^{92,98}Mo$(d,\alpha)^{90,96}$Nb reactions and particle-hole multiplets*

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(Received 19 September 1974)

The ^{92,98}Mo$(d,\alpha)^{90,96}$Nb reactions have been studied at a bombarding energy of 17 MeV. Attention is given to the states previously observed in (^{3}He,t) reactions that are thought to arise from simple particle-hole multiplets. Distorted-wave (DW) calculations were carried out with spectroscopic amplitudes calculated from assumed simple configurations. The DW calculations are found to be sensitive to variations in a number of parameters. It is found that the normalization constants needed to fit the observed cross sections vary widely for members of a single multiplet. In addition, the average normalization constants needed for states of the $(g_{9/2})^2$ multiplet in ^{90}Nb and the $(g_{9/2}d_{5/2})$ multiplet in ^{96}Nb differ by a factor of 4.

[NUCLEAR REACTIONS ^{92,98}Mo$(d,\alpha)^{90,96}$Nb, $E=17$ MeV; measured $\sigma(E_\alpha,\theta)$, $\theta=11-90°$, $\Delta\theta=4-10°$; enriched targets; resolution 25–30 keV. Deduced J^π, L, multiplet assignments. DWBA analysis.]

I. INTRODUCTION

Nuclei in the region of $A=90$ are interesting for understanding the shell model since the simplest description provides at least three doubly-closed-shell nuclei—^{88}Sr, ^{90}Zr, and ^{96}Zr. Thus, there are available over a small mass range many single-particle, single-hole, two-particle, and particle-hole spectra. Extensive information is therefore available for single-particle energies and two-body residual interactions.

Values for the two-body matrix elements are available, for example, in particle-hole multiplets of the isobars of the closed-shell nuclei. The nuclei ^{88}Y, ^{90}Nb, and ^{96}Nb have recently been investigated[1-4] with the charge-exchange (^{3}He, t) reaction. Among the low-lying states are the multiplets $(p_{1/2}g_{9/2}^{-1})$, $(g_{9/2}g_{9/2}^{-1})$, and $(g_{9/2}d_{5/2}^{-1})$. Even though it has been generally possible to assign dominant configurations to the energy levels and to make reasonable spin and parity assignments, difficulties in the interpretation of the (^{3}He, t) reaction mechanism[5,6] have precluded firm conclusions about the configuration purity of the states.

These particle-hole nuclei cannot generally be populated by single-particle transfer reactions nor, in any case, could all the multiplets of interest be reached. The (d,α) reaction is, however, a valuable spectroscopic probe. Like the (^{3}He, t) reaction, the microscopic description is sensitive, through coherence, to mixed configurations. Several detailed studies[7,8] have illustrated its utility in the $1f$-$2p$ shell and investigations have recently been extended to the region of $A=90$.[9,10]

The usefulness of the (d,α) reaction results primarily from its selectivity. If the transferred nucleon pair is in a relative s state, the reaction process is restricted to having $T=0$ and $S=1$ and the cross sections are sensitive only to the coherence of different microscopic components in the wave functions (neglecting spin-orbit effects). This restriction simplifies the calculations and the interpretation, especially in the present case in which simple shell-model configurations might be assumed for the relevant nuclei.

In this respect the (d,α) reaction has a very important selection rule: If both transferred nucleons come from the same shell-model orbital, a transfer of even total angular momentum is forbidden. Thus, for example, the (d,α) reaction on the even-even target ^{92}Mo$[\pi(g_{9/2})^2\nu(g_{9/2})^{10}]$ cannot populate the even-J members of the $(g_{9/2}g_{9/2}^{-1})$ multiplet in ^{90}Nb. In a direct reaction as described above, any observed cross section to these states, therefore, measures the amount of pickup from nonidentical orbits and thus gives information con-

TABLE I. The energies (MeV) of neutron and proton single-particle orbitals relative to the lowest unfilled (proton) or filled (neutron) orbital.

Proton Orbital	Proton Energy	Neutron Orbital	Neutron Energy
	^{90}Zr		
$g_{9/2}$	0.0	$g_{9/2}$	0.0
$p_{1/2}$	0.1	$p_{1/2}$	0.6
$p_{3/2}$	1.6	$p_{3/2}$	1.1
$f_{5/2}$	1.8	$f_{5/2}$	1.5
$d_{5/2}$	~3.0		
	^{96}Zr		
$g_{9/2}$	0.0	$d_{5/2}$	0.0
$p_{1/2}$	0.7	$s_{1/2}$	1.0
$p_{3/2}$	1.2	$g_{9/2}$	3.0
$f_{5/2}$	1.5		
$d_{5/2}$	2.5		

cerning the degree of configuration mixing in the initial and final wave functions.

Such information is the principal motivation for the present work. Multiplets in two of the particle-hole nuclei mentioned above were populated by the (d, α) reaction on ^{92}Mo and ^{98}Mo targets. The possible multiplets are carefully enumerated and, in the absence of detailed model calculations, the simplest assumptions are made for the mixing of these configurations. The two-particle-transfer spectroscopic amplitudes can thereby be calculated and are used for comparison with the observed cross sections.

Since it is well known[7-9] that both the magnitudes and shapes of (d, α) angular distributions as computed in the distorted-wave (DW) mechanism are very sensitive to the optical potential for the α particles, the effects of several potentials are investigated. The effects of finite range and nonlocality factors are also considered.

TABLE II. Multiplets and (d, α) transition strengths $[\beta^2/(2J+1)]$ in ^{90,96}Nb. The strength d^2 represents the intensity of a $(d_{5/2}^{-2} s_{1/2}^{2})$ admixture in the ^{98}Mo ground state.

Multiplet	Configuration $(j^n)_{JT}$	E_x	J	π	Num.	$\pi(j_1)\nu(j_2)$	Strength	Phase	$\pi(j_2)\nu(j_1)$	Strength
				^{90}Nb						
$(g_{9/2})^2$	$(g_{9/2}^{10})_{04}$	0.0	0–9	+	1	$g_{9/2}g_{9/2}$	1/5			
$g_{9/2}p_{1/2}$	$(p_{1/2}^{3})_{1/2,1/2}(g_{9/2}^{11})_{9/2,9/2}$	0.9	4–5	−	1	$g_{9/2}p_{1/2}$	1/10	−	$p_{1/2}g_{9/2}$	1/90
$p_{1/2}g_{9/2}$	$(p_{1/2}^{3})_{1/2,1/2}(g_{9/2}^{11})_{9/2,7/2}$	0.6	4–5	−	1	$p_{1/2}g_{9/2}$	22/45			
$(p_{1/2})^2$	$(p_{1/2}^{2})_{00}(g_{9/2}^{12})_{04}$	1.2	0–1	+	1	$p_{1/2}p_{1/2}$	1			
$p_{3/2}g_{9/2}$	$(p_{3/2}^{7})_{3/2,1/2}(g_{9/2}^{11})_{9/2,7/2}$	1.6	3–6	−	1	$p_{3/2}g_{9/2}$	22/45			
$g_{9/2}p_{3/2}$	$(p_{3/2}^{7})_{3/2,1/2}(g_{9/2}^{11})_{9/2,9/2}$	1.8	3–6	−	1	$g_{9/2}p_{3/2}$	1/10	−	$p_{3/2}g_{9/2}$	1/90
$f_{5/2}g_{9/2}$	$(f_{5/2}^{11})_{5/2,1/2}(g_{9/2}^{11})_{9/2,7/2}$	1.8	2–7	−	1	$f_{5/2}g_{9/2}$	22/45			
$g_{9/2}f_{5/2}$	$(f_{5/2}^{11})_{5/2,1/2}(g_{9/2}^{11})_{9/2,9/2}$	2.0	2–7	−	1	$g_{9/2}f_{5/2}$	1/10	−	$f_{5/2}g_{9/2}$	1/90
$p_{1/2}p_{3/2}$	$(p_{3/2}^{7})_{3/2,1/2}(p_{1/2}^{3})_{1/2,1/2}$	2.2	1–2	+	2	$p_{1/2}p_{3/2}$	$\sum$=9/10	±	$p_{3/2}p_{1/2}$	$\sum$=9/10
$p_{1/2}f_{5/2}$	$(f_{5/2}^{11})_{5/2,1/2}(p_{1/2}^{3})_{1/2,1/2}$	2.5	2–3	+	2	$p_{1/2}f_{5/2}$	$\sum$=9/10	±	$f_{5/2}p_{1/2}$	$\sum$=9/10
				^{96}Nb						
$g_{9/2}d_{5/2}$	$(g_{9/2}^{11})_{9/2,9/2}(d_{5/2}^{5})_{5/2,5/2}$	0.0	2–7	+	1	$g_{9/2}d_{5/2}$	1/10			
$p_{1/2}d_{5/2}$	$(p_{1/2}^{3})_{1/2,1/2}(g_{9/2}^{12})_{04}(d_{5/2}^{5})_{5/2,5/2}$	0.7	2–3	−	1	$p_{1/2}d_{5/2}$	1/2			
$g_{9/2}s_{1/2}$	$(g_{9/2}^{11})_{9/2,9/2}(d_{5/2}^{4})_{02}(s_{1/2}^{1})_{1/2,1/2}$	1.0	4–5	+	1	$g_{9/2}s_{1/2}$	$(1/10)d^2$			
$p_{3/2}d_{5/2}$	$(p_{3/2}^{7})_{3/2,1/2}(g_{9/2}^{12})_{04}(d_{5/2}^{5})_{5/2,5/2}$	1.2	1–4	−	1	$p_{3/2}d_{5/2}$	1/2			
$f_{5/2}d_{5/2}$	$(f_{5/2}^{11})_{5/2,1/2}(g_{9/2}^{12})_{04}(d_{5/2}^{5})_{5/2,5/2}$	1.5	0–5	−	1	$f_{5/2}d_{5/2}$	1/2			
$p_{1/2}s_{1/2}$	$(p_{1/2}^{3})_{1/2,1/2}[(g_{9/2}^{12})_{04}(d_{5/2}^{6})_{02}]_{06}(s_{1/2}^{1})_{1/2,1/2}$	1.7	0–1	−	1	$p_{1/2}s_{1/2}$	$(1/2)d^2$			
$p_{3/2}s_{1/2}$	$(p_{3/2}^{7})_{3/2,1/2}[(g_{9/2}^{12})_{04}(d_{5/2}^{6})_{02}]_{06}(s_{1/2}^{1})_{1/2,1/2}$	2.2	1–2	−	1	$p_{3/2}s_{1/2}$	$(1/2)d^2$			
$f_{5/2}s_{1/2}$	$(f_{5/2}^{11})_{5/2,1/2}[(g_{9/2}^{12})_{04}(d_{5/2}^{6})_{02}]_{06}(s_{1/2}^{1})_{1/2,1/2}$	2.5	2–3	−	1	$f_{5/2}s_{1/2}$	$(1/2)d^2$			
$(g_{9/2})^2$	$(g_{9/2}^{10})_{04}(d_{5/2}^{6})_{03}$	3.0	0–9	+	1	$g_{9/2}g_{9/2}$	1/5			

II. SPECTROSCOPY

The (d,α) transition strengths are computed here for simple particle-hole multiplets. These multiplets are carefully identified, subject, whenever possible, to the assumptions of good closed shells, lowest seniority configurations, and good isospin and seniority quantum numbers. Details of these considerations are given in the Appendix.

The ground state of a particle-hole nucleus is a member of the multiplet formed by transferring a neutron in the highest (least-bound) orbital to the lowest empty proton orbital. The centroid excitation energies of other particle-hole multiplets can be estimated by adding appropriate energy differences between the various orbitals. Such energy differences may contain small fluctuations with mass number. By using the observed energies and strengths for population of the orbitals in single-particle-transfer reactions on nearby closed-shell nuclei,[11-21] relative single-particle energies for the nuclei of interest can be estimated. They are listed in Table I.

The degeneracies of each multiplet will be removed by residual interactions that will also cause some configuration mixing. If the splitting is large for the ground-state multiplet, the centroid excitation energies must be readjusted to account for the shift of the ground state. This will be the case in ^{90}Nb where the centroid energy of the $(g_{9/2}\,g_{9/2}^{-1})$ configuration is near 500 keV excitation.[2] The centroid energies of the excited multiplets have been increased by this amount in ^{90}Nb.

Not all multiplets in these nuclei have unique orbit representations. Some may have $T_>$(analog) or $T_<$(antianalog) designations. There may also be two $T_<$ multiplets with the same designation, but distinguished by the partial isospin of the participating orbits. In addition, there may be even more than one multiplet with the same orbit and isospin configuration but differing by the relative amplitudes and phases of simple shell-model components. A full discussion is given in the Appendix.

The spectroscopic amplitudes β for the (d,α) reaction may be evaluated for all of the expected multiplets, as is also discussed in the Appendix. The cross sections are proportional to β^2 and in every case $\beta^2 \propto (2J+1)$, where J is the total angular momentum transferred in the reaction.

Table II itemizes the expected multiplets in ^{90}Nb and ^{96}Nb and identifies their full (J,T) configurations, expected centroid energies, J^π values, the p-n pairs transferred in the (d,α) reaction for populating the multiplets, and their spectroscopic intensities $\beta^2/(2J+1)$. For the unique antianalogs in ^{90}Nb the relative phases of the component terms are given. Such phases are not known *a priori* for the double antianalog multiplets and only the total strengths are given for these.

III. DATA

A. Data acquisition

Data for the reactions ^{92,98}Mo$(d,\alpha)^{90,96}$Nb were obtained with a 17-MeV deuteron beam from the Argonne National Laboratory FN tandem accelerator. The targets were self-supporting rolled foils, each at least 98% isotopically pure, with thicknesses between 210 and 240 μg/cm^2.

The α particles were detected in photographic plates placed in the focal plane of the Enge split-pole spectrograph.[22] Discrimination against other particles was achieved by use of low-sensitivity Ilford $K-1$ emulsions which were then underdeveloped. The plates were scanned manually. Data were obtained for 11 angles between 20° and 90°.

It was found impossible to obtain data at angles less than 20° with photographic plates because the large number of scattered deuterons fogged the emulsions. For such angles, a position-sensitive silicon surface-barrier detector was placed in the focal plane of the spectrograph. The depletion depth was about 450 μ and its length was about 5 cm. The energy signal was passed through a discriminator that was set very high (since the α particles lose the greatest energy in the detector) and used to gate the energy-times-position signal in a multichannel analyzer. The relative efficiency of the detector was evaluated and found to be constant over the region of interest.

Spectra were obtained at angles of 11° and 15°. Additional spectra were also obtained at 20° and 25° in order to compare the consistency of the detector and emulsion data. As a whole, the data were found to be in good accord and did not require relative renormalization. Since the detector could not span the entire excitation energy region of interest, several runs were taken with separate settings of the magnetic field.

The emulsion data showed resolutions of about 30 and 25 keV on the ^{92}Mo and ^{98}Mo targets, respectively, due primarily to target thickness. The detector data also had about 30-keV resolution, but the spectra had significant backgrounds.

B. Data analysis

Levels in the residual nuclei that were known from previous experiments[2-4,23,24] were readily identified. The computer programs QPLOT and AUTOFIT[25] were used to identify many new levels in these nuclei and to extract the number of counts in each peak in the spectra.

Absolute cross sections were calculated from the experimental yields and knowledge of the target

thicknesses, total beam charge, and solid angles. Such calculations were done independently for the emulsion and detector data and, as mentioned above, did not appear to require renormalization. The absolute cross sections are thus believed to be accurate to within about 15%.

C. General interpretation

Representative spectra for the reactions ^{92}Mo-$(d, \alpha)^{90}$Nb and ^{98}Mo$(d, \alpha)^{96}$Nb are shown in Figs. 1 and 2. Arrows indicate the position of some levels known from earlier studies of the niobium nuclei.[2-4,23,24]

Levels in ^{90}Nb believed to arise from the $(g_{9/2}\, g_{9/2}^{-1})$ configuration[2,3] are indicated by the lower group of arrows in Fig. 1. If these levels do not have mixed configurations, and if ^{92}Mo is a simple shell-model nucleus as assumed in Sec. II, then the even-J states should not be populated by the (d, α) reaction. The $J^\pi = 8^+$ ground state is indeed very weak, having a maximum cross section of about 2 μb/sr. The 6^+ state is unresolvable from a known[23] 4^- level. The 4^+ state is as weakly populated as the ground state and could not be clearly distinguished from the tail of the 5^+ state at most angles. The 2^+ state, which could only be resolved from the neighboring 9^+ state by the use of the program AUTOFIT,[24] is not so weakly populated and has a maximum cross section of about 40 μb/sr. The 0^+ state is the analog state near 6-MeV excitation energy. Thus, with the possible exception of the 2^+ state, an initial consideration of these even-J states of ^{90}Nb suggests that they may not have substantial configuration mixing.

Other known[23,24] levels in ^{90}Nb are indicated in Fig. 1 by the upper group of arrows. These were seen very weakly or not at all in the $(^3\mathrm{He}, t)$ reaction.[2,3] The levels at 1.78, 1.84, 2.31, and 2.35 MeV were assigned $J^\pi = (0, 1)^+$ in the study[23] of the β decay of ^{92}Mo. The $(p, n\gamma)$ data of Yoshida *et al.*,[24] support $J^\pi = 1^+$ for the first three states and dispute the presence of the fourth. Levels near these energies are seen in the (d, α) reaction and $J^\pi = 0^+$ assignments should therefore be excluded since 0^+ states are forbidden by the $S = 1$ selection rule. However, not all of the angular distributions have a distinctive $J^\pi = 1^+$ pattern. A low-spin positive-parity level proposed at 1.324 MeV in Ref. 23 is not seen in the (d, α) reaction. If such a level exists, it could likely have a 0^+ assignment. Its existence is disputed in Ref. 24.

In the ^{96}Nb spectrum in Fig. 2, the levels attributed[4] to the $(g_{9/2}d_{5/2}^{-1})$ multiplet are indicated by the lower group of levels. Spins were tentatively assigned in the $(^3\mathrm{He}, t)$ study,[4] and they appear to be confirmed by recent $(p, n\gamma)$ data.[26]

Even though there is no specific selection rule prohibiting transitions to the even-J states, the 2^+, 4^+, and 6^+ states are very weakly populated with maximum cross sections near 10, 5, and 4 μb/sr, respectively. This inhibition can be traced to the 9-j symbol which appears in the transition amplitude.

Three other levels were weakly populated by the $(^3\mathrm{He}, t)$ reaction[4] and they are designated by the upper group of arrows. Two of them are strongly populated by the (d, α) reaction. They can be attributed to the $(p_{1/2}d_{5/2})$ multiplet in Table II.

The significance of the very strong levels seen near 1.5 MeV excitation in both ^{90}Nb and ^{96}Nb will be discussed later.

IV. DISTORTED-WAVE CALCULATIONS

The mechanism of the present (d, α) reactions is treated here according to the usual distorted-wave (DW) methods.[27-29] Previous work with 12-MeV

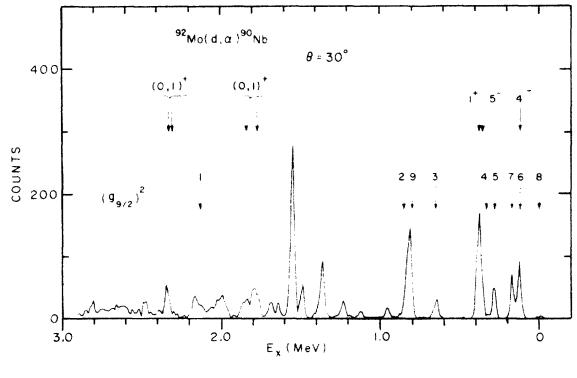

FIG. 1. Spectrum for the (d, α) reaction on ^{92}Mo. Arrows indicate the positions of some states with previously known spin-parity assignments. The lower group gives locations of the supposed positive-parity states of the $(g_{9/2})^2$ configuration.

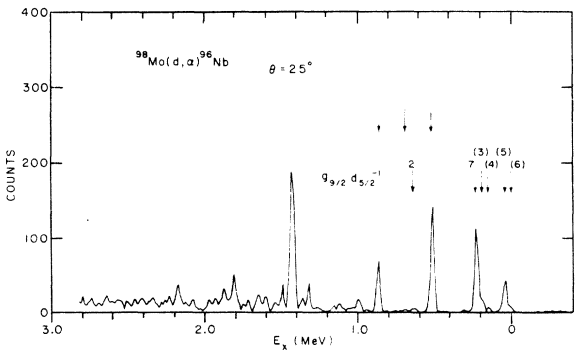

FIG. 2. Spectrum for the (d, α) reaction on ^{98}Mo. Arrows indicate the positions of states previously known in ^{96}Nb. States associated with the lower group are believed to arise from the $(g_{9/2}d_{5/2}^{-1})$ configuration.

deuterons in the $1f$-$2p$ shell[7,8] has shown the fruitfulness of interpreting the (d,α) reaction as proceeding by a simple direct process. This has also been the case in the recent studies[9,10] of the ^{90}Zr-$(d,\alpha)^{88}$Y reaction with 15- and 17-MeV deuterons. Angular distributions taken out to 150° in Ref. 9 gave no indication of competing compound-nucleus processes.

A. Cross sections and form factors

The direct DW transition amplitude for the reaction $A(d,\alpha)B$ contains, in the zero-range approximation, integrals of the form[27-29]

$$I=\int \chi_\alpha^{(-)}(k_\alpha,R)F_{LSJ}(j_1j_2,R)\chi_d^{(+)}\left(k_d,\frac{M_B}{M_A}R\right)dR\,, \tag{1}$$

where $\chi_\alpha^{(-)}$ and $\chi_d^{(+)}$ are α-particle and deuteron distorted waves and $F_{LSJ}(j_1j_2,R)$ is the two-nucleon form factor. This form factor contains a sum over the microscopic configurations with spectroscopic amplitudes given by β in Eq. (A6) of the Appendix, integrals over the internal wave functions of the incident and outgoing particles, and the $LS-jj$ transformation factors

$$\begin{bmatrix} l_1 & s_1 & j_1 \\ l_2 & s_2 & j_2 \\ L & S & J \end{bmatrix}=[(2S+1)(2L+1)(2j_1+1)(2j_2+1)]^{1/2} \times \begin{Bmatrix} l_1 & s_1 & j_1 \\ l_2 & s_2 & j_2 \\ L & S & J \end{Bmatrix}, \tag{2}$$

where the quantity in braces is a 9-j symbol.

The calculations were carried out with the computer program DWUCK.[30] The form factor was calculated by the method of Bayman and Kallio[31] who transform the product two-nucleon form factor directly into relative and center-of-mass coordinates. A relative s state is assumed for the transferred pair. The cross sections are given by

$$\frac{d\sigma}{d\Omega}=N\frac{\sigma(\mathrm{DWUCK})}{2J+1}, \tag{3}$$

where N is a normalization factor.[27-29] It includes the overlap of the light-mass particles and its absolute value is not an important concern in the present study. All spins are included in the DWUCK calculations.

B. Troubles

The calculation of cross sections for (d,α) reactions is subject to many uncertainties and ambiguities which have been discussed in several articles.[7,9,32] The issues involve the sensitivity of shapes and cross sections of (d,α) reactions to different prescriptions of the separation-energy requirements, variations in the optical and bound-state potentials, and corrections for nonlocality and finite-range effects. In the present case, these must be known as a function of target, Q value, configuration, and total angular momentum. A set of eight simple (d,α) transitions were thus chosen for an examination of the issues in some additional detail.

The uncertainty concerning the separation-energy procedure is complex in principle, but may not be very important in practice. In constructing the microscopic form factor from single-particle wave functions, it is not obvious whether the neutron and proton separation energies should sum to the (γ,d) Q value (corresponding to a pickup of a deuteron cluster) or to the (γ,np) Q value (corresponding to the total energy difference between the nuclear states), nor is it clear whether they should be equal or whether they should reflect the real separation energies.[7] In fact, as is made clear by Jaffe and Gerace,[33] none of these methods is correct. The total separation energy is not necessarily independent of the radial coordinate, but varies in response to the residual interaction of the transferred nucleons. This interaction also introduces correlations to the form factor that cannot be included easily in zero-range DW calculations.

Apart from the neglect of the residual interaction, there is not much difference between the various prescriptions. The calculations are almost completely insensitive to the division of separation energy between the nucleons, provided that the sum remains constant. There is some dependence on whether or not the deuteron binding energy of 2.225 MeV is included in the sum, but the effect appears to be less than 20% at most and commonly less than 10% in cross section. In all subsequent calculations we have taken $E_n=E_p=\frac{1}{2}[Q(\gamma,np)]$, in agreement with common practice. The nucleons are each bound to the residual nucleus (for pickup reactions) as is required by standard distorted-wave Born-approximation (DWBA) programs.

The examination of the dependence on optical-model potentials was made with potential sets taken directly from the literature. Since these are normally adjusted to fit elastic scattering data, different choices of these potentials should reflect the relative uncertainty to be attached to the present (d,α) calculations.

Some of the potential sets are listed in Table III. For the deuterons, Set D1 is the best six-parameter fit[34] to elastic scattering data of 15-MeV deuterons from ^{90}Zr, while Set D4 is the average Set D of Perey and Perey.[34] It was used in Ref. 14.

TABLE III. The optical-model and bound-state potential sets used for the DW calculations. The potentials have the usual Woods-Saxon forms (volume and surface derivative) that are defined explicitly in the references. The DWUCK program requires the use of $W' = 4W_D$. Potentials have units MeV and lengths have units fm.

Set	V	r_0	a	W	W_D	r'	a'	r_c	Ref.
					Deuterons				
D1	98.1	1.127	0.848	0	14.87	1.394	0.655	1.127	34
D4	79.0	1.30	0.73	0	24.0	1.34	0.65	1.30	34,14
D5	90	1.20	0.75	0	16.6	1.30	0.70	1.30	32
					α particles				
A4	187.3	1.444	0.523	22.3	0	1.444	0.523	1.30	35
A5	228	1.366	0.557	23.3	0	1.242	0.557	1.40	36
A7	177.2	1.443	0.514	19.84	0	1.459	0.445	1.40	9
A8	181.3	1.20	0.75	15.0	0	1.70	0.60	1.30	32
					Bound nucleons				
BS1	a	1.25	0.65		(λ=25)				
BS2	a	1.17	0.75		(λ=25)				37
BS3	a	1.25	0.75		(λ=25)				10

[a] Adjusted to reproduce the proper separation energy.

For α particles Set A4 is the best four-parameter fit potential (No. 3 of McFaddon and Satchler[35]) for the interaction of 24.7-MeV α particles with ^{90}Zr. Set A7 is a six-parameter revision of Set A4 by Park, Jones, and Bainum.[9] Set A5 was quoted by Fou, Zurmühle, and Joyce[36] as best representing the scattering of 34.7-MeV α particles from ^{90}Zr and is based on unpublished results from the Oak Ridge National Laboratory. The potential Sets D5 and A8 are a special combination obtained from the well-matching procedures of DelVecchio and Daehnick.[32]

For calculating the single-nucleon wave functions, Set BS1 in Table III has been a common choice and Set BS2 is based on the extensive analysis of proton elastic scattering by Becchetti and Greenlees.[37] Set BS3 is used only with the combination D5-A8.

Not all possible combinations of the potentials were investigated. The effects on cross sections for some of them are shown in Table IV. For reasons that are given below, we have taken the potential combination D1-A5-BS1 in the zero-range and local (ZRL) approximations as standard. The numbers in Table IV show the relative cross sections of the eight representative transitions for other

TABLE IV. Relative cross sections for specified (d,α) transitions as a result of variations in the parameters of the calculations. The combination D1-A5-BS1, with type ZRL, is taken as standard. The numbers in brackets for the first transition indicate the change in absolute scale from the reference calculation. (The angular distributions for states in ^{88}Y were obtained along with the others. They agree with those in Ref. 10 and are not considered in detail in this article.)

		Potentials			D4-A5	D1-A4	D1-A7	D1-A5	D1-A5	D1-A5	D5-A8
		Type			ZRL	ZRL	ZRL	ZRL	ZRNL	FRNL	FRNL
		Bound State		Geom.	BS1	BS1	BS1	BS2	BS1	BS1	BS3
Nucleus	Config.	J	L	Q							
^{96}Nb	$g_{9/2}d_{5/2}$	3	2	8.246	1.0	1.0	1.0	1.0	1.0	1.0	1.0
					(0.85)	(~4.0)	(~12.0)	(0.50)	(1.0)	(1.4)	(3.3)
		7	6	8.246	1.0	~0.40	~0.33	1.0	1.0	1.3	0.65
^{90}Nb	$(g_{9/2})^2$	3	2	6.567	0.92	~0.80	~0.70	1.0	1.0	1.0	1.0
		7	6	6.567	0.95	~0.40	~0.33	1.0	1.0	1.3	0.70
^{96}Nb	$p_{3/2}d_{5/2}$	4	3	7.046	1.0	~0.40	~0.40	1.1	1.0	1.2	0.60
^{90}Nb	$p_{1/2}g_{9/2}$	4	3	5.967	1.0	−0.55	~0.55	1.1	1.0	1.2	0.70
^{88}Y	$p_{1/2}g_{9/2}$	4	3	6.214	1.0	~0.50	~0.40	1.1	1.0	1.2	0.70
	$p_{3/2}g_{9/2}$	4	3	4.814	1.0	~0.40	~0.35	1.1	1.0	1.2	0.70

combinations. The change in the absolute scale [i.e., the variation of N in Eq. (3)] for each column is shown for the first transition. Since there were also occasionally quite sizable changes in shape for the angular distributions, the numbers should not be taken too rigorously.

Included in Table IV are the effects of nonlocality (NL) and finite-range (FR) corrections. They were calculated by the methods of Rost and Kunz[38] that are available in DWUCK3.[30] The nonlocality factors were applied only to the optical channels and employed $\beta_d = 0.54$ and $\beta_\alpha = 0.20$. The finite-range parameter was $R = 0.4$ fm. For the calculations in the last column, all the parameter values are as specified by Daehnick and Bhatia.[10]

It is clear from Table IV that the changes in the deuteron or bound-state potentials, or the nonlocality corrections, do not affect significantly the relative cross sections, although the absolute scale could change by a factor of 2. The effects of finite-range corrections do not appear to exceed about 30%. On the other hand, the calculations are quite sensitive to the choice of the α-particle potential. By changing this potential, one can obtain variations of a factor of 3 in relative cross section and more than an order of magnitude in absolute scale. It should be noted that these differences are also accompanied by sharp changes in the shapes of the angular distributions. In fact, Set A5 was the only potential (except the combination D5-A8) that produced even approximate agreement with the data, at least in the ZRL calculations.

Our choice of the combination D1-A5-BS1 is based primarily on the fact that in ZRL calculations it best represents the data. It provided the necessary rate of decrease in cross section with angle that other combinations could not. In addition, the nonlocality corrections are negligible and the finite-range corrections are small with this combination. The combination D1-A7 was most favored in the extensive analysis by Park *et al.*,[9] but in this case the FRNL corrections are necessary and quite sizable. The well-matched combination D5-A8 gave results that did not differ greatly from those of Set D1-A5. Either combination could give slightly superior fits to some experimental transitions. On the whole, D1-A5 appeared to do slightly better. Since the FRNL variations are small for the Set D1-A5, we have preferred to use only ZRL calculations until a complete finite-range calculation can be done. For

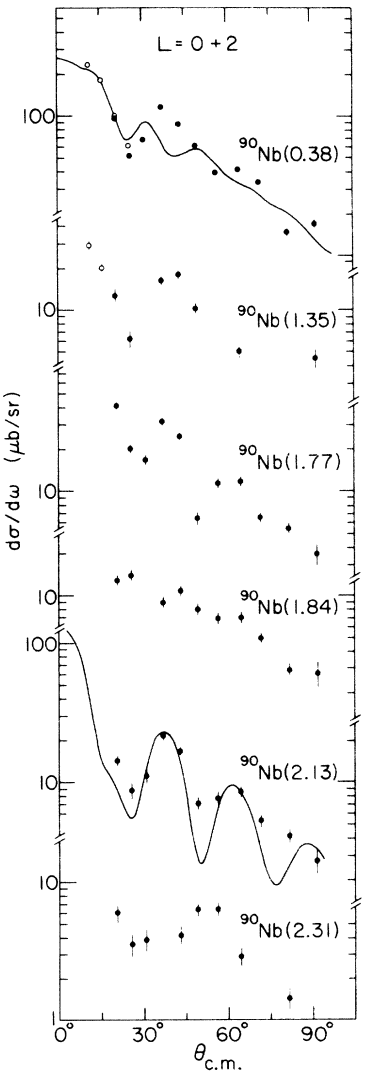

FIG. 3. Angular distributions for known or probable $J^\pi = 1^+$ states in ^{90}Nb. The curves are DW calculations for the states at 0.38 and 2.13 MeV, assuming pure $(p_{1/2}{}^2)$ and $(g_{9/2}{}^2)$ configurations, respectively.

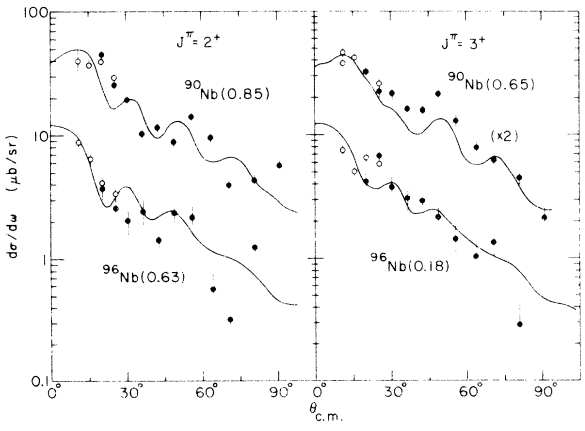

FIG. 4. Angular distributions for known $J^\pi = 2^+$ and 3^+ states. Each state is identified by nucleus and excitation energy in MeV. The curves are DW calculations.

completeness we note that we use a radius r_{rms} = 1.70 fm for the α particle, consistent with the value of Ref. 28. The calculations are not very sensitive to changes in this parameter.

V. DISCUSSION

A. L-value assignments

Irrespective of the ambiguities in the detailed DW calculations, significant information is obtainable from the present (d, α) data. Some L values can be assigned for states that have angular distributions similar to the experimental shapes of states with known L assignments. Previous tentative J^π assignments can be compared for consistency with the (d, α) data and, in some cases, new J^π values and configuration assignments can be suggested.

1. Even-L values

Angular distributions for known or probable 1^+ states are shown in Fig. 3. Most of these have similar and distinctive patterns that can be fitted by the DW calculations. The levels at 0.38 and 2.13 MeV are known[2, 23, 24] to have $J^\pi = 1^+$. The 1.35-MeV state may be the same as the 1.344-MeV state in Ref. 24, for which 1^+ is favored. Tentative $(0, 1)^+$ assignments[23] or $(1)^+$ assignments[24] have been given to states very near to the other levels shown. Since 0^+ states are unobservable in a direct (d, α) reaction, these should also have $J^\pi = 1^+$ if they are indeed the same states. The angular distributions for the 1.84- and 2.31-MeV states are somewhat unlike the others. The former undoubtably comes from an unresolved doublet,[24] and so might also the latter. The DW calculation for the 0.38-MeV state is for a $(p_{1/2})^2$ configuration. It is dominated by $L = 2$, in contrast with the data which show an appreciable $L = 0$ component.

Angular distributions for states that have been assigned[2-4, 23] 2^+ and 3^+ are shown in Fig. 4. Since the 2^+ state in ^{90}Nb is thought[2, 3] to be a member of the $(g_{9/2})^2$ multiplet, it should not be populated by the (d, α) reaction. However, the data may contain contributions from three other states.[24] The DW calculation is for a $(p_{1/2} p_{3/2})$ admixture in the form factor. The two 2^+ states in Fig. 4 have very similar angular distributions except at very forward angles. Backgrounds in the position-sensitive-detector data or difficulties in resolving the level in ^{90}Nb from other states may have caused some discrepancies. The two 3^+ states also have very similar shapes, but the cross sections appear to decline less steeply with angle, suggesting the presence of $L = 4$.

Figure 5 shows angular distributions for states previously assigned[2-4, 23] 5^+, 7^+, and 9^+. They are reasonably well fitted by the DW calculations, the largest discrepancies being at forward angles and

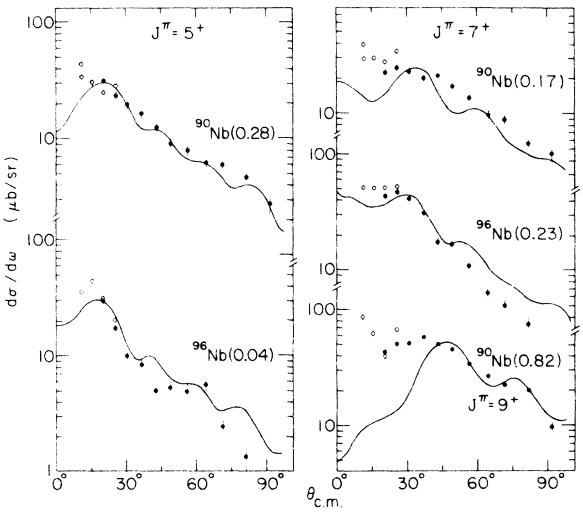

FIG. 5. Angular distributions for known $J^\pi = 5^+$, 7^+, and 9^+ states. See also the caption for Fig. 4.

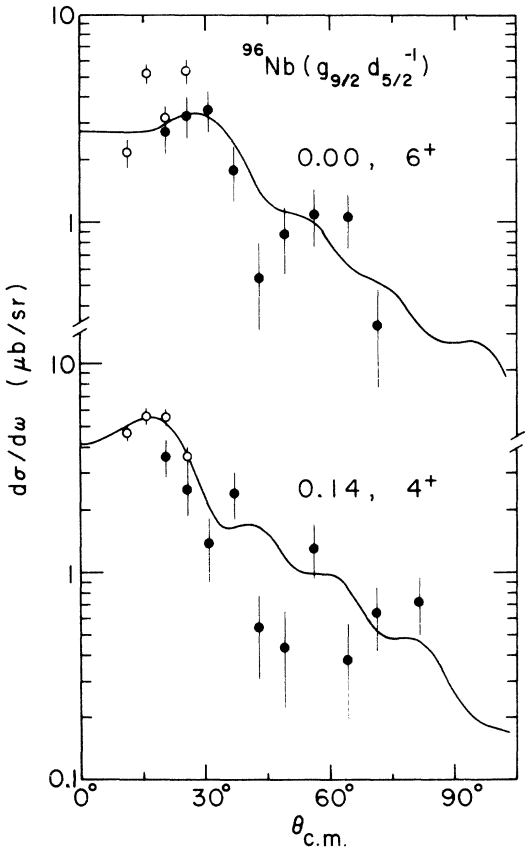

FIG. 6. Angular distributions for the $J^\pi = 6^+$ and 4^+ states in ^{96}Nb. See also the caption for Fig. 4.

for the highest spins. Some of the difficulty with the 9^+ state may be due to unresolved levels.[24] The angular distribution for this state is similar to that for the 1.75-MeV state in ^{88}Y, shown in Ref. 10. Although $L=3$ was preferred by Daehnick and Bhatia,[10] there is good evidence[1,39] that it is rather a 9^+ state.

The 5^+ state in ^{96}Nb appears to have a steeper decrease of cross section with angle than does the one in ^{90}Nb. It supposedly has a $(g_{9/2}d_{5/2}^{-1})$ configuration[4] whereas the other has a $(g_{9/2})^2$ configuration.[2,3] The difference in shapes could reflect the differences in the $L=4$ and $L=6$ contributions for the two configurations. Indeed, the 9-j symbol in Eq. (2) provides an enhancement of a factor of 4 in the ratio of $L=4$ to $L=6$ strengths between the $(g_{9/2}d_{5/2}^{-1})$ and $(g_{9/2})^2$ configurations, respectively. A similar situation occurs for the 7^+ states where an $L=8$ component is forbidden in ^{96}Nb. However, the present DW calculations show that the contributions from the higher-L values are negligible and that this effect actually arises from Q dependence.

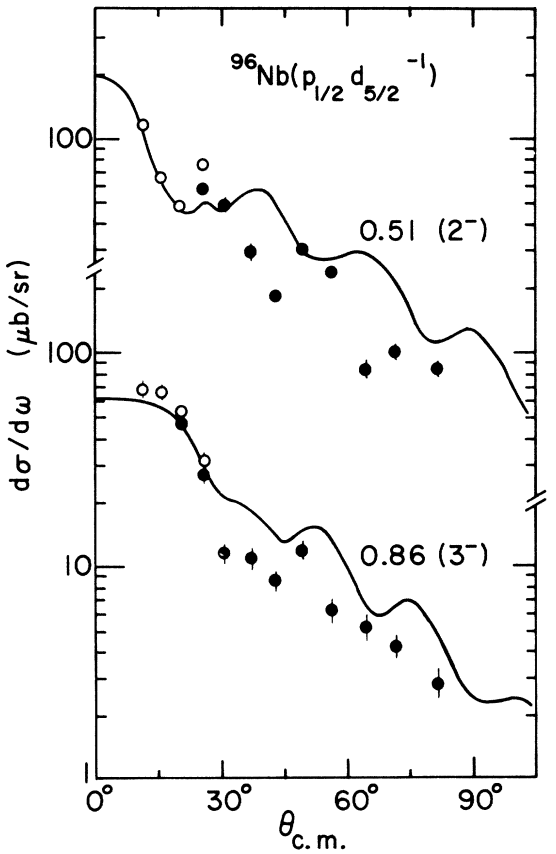

FIG. 7. Angular distributions for the states at 0.51 and 0.86 MeV in ^{96}Nb. The preferred J^π assignments are given in parentheses. The curves are DW calculations.

Most of these even-parity states have odd values of J so that two values of L are usually allowed. Known even-J states in ^{90}Nb frequently have dominant j^2 configurations, and so are weak; others were unresolvable from neighboring states. Those in ^{96}Nb were weakly populated and had poorly defined angular distributions. These angular distributions are shown in Fig. 6.

2. *Odd-L values*

The states at 0.51 and 0.86 MeV in ^{96}Nb were strongly populated and their angular distributions are shown in Fig. 7. A doublet from the $(p_{1/2}d_{5/2}^{-1})$ configuration is expected near those energies and

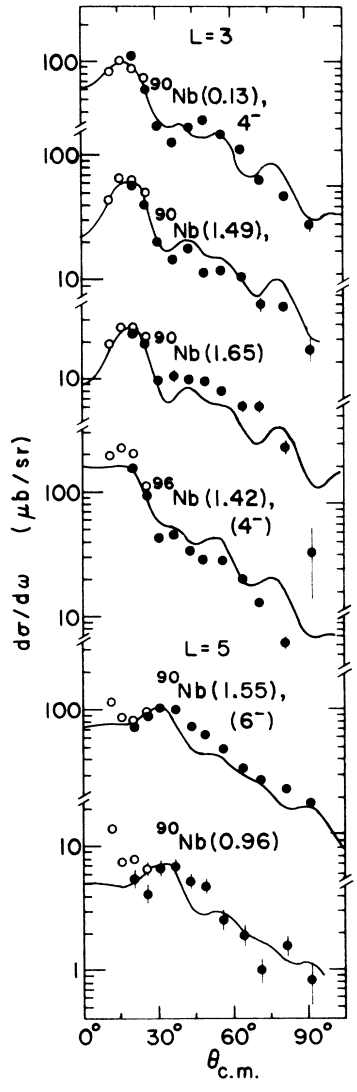

FIG. 8. Angular distributions dominated by $L=3$ and $L=5$. See also the caption for Fig. 4. The state at 0.13 MeV in ^{90}Nb has a known 4^- assignment. Preferred assignments are given for other states in parentheses.

the DW calculations for such states agree with the systematics of the data. We thus assign 2^- and 3^- to these states, respectively. This spin sequence agrees with the sequence[40] for the $(p_{1/2}d_{5/2})$ multiplet in ^{90}Y.

Angular distributions for two other groupings of levels are shown in Fig. 8. The 0.13-MeV state of ^{90}Nb is known[23,24] to have $J^\pi = 4^-$. The next 3 states have very similar shapes which are primarily $L = 3$, and hence, should have $J^\pi = 3^-$ or 4^-.

Table II shows that a $(p_{3/2}g_{9/2}^{-1})$ multiplet should occur near 1.6 MeV in ^{90}Nb. The 1.49- and 1.65-MeV states could easily be the 3^- and 4^- members of such a multiplet. The DW calculations predict nearly equal cross sections for these states, and so the sharp difference in yields is evidence for some configuration mixing. In ^{96}Nb, the $(p_{3/2}d_{5/2}^{-1})$ multiplet should lie near 1.2 MeV. The 4^- member is calculated to have a very strong cross section —nearly a factor of 5 over that for the 3^- member. This can be traced to the 9-j symbol in Eq. (2) (both orbits have $j = l + \frac{1}{2}$ and they are coupled to maximum J for the 4^- state). Since the 1.42-MeV state is one of the most intensely populated levels in ^{96}Nb, it is an excellent candidate for this 4^- state.

States with possible $L = 5$ angular distributions ($J^\pi = 5^-$ or 6^-) are also shown in Fig. 8. The shapes are very similar to those for the known 5^- state at 0.23 MeV in ^{88}Y.[10] A $(p_{3/2}g_{9/2}^{-1})$ multiplet should occur near 1.5 MeV in ^{90}Nb. The 6^- member is again calculated to have large cross sections (more than a factor of 10 over the 5^- member). Since the 1.55-MeV state in ^{90}Nb is so prominent near the expected energy, it is most likely a 6^- state. The 0.96-MeV level in ^{90}Nb appears to have $L = 5$ and could be the 5^- member of one of the $(p_{1/2}g_{9/2}^{-1})$ multiplets.

TABLE V. Experimental results for ^{90}Nb. The excitation energies are in MeV. The J^π, L, and configuration assignments are based on previous work and the present data (see text). The normalization factors N refer to Eq. (3) of the text.

E_x	J^π	L	Config.	N
0.000	8^+		$(g_{9/2})^2$	
0.129	4^-	3	$p_{1/2}g_{9/2}$	8000
	6^-		$(g_{9/2})^2$	
0.176	7^+	6	$(g_{9/2})^2$	20 000
0.286	5^+	4	$(g_{9/2})^2$	35 000
0.381	1^+	0, 2	$(p_{1/2})^2$	13 000
	and 5^-?		$p_{1/2}g_{9/2}$	
0.655	3^+	2, 4	$(g_{9/2})^2$	35 000
0.819	9^+	8	$(g_{9/2})^2$	19 000
0.848	2^+	2	$p_{1/2}p_{3/2}$	1500
0.958	(5^-)	(5)	$p_{1/2}g_{9/2}$	(1100)
1.131				
1.194				
1.231				
1.255				
1.289				
1.350	1^+	0, 2		
1.370	(2^-)			
1.415				
1.492	$(3, 4)^-$	3	$p_{3/2}g_{9/2}$	(6600)
1.552	$(6)^-$	5	$p_{3/2}g_{9/2}$	(7100)
1.647	$(3, 4)^-$	3	$p_{3/2}g_{9/2}$	(3000)
1.691				
1.776	1^+	0, 2		
1.804				
1.842	(1^+)			
1.873				
1.974				
2.002				
2.130	1^+	0, 2	$(g_{9/2})^2$	65 000
2.172				
2.311	(1^+)			
2.349				
2.479				

B. Spectroscopic assignments

Summaries of the data are given in Tables V and VI. The excitation energies are averages over several angles and are accurate to 5–10 keV. The J^π assignments and configurations are generally taken from earlier studies, augmented in a few cases by the discussion above. The normalization factors N are those values required in Eq. (3) in

TABLE VI. Experimental results for ^{96}Nb. See also the caption for Table V.

E_x	J^π	L	Config.	N
0.000	6^+	6	$g_{9/2}d_{5/2}$	20 000
0.034	5^+	4	$g_{9/2}d_{5/2}$	6000
0.137	4^+	4	$g_{9/2}d_{5/2}$	10 000
0.177	3^+	2, 4	$g_{9/2}d_{5/2}$	4500
0.217	7^+	6	$g_{9/2}d_{5/2}$	3500
0.511	2^-	1, 3	$p_{1/2}d_{5/2}$	40 000
0.633				
0.699	3^-	3	$p_{1/2}d_{5/2}$	10 000
0.865				
0.989				
1.038				
1.120				
1.161				
1.260				
1.317				
1.365				
1.423	$(4)^-$	3	$p_{3/2}d_{5/2}$	10 000
1.489				
1.536				
1.603				
1.650				
1.720				
1.809				
1.870				

order to reproduce the cross sections for the states, assuming that they have the pure configurations given. The appropriate spectroscopic amplitudes are taken into account.

It is apparent that there is considerable fluctuation in the normalization factors. Some of this could result from other reaction processes, particularly for states with small cross sections. It undoubtedly reflects also some degree of configuration mixing. Larger values of N signify larger cross sections than implied by the DW calculations for the pure configurations cited.

In attempting to assess the validity of the configuration assignments, it may be useful to inquire how the normalization factors vary with respect to (a) different states of the same multiplet, (b) between multiplets in the same nucleus, and (c) between multiplets in different nuclei.

1. Variations within a multiplet

In ^{90}Nb, the $J=1,3,5,7,9$ states of the presumed $(g_{9/2})^2$ multiplet have N values of 65 000, 35 000, 35 000, 20 000, and 19 000, respectively. From the multiplets listed in Table II, only the first two states could have mixed configurations. The structure of the 1^+ state is undoubtedly quite complicated. Since it is possible that the ground state of ^{92}Mo may have an admixture of $\nu(g_{9/2}{}^{-2})(d_{5/2}{}^2)$, there could be contributions from $(\pi g_{9/2}\nu d_{5/2})$ pickup to the 5^+ and 7^+ states. The DW cross sections for this process are a factor of 10 larger than for $(g_{9/2}{}^2)$ pickup. The 9^+ state has no possibilities for admixtures unless the seniority restrictions are relaxed. An over-all normalization for a pure configuration would thus seem to be about 20 000. The data are in general agreement with the assumption of a dominant $(g_{9/2}{}^2)$ configuration, except for the 1^+ state where admixtures are expected. The even-J states seem to be generally absent except for the 2^+ state. An admixture of about 10% in intensity of a $(p_{1/2}p_{3/2})$ configuration could account for its cross section.

In ^{96}Nb, the $J=2-7$ states of the proposed $(g_{9/2}d_{5/2})$ multiplet have N values of 5000, 4500, 10 000, 6000, 20 000, and 3500, respectively. Except for the 4^+ and 6^+ states, these numbers are in reasonable accord with each other. The exceptional states have very small cross sections so that

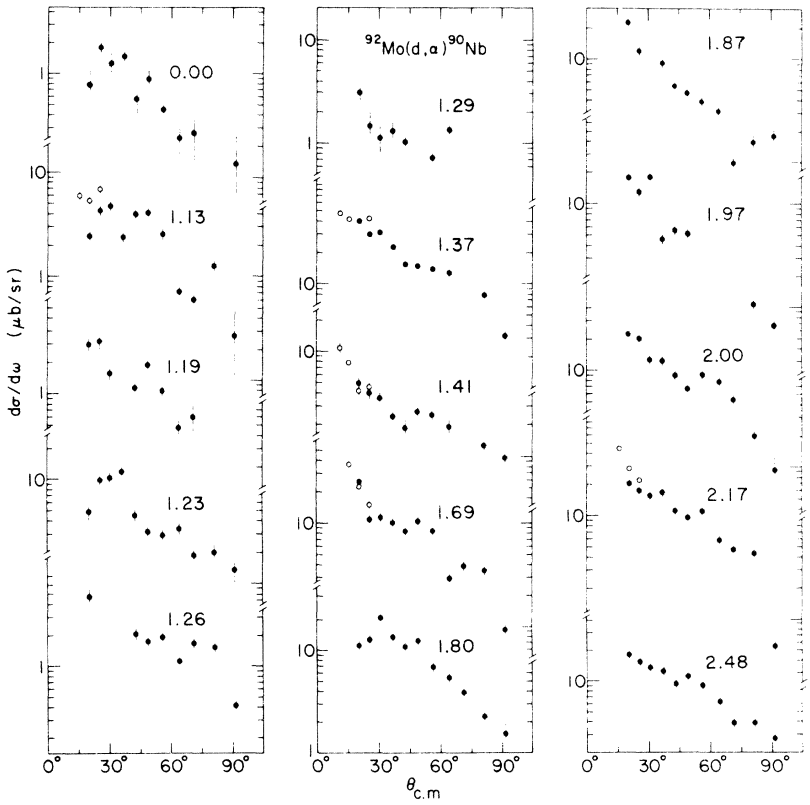

FIG. 9. Angular distributions for states in ^{90}Nb that do not have known or favored J^π assignments and for which L values cannot be given. The states are labeled by excitation energy in MeV. The ground state is known, however, to have $J^\pi=8^+$ but its population is forbidden to direct (d,α) reactions.

compound-nucleus or multistep contributions may not be negligible. Table II indicates that the 4^+ and 5^+ states could have some contribution from $(g_{9/2}s_{1/2})$ pickup, arising from an impurity in the ^{98}Mo ground state. All states could have admixtures from $(g_{9/2}^2)$ pickup. However, the centroid for these states is expected to be near 3 MeV excitation, and the intrinsic cross sections are much lower. Again, the data are not in disagreement with the assumption of a reasonably pure configuration.

The two supposed members of the $(p_{1/2}d_{5/2})$ multiplet in ^{96}Nb have N values of 40 000 and 10 000 for $J=2$ and 3. Mixing with $(p_{3/2}d_{5/2})$ and $(f_{5/2}d_{5/2})$ multiplets is undoubtedly present.

2. *Variations between multiplets and nuclei*

It is apparent from the preceding discussion that the normalization factor for the $(g_{9/2}^2)$ multiplet in ^{90}Nb, which is about 20 000, is much larger than the factor for the $(g_{9/2}d_{5/2})$ multiplet in ^{96}Nb, which is about 4000. The factor for a pure $(p_{1/2}d_{5/2})$ multiplet in ^{96}Nb cannot be assessed since the observed states must have complicated wave functions. Recalling the order-of-magnitude difference in the intrinsic DW cross sections, it would be attractive to ascribe the discrepancy to the presence of a relatively small $(g_{9/2}d_{5/2})$ impurity for all the ^{90}Nb states. Unfortunately, this is not possible for the 9^+ state. Neither can it be ascribed to the choice of optical potentials. As shown in Table IV, these can affect greatly the absolute cross sections, but not very much the relative ones, at least for the same value of L.

The DW calculations also show that the mass and Q-value effects are very small and cannot account for the over-all discrepancy. This is supported by experimental evidence. The $(g_{9/2}^2)$ states of ^{88}Y were studied by Daehnick and Bhatia.[10] The 5^+ state is well separated from other levels and the 9^+ state is probably the level at 1.475 MeV.[1,39] When the data of Ref. 10 are reanalyzed

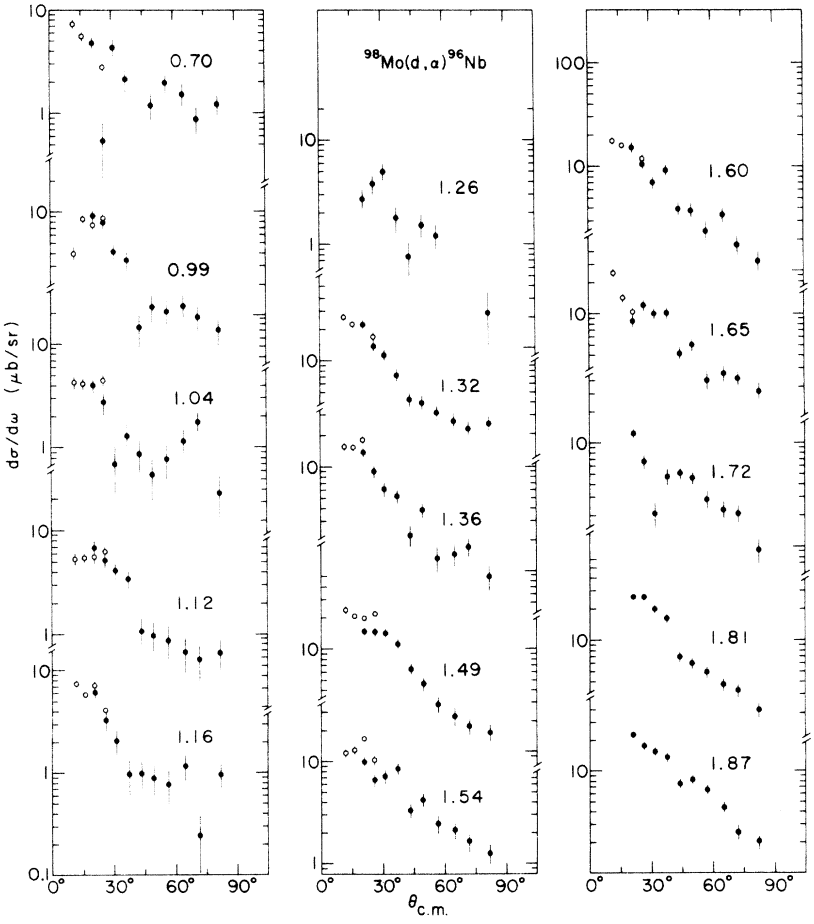

FIG. 10. Angular distributions for states in ^{96}Nb that do not have known or favored J^π assignments and for which L values cannot be given. The states are labeled by excitation energy in MeV.

with the DW calculations described in Sec. IV B, we obtain normalization factors of about 40 000 and 25 000, respectively, in rather good agreement with the results for ^{90}Nb. (Allowance is made for the configuration mixing in the ^{90}Zr target ground state.)

It is not clear why the normalization factors needed for ^{90}Nb and ^{96}Nb are so different. It may well be that the standard DW treatment of (d,α) reactions is inadequate—either because the reaction mechanism is more complicated than simultaneous n-p pickup or because the usual treatment of the form factor is too simple. In the latter regard, the order-of-magnitude difference in predicted cross sections for $(g_{9/2}{}^2)$ and $(g_{9/2}d_{5/2})$ pickup seems excessive since both orbits are in the same major oscillator shell.

VI. SUMMARY AND CONCLUSIONS

The data presented here illustrate the utility of (d,α) reactions in interpreting nuclear states despite difficulties in assessing reaction mechanisms. Support is provided for the J^π assignments previously given[2-4,23,24] to states of ^{90,96}Nb. The data are not inconsistent with the view[2-4] that the states of the $(g_{9/2}{}^2)$ and $(g_{9/2}d_{5/2})$ multiplets have reasonably pure configurations, at least those states with the highest spins.

It must be emphasized, however, that the interpretation given here is made on the basis of the DW calculations that were finally chosen. Variations in the normalization factors were attributed either to configuration admixtures or to a uniform problem involving the treatment of the form factor. The conclusions about individual states, or the magnitude of the possible problem, could be modified with a different prescription of the DW calculations for a different treatment of the reaction mechanism.

Table IV shows quite clearly that the normalization factors not only have a very sharp dependence in absolute scale on the DW prescription, but also a very significant relative dependence on the configuration and/or spin. Whereas the cross sections should be calculated with more realistic form factors when better calculations of the shell-model wave functions are eventually made, it is unlikely that a fair test with experiment could be made until the reaction mechanism is better understood.

In the latter regard, calculations of the two-step contributions to the ^{98}Mo$(d,\alpha)^{96}$Nb reaction, for the $(g_{9/2}d_{5/2})$ multiplet, have recently become available.[41] When added coherently to the one-step DW mechanism, with a single normalization factor, they produce excellent agreement with the data for all states. Although this certainly signifies that careful attention must be paid to the reaction mechanism, the results in this case would enhance the belief that the $(g_{9/2}d_{5/2})$ multiplet in ^{96}Nb has high configuration purity.

ACKNOWLEDGMENTS

We thank the staff of the Argonne tandem accelerator for much assistance in taking the data, James W. Connelley for help in the analysis, and Dave Kurth for drawing and redrawing several figures. We are also grateful to R. Lawson, M. Macfarlane, and D. Robson for reading portions of the manuscript and providing helpful suggestions and comments. The Ohio University Computer Center generously provided support for some of the final calculations.

APPENDIX

Considering ^{90}Zr and ^{96}Zr as good closed-shell nuclei, the simplest description of the particle-hole multiplets in ^{90}Nb and ^{96}Nb would correspond to $\pi(j_1)\nu(j_2{}^{-1})$. For some of these multiplets, however, the T^+ operator will not yield a zero expectation value as would be required for the lowest-isospin states. The situation is remedied by introducing isotopic spin into the description of the multiplets. This allows some states to have both $\pi(j_1)\nu(j_2{}^{-1})$ and $\pi(j_2)\nu(j_1{}^{-1})$ components in their p-n representation. It may also increase the total number of multiplets which could be characterized by the same orbit representation (j_1,j_2).

The construction of particle-hole multiplets with good isospin is discussed in the following section. For simplicity, partial isospin (i.e., the isospin of a single orbital) is also assumed to be a good quantum number. Only the lowest seniority components are considered. The possible effects of relaxing these restrictions are considered in Sec. B. Finally, in Sec. C, the spectroscopic amplitudes for the (d,α) reaction are evaluated.

A. Multiplet construction

In the notation $\lfloor(j_1{}^n)_{J_1T_1}(j_2{}^m)_{J_2T_2}\rfloor_{JT}$, the ^{90}Zr ground state may be represented by

$$|^{90}\mathrm{Zr(g.s.)}\rangle = a\lfloor(p_{1/2}{}^4)_{00}(g_{9/2}{}^{10})_{05}\rfloor_{05} + b\lfloor(p_{1/2}{}^2)_{01}(g_{9/2}{}^{12})_{04}\rfloor_{05}, \quad \text{(A1)}$$

where $|a|=0.8$ and $|b|=0.6$ appear to be good estimates for the strength of each term.[18,21,42] Probable admixtures of terms containing the $p_{3/2}$ orbit[43,44] are ignored here.

Apart from 0^+ states, which should be unobservable in the (d,α) reaction, the identical-orbit configurations $(g_{9/2}g_{9/2}{}^{-1})$ and $(p_{1/2}p_{1/2}{}^{-1})$ in ^{90}Nb are

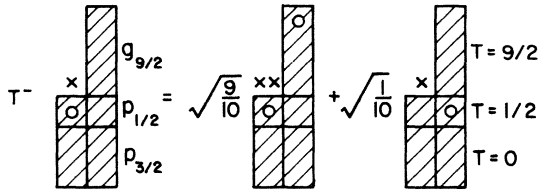

FIG. 11. Schematic representation of the analog configuration in ^{90}Nb of the $(p_{1/2}^{-1}g_{9/2})$ parent configuration in ^{90}Zr. The partial isospin of each shell-model orbital is given on the right. The T^- is an isospin-lowering operator.

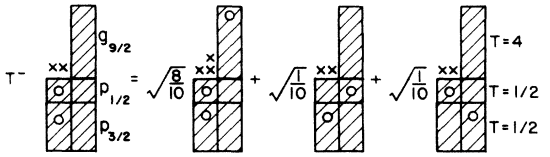

FIG. 12. Schematic representation of the analog configuration in ^{90}Nb of the $(p_{3/2}^{-1}p_{1/2})$ parent configuration in ^{90}Zr. See also the caption for Fig. 11.

expressed by unique proton and neutron representations. This is not the case for nonidentical-orbit configurations.

Considering only the first term in Eq. (A1), the $(p_{1/2}^{-1}g_{9/2})$ multiplet in ^{90}Zr has an analog in ^{90}Nb which is represented schematically in Fig. 11. We may therefore define one antianalog multiplet which, itemizing neutrons and protons separately, may be written

$$^{90}\mathrm{Nb}[(p_{1/2}^{3})_{1/2,\,1/2}(g_{9/2}^{11})_{9/2,\,9/2}]_{J4} = \sqrt{\tfrac{9}{10}}[\pi(p_{1/2}^{2}g_{9/2})\nu(p_{1/2}^{-1})]_{J4} - \sqrt{\tfrac{1}{10}}[\pi(p_{1/2}^{-1}g_{9/2}^{2})\nu(g_{9/2}^{-1})]_{J4}. \quad \text{(A2)}$$

In lowest seniority there is one additional multiplet in ^{90}Nb which schematically looks very similar to the first term in Fig. 11, but for which the $g_{9/2}$ particles couple to partial isospin $T(g_{9/2}) = \frac{7}{2}$, namely

$$|^{90}\mathrm{Nb}(p_{1/2}g_{9/2}^{-1})\rangle = [(p_{1/2}^{3})_{1/2,\,1/2}(g_{9/2}^{11})_{9/2,\,7/2}]_{J4}. \quad \text{(A3)}$$

The same arguments also apply to the $(p_{3/2}^{-1}g_{9/2})$ and $(f_{5/2}^{-1}g_{9/2})$ multiplets.

The situation is more complicated for the $(p_{3/2}^{-1}p_{1/2})$ multiplet, for which we must consider only the second term in Eq. (A1). The analog multiplet in ^{90}Nb is shown schematically in Fig. 12. There are now two T-lower multiplets with the same orbit representations (antianalogs). The relative sizes and phases of the terms of these antianalogs cannot be determined without a more detailed model. The (d, α) reaction can reach only the portions which are like the second and third terms of the analog configuration in Fig. 12. Limits on the population intensities can therefore be evaluated. These antianalog multiplets have the configurations $[(p_{3/2}^{7})_{3/2,\,1/2}(p_{1/2}^{3})_{1/2,\,1/2}(g_{9/2}^{12})_{04}]_{J4}$ and there are no other multiplets that can be reached by the (d, α) reaction. The same considerations hold for the $(f_{5/2}^{-1}p_{3/2})$ configuration.

A much simpler nucleus is ^{96}Nb in that all of the multiplets which are expected to lie below 3 MeV have unique proton and neutron representations. The $(p_{1/2}^{-1}g_{9/2})$ multiplet is the lowest configuration which could present complications and it is expected to be above 3 MeV. Considerations similar to the foregoing show that there should be two antianalog multiplets in ^{96}Nb with $T(g_{9/2}) = \frac{9}{2}$ and one multiplet with $T(g_{9/2}) = \frac{7}{2}$.

It is, however, necessary to consider possible multiplets formed by a $\nu(d_{5/2}^{-2}s_{1/2}^{2})$ admixture in the ^{98}Mo ground state. While such admixture should be small,[14] it may be possible to see evidence for it in the (d, α) reaction.

B. Multiplet mixing

The particle-hole multiplets which were constructed in Sec. A not only have good isospins, but it was further assumed that the partial isospins of each orbit are also good quantum numbers. While this is convenient for enumerating the number of available states, it is unrealistic since the nuclear force will cause the multiplets so enumerated to mix. As an example we compute the off-diagonal matrix elements between the two $(p_{1/2}g_{9/2})$ configurations in ^{90}Nb. These differ by the partial isospin of the $g_{9/2}$ orbit, namely

$$[(p_{1/2}^{3})_{1/2,\,1/2}(g_{9/2}^{11})_{9/2,\,9/2}]_{J4}$$

and

$$[(p_{1/2}^{3})_{1/2,\,1/2}(g_{9/2}^{11})_{9/2,\,7/2}]_{J4},$$

where J may be 4 or 5. We assume here that these configurations have lowest seniority.

By use of standard techniques, and in particular the use of Eq. (37.19) of de-Shalit and Talmi,[45] the matrix element may be reduced to an expression involving the two-body matrix elements $V(j_1 j_2, J'T')$. Setting

$$X = [V(J'=4, T'=0) - \tfrac{1}{2}V(J'=4, T'=1)] - [V(J'=5, T'=0) - \tfrac{1}{2}V(J'=5, T'=1)], \quad \text{(A4)}$$

we obtain

$$\text{M.E.}(J=4) = -0.699X,$$
$$\text{M.E.}(J=5) = +0.572X. \tag{A5}$$

The necessary one-particle coefficients of fractional parentage (cfps) were taken from Grayson and Nordheim.[46] The phases of the cfps for the $g_{9/2}$ orbit are important.

Approximate values of the $p_{1/2}$-$g_{9/2}$ two-body matrix elements were obtained from Kuo.[47] With these values we obtain total off-diagonal matrix elements of about +600 keV for $J=4$ and −500 keV for $J=5$. Since the centroids of the unperturbed configurations are estimated to be about 300 keV apart, as seen in Table II, it is clear that considerable mixing will occur. In fact, ignoring all other mixings, the lowest 4^- state is expected to be near 200-keV excitation energy, rather than near 600 keV, and the states of the same J should be separated by more than 1 MeV. The lowest 4^- state in ^{90}Nb is at 120 keV, not far from our rough estimate.

As mentioned above, seniority was assumed to be a good quantum number for these calculations. Since both unperturbed configurations have components with two $g_{9/2}$ protons, it is possible that these could couple to angular momenta 2, 4, 6, or 8 and result in seniority $v=3$ components. This would also increase the enumeration of available states and allow the spectroscopic strengths for the (d,α) reaction to drain from the states of principle interest. Fortunately, there is evidence that seniority is a very good quantum number for several nuclei in the mass-90 region.[48] Calculations of seniority mixing were not made here.

C. Spectroscopic amplitudes

The (d,α) spectroscopic amplitudes for the various multiplets in Table II can be readily evaluated. General expressions have been given elsewhere.[27-29] Those derived by Towner and Hardy[29] in the isospin representation may be reduced to forms directly applicable to the reactions of interest here.

We shall assume the simplest shell-model representation for the even-even target nuclei. Thus the ground states of ^{92}Mo and ^{98}Mo have the configurations $[(g_{9/2}{}^{12})]_{04}$ and $[(g_{9/2}{}^{12})_{04}(d_{5/2}{}^{6})_{03}]_{07}$, respectively. Even though single-particle-transfer reactions indicate that these assumptions are only approximately valid,[49-51] more realistic spectroscopic amplitudes must await detailed shell-model calculations.

The reaction $A(d,\alpha)B$ transfers total angular momentum J with no change in isospin. We may write the initial and final wave functions as

$$\psi_A = |(j_1{}^{n_1})_{00}(j_2{}^{n_2})_{0T_2}(j_3{}^{n_2})_{0T_3};0T\rangle$$

and

$$\psi_B = |(j_1{}^{n_1'})_{J_1'T_1'}(j_2{}^{n_2'})_{J_2'T_2'}(j_3{}^{n_3'})_{J_3'T_3'};JT\rangle,$$

where n_1, n_2, and n_3 are even integers and $n_1'+n_2'+n_3'=n_1+n_2+n_3-2$. The spectroscopic amplitude is defined by

$$\beta = \binom{A}{2}^{1/2}\langle\psi_A|[\psi_B\otimes\varphi(j_1j_2)]_A\rangle, \tag{A6}$$

where the square bracket designates vector coupling of the residual state with the two transferred nucleons to form the target ground state. The coefficient in front of the overlap integral is an antisymmetrization factor.[27-29]

There are three particular cases of interest. Since we are not concerned here with configuration mixing, the relative phases of the amplitudes β are unimportant and only the intensities shall be given.

Case I. Both particles are removed from the same orbit (j_1, j_2, or j_3).

$$\beta^2 = \frac{n(n-1)}{2}[j^n(0T)\{|j^{n-2}(JT),j^2(J0)]^2, \tag{A7}$$

where the bracket is a two-particle coefficient of fractional parentage.

Case II. The particles are transferred from orbits j_3 and j_2.

$$\beta^2 = n_2n_3[j_2{}^{n_2}(0T_2)\{|j_2{}^{n_2-1}(j_2T_2'),j_2]^2 \times[j_3{}^{n_3}(0T_3)\{|j_3{}^{n_3-1}(j_3T_3'),j_3]^2 \times\left[\frac{2J+1}{(2j_2+1)(2j_3+1)}\right]\left[\frac{(2T_2+1)(2T_3+1)}{2}\right] \times\begin{Bmatrix} T_2 & T_3 & T \\ T_3' & T_2' & \frac{1}{2}\end{Bmatrix}^2 \tag{A8}$$

If there is only one unfilled orbit in the target nucleus, i.e., $T_2=0$, or if the particles are removed from orbits j_3 and j_1, then this expression reduces to a form identical with that for case III.

Case III. The particles are transferred from orbits j_1 and j_2.

$$\beta^2 = n_1n_2[j_1{}^{n_1}(00)\{|j_1{}^{n_1-1}(j_1\tfrac{1}{2}),j_1]^2 \times[j_2{}^{n_2}(0T_2)\{|j_2{}^{n_2-1}(j_2T_2'),J_2]^2 \times\frac{1}{4}\left[\frac{(2J+1)}{(2j_1+1)(2j_2+1)}\right]. \tag{A9}$$

The two-particle cfp in Eq. (A7) is given for lowest seniority states by[29]

$$[j^n(0T)\{|j^{n-2}(JT),j^2(J0)]^2 = \frac{(n-2T)(n+2T+2)(2J+1)}{4n(n-1)(2j+1)(j+1)}. \tag{A10}$$

The one-particle cfps in Eqs. (A8) and (A9) are given for lowest seniority states by[45,46]

$$\lfloor j^n(0T)\{\lfloor j^{n-1}(jT_1), j\rfloor^2 = \begin{cases} \dfrac{(n-2T)(T+1)}{n(2T+1)}, & T_1 = T+\frac{1}{2} \\ \dfrac{(n+2T+2)T}{n(2T+1)}, & T_1 = T-\frac{1}{2}. \end{cases} \tag{A11}$$

It is clear that in every case $\beta^2 \propto 2J+1$. Values of the (d, α) transition strengths $\lfloor \beta^2/(2J+1)\rfloor$ are given in Table II for the multiplets of interest. Since the distorted-wave programs require a specification of the two-particle transfer in terms of neutrons and protons, these are also identified for each configuration. Some multiplets have two (antianalog) configurations and only the total strengths are given for these.

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*Supported in part by the U.S. Atomic Energy Commission at Argonne National Laboratory and Princeton University, and also in part by the National Science Foundation at the University of Pennsylvania.

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