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Semiclassical model for pion production by neutrons on nuclei

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The semiclassical model for pion production by protons has been extended to neutron production. Calculations of the production by both neutrons and protons incident upon Al, Cu, and Pb agree well with experiment. The pion absorption cross sections in nuclear matter deduced from each of the six experiments are reasonably consistent with one another.

I. INTRODUCTION

Pion production by nucleons incident upon complex nuclei is particularly interesting because the over-all features may be understood with a simple semiclassical model while some effects of the nuclear medium are revealed in the details of the observed cross sections. Data for the doubly differential pion production cross section, $d^2\sigma/d\Omega dT$, for 740 MeV protons[1] have been analyzed using the semiclassical approach.[2-5] Neutron production data at 600 MeV exist also.[6] It is the purpose of this paper to extend the previous analyses[2,3] to production by neutrons and to extract the pion absorption cross section in nuclear matter. This cross section is a necessary input for various nuclear processes including the production of pions by leptons.[7]

In Sec. II the model is briefly discussed. In Sec. III the calculation of the input cross sections is described. The results for total π^+ and π^- production at 600 and 740 MeV are presented in Sec. IV A. In Sec. IV B the deduced pion absorption is presented with comments on the "theoretical errors" in this quantity. Section V contains the summary and conclusions.

II. SEMICLASSICAL MODEL

The details of the algebra involved in the calculation of the production cross sections have been presented earlier.[2,3] Here we will briefly discuss the assumptions, and treat in detail only those features of the calculation which have been changed.

The basic picture is illustrated in Fig. 1(a). A high energy nucleon enters the nucleus and travels through it, at some point producing a pion traveling at a certain angle θ with a given kinetic energy T. This production is assumed to proceed via formation of the $T=\frac{3}{2}$, $J=\frac{3}{2}$ resonance, the $\Delta(1231)$. The pion then exits the nucleus along a straight path. The actual process is, of course, a complicated series of scatterings, first of the nucleons and later of the pions. The most important of these effects, charge exchange and absorption of the incoming nucleons and the outgoing pions, may be incorporated without abandoning the picture of straight line propagation. Nucleons are not absorbed in the sense that they disappear. However, by scattering they may lose appreciable kinetic energy. Since this severely decreases the probability of pion production, the nucleons are absorbed so far as the production is concerned.

There are three assumptions that underlie this view of pion production. The first assumption is that the pion nucleon interaction is dominated by the (3, 3) resonance. Second, a full multiple scattering treatment of the incident nucleon and outgoing pion is not necessary in understanding the main features of the data. This assumption will tend to overestimate the production of small angle high energy pions and generally underestimate the production of low energy pions; however, total production cross sections should not be sensitive to this approximation. Finally, although the nuclear medium corrections such as Fermi motion and the Pauli principle are not entirely negligible, the primary difference between nucleon-nucleon and nucleon-nucleus production is the absorption and charge exchange of incident nucleons and outgoing pions.

Within the framework of these assumptions one can compute the pion production in terms of the cross sections labeling the paths in Fig. 1(b). All but the pion absorption cross sections may be inferred from other experimental data. Nucleons enter the nucleus and, traveling in a straight line, are depleted or mixed by absorption, pion creation, and charge exchange. The transport equation is of the form

$$\frac{dN}{dx}=\underline{B}\rho(x)N, \tag{1}$$

where $\underline{B}$ is a constant square matrix, ρ is the nuclear density, and N is a column matrix containing

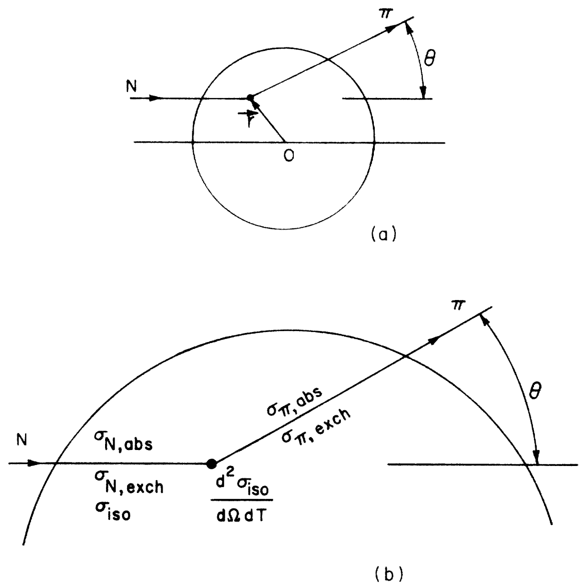

FIG. 1. (a) Schematic representation of the semiclassical model for pion production. (b) Expanded view of (a) with the nucleon and pion paths labeled with the relevant cross sections. Note that $\sigma_{\pi,\,\mathrm{abs}}$ and $\sigma_{\pi,\mathrm{exch}}$ each depend on (T).

the fluxes for the two nucleon charge states. The isobar model[8] is used to determine the relative probabilities p_j of production of pions in the charge states $j=+,\ 0,\ -$. A second transport equation describes the propagation of the outgoing pions. Its solution gives the probability $M_{ij}(T,\theta,\vec{r})$ that a pion produced at point $\vec{r}$ in charge state j will escape in charge state i. All that remains to compute the nuclear production cross section is to weight the production sites in the nucleus. This gives[3]

$$\frac{d^2\sigma}{d\Omega dT}(\pi^i)=\frac{d^2\sigma_{\mathrm{iso}}}{d\Omega dT}\sum_j\int d^3\vec{r}\;\rho(\vec{r})M_{ij}(T,\theta,\vec{r})P_j(\vec{r})\,. \tag{2}$$

III. INPUT CROSS SECTIONS

In order to compute the pion production from Eq. (2) we need five cross sections: the isobar production cross section, and the charge exchange and absorption cross sections for nucleons and for pions. The isobar production cross section may be determined from the free nucleon-nucleon production data. The nucleon charge exchange is obtained from the large angle nucleon-nucleon scattering.[3] The nucleon "absorption" is determined by the medium angle elastic scattering data. Assuming dominance by the (3, 3) resonance, the pion charge exchange satisfies

$$\sigma_{\pi,\,\mathrm{exch}}=\tfrac{2}{9}\sigma(\pi^+,p)\,.$$

Accurate experimental values of $\sigma(\pi^+,p)$ are available.[9] The only cross section which is not determined by existing experimental results is the pion absorption cross section. As before[2,3] this is determined by fitting to the proton induced π^+ production on Pb at 15°.

This is the simplest form in which the calculation may be done, and is essentially the procedure used in Ref. 2. The effects of the nuclear medium will alter the probabilities of various events, such as charge exchange or pion creation; however, these changes are fairly unimportant for the over-all cross sections. Unlike Ref. 3, the "standard" calculations presented here will not include these effects.

Our estimate of the nucleon absorption differs from that of Ref. 3 in three particulars. Previously, this cross section was estimated by using a small angle exponential fit to the p-p cross section in

$$\sigma_{N,\,\mathrm{abs}}=\int_{-\infty}^{t_{\max}}\left(\frac{d\sigma}{dt}\,pp\right)dt\,.$$

Here $t_{\max}$ corresponds to an energy loss of 180 MeV by the projectile. Now we have included the correct angular and charge dependences, and have allowed for multiple small angle scatterings.

Computing the absorption cross section using

$$\begin{aligned}\sigma_{N,\,\mathrm{abs}}&=\int_{\theta_{\max}}^{180^\circ-\theta_{\max}}\frac{d\sigma}{d\Omega}\,pp\,d\Omega\\&\approx 2\int_{\theta_{\max}}^{90^\circ}\frac{d\sigma}{d\Omega}\,pp\,d\Omega\end{aligned} \tag{3}$$

increases the absorption about 40% from 5.6 to 8.0 mb. Since the n-p cross section is smaller than the p-p cross section in this energy range, using the correctly weighted average cross section *decreases* the absorption. This reduces the absorption cross section for an $N=Z$ nucleus to 6.1 mb, and to 5.7 mb for protons and 6.5 mb for neutrons in lead.

In addition to losing energy in a single collision, the incident nucleon can lose energy as a result of two or more small angle collisions. Since the nucleon-nucleon cross section is fairly strongly forward peaked this is an important effect. Detailed calculations show that the influence of double scattering on "absorption" are accurately represented by

$$\sigma_{\substack{N,\,\mathrm{abs}\\ \mathrm{double}}}\approx\tfrac{1}{2}\int_{t_{\max/2}}^{t_{\max}}\left(\frac{d\sigma}{dt}\right)dt\,. \tag{4}$$

The over-all factor of $\frac{1}{2}$ results from the fact that after a single collision in this range of momentum transfer the nucleon still has sufficient energy to produce a pion. The effect of including the double

TABLE I. The effects of nuclear corrections.

σ^+	% change	σ^-	% change	Ratio	% change
Experiment					
104.2 ± 5.8	···	53.7 ± 4.9	···	1.94 ± 0.21	···
Standard					
148.7	···	56.6	···	2.63	···
$\sigma_{\pi,\text{exch}}$ corrected for Pauli principle					
156.2	+5.1	47.6	−15.8	3.28	+24.8
$d^2\sigma_{\text{iso}}/d\Omega dT$ corrected for Fermi motion					
121.5	−18.3	46.4	−18.0	2.62	−0.4
$\sigma_{N,\text{exch}}$ corrected for Pauli principle					
150.9	+1.5	53.6	−5.3	2.81	+7.1

TABLE II. Comparison of calculated and experimental total production cross sections for charged pions. Also given are the ratios $\gamma_{\text{nuclear}} = \sigma(\pi^>)/\sigma(\pi^<)$ and the theoretical ratios γ_0 calculated assuming no charge exchange.

Projectile target	$\sigma(\pi^+)$	$\sigma(\pi^-)$	γ_{nuclear}	γ_0
p-Al (exp)	53.1 ± 2.9	13.2 ± 0.9	4.02 ± 0.35	10.3
(calc)	56.6	11.9	4.75	
p-Cu	77.3 ± 4.3	25.2 ± 2.0	3.07 ± 0.3	9.5
	89.8	22.5	3.98	
p-Pb	104.2 ± 5.8	53.7 ± 4.9	1.94 ± 0.21	7.5
	148.7	56.6	2.63	
n-Al	13.1 ± 0.4	73.0 ± 2.5	5.57 ± 0.3	11.8
	14.1	76.8	5.44	
n-Cu	21.0 ± 0.15	115 ± 4	5.48 ± 0.5	12.7
	23.9	132.5	5.54	
n-Pb	29.7 ± 3	220 ± 23	7.3 ± 1.4	16.4
	39.5	278.2	7.1	

scattering is to *increase* the nucleon absorption by about 80%.

Including all three corrections *increases* the nucleon absorption to about double the estimate of Ref. 3; it is slightly under half the fitted value of Ref. 2. For an $N=Z$ nucleus, $\sigma_{N,\text{abs}} = 11.2$ mb. For ^{208}Pb, the values for protons and neutrons are 10.6 and 11.8 mb, respectively.

IV. RESULTS

A. Total cross sections

As noted, this calculation was motivated by the desire to apply the semiclassical model to neutron production, and to deduce the pion absorption cross section. Considering neutron as well as proton production data will provide a better test of the isobar production model than considering proton production alone. Also, our confidence in the procedures used is increased to the extent that similar results for the pion absorption are obtained for the different targets and projectiles. For simplicity, we will concentrate on total cross sections in comparing the calculations and the data. Agreement of the differential cross sections is similar to that of Ref. 2.

For checking the isobar model, the nucleon and pion charge exchange were computed as in Ref. 3, but without nuclear corrections in these or in the isobar cross section. The nucleon absorption was computed as outlined above, and the pion absorption was again adjusted so the model would predict the proton π^+ production on Pb at 15°. This procedure defines our "standard calculation."

The validity of neglecting the nuclear corrections was checked by turning on each of the corrections, one at a time. The predictions for total π^+ and π^- production and percentage changes relative to the standard calculation are given in Table I. The results are generally similar to those of Table II in Ref. 3, with the exception of the Fermi correction to $d^2\sigma_{\text{iso}}/d\Omega dT$. In Ref. 3 this effect was computed incorrectly, and the percentage changes given in the present paper are quite close to the correct changes for the standard calculation in that work.

The effect of the corrections to $\sigma_{\pi,\text{exch}}$ is approximately twice as large for the present standard calculation as for that of Ref. 3. This can be traced to the different pion absorption cross sections, and indicates a strong coupling between the detailed energy dependence of the pion charge exchange and absorption. This coupling exists because both cross sections rise rapidly as the pion kinetic energy increases from 100 to 200 MeV. In this model, since the pion loses no energy upon charge exchange with a nucleon, the predicted $\pi^>/\pi^<$ ratio is quite sensitive to which cross section rises fastest. (For proton production, $\pi^>$ is π^+ and $\pi^<$ is π^- and vice versa for neutron production.) This sensitivity is somewhat unphysical, and probably would not persist if the pion energy loss during charge exchange were included.

Table II presents the predictions for 740 MeV protons and 600 MeV neutrons on Al, Cu, and Pb of the total production cross sections for π^+ and π^- mesons, and of the ratio $\gamma = \pi^>/\pi^<$; also given are the γ_0 ratios obtained from the isobar model without any exchange corrections.[4,8] The over-all agreement is quite reasonable, and the discrepancies that exist appear as systematic trends in the agreement with experiment. The systematic trends are: (1) The larger the ratio γ_0/γ, the larger the error in predicting it. (2) The sum of total π^+ and π^- production cross sections is too large for high A. Each of these effects is directly related to a specific simplification employed in the

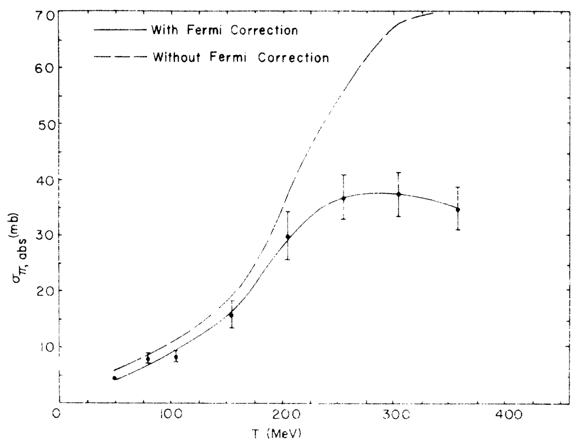

FIG. 2. Pion absorption cross sections deduced from p-Pb with and without Fermi averaging of $d^2\sigma/d\Omega dT$. Error bars are derived from the p-Pb data.

calculation.

The ratio $\pi^>/\pi^<$ is decreased in complex nuclei relative to free production by charge exchange, particularly that of pions. The observed $d\sigma/dT(\pi^<)$ spectrum peaks at much lower energy than the $\pi^>$ spectrum. This is because of the energy lost in the charge exchange reaction which created many of the $\pi^<$ mesons from π^0's. As mentioned above, the pion charge exchange and absorption cross sections are similar rapidly rising functions of T. Since this model neglects the energy loss of the charge exchanged pions, it predicts that most of the pions created by charge exchange from 150–200 MeV neutral pions are absorbed. In fact many of these pions have lost sufficient energy to escape from the nucleus, which is nearly transparent to pions in the 0–100 MeV energy range. Thus the survival of $\pi^<$ produced via pion charge exchange is systematically underestimated, leading to the systematic trend (1).

A simple explanation also exists for the second discrepancy. Pion absorption in nuclei is believed to occur on two nucleons, and is therefore sensitive to the density squared. Thus absorption takes place predominantly in the center of the nucleus, not at the edge. Assuming the absorption is proportional to ρ overweights the surface region, and predicts too much absorption for light nuclei relative to heavy nuclei. Calculations are in progress to see if a ρ^2 absorption will in fact remove this A dependence.

B. Pion absorption cross section

Although the nuclear corrections are not important in a simple check of the isobar model using total production cross sections, they can be important if more detailed information is sought. In particular, for deducing the pion absorption

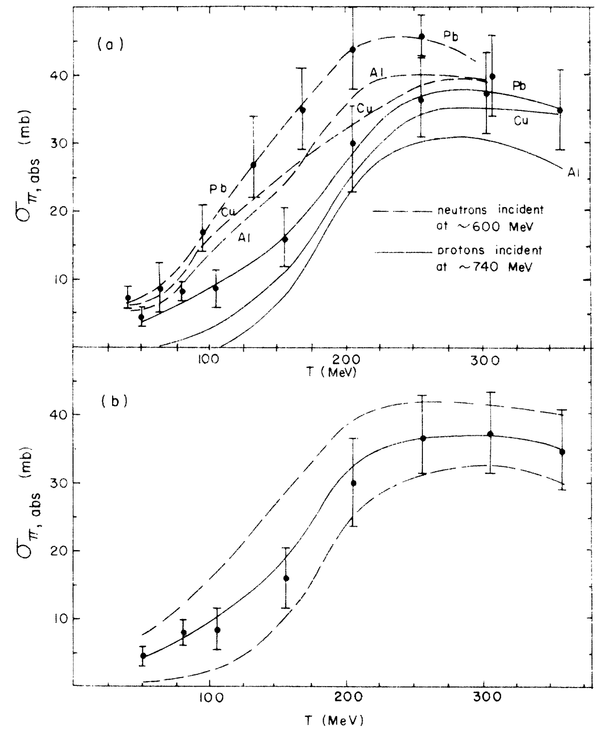

FIG. 3. (a) Six deduced pion absorption cross sections obtained by fitting the $\pi^>$ 15° data. The points and error bars for the Pb cross sections are the values of $\sigma_{\pi,\text{abs}}$ for the specific kinetic energies at which 15° production data exists. The p-p and p-n errors have not been folded in. (b) The average of the six pion absorption cross section curves in (a). The dashed lines indicate one standard deviation. The data points shown are the p-Pb deduced cross sections; the error bars include both the p-p and p-Pb experimental errors.

cross section in nuclei the Fermi motion must be properly included. This is because although the Fermi motion has only an 18% effect on the total cross sections, the effect is concentrated in the production of high energy pions, above around 200 MeV. Including this correction gives a much smaller and more plausible phenomenological absorption cross section as a function of T (Fig. 2).

In order to determine exactly how uniquely the procedure of fitting to the forward production for $\pi^>$ determines this cross section, the fitting was repeated for all six cases we have considered so far. This will test the internal consistency of the over-all approach and be useful in estimating how well the pion absorption cross section has been determined in this model. Figure 3(a) shows the absorption cross sections for the six cases, with the Fermi motion included. The error bars on the Pb deduced curves are derived from the nuclear production data; the uncertainties deriving from the free production have not been included.

The discrepancies among the curves are comparable to the experimental errors involved.

Two general features stand out. First, larger nuclei require larger absorption cross sections. This is partly due to the overweighting of the surface region, as discussed above. Below 100 MeV the p-Al data require a "creation" rather than an absorption cross section. Since the low energy free production is peaked at large angles rather than at small ones, this is probably due to multiple scattering effects which enhance rather than diminish the forward production. Calculations are in progress to test this hypothesis.

The second feature of note is that for a given nucleus the neutron production requires more absorption than the proton production. This may be due to different systematic errors in the two data sets. Averaging over the 130 MeV spread of the neutron beam should certainly be included. This may be comparable in importance to the Fermi averaging over the target nucleon momenta, which we saw gave large effects in the pion absorption cross section.

Figure 3(b) shows the pion absorption cross section obtained by taking an unweighted average of the results from the six forward angles fits. The dashed lines indicate a one standard deviation from the average. The major contribution to the "error" comes from the *systematic* differences between the proton and neutron derived absorption. The "data points" are the cross sections deduced from the p-Pb production with the combined errors from both the p-p and p-Pb measurements indicated.

There are two encouraging aspects to this plot. First, except in the region between 100 and 150 MeV, the differences among the various deduced absorption cross sections are consistant with the uncertainties in the data. Second, the p-Pb deduced absorption cross section used here and previously does as good a job as is currently possible.

So far as a precise determination of the absorption cross section is concerned, these results indicate roughly what may be expected from more detailed analyses. There are three major effects that clearly must be considered. These are the beam momentum averaging for the neutron calculations, pion absorption which goes as ρ^2 rather than as ρ, and most difficult, multiple scattering of the outgoing pion.

For the reasons discussed above, the first two of those effects are expected to improve the agreement among the six curves in Fig. 3(a). On the other hand, a multiple scattering approach which takes into account the energy degradation of the outgoing pions will certainly increase the deduced absorption at low energies and decreases it at high energies. A comparison with the reaction cross section on ^{12}C measured by Binon *et al.*,[10] suggests that the true absorption deduced here is too large for large T.

V. SUMMARY AND CONCLUSIONS

The semiclassical model for pion production by both neutrons and protons on nuclei is found to be successful at predicting total charged pion production cross sections. The extension to neutron data adds support to the isobar production model. The detailed structure of $d^2\sigma/d\Omega dT$ is predicted about as well for neutrons as for protons.[2] It is hypothesized that the remaining discrepancies between theory and experiment could be largely accounted for by using a pion absorption cross section depending quadratically instead of linearly on the density, and allowing for the energy loss of the outgoing pions during charge exchange with target nucleons. Calculations are in progress to test these effects.

Pion absorption cross sections deduced from the six experiments considered here agree reasonably well with one another. A best absorption cross section is determined, but uncertainties on the order of 5–10 mb persist, not including systematic errors associated with the model which are difficult to estimate. The pion absorption cross section may ultimately be well established from a careful analysis of pion production data. It will be crucial in any such analysis to treat more fully the effects of scattering of the pions leaving the nucleus.

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