theta)$ ; calculated  $\sigma(E_{p'}, \theta)$  microscopic DWBA;  $^{92}\text{Zr}$  level measured, deduced  $J^\pi$ .

### I. INTRODUCTION

In order to study the effective nucleon-nucleon interaction it is necessary to separate effects due to unknown nuclear structure from those due to the projectile-nucleon interaction.  $^{90}\text{Zr}$  with its closed neutron shell and well-known proton structure is especially well suited to this purpose. Studies of the effective interaction in proton inelastic scattering on  $^{90}\text{Zr}$  to the low-lying natural parity states of the  $2p_{1/2}$ ,  $1g_{9/2}$  proton configurations have been carried out at incident energies of  $12.7$ ,<sup>1</sup>  $18.8$ ,<sup>2</sup>  $40.0$ ,<sup>3</sup> and  $61.2$ <sup>4</sup> MeV. Transitions to these states, the  $2^+$  (2.18 MeV),  $4^+$  (3.07 MeV),  $6^+$  (3.45 MeV),  $8^+$  (3.60 MeV) and  $5^-$  (2.32 MeV) states, have been examined. The transition to the unnatural parity  $4^-$  state has not been observed in these works. This state is known to exist at an excitation energy of  $2.738 \text{ MeV}$ <sup>5, 6</sup> and it was not observed in the earlier  $(p, p')$  work presumably due to the proximity of the strongly excited  $3^-$  state at  $2.748 \text{ MeV}$ .

Because of the strong spin-independent central component in the effective interaction, analysis of the excitation to natural parity levels will, in general, be relatively insensitive to the weaker spin-dependent components. The excitation of  $4^-$  states, on the other hand, can proceed only by a "spin flip" transition and hence the direct contribution to this state is sensitive only to the spin-dependent components of the effective interaction.

In this paper, we present the results of our study of the strength with which the  $4^-$  state in  $^{90}\text{Zr}$  is excited in the inelastic scattering of  $17.2 \text{ MeV}$  protons. Our results are compared with distorted-wave Born-approximation (DWBA) calculations of the cross section. (Similar analysis of proton inelastic scattering to low-lying unnatural parity states in  $^{88}\text{Sr}$  is presented in the companion paper.) In addition, a  $4^-$  state of  $^{92}\text{Zr}$  whose proton structure is similar to that of the

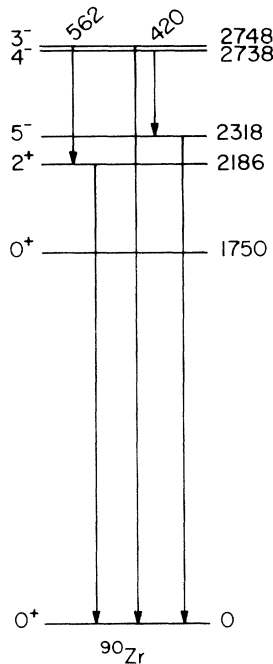
$^{90}\text{Zr}$  state is identified in the reaction  $^{93}\text{Nb}(d, ^3\text{He})$ . The feasibility of studying this state in inelastic scattering is discussed.

### II. EFFECTIVE INTERACTION

The effective interaction in its usual form is written as

$$V(r) = V_0(r) + V_\sigma(r)(\sigma \cdot \sigma) + V_\tau(r)(\tau \cdot \tau) + V_{\sigma\tau}(r)(\sigma \cdot \sigma)(\tau \cdot \tau) + V_{LS}(r) + V_T(r),$$

where  $\sigma$  and  $\tau$  are twice the usual spin and isospin operators  $S$  and  $T$ . Since the population of a  $4^-$  state involves a spin-flip in the direct process, the only central terms which may contribute are the spin-dependent terms  $V_\sigma$  and  $V_{\sigma\tau}$ , and since in the present instance the wave function consists almost purely of proton-proton hole configurations, the combination  $V_\sigma + V_{\sigma\tau}$  dominates the excitation of this state. In general, all central components can contribute to the exchange process; however, if the interaction is completely even or completely odd, the exchange amplitudes from each component in the effective interaction are proportional to the direct amplitude from the same component. The interaction we use here is similar to the Serber force ( $V_0 : V_\sigma : V_\tau : V_{\sigma\tau} = -3 : 1 : 1 : 1$ ), which is an even state force. Hence exchange amplitudes from  $V_0$  and  $V_\tau$  are small compared to amplitudes from  $V_\sigma$  and  $V_{\sigma\tau}$ . The tensor ( $V_T$ ) and spin-orbit ( $V_{LS}$ ) terms also contribute to the  $4^-$  cross section. The contribution to the excitation of the  $4^-$  state of the tensor force is important, but fortunately its strength has been measured, for example, by Austin<sup>7</sup> in the  $^{14}\text{N}(p, p')$  reaction to the  $2.31 \text{ MeV } T=1 \ 0^+$  state, for which the tensor dominates. This tensor strength is energy dependent, and extrapolated to  $17.2 \text{ MeV}$  gives  $V_T = 20 \text{ MeV}$  with a range of  $0.816 \text{ fm}^{-1}$ . The spin-orbit term is that used by Hinrichs *et al.*<sup>3</sup> The range and strength parameters were chosen to

FIG. 1.  $^{90}\text{Zr}$  level diagrams.

match the low momentum components of the corresponding part of the Hamada-Johnston<sup>8</sup> potential.

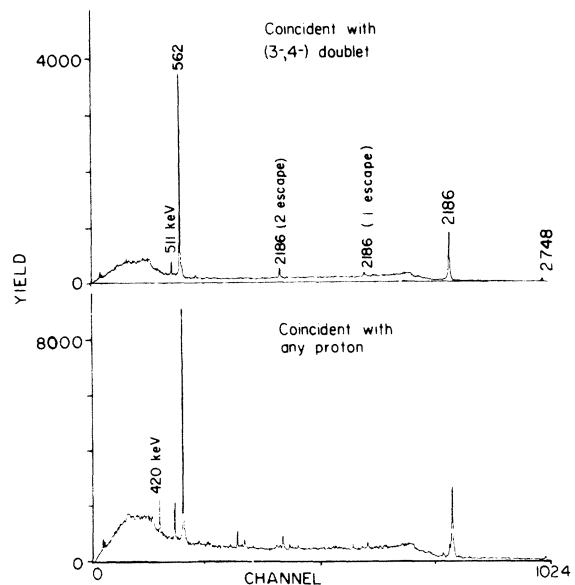
### III. EXPERIMENT

#### A. $4^-$ state in $^{90}\text{Zr}$

The  $4^-$  state in  $^{90}\text{Zr}$  is not easily resolvable in inelastic proton scattering from the strong collective  $3^-$  state at 2.748 MeV excitation. However, it is known<sup>5</sup> that the  $4^-$  state undergoes  $\gamma$  decay exclusively to the  $5^-$  isomeric state at 2.318 MeV, and that the  $3^-$  state undergoes  $\gamma$  decay primarily to the  $2^+$  state at 2.182 MeV, as indicated in Fig. 1. Accordingly we made a measurement of the strengths of the 420 and 562 keV  $\gamma$  rays in coincidence with the excitation in  $(p, p')$  of the doublet at 2.74 MeV.

Data were collected with a solid-state particle detector at  $45^\circ$ , subtending a solid angle of  $\approx 100$  msr, and a Ge(Li) detector at  $90^\circ$ , subtending a solid angle of  $\approx 500$  msr. The 17.2 MeV protons from the Stony Brook FN tandem bombarded a thick rolled foil of partially enriched  $^{90}\text{Zr}$ . The data were taken in threefold coincidence with the  $\gamma$  energy, proton energy, and a signal representing the relative timing of the  $\gamma$  and proton events analyzed by a PACE-8 analog-to-digital converter system coupled to a PDP-9 computer. The data could be viewed on line and were also written event-by-event on magnetic tape for further analysis. Data were collected for approximately 48 hours.

Figure 2 shows the  $\gamma$  spectrum coincident with any proton and the  $\gamma$  spectrum coincident with the  $(4^-, 3^-)$  doublet. The energies of the levels seen in these spectra were measured by comparison to spectra of  $\gamma$  rays of known energies from  $^{60}\text{Co}$  and  $^{22}\text{Na}$  sources. The energy calibration thus measured was consistent, to within the 1 or 2 keV obtainable accuracy, with the calibration derived under the assumption that the strong lines in the spectra were the expected 562 keV and 2186 keV  $\gamma$  rays. No yield of 420 keV  $\gamma$  rays was observed in the spectrum coincident with the  $(4^-, 3^-)$  doublet. An upper limit to this yield was determined to be  $\approx 60$  counts. The peak shape and position were determined from the 420 keV  $\gamma$ -ray peak visible in the spectrum coincident with any proton. (It should be noted that in the spectrum containing the 420 keV peak, the  $4^-$  state is populated by undergoing  $\gamma$  decay of higher-lying states.) The yield of the 562 keV  $\gamma$  ray in coincidence with the  $(4^-, 3^-)$  doublet was found to be  $7300 \pm 90$ . All  $\gamma$  rays observed were identified as belonging to either  $^{90}\text{Zr}$  or  $^{91}\text{Zr}$ , the only observed target contaminant, except for the 659 keV  $\gamma$  ray. It is thought to come from the reaction  $^{90}\text{Zr}(p, \alpha\gamma)^{87}\text{Y}$  or  $^{91}\text{Zr}(p, \alpha\gamma)^{88}\text{Y}$ . The level structure of each of these residual nuclei is not well known. Due to the need for a large solid angle, no particle identification was used. However, from Q-value considerations, only the  $(p, ^4\text{He})$  reaction mentioned above could provide  $\gamma$  rays in coincidence with particles in the energy range of interest. Particle events with an excitation energy of greater than 8 MeV were not processed.

FIG. 2.  $\gamma$  spectra from  $^{90}\text{Zr}(p, p'\gamma)$ .

The 2.748 MeV  $\gamma$ -ray peak has 260 counts, which represent a previously unmeasured 14% ground state branch of the  $3^-$  state. The sum peak contribution to the 2.748 MeV peak is  $\approx 10$  counts, or about 4% of the measured yield.

A cross section upper limit for the  $4^-$  state was derived from the above  $\gamma$ -decay data together with the  $(p, p')$   $3^-$  differential cross section. Inelastic cross sections of the  $3^-$  state as well as the  $5^-$  state at 2.32 MeV and the  $2^+$  state at 2.18 MeV excitation were measured at 17.2 MeV incident energy. Data were collected at 24 angles between 20 and 95° using an uncooled 2000  $\mu\text{m}$  surface barrier silicon detector. A monitor counter provided relative normalizations, and Rutherford scattering was performed to establish an absolute normalization. The elastic angular distribution is plotted in Fig. 3 as the ratio to Rutherford scattering, as is the optical model prediction using the parameters of Becchetti and Greenless.<sup>9</sup> The data for the  $2^+$ ,  $5^-$ , and  $3^-$  states, as well as the  $L=2$ ,  $L=5$ , and  $L=3$  angular distributions calculated with macroscopic form factors, are shown in Fig. 4. All of the distorted wave predictions were calculated with the DWBA code DWUCK<sup>10</sup> and no fitting was performed. The cross section of the  $3^-$  state, averaged over the solid angle of the proton detector used in the coincidence experiment, is 3 mb/sr. A  $(p, p' \gamma)$  angular correlation calculation was carried out for both the  $3^- - 2^+$

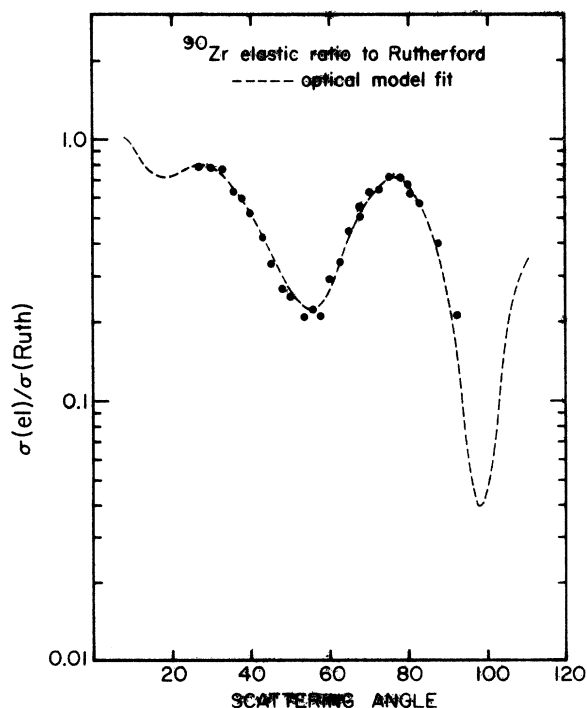


FIG. 3.  $^{90}\text{Zr}$  elastic scattering data.

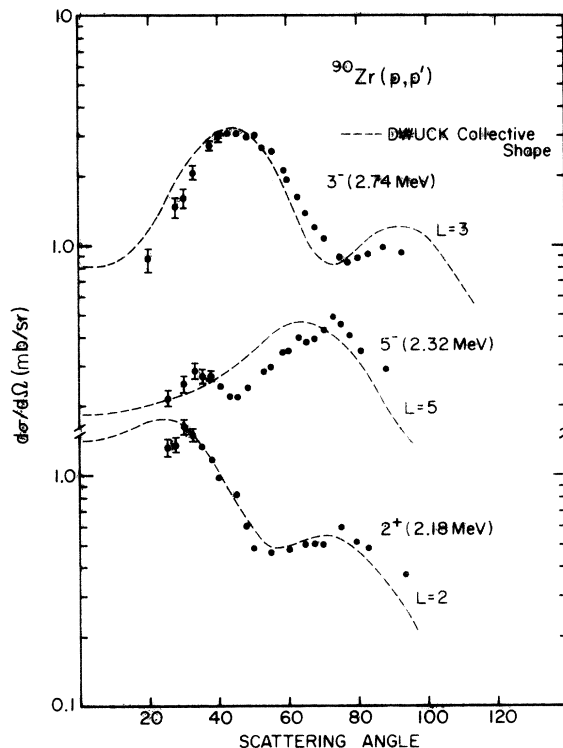


FIG. 4. Data and angular distributions calculated with macroscopic form factor for the low-lying  $3^-$ ,  $5^-$ , and  $2^+$  states in  $^{90}\text{Zr}$ .

$E1$  and  $4^- - 5^- M1$  transitions using amplitudes from the DWBA code DWBA70<sup>11</sup> to construct the statistical tensor.<sup>12</sup> After averaging over the solid angle of the proton detector, no anisotropies greater than a few percent were found in either angular correlation. Taking into account the cross section of the  $3^-$  state, the yields of the 562 and 420 keV  $\gamma$  rays, the measured Ge(Li) relative efficiency, and the 14% ground state branch of the  $3^-$  state, the ratio  $\sigma(4^-)/\sigma(3^-)$  is found to be less than  $2.6 \times 10^{-3}$ . Hence the cross section of the  $4^-$  state at 2.738 MeV excitation in  $^{90}\text{Zr}$ , averaged over the solid angle of the proton detector, is  $< 7.5 \mu\text{b/sr}$ .

A naive prediction of the expected cross section of the  $4^-$  state was made by calculating the cross sections of the  $4^-$  and  $5^-$  states with the code DWUCK, making the proper ground state correlation correction, and adjusting the strength of the Serber interaction to match the  $5^-$  observed cross section. The Serber interaction used has a Yukawa radial dependence with a range of 1 fm. The  $4^-$  and  $5^-$  states were described as pure  $(p_{1/2}g_{9/2})^{j^-}$  proton configurations, and the effect of the ground state correlations is to enhance or diminish the amplitude of the transition by the factor  $[a + (-1)^s b / \sqrt{5}]$ ,<sup>13</sup> where  $a$  and  $b$  are the  $p_{1/2}$  and  $g_{9/2}$  admixtures in the ground state, well known to be

$a = 0.8$  and  $b = 0.6$ .<sup>13, 14</sup> The correlation is destructive in the  $4^-$  case which must proceed by  $S=1$ , and constructive in the  $5^-$  case, where the dominant transition is  $S=0$ . The correction factors are 0.53 and 1.07 for the  $4^-$  and  $5^-$  cases, respectively. The proton optical model parameters were again those of Becchetti and Greenlees. The strength necessary to match the observed  $500 \mu\text{b/sr}$  cross section of the  $5^-$  state at  $55^\circ$  was  $V_0 = 205$  MeV. The same central strength was found by Gray *et al.*<sup>2</sup> in fitting 18.8 MeV inelastic proton scattering to the  $4^+$ ,  $6^+$ , and  $5^-$  states in  $^{90}\text{Zr}$ . Inserting this strength in the  $4^-$  calculation yields an expected maximum cross section of  $250 \mu\text{b/sr}$  at  $34^\circ$ , more than 30 times the measured upper limit.

Calculations of the cross sections to the  $4^-$  and  $5^-$  states were then made with more realistic wave functions using the central and spin-orbit part of the interaction of Hinrichs *et al.*,<sup>3</sup> which fitted natural parity transitions in  $^{90}\text{Zr}$  in 40 MeV inelastic proton scattering, and the tensor strength of Austin and Fox.<sup>7</sup>

Recently the relatively large cross section to the  $4^-$  state at 3.48 MeV excitation in  $^{208}\text{Pb}$  in 35 MeV proton inelastic scattering has been described by similar microscopic analysis<sup>15</sup> using a Serber central force with  $V_0 = 30$  MeV, the spin-orbit force used here, and the tensor strength

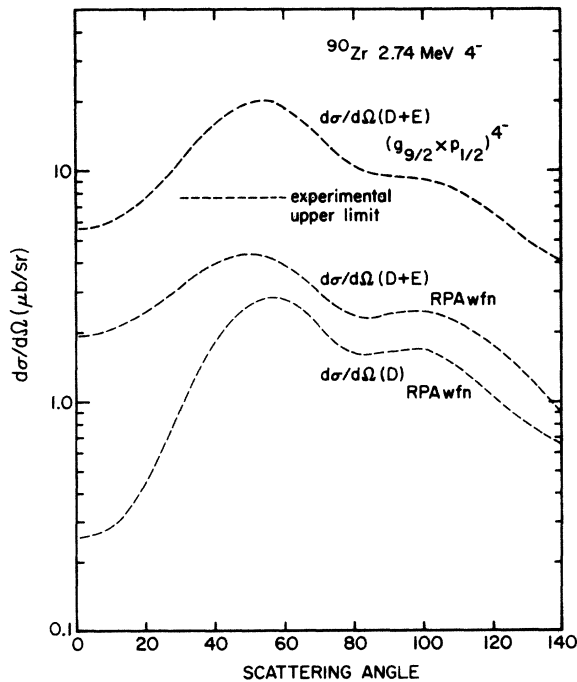


FIG. 5. Calculated angular distribution for the  $4^-$  state using simple and RPA functions. ( $D+E$ ) indicates a calculation including exchange; and ( $D$ ), a calculation including the direct term only.

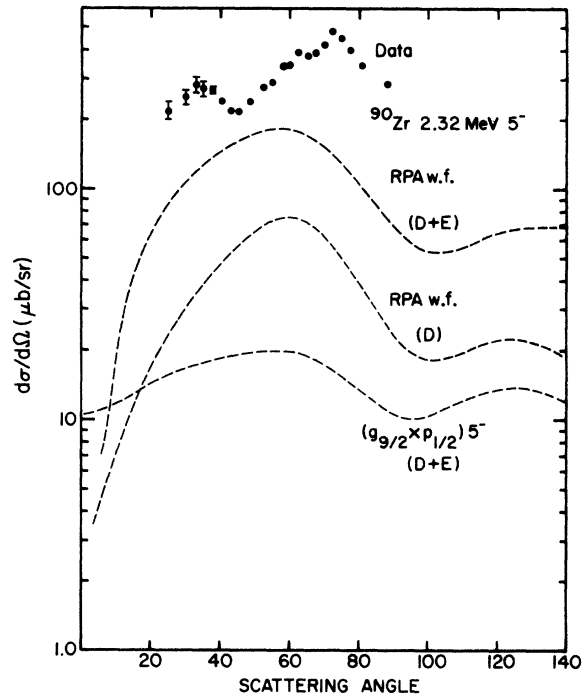


FIG. 6. Calculated angular distribution for the  $5^-$  state; notation as in Fig. 5.

of Austin, evaluated at 35 MeV. There the tensor term was found to dominate the excitation. The optical model parameters used in our calculation were again those of Becchetti and Greenlees.<sup>8</sup>

Spectroscopic amplitudes of the various one particle-one hole configurations which comprise the  $4^-$  and  $5^-$  states were calculated in the random phase approximation (RPA) using the Kuo-Brown matrix elements<sup>16</sup> for the mass 90 region and Kuo's RPA code. RPA calculations were made first over a small shell model subspace including only the  $1f-2p$  and  $1g-2d-3s$  shells, then over an expanded space which included the  $1d-2s$  and  $1h-2f-3p$  shells. For the RPA calculations on the  $5^-$  state, expanding the model space not only added configurations with considerable amplitudes, but also markedly increased the amplitudes of configurations in the smaller shell model space. The spectroscopic amplitudes from the more complete calculation are given in Table I. The cross sections for the  $4^-$  and  $5^-$  states as calculated by the code DWBA70 are shown in Figs. 5 and 6, respectively. The results of calculations including exchange with simple and RPA wave functions, as well as the results of calculations of only the direct term for the RPA wave functions, are shown.

The cross section computed with the RPA wave functions still falls far short of the  $500 \mu\text{b/sr}$  measured maximum cross section of the  $5^-$  state, although the shape of the angular distribution

TABLE I.  $^{90}\text{Zr}$  RPA spectroscopic amplitudes.

Particle-hole configuration	X	Y	Particle-hole configuration	X	Y
4 <sup>-</sup> proton			5 <sup>-</sup> proton		
1 $g_{9/2}$ 1 $f_{7/2}$	0.026	0.012	2 $d_{3/2}$ 1 $f_{7/2}$	0.047	0.028
1 $g_{9/2}$ 2 $p_{3/2}$	0.134	0.012	1 $h_{1/2}$ 2 $s_{1/2}$	0.020	0.022
1 $g_{9/2}$ 1 $f_{5/2}$	-0.046	-0.021	1 $i_{13/2}$ 1 $f_{7/2}$	0.032	0.032
1 $g_{9/2}$ 2 $p_{1/2}$	0.53	(see text)	4 <sup>-</sup> neutron		
2 $d_{5/2}$ 1 $f_{7/2}$	0.005	0.006	1 $g_{7/2}$ 2 $p_{1/2}$	0.022	-0.005
2 $d_{5/2}$ 2 $p_{3/2}$	0.014	0.011	5 <sup>-</sup> neutron		
2 $d_{5/2}$ 1 $f_{5/2}$	-0.011	-0.006	2 $d_{5/2}$ 1 $f_{7/2}$	-0.035	-0.034
1 $g_{7/2}$ 1 $f_{7/2}$	0.008	-0.002	2 $d_{5/2}$ 1 $f_{5/2}$	-0.116	-0.086
3 $s_{1/2}$ 1 $f_{7/2}$	0.005	0.010	1 $g_{7/2}$ 1 $f_{7/2}$	0.060	0.051
2 $d_{3/2}$ 1 $f_{5/2}$	0.011	0.001	1 $g_{7/2}$ 2 $p_{3/2}$	0.140	0.088
1 $h_{11/2}$ 1 $d_{3/2}$	0.008	0.006	1 $g_{7/2}$ 1 $f_{5/2}$	-0.089	-0.064
1 $i_{13/2}$ 1 $f_{5/2}$	0.014	0.007	2 $d_{3/2}$ 1 $f_{7/2}$	0.055	0.052
5 <sup>-</sup> proton			1 $h_{11/2}$ 1 $d_{5/2}$	0.034	0.023
1 $g_{9/2}$ 1 $f_{7/2}$	-0.080	-0.054	1 $h_{11/2}$ 2 $s_{1/2}$	0.046	0.028
1 $g_{9/2}$ 2 $p_{3/2}$	-0.231	-0.088	1 $h_{11/2}$ 1 $d_{3/2}$	0.025	0.017
1 $g_{9/2}$ 1 $f_{5/2}$	-0.239	-0.123	1 $h_{11/2}$ 1 $g_{9/2}$	-0.109	-0.067
1 $g_{9/2}$ 2 $p_{1/2}$	-1.07	(see text)	1 $h_{9/2}$ 2 $s_{1/2}$	-0.028	-0.026
2 $d_{5/2}$ 1 $f_{7/2}$	-0.027	-0.023	1 $h_{9/2}$ 1 $g_{9/2}$	0.048	0.038
2 $d_{5/2}$ 1 $f_{5/2}$	-0.053	-0.066	1 $i_{13/2}$ 1 $f_{7/2}$	0.034	0.023
1 $g_{7/2}$ 1 $f_{7/2}$	0.051	0.028	1 $i_{13/2}$ 2 $p_{3/2}$	0.065	0.036
1 $g_{7/2}$ 2 $p_{3/2}$	0.098	0.045	1 $i_{13/2}$ 1 $f_{5/2}$	0.024	0.017
1 $g_{7/2}$ 1 $f_{5/2}$	-0.053	-0.038			

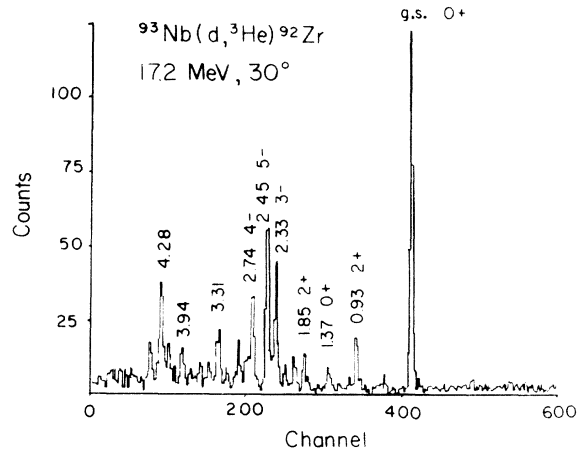
matches the experimental data as well as the calculation using a macroscopic form factor. Our 5<sup>-</sup> calculation agrees with a calculation of the excitation of the 5<sup>-</sup> state in 18.8 MeV ( $p, p'$ ) by Petrovich.<sup>17</sup>

The prediction for the cross section of the 4<sup>-</sup> state is lower than the measured upper limit, and hence, is not inconsistent with the experiment. Increasing the strength of either the tensor force or the combination  $V_{\sigma} + V_{\sigma\tau}$  does increase the predicted maximum cross section as one would expect; unfortunately, on the basis of the measurement of an upper limit, it would be meaningless to attempt to modify either of the above spin-dependent terms to obtain a better "fit."

Since the RPA wave functions did not adequately describe the 5<sup>-</sup> cross section, considerations with respect to the adequacy of the 4<sup>-</sup> calculations are in order. Firstly, in the sense that the RPA represents a first correction to the Tamm-Dancoff approximation (TDA), if the spectroscopic amplitudes calculated in the RPA differ even moderately from the corresponding TDA amplitudes, then higher order corrections are necessary.<sup>18</sup> In the 5<sup>-</sup> calculation this was the case. Secondly, as discussed above, expanding the model space had a disquietingly large effect on the RPA amplitude

in the 5<sup>-</sup> calculation. In the 4<sup>-</sup> calculation neither of these two effects were evident; hence, we feel that the 4<sup>-</sup> calculation is adequate in spite of the shortcomings of the 5<sup>-</sup> calculation.

In view of the small cross section of the 4<sup>-</sup> state we have made a simplified Hauser-Feshbach calculation using the few open channels ( $p, p'$ ), ( $p, n$ ), and ( $p, {}^4\text{He}$ ) to estimate the compound cross section to the 2.748 MeV 4<sup>-</sup> state. The transmission

FIG. 7.  $^{92}\text{Zr}$  spectrum from  $^{93}\text{Nb}(d, {}^3\text{He})^{92}\text{Zr}$ .

coefficients were taken as step functions in  $l$  space with the cutoff at the  $T_l = \frac{1}{2}$  point calculated by the optical model code ABACUS. Using the approximate method described in Ref. 19, an estimate of the compound cross section was calculated to be less than  $2.0 \mu\text{b}/\text{sr}$ . Hence the compound contribution to the cross section of the  $4^-$  state is much lower than the calculated direct contribution, and we conclude that the measured upper limit on the cross section is not due to any possible destructive interference between direct and compound amplitude.

#### B. $4^-$ state in $^{92}\text{Zr}$

$^{92}\text{Zr}$  will have a  $4^-$  state of the same basic ( $p_{1/2}g_{9/2}$ ) proton configuration built on essentially the same ground state proton configuration as was the  $4^-$  state in  $^{90}\text{Zr}$ . We searched for this state using the pickup reaction  $^{93}\text{Nb}(d, ^3\text{He})^{92}\text{Zr}$ . Data were collected using cooled silicon surface barrier detector telescopes and the usual particle identification technique. A monitor counter was used to establish relative normalizations and Rutherford scattering was performed to establish an absolute normalization. A typical spectrum is shown in Fig. 7. We assign  $J^\pi = 4^-$  to the strongly excited state at  $2.740 \pm 0.010$  MeV both because its angular distribution has an  $L = 1$  shape similar to the angular distribution of the previously known  $5^-$  state at 2.45 MeV (see Fig. 8), and because these two states exhaust the  $p_{1/2}$  pickup strength. The calculated spectroscopic factor sum is 2.08, which is to be compared to the theoretical value of 2.0. As is apparent from Fig. 8, spectroscopic factors for the 2.45 and 2.74 MeV levels deduced from the measured angular distributions are determined to an accuracy no better than about 20%. The ground state  $g_{9/2}$  spectroscopic factor computed from the data is 10.8. This good agreement with the expected value of 10.0 confirms the overall normalization. The data and theoretical calculations for the ground state, 2.45 MeV  $5^-$  state, and 2.74 MeV  $4^-$  state are shown in Fig. 8. The pickup calculations were performed using the code DWUCK with the deuteron optical model parameters taken from the compilation and parametrization of Perey and Perey<sup>22</sup> and the  $^3\text{He}$  optical model parameters taken from a  $^3\text{He}$  inelastic scattering measurement on  $^{92}\text{Zr}$  at 16 MeV incident energy.<sup>23</sup> The calculations were insensitive to the  $^3\text{He}$  parameters.

There have been measurements of inelastic proton scattering on  $^{92}\text{Zr}$  at various energies,<sup>21</sup> but in each measurement the yield of protons in the region of 2.74 MeV excitation is attributable to excitation of the  $3^-$  state of  $^{90}\text{Zr}$  due to target impurity. A 1%  $^{90}\text{Zr}$  impurity would contribute

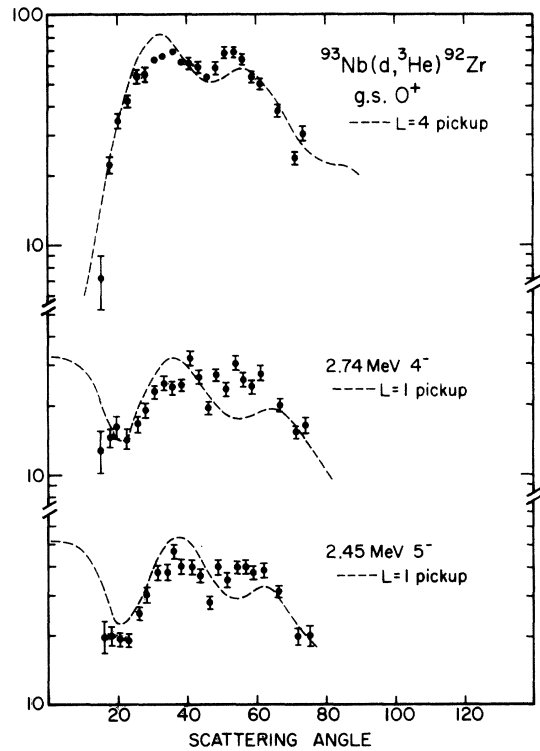


FIG. 8. Data and visually fitted pickup angular distributions of the states of interest in  $^{92}\text{Zr}$ .

an apparent  $30 \mu\text{b}/\text{sr}$  cross section at 2.74 MeV excitation in 17.2 MeV proton inelastic scattering. Hence extremely pure  $^{92}\text{Zr}$  would be necessary to study the population of the  $4^-$  state in inelastic scattering.

#### IV. CONCLUSION

The small measured upper limit on the forward angle cross section of the  $4^-$  state is inconsistent with a naive DWBA calculation assuming a pure  $(g_{9/2}p_{1/2})^{4^-}$  configuration and a Serber interaction. On the other hand, a DWBA calculation including full one particle-one hole RPA shell model wave functions and a realistic nucleon-nucleon effective interaction is consistent with the experimental upper limit. It is encouraging to see that this interaction derived from the Hamada-Johnston potential describes  $(p, p')$  inelastic scattering to both natural parity states<sup>3</sup> and unnatural parity state in  $^{90}\text{Zr}$ . As discussed above, these two different types of excitations are sensitive to different components of the interaction. Repeating the calculations for a proton bombarding energy of 40 MeV, we find that the predicted maximum cross section of the  $4^-$  state is  $63 \mu\text{b}/\text{sr}$  at a scattering angle of  $35^\circ$ . Thus the particle  $\gamma$  coincidence technique employed in the present work might yield a positive result if carried out at the higher

incident energy of 40 MeV, and a better determination of the tensor and  $V_{\sigma} + V_{\sigma\tau}$  components of the effective interaction ought to be possible.

#### ACKNOWLEDGMENTS

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