# 17.2-MeV proton inelastic scattering from low-lying unnatural-parity states in $^{88}Sr$†

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The differential cross sections to the $1^+$ (3.487-MeV) and $3^+$ (3.635-MeV) levels in $^{88}Sr$ have been measured between laboratory angles of 20° and 110° in the inelastic scattering of 17.2-MeV protons. The measured cross sections are reasonably well predicted by distorted-wave Born-approximation calculations assuming realistic interactions and shell model wave functions spanning a large basis space. It is found that the calculated cross sections are dominated by the components in the wave functions involving the valence nucleons and that the core polarization is relatively unimportant.

[NUCLEAR REACTIONS $^{88}Sr(p, p')$, $E = 17.2$ MeV; measured $\sigma(E_{p'}, \theta)$; enriched target, microscopic DWBA analysis, $\theta = 20°-110°$.]

## I. INTRODUCTION

The recent years have seen considerable effort devoted to the understanding of inelastic scattering of protons by nuclei. One aspect of this effort has been the description of these processes in terms of shell model wave functions for initial and final nuclear states, an effective nucleon-nucleon interaction, and the distorted wave Born approximation (DWBA) for direct nuclear reactions. One difficulty associated with this approach is that an empirical determination of the effective interaction requires an *a priori* knowledge of the particular wave functions. Consequently, most of such determinations derive from the study of the excitation of states in light nuclei where, for instance, level densities and available shell model orbitals are minimal, facilitating both experiment and theoretical analysis.[1,2] On the other hand, one may simply assume a "realistic" effective interaction which approximates the free nucleon-nucleon interaction. In either case, if the pedagogical value of the microscopic approach to inelastic scattering (e.g., as a probe of nuclear wave functions) is to be fully realized, then the effective interaction must remain relatively independent of, or at least be parametrizable in terms of, such ingredients as projectile bombarding energy, and where in the Periodic Table the experiment takes place.

There are, unfortunately, few excited states in medium to heavy nuclei which can be described, for purposes of the present discussion, in terms of very simple shell model wave functions. Indeed, recent analyses of inelastic proton scattering to low-lying states of nuclei in the zirconium[3,4] and lead[5,6] regions, in which realistic effective interactions were assumed, have demonstrated the fact that the contributions to the cross sections arising from core polarization invariably dominate the contribution from those components in the wave function involving only valence nucleons.

In this work we show that the excitation in $(p,p')$ to two unnatural-parity states in $^{88}Sr$ can, in contrast to the results mentioned above, be described through DWBA calculations assuming realistic interactions and very simple shell model wave functions. However, in view of the important role generally played by core polarization in medium and heavy nuclei, we feel that it is not sufficient merely to obtain a reasonable fit assuming a simple wave function. To do so would invite a fortuitous conspiracy between a "wrong" interaction and "wrong" wave functions. Consequently we have calculated the cross sections using wave functions which include the core polarization that is predicted assuming a large shell model basis space. We show that these cross sections are very similar in size and shape to the cross sections calculated assuming simple wave functions. The two states which we studied are at excitation energies of 3.487 and 3.635 MeV and have been tentatively identified in a recent compilation[7] as having spin parities of $1^+$ and $3^+$, respectively. Neither of these states were reported in earlier studies of the inelastic scattering of protons by $^{88}Sr$ at 19[8] and 20.2 MeV.[9] The spin assignment of 1 to the 3.487-MeV level has since been verified by Metzger.[10] Harrison and Hiebert[11] verify the spin assignments to both levels. In addition, the excitation energies of these states are in reasonable agreement with recent theoretical predictions.[12,13] For the purposes of this work, we will, therefore, assume these states to have $J^\pi = 1^+$ and $3^+$. In Sec. II we present a discussion of the experimental techniques employed in the measurements of the cross sections to these states. In Sec. III we discuss the DWBA predictions of the cross sections, including a discussion of the wave functions of the

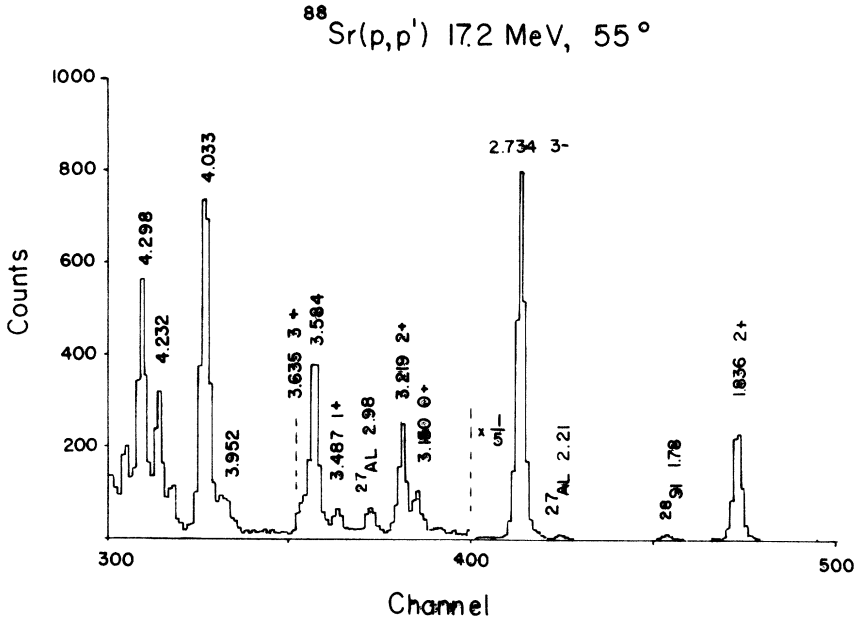


FIG. 1. Energy spectrum of protons inelastically scattering from $^{88}$Sr.

ground and excited states and the interactions employed. Our conclusions are presented in Sec. IV.

## II. EXPERIMENT

The measurement of the cross sections to the $1^+$ and $3^+$ states in $^{88}$Sr was carried out in a relatively straightforward manner. The target consisted of reduced metallic Sr, isotopically enriched to 98% $^{88}$Sr, evaporated onto a 50-$\mu$g/cm$^2$ carbon foil. The target was transferred *in vacuo* from the evaporation bell jar to the scattering chamber. The target was bombarded with the 17.2-MeV proton beam from the Stony Brook FN tandem. Scattered charged particles were detected in a telescope consisting of two 1000-$\mu$m Ortec surface barrier silicon detectors. A single 2000-$\mu$m detector was used to monitor the yield of Sr elastic scattering of a fixed angle, establishing a relative normalization to the inelastic data. The defining solid angle for the telescope consisted of a thin tantalum collimator located in the telescope package *between* the two detectors. Using this arrangement, the particle identification technique of Garvey, Haight, and Lynch[14] was employed to reduce the effect of slit scattering. With this technique, the energy spectra of inelastically scattered protons were measured between laboratory angles of 20° and 110°. One of the better spectra is shown in Fig. 1, where the total energy resolution is seen

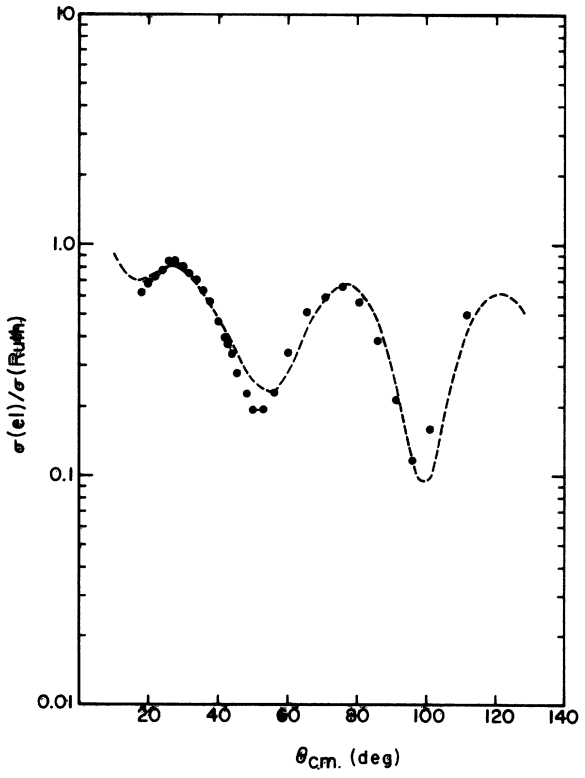


FIG. 2. Comparison of measured elastic angular distribution and DWBA predicted angular distribution (both divided by Rutherford).

to be about 25 keV. In addition to the usual contaminant states of $^{12}C$ and $^{16}O$, excited states of $^{13}C$, $^{27}Al$, and $^{28}Si$ were observed. At certain angles, these states prevented the measurement of the yields to the $1^+$ and $3^+$ levels. As can be seen, the $1^+$ state at 3.487 MeV is cleanly resolved and the $3^+$ state at 3.635 MeV is partially resolved from the 3.584-MeV level. Standard computerized fitting routines were used to extract the yield to these levels. In order to minimize the uncertainty in the extracted yields, the channel numbers of the peak centroids were predetermined using a calibration derived from several cleanly resolved, strongly excited levels, such as the 4.033-, 2.734-, and 1.836-MeV levels. For the same reason, the peak shapes were taken from the strong, isolated levels. Thus, only the yields of the $1^+$ and $3^+$ levels were allowed to vary in the fitting routine.

A measurement of the absolute differential cross section for the excitation of the states was obtained in terms of the yields by normalizing the ground state angular distribution to that predicted in DWBA using the code DWUCK[15] and the Becchetti-Greenlees[16] proton optical model parameters. The measured and predicted elastic angular distributions (both divided by the Rutherford cross section) are shown in Fig. 2. This method of normalization is accurate to about 10%. The target thickness (about 600 $\mu g/cm^2$) deduced from this normalization and the measured solid angle of the telescope collimator is consistent with the thickness measured to an accuracy of about 15% using the 5.48-MeV $\alpha$ particle from an $^{241}Am$ source.

## III. ANALYSIS

### A. Excited state wave functions

Random phase approximation (RPA) wave functions for the $1^+$ and $3^+$ states were obtained by diagonalizing the Hamiltonian matrix of one particle-one hole states built upon a $^{88}Sr$ core for which the protons close the $2p_{3/2}$ orbital and the neutrons close the $1g_{9/2}$ orbital. The shell model basis space was constructed from particles and holes in the $2s-1d$, $2p-1f$, $3s-2d-1g$, and $3p-2f-1h$ major harmonic oscillator shells. The Kuo-Brown matrix elements for the mass 90 region[17] were used in constructing the Hamiltonian matrix. The unperturbed energies of the neutron and proton single particle orbitals are given in Table I. In Table II the amplitudes predicted by the RPA calculation for the available particle hole components, the $X_{ph}$ and $Y_{ph}$ amplitudes, are given. A description of the definition of these amplitudes, in the general case is given elsewhere.[18] The next predicted states are predominantly the neutron-neutron hole configurations $(1g_{9/2}^{-1}\text{-}1g_{7/2})^{1^+}$ and $(1g_{9/2}^{-1}\text{-}2d_{5/2})^{3^+}$. These states are predicted at excitation energies of 8.481 and 4.773 MeV, respectively.

TABLE I. Unperturbed energies of single particle orbitals.

| $nlj$ | Proton energy | Neutron energy |
|---|---|---|
| $1d_{5/2}$ | −18.00 | −18.00 |
| $2s_{1/2}$ | −18.00 | −18.00 |
| $1d_{3/2}$ | −18.00 | −18.00 |
| $1f_{7/2}$ | −10.00 | −12.00 |
| $2p_{3/2}$ | −4.20 | −7.00 |
| $1f_{5/2}$ | −4.50 | −7.30 |
| $2p_{1/2}$ | −0.68 | −5.30 |
| $1g_{9/2}$ | −0.22 | −4.70 |
| $2d_{5/2}$ | 3.10 | 0.0 |
| $1g_{7/2}$ | 3.60 | 2.74 |
| $3s_{1/2}$ | 5.20 | 2.11 |
| $2d_{3/2}$ | 5.30 | 2.50 |
| $1h_{11/2}$ | 8.10 | 5.00 |
| $1h_{9/2}$ | 12.00 | 12.00 |
| $2f_{7/2}$ | 12.00 | 12.00 |
| $3p_{3/2}$ | 12.00 | 12.00 |
| $2f_{5/2}$ | 12.00 | 12.00 |
| $3p_{1/2}$ | 12.00 | 12.00 |

In view of the fact that the observed $1^+$ and $3^+$ states are predominantly proton-proton hole states as evidenced by the fact that they are strongly excited in the pickup reaction described in Ref. 11, and they were *not* observed in the reaction $^{87}Sr(d,p)^{88}Sr$,[19] we associate the observed states with the predicted states given in Table II.

### B. Ground state wave functions

The spirit of the RPA as opposed to that of the TDA (Tamm-Dancoff approximation) is to include, in an approximate manner, multiparticle multihole correlations in the ground state.[18] Transitions to unnatural-parity states from the ground states of even-even nuclei are very sensitive to such correlations and their effect on the present calculation must be investigated. Thus we calculate the matrix element of a one-body operator between an RPA excited state and the correlated ground state:

$$\langle J0 \mid T^{LSJ,0} \mid \tilde{0}\rangle = \langle J0 \mid \sum_{\substack{j_p m_p \\ j_h m_h}} \{\langle j_p m_p \mid T^{LSJ,0} \mid j_h m_h\rangle a^\dagger_{j_p m_p} a_{j_h m_h} + \langle j_h m_h \mid T^{LSJ,0} \mid j_p m_p\rangle a^\dagger_{j_h m_h} a_{j_p m_p}\} \mid \tilde{0}\rangle . \quad (1)$$

TABLE II. RPA wave function components for lowest predicted $1^+$ and $3^+$ levels in $^{88}$Sr.

| $(nlj)_{part}$ | $(nlj)_{hole}$ | $(X^{1^+}_{ph})_{prot}$ | $(Y^{1^+}_{ph})_{prot}$ | $(X^{1^+}_{ph})_{neut}$ | $(Y^{1^+}_{ph})_{neut}$ | $(X^{3^+}_{ph})_{prot}$ | $(Y^{3^+}_{ph})_{prot}$ | $(X^{3^+}_{ph})_{neut}$ | $(Y^{3^+}_{ph})_{neut}$ |
|---|---|---|---|---|---|---|---|---|---|
| $2p_{1/2}$ | $1f_{7/2}$ | | | | | −0.012 | 0.000 | | |
| $2p_{1/2}$ | $2p_{3/2}$ | −1.000 | 0.086 | | | | | | |
| $2p_{1/2}$ | $1f_{5/2}$ | | | | | 0.999 | −0.011 | | |
| $1g_{9/2}$ | $1d_{5/2}$ | | | | | −0.002 | −0.001 | | |
| $1g_{9/2}$ | $1d_{3/2}$ | | | | | −0.009 | −0.001 | | |
| $2d_{5/2}$ | $1d_{5/2}$ | −0.004 | −0.004 | −0.002 | −0.003 | −0.017 | −0.004 | −0.008 | 0.002 |
| $2d_{5/2}$ | $2s_{1/2}$ | | | | | −0.011 | 0.000 | −0.003 | 0.006 |
| $2d_{5/2}$ | $1d_{3/2}$ | −0.010 | 0.006 | 0.004 | 0.009 | 0.007 | −0.004 | 0.005 | −0.010 |
| $2d_{5/2}$ | $1g_{9/2}$ | | | | | | | 0.015 | −0.004 |
| $1g_{7/2}$ | $1d_{5/2}$ | −0.002 | −0.015 | 0.016 | −0.008 | 0.008 | 0.008 | 0.001 | 0.006 |
| $1g_{7/2}$ | $2s_{1/2}$ | | | | | 0.009 | 0.009 | 0.005 | 0.004 |
| $1g_{7/2}$ | $1d_{3/2}$ | | | | | 0.003 | 0.002 | −0.003 | −0.003 |
| $1g_{7/2}$ | $1g_{9/2}$ | | | −0.063 | 0.000 | | | 0.007 | −0.003 |
| $3s_{1/2}$ | $1d_{5/2}$ | | | | | −0.003 | −0.005 | 0.010 | 0.000 |
| $3s_{1/2}$ | $2s_{1/2}$ | 0.019 | 0.011 | −0.006 | −0.007 | | | | |
| $3s_{1/2}$ | $1d_{3/2}$ | −0.002 | −0.006 | 0.013 | −0.004 | | | | |
| $2d_{3/2}$ | $1d_{5/2}$ | 0.010 | 0.002 | −0.009 | −0.005 | 0.005 | −0.002 | 0.011 | 0.000 |
| $2d_{3/2}$ | $2s_{1/2}$ | 0.013 | −0.011 | 0.025 | −0.022 | | | | |
| $2d_{3/2}$ | $1d_{3/2}$ | 0.002 | 0.001 | 0.002 | 0.002 | 0.006 | 0.001 | 0.000 | 0.002 |
| $2d_{3/2}$ | $1g_{9/2}$ | | | | | | | −0.015 | 0.004 |
| $1h_{11/2}$ | $1f_{7/2}$ | | | | | −0.005 | −0.004 | −0.004 | −0.001 |
| $1h_{11/2}$ | $1f_{5/2}$ | | | | | −0.004 | 0.001 | 0.005 | 0.009 |
| $1h_{9/2}$ | $1f_{7/2}$ | 0.003 | −0.011 | 0.017 | −0.005 | 0.008 | 0.006 | 0.002 | 0.003 |
| $1h_{9/2}$ | $2p_{3/2}$ | | | | | 0.012 | 0.013 | 0.006 | 0.005 |
| $1h_{9/2}$ | $1f_{5/2}$ | | | | | 0.001 | 0.000 | −0.006 | −0.008 |
| $2f_{7/2}$ | $1f_{7/2}$ | −0.002 | −0.002 | −0.001 | −0.001 | −0.006 | −0.002 | 0.001 | 0.002 |
| $2f_{7/2}$ | $2p_{3/2}$ | | | | | −0.001 | 0.005 | 0.002 | 0.007 |
| $2f_{7/2}$ | $1f_{5/2}$ | −0.009 | −0.001 | 0.006 | 0.008 | 0.009 | 0.000 | 0.003 | −0.009 |
| $2f_{7/2}$ | $2p_{1/2}$ | | | | | | | −0.002 | −0.001 |
| $3p_{3/2}$ | $1f_{7/2}$ | | | | | 0.002 | −0.002 | 0.008 | 0.000 |
| $3p_{3/2}$ | $2p_{3/2}$ | 0.025 | 0.015 | 0.003 | 0.004 | −0.001 | 0.002 | −0.001 | 0.002 |
| $3p_{3/2}$ | $1f_{5/2}$ | −0.002 | −0.001 | 0.008 | 0.002 | 0.001 | 0.002 | −0.005 | 0.000 |
| $3p_{3/2}$ | $2p_{1/2}$ | | | −0.004 | 0.018 | | | | |
| $2f_{5/2}$ | $1f_{7/2}$ | 0.009 | 0.002 | −0.003 | −0.005 | −0.001 | −0.005 | 0.004 | −0.003 |
| $2f_{5/2}$ | $2p_{3/2}$ | 0.010 | −0.010 | 0.028 | −0.006 | 0.002 | 0.000 | −0.003 | −0.001 |
| $2f_{5/2}$ | $1f_{5/2}$ | 0.003 | 0.000 | −0.001 | −0.002 | 0.004 | 0.001 | −0.005 | 0.000 |
| $2f_{5/2}$ | $2p_{1/2}$ | | | | | | | −0.002 | 0.002 |
| $3p_{1/2}$ | $1f_{7/2}$ | | | | | −0.008 | −0.005 | −0.004 | −0.003 |
| $3p_{1/2}$ | $2p_{3/2}$ | 0.029 | 0.009 | −0.010 | −0.001 | | | | |
| $3p_{1/2}$ | $1f_{5/2}$ | | | | | −0.002 | 0.001 | 0.001 | −0.001 |
| $3p_{1/2}$ | $2p_{1/2}$ | | | 0.027 | 0.005 | | | | |

This matrix element may be written in terms of the RPA $X$ and $Y$ coefficients by noting

$$X^J_{ph} = \langle J0 \mid A^\dagger(ph;J) \mid \tilde{0} \rangle ,$$

$$Y^J_{ph} = \langle J0 \mid A(ph;J) \mid \tilde{0} \rangle$$

with

$$A^\dagger(ph;J) = \sum_{m_p m_h} \{ \langle j_p j_h m_p - m_h \mid J0 \rangle \times (-1)^{j_h + m_h} a^\dagger_{j_p m_p} a_{j_h m_h} \}$$

and $A$ (ph; $J$) the Hermitian adjoint of $A^\dagger$(ph; $J$). Inverting the expression for $A^\dagger$ and $A$ in terms of $a^\dagger_{j_p m_p} a_{j_h m_h}$ and $a^\dagger_{j_h m_h} a_{j_p m_p}$ and substituting into (1) we find that a given term in the sum in (1) is proportional to

$$X^J_{ph} \langle j_p m_p \mid T^{LSJ,0} \mid j_h m_h \rangle + Y^J_{ph} \langle j_h m_h \mid T^{LSJ,0} \mid j_p m_p \rangle ,$$

which, reversing the order of the second single particle matrix element, is equal to

$$\langle j_p m_p \mid T^{LSJ,0} \mid j_h m_h \rangle [ X^J_{ph} + (-1)^{L+S+J} Y^J_{ph} ] . \qquad (2)$$

Thus the ground state correlation is accounted for by merely correcting the matrix element calculated assuming a closed ground state by the factor $[ X^J_{ph} + (-1)^{L+S+J} Y^J_{ph} ]$.

For purposes of comparison with calculation and experiment in which ground state wave functions are written explicitly in terms of multiparticle-

multihole additions to the bare ground state, we relate the RPA coefficients to the amplitudes of the correlated terms in the ground state: Recalling

$$Y^J_{\rm ph} = \langle J0 \,|\, A(\mathrm{ph}; J) \,|\, \tilde{0}\rangle$$

we assume

$$|\,\tilde{0}\rangle = \epsilon_0 \,|\, 0\rangle + \sum_{\rm ph} \sum_{m_p m_h} \frac{\epsilon_{\rm ph}}{2} (-1)^{2j_h} \langle j_p m_p j_p - m_p \,|\, 00\rangle \langle j_h m_h j_h - m_h \,|\, 00\rangle \, a^\dagger_{j_p m_p} a^\dagger_{j_p - m_p} a_{j_h m_h} a_{j_h - m_h} \,|\, 0\rangle \tag{3}$$

and to first order in $\epsilon_{\rm ph}$, we may neglect the $Y$ term in the excited state. Thus,

$$\langle J0 \,| \approx \sum_{\rm ph} \sum_{m_p m_h} X_{\rm ph} \langle 0 \,|\, a_{j_p m_p} a^\dagger_{j_h m_h} (-1)^{j_h - m_h} \langle j_p m_p j_h m_h \,|\, J0\rangle , \tag{4}$$

where the factor of 2 in the denominator of expression (3) obtains from antisymmetrizing the two particles in the same orbit and the two holes in the same orbit. These expressions together with the definition of $A(\mathrm{ph}; J)$ given above allows $Y^J_{\rm ph}$ to be explicitly evaluated in terms of the amplitude $\epsilon_{\rm ph}$. This is done in the usual manner by contracting the matrix element of the occupation representation operators and summing over the $m$ substates. So doing we find,

$$Y^J_{\rm ph} = -\frac{2\epsilon_{\rm ph} X^J_{\rm ph}}{[(2j_p+1)(2j_h+1)]^{1/2}} . \tag{5}$$

Substituting this into expression (2) we are in agreement with a direct calculation of the effect of a two particle-two hole ground state component on a transition to a one particle-one hole excited state.[20]

If we approximate the correlated ground state of $^{88}$Sr as

$$|\,\tilde{0}\rangle \simeq (\alpha + \beta p_{3/2}{}^{-2} p_{1/2}{}^2 + \gamma f_{5/2}{}^{-2} p_{1/2}{}^2) \,|\, 0\rangle \tag{6}$$

with $|\,0\rangle$ the bare closure of the $p_{3/2}$ orbit, then, in Table III, we compare the ground state wave function calculated with expression (5) and the RPA coefficients for the $(f_{5/2}{}^{-1}\text{-}p_{1/2})^{3^+}$ and $(p_{3/2}{}^{-1}\text{-}p_{1/2})^{1^+}$ components as given in Table II to a recent calculation of the $^{88}$Sr ground state[13] and to recent experimental determinations thereof.[11,21] The effect of the correlated components in the ground state on the transitions to the $1^+$ and $3^+$ states are also given in Table III. It is seen that the RPA calculations significantly underestimate the effect of the ground state correlation compared to the other wave functions given in Table III. For the components $\alpha$, $\beta$, and $\gamma$ in expression (6) we assume the correction factor derived from Hughes's[13] wave function. For the weaker components, we assume the present RPA calculation.

TABLE III. Amplitudes of ground state wave function components and corresponding matrix element correction factors.

| | Ground state amplitude [a] | | | Correction factor [b] | |
|---|---|---|---|---|---|
| | $\alpha$ | $\beta$ | $\gamma$ | $1^+$ | $3^+$ |
| Ref. 13 | 0.83 | 0.46 | 0.21 | 0.51 | 0.67 |
| Ref. 11 | 0.86 | 0.45 | 0.31 | 0.54 | 0.60 |
| Ref. 21 | 0.89 | 0.40 | 0.31 | 0.62 | 0.67 |
| Present work | 0.99 | 0.12 | 0.01 | 0.91 | 0.01 |

[a] See expression (6).

[b] Calculated with the correction factor in expression (2) together with expression (5) where appropriate.

### C. DWBA Analysis

The DWBA calculations were performed with the code DWBA70,[22] using the Becchetti-Greenlees proton optical model parameters. The form factors included contributions from those terms in the RPA wave functions for which the net amplitudes $X^J_{\rm ph} + Y^J_{\rm ph}$ are greater in absolute magnitude than 0.01. [Note that the phase factor in expression (2) $(-1)^{L+S+J}$ is equal to +1 for the transitions to the $1^+$ and $3^+$ states from the ground state.] The "knock on" exchange contributions to the cross sections were included. The interaction between the projectile proton and the target nucleon is taken to be the most general two-body interaction (neglecting terms quadratically dependent on the nucleons relative momentum.) Following the notation of Ref. 3

$$V_{pq} = V^0_{pq}(r_{pq}) + V^1_{pq}(r_{pq})(\vec{\sigma}_p \cdot \vec{\sigma}_q) + V^T_{pq}(r_{pq}) r_{pq}{}^2 S_{pq} + V^{LS}_{pq}(r_{pq}) \vec{L} \cdot \vec{\sigma}_{pq} \tag{7}$$

with

$$S_{pq} = 3(\vec{\sigma}_p \cdot \hat{r}_{pq})(\vec{\sigma}_q \cdot \hat{r}_{pq}) - \vec{\sigma}_p \cdot \vec{\sigma}_q ,$$

$$\vec{L}_{pq} = (\vec{r}_p - \vec{r}_q) \times (\vec{p}_p - \vec{p}_q) ,$$

$$\vec{\sigma}_{pq} = \vec{\sigma}_p + \vec{\sigma}_q ,$$

where $q = p$ or $n$.

The radial dependence of each term is taken to

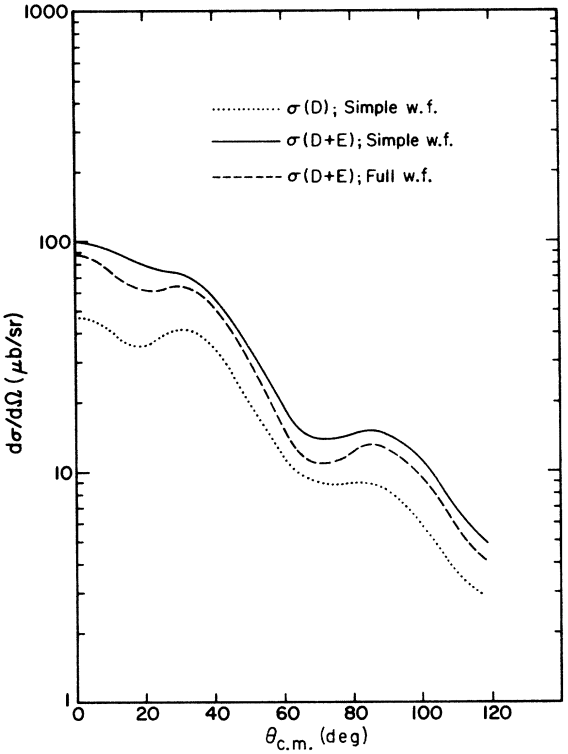


FIG. 3. Predicted angular distribution of the $1^+$ state assuming force A from Table IV. $\sigma(D)$ denotes calculation including only the direct term in the transition amplitude while $\sigma(D+E)$ includes the direct and exchange terms. The wave functions are described in the text.

be a sum of Yukawa functions:

$$V^i_{pq}(r_{pq}) = \sum_j V^{ij}_{pq} \frac{\exp[-(r_{pq}/\mu_{ij})]}{r_{pq}/\mu_{ij}} . \quad (8)$$

The differential cross sections for the excitation of the $1^+$ and $3^+$ states were calculated for five different sets of forces. The range and strength parameters characterizing these forces are given in Table IV. The central and *LS* components of force A are those of Hinrichs *et al.*[3] The strength of the tensor term is the energy dependent, empirically determined tensor strength of Austin and Fox[2] extrapolated to a bombarding energy of 17.2 MeV. Force B was derived by Borysowicz, McManus, and Bertsch[23] by fitting to the harmonic oscillator matrix elements of the Reid potential. Force C is the central part of force A. In force D, the range of the central spin dependent term is set equal to 1.4 fm and its strength, and the strength of the tensor term are adjusted to give the best fit to the measured $1^+$ angular distribution. Force E includes the central and tensor terms of force A, but the *LS* term is set equal to zero.

In order to determine whether the calculated cross sections are sensitive mainly to the dominant components in the wave functions, we examine the calculation of the $1^+$ cross section assuming force A from Table IV. The details of this calculation are shown in Fig. 3. In this figure, the simple wave function refers to the calculation in which we assume the wave function of the $1^+$ state to be $(2p_{3/2}{}^{-1}\text{-}2p_{1/2})$. The correction for the ground state correlation between the $2p_{3/2}$ and $2p_{1/2}$ orbits was made using the factor in expression (2) where instead of taking the value of $Y_{\rm ph}$ from Table II, $Y_{\rm ph}$ is calculated using expression (5) and the ground state wave function of Hughes. The exchange contribution is seen to be important, as in the calculations of Hinrichs *et al.*[3] In the present case, its inclusion raises the cross section by about a factor of 2. However, including all the components in the excited wave function as described in Sec. III B, we see that the cross section is diminished by only about 10%. The same results obtain for the $3^+$ case; including the exchange increases the cross section by about a factor of $2\frac{1}{2}$, while the cross section calculated with the full

TABLE IV. Parameters of interaction employed in DWBA analysis. The strengths are given in MeV and the radii in fm. The parameters are those presented in expression (7) and (8).

| Force | $j$ | $V^{0j}_{pp}$ | $V^{0j}_{pn}$ | $\mu_{0j}$ | $V^{1j}_{pp}$ | $V^{1j}_{pn}$ | $\mu_{1j}$ | $V^{Tj}_{pp}$ | $V^{Tj}_{pn}$ | $\mu_{Tj}$ | $V^{LSj}_{pp}$ | $V^{LSJ}_{pn}$ | $\mu_{LSj}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 1 | −14.5 | −36.4 | 1.06 | 14.6 | 2.4 | 1.06 | −24.0 | +24.0 | 0.816 | 29.1 | 20.1 | 0.557 |
| | 2 | 0.0 | 0.0 | | 0.0 | 0.0 | | 0.0 | 0.0 | | 1490 | 752 | 0.301 |
| B | 1 | 1323.6 | 5177 | 0.2 | −1323.6 | 843.3 | 0.2 | 3884 | −12 416 | 0.2 | −6700 | 3604 | 0.2 |
| | 2 | 752 | 1417 | 0.4 | −752 | −28.9 | 0.4 | −246 | 146 | 0.4 | −49 | −105 | 0.4 |
| | 3 | −605 | −1432 | 0.5 | 605 | −74.2 | 0.5 | 210 | −105 | 0.5 | 9.7 | 17.9 | 0.5 |
| | 4 | 0.0 | 0.0 | 0.7 | 0.0 | 0.0 | 0.7 | 0.0 | 0.0 | 0.7 | 0.0 | 0.0 | 0.7 |
| C | 1 | −14.5 | −36.4 | 1.06 | 14.6 | 2.4 | 1.06 | 0.0 | 0.0 | | 0.0 | 0.0 | |
| D | 1 | −14.5 | −36.4 | 1.06 | 6.0 | 1.0 | 1.4 | −18.0 | 18.0 | 0.816 | 0.0 | 0.0 | |
| E | 1 | −14.5 | −36.4 | 1.06 | 14.6 | 2.4 | 1.06 | −24.0 | 24.0 | 0.816 | 0.0 | 0.0 | |

wave function is about 6% less than that assuming only the configuration $(1f_{5/2}^{-1}\text{-}2p_{1/2})^{3^+}$

All calculations henceforth referred to will be with the exchange contribution included and for the full wave functions. As with force A, by including the full wave functions in the calculations assuming the other forces, the cross sections were diminished less than 10% relative to the corresponding calculations assuming the simple wave functions.

The calculated cross sections for forces A, B, and C are compared to the measured cross sections in Figs. 4 and 5. Only the calculation with force A gives a reasonable fit to both measured angular distributions. The 25% discrepency between force A calculation and the $1^+$ data can be reduced by small variations in the ground state correlation coefficients $\alpha$ and $\beta$ in Table III. It should be noted, however, that since the calculated cross sections in Fig. 4 assumed the ground state wave function of Hughes,[13] by using the other deduced ground state wave functions in Table III, the calculated cross sections will increase (by about 35% in the case of the ground state wave function of Comfort, Duray, and Braithwaite[21]).

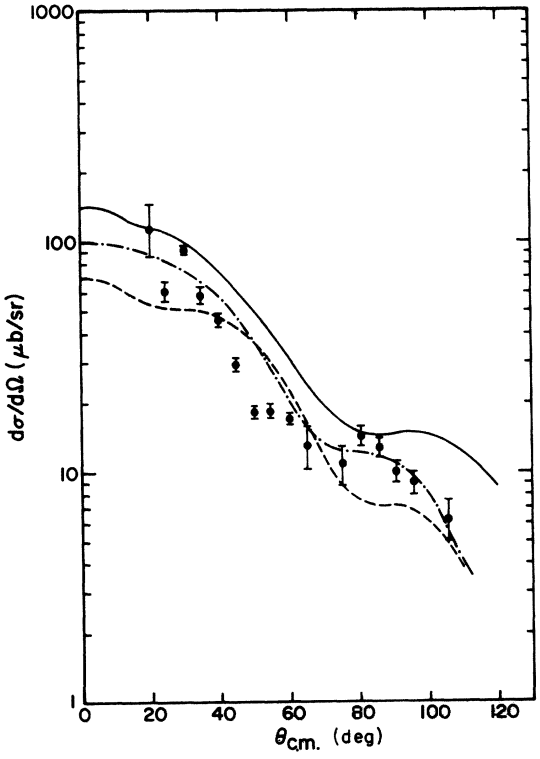


FIG. 4. A comparison of the measured angular distribution of the $1^+$ state and predicted angular distributions assuming force A (the solid curve), force B (the dash curve), and force C (the dash-dot curve). The forces are given in Table IV.

The good fit to the $1^+$ and the poor fit to the $3^+$ by force C (the purely central force, the dash-dot curves in the figures) indicates that the excitation of the $1^+$ is dominated by the central force and that the $3^+$ is dominated by the tensor terms. (Excluding the *LS* forces, force E versus force A, changes neither the $1^+$ or $3^+$ predicted cross sections by more than a few percent.) On the basis of these observations, it is reasonable to assume that if the strengths of the tensor terms in force B (the fit to the Reid potential) were increased, then the relatively good fit to the $1^+$ data would not be seriously disturbed while the $3^+$ might be substantially improved.

Finally, in Fig. 6, the cross sections calculated with force D are presented. It is seen that while this calculation gives an excellent fit to the $1^+$ data, the fit to the $3^+$ is worsened somewhat in comparison to the calculation assuming force A.

Throughout, we have assumed only direct contributions to the cross sections. We wish to address ourselves to this assumption. To estimate the compound contribution, we have performed an approximate Hauser-Feshbach calculation. In this calculation, only the channels $(p,p')$, $(p,n)$, and $(p,{}^4\text{He})$ were considered. The transmission coef-

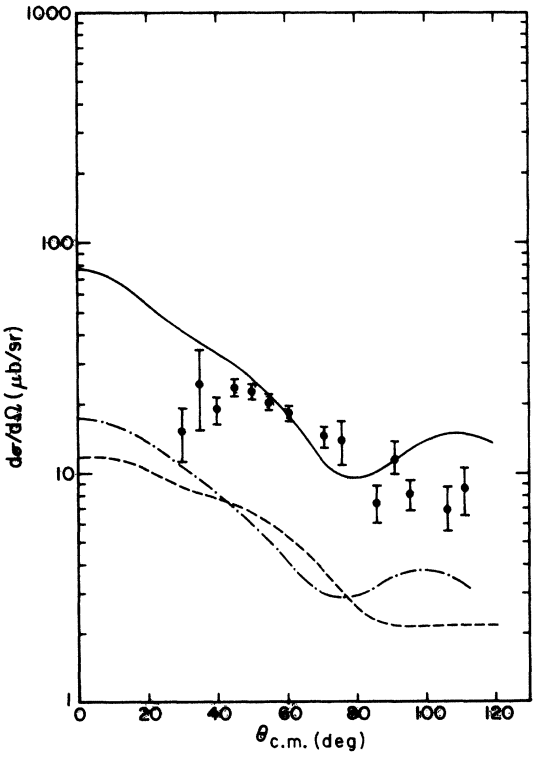


FIG. 5. A comparison of the measured and predicted angular distributions of the $3^+$ state. The forces are designated as in Fig. IV.

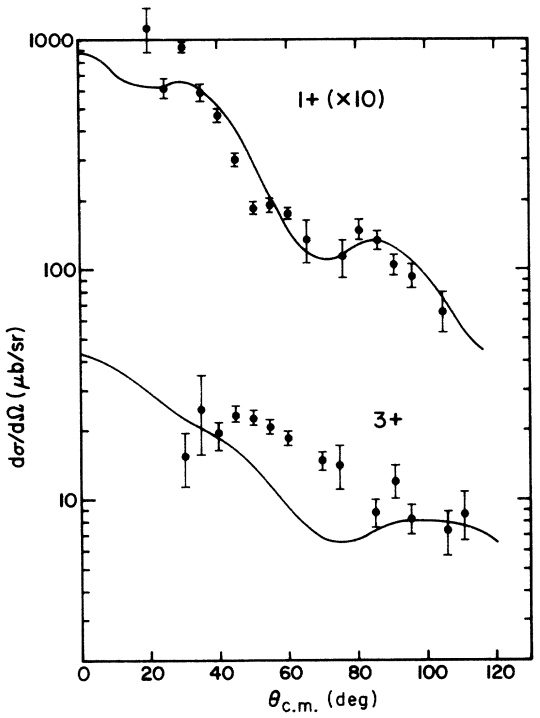


FIG. 6. A comparison of the measured angular distribution of the $1^+$ and $3^+$ levels and the angular distribution predicted assuming force D from Table IV.

ficients were approximated by step functions in $l$ space with the cutoff at $T_l = \frac{1}{2}$ as calculated by ABACUS.[24] An example of such an approximate calculation, including the level density expression used in the present calculation, is given elsewhere.[25] We thus calculate an upper limit on the compound contribution to the $1^+$ and $3^+$ levels to be about 2 $\mu$b/sr; well below the measured cross sections. The $(p,d)(d,p)$ two-step contribution will similarly be negligible since the wave functions of the $1^+$ and $3^+$ states are predominantly proton-proton hole. As mentioned earlier, Cosman and Slater[18] did not observe the $1^+$ or $3^+$ levels in $^{87}$Sr$(d,p)^{88}$Sr at $E_d = 7.5$ MeV. If we take the cross section for the two-step process as no larger than a few percent of the cross section for either of the two one-step processes,[26] then the results of Cosman *et al.*[18] in which levels other than the $1^+$ and $3^+$ *were* reported with cross sections of a few tens of microbarns per steradian, strongly suggest that the two-step $(p,d)(d,p)$ process will contribute no more than one or two microbarns per steradian to the cross sections measured in the present experiment. We were unable to make a reasonable estimate of the contributions for other two-step processes, such as $(p,2p)(2p,p)$ or $(p,n)(n,p)$, to the cross sections of the $1^+$ and $3^+$ states. We have not considered other nondirect contributions to the cross sections of the $1^+$ and $3^+$ states.

## IV. CONCLUSIONS

The evidence presented in this work indicates that the $1^+$ and $3^+$ levels can be reasonably well described through the use of very simple wave functions. This corroborates an earlier result[12] in which the ground state $M1$ transition width of the $1^+$ level is accurately predicted through the use of a wave function which is practically identical to that employed in the present work.

The adequacy of these simple wave functions derives as much from the smallness (typically a few percent) of the amplitudes of the core excitation components in the wave functions as from the fact that there is very little coherence among these terms. Indeed, if *all* the terms in the predicted $1^+$ wave function given in Table III were to enter the calculation of the cross section with total coherence, then the predicted cross section would be increased by about a factor of 3 relative to the calculation assuming only the valence terms in the wave function. Such a threefold enhancement must be compared to the actual calculated 10% diminution noted in Fig. 3.

On the basis of the apparent simple shell model structure of these states, it would be of interest to study inelastic scattering to them with projectiles and at bombarding energies other than was done in the present work. For example, one might study whether the strength of the tensor component in the interaction exhibits the same dependence on bombarding energy as was observed by Austin and Fox.[2] In addition, it would be of interest to carry out an investigation similar to that reported here on the $1^+$ and $3^+$ levels in $^{206}$Pb at 1.704 and 1.341 MeV,[27] respectively. Theoretical investigations[28] of the structure of $^{206}$Pb suggest that the dominant terms in the wave functions of these states are $(3p_{1/2}\text{-}3p_{3/2}{}^{-1})^{1^+}_n$ and $(3p_{1/2}\text{-}2f_{5/2}{}^{-1})^{3^+}_n$; analogous to the levels studied in the present work.

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