Pion-nucleus charge exchange reactions in the isobar-doorway model*

M. A. Nagarajan and W. L. Wang

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

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The isobar-doorway model, previously applied to the study of pion-nucleus elastic scatterings, is generalized to the treatment of single and double charge exchange reactions. The charge exchange amplitudes are calculated within the framework of distorted-wave Born approximation, where the resonant distortion of the elastic scattering wave functions in the initial and final channels as well as the resonant part of the charge exchange interaction are explicitly taken into account. In the isobar-doorway model, it is shown that the transition amplitude depends upon quantities related to the elastic scattering and to a charge exchange amplitude which contains nonresonant initial and final pion wave functions. The strong energy dependence in various components of single and double charge exchange amplitudes is clearly displayed in the model; this energy dependence due to the (3,3) isobar in the reaction mechanism (including initial- and final-state interactions) may also be separated out from the nuclear structure information as contained in the nuclear form factors. This separation is obtained through the static and the closure approximations which are in the isobar-doorway model.

[NUCLEAR REACTIONS Pion-nucleus charge exchange; (3,3) resonance; isobar-doorway model.]

I. INTRODUCTION

During recent years, pion-nucleus scattering has received a great deal of attention. A general review of the subject can be found in the article by Koltun.[1] In this paper, we shall concentrate only on the charge exchange reactions, the single and double charge exchange reactions being referred to as SCX and DCX, respectively. We shall be mainly interested in these reactions in the vicinity of the (3, 3) resonance. In this energy region, we expect the interaction mechanism to considerably simplify due to the dominance of the $\Delta(1231)$ resonance.

There have been several investigations of the mechanism of the charge exchange reaction. A general review may be found in Becker and Batusov.[2] The simplest calculations of the charge exchange process have been in terms of the production and decay of the Δ particle. In SCX, the pion absorbs a nucleon of the target to form a Δ which decays by emitting a pion of different charge and forming the analog nucleus, while in DCX, two nucleons are converted into Δ particles which decay into the "double analog" nucleus. The work of Parsons, Trefil, and Drell,[3] Becker and Marć,[4] Becker and Schmidt,[5] and Barshay and Brown[6] fall under the above category. The effect of the $\Delta(1231)$ resonance in the initial and final channels have, however, not been considered in these impulse approximation calculations.

The initial- and final-state interaction effects have been considered by various authors. Kaufman and Hower[7] used a simple absorption reduction factor. Lucci and Picchi,[8] and Bjørnenak *et al.*[9] have used the Glauber approximation. Recently, Kaufmann, Jackson, and Gibbs[10] have calculated the SCX and DCX amplitudes using a multiple scattering theory. There are also optical model calculations by Koren,[11] and by Kerman and Logan.[12] There also exist distorted-wave Born-approximation (DWBA) calculations due to Charlton, Eisenberg, and Jones,[13] and Rost and Edwards.[14] Recently, Miller and Spencer have done a coupled-channel calculation for these reactions.[15] The initial and final distorted-wave functions utilized in the above calculations are usually calculated from a first order approximation to the optical potential. These ensure the asymptotic form of the elastic scattering wave functions, but a calculation of the SCX and DCX amplitudes require also the wave functions at small distances. The study of various improvements on these theories has received much attention.

In this work, we shall discuss the application of the isobar-doorway model[16] to the charge exchange reactions. It is appropriate to recall the main ideas of the model. One separates the pion-nucleon interaction into a resonant and a nonresonant part. One assumes that the resonant part of the interaction creates an isobar compound state, i.e., a nucleon hole and a Δ particle in the target. One further assumes that the scattering and reaction phenomena are largely determined by the de-

tailed properties of the isobar compound system. In an elastic scattering process, one assumes that the isobar is formed in the elastic channel and then decays into both elastic and inelastic channels. The doorway state picture emerges if one further assumes that the coupling between the elastic and inelastic channels is solely through the isobar compound state.

In this paper, we do not discuss coupled-channel effects. We do believe that the coupled-channel effects should be taken into account. We shall discuss it in a later publication. We restrict our present formulation within the framework of the DWBA. We shall consider an extension of the isobar-doorway model, wherein we shall replace the elastic scattering by the elastic and charge exchange channels. In the extended isobar-doorway model, we shall show that the transition amplitude contains strong energy dependence due to the resonance effects in the initial and final channel wave functions as well as in the charge exchange interaction.

The formalism we present in the following section will be applicable to elastic (leading to the ground state of the final nucleus) as well as inelastic charge exchange reactions, leading to quasi-bound excited states of the final nucleus. Breakup of the final nucleus is considered only through a parametrization of the omitted channels. The formalism is within the spirit of the usual DWBA calculations where the initial and the final states wave functions are assumed to be determined from the elastic scattering experiments in the respective channels. In our model, we obtain such elastic scattering wave functions from the isobar-doorway model.

II. ISOBAR-DOORWAY FORMALISM

We shall consider both the single charge exchange and double charge exchange reactions in the isobar-doorway model. For simplicity, we give detailed formulation for the SCX reactions, since the extension to the DCX reactions is straightforward and the results will be given only at the end of this section.

For SCX reaction, we consider the following process:

$$\pi_1 + A_1 \rightarrow \pi_2 + A_2 , \tag{1}$$

where 1 and 2 denote the initial and final charge states of the pions. The incident channel has a π_1 (π^+ or π^-) interacting with the target nucleus A_1; the outgoing channel has a π_2 (generally π^0 for SCX reactions) with the residual nucleus A_2. The Hamiltonian of the system may be written as

$$H = H_b + K_\pi + V, \tag{2}$$

where $H_b(r_1, r_2 \cdots r_A)$ is the *baryon* Hamiltonian with baryon coordinates $r_1, \cdots r_A$. The baryon Hamiltonian also describes the motion of the excited state of the nucleon (i.e., isobar). We allow at most one isobar in the system. In Eq. (2), the pion kinetic energy operator is K_π and the π-nucleus interaction is V. This π-nucleus interaction V describes the elastic scattering as well as the charge exchange process, along with all other reactions. We may separate this interaction into two parts:

$$V = V_0 + V_R , \tag{3}$$

where the nonresonant interaction V_0 contains operators of the following form

$$V_0 = f_{\pi N} a_\pi^\dagger a_N^\dagger a_N a_\pi \tag{4}$$

with $f_{\pi N}$ as the strength of the interaction ($a_\pi^\dagger$ and $a_N^\dagger$ are creation operators of a pion and a nucleon, respectively). The Coulomb interaction is included in V_0. The resonant interaction V_R has the following form

$$V_R = G_{\pi N \Delta} a_\Delta a_N^\dagger a_\pi^\dagger + \text{H.c.}, \tag{5}$$

where $G_{\pi N \Delta}$ depends on the coupling strength and the quantum numbers of π, N, and Δ. For details of the resonant interaction, we refer to the nonrelativistic form of Kerman and Kisslinger[17] and a field theory calculation of Δ exchange.[18] However, in the following discussion, we do not need the explicit form for the interaction. The nonresonant interaction contains the π-nucleon s-wave and $T = \frac{1}{2}$ p-wave interactions. The resonant interaction is the $T = \frac{3}{2}$ p-wave interaction.

We may now define the various isobar-nucleus states in terms of the eigenstates of the baryon Hamiltonian H_b: (1) The initial target state $|\Phi_1\rangle$ and the final residual nuclear state $|\Phi_2\rangle$ may be defined as

$$H_b(r_1 \cdots r_A) \, |\Phi_i(r_1 \cdots r_A)\rangle = E_i \, |\Phi_i(r_1 \cdots r_A)\rangle , \tag{6}$$

where $i = 1$ or 2. The coordinates $(r_1 \cdots r_A)$ refer to those of the nucleons of the nucleus; (2) the isobar-doorway states $|\Phi_{n\alpha}\rangle$ may be defined as

$$H_b(r_1 \cdots r_A) \, |\Phi_{n\alpha}(r_1 \cdots r_{A-1}, \Delta_\alpha)\rangle = (E_n + \epsilon_\alpha) \, |\Phi_{n\alpha}(r_1 \cdots r_{A-1}, \Delta_\alpha)\rangle , \tag{7}$$

where $(r_1 \cdots r_{A-1})$ refers to the coordinates of the $(A-1)$ nucleons with energy E_n, and Δ_α denotes the coordinate of the isobar with a single-particle energy ϵ_α in the nucleus. The isobar in the state $|\Phi_{n\alpha}(r_1 \cdots r_A, \Delta_\alpha)\rangle$ is assumed to be a stable particle (the narrow resonance approximation); the width of the resonance is restored latter by coupling the doorway states $|\Phi_{n\alpha}\rangle$ to the inelastic and

the open channels under our consideration.

To formulate our problem, it is useful to use the projection-operator techniques of Feshbach.[19] We define the elastic pion scattering states as the P space (the π-continuum space). We have two charge states in the P space, so we separate the P space into two parts: P_1 and P_2. We define the P_1 and P_2 operators as

$$P_1 = |\Phi_1(r_1 \cdots r_A)\rangle\langle\Phi_1(r_1 \cdots r_A)| \tag{8}$$

and

$$P_2 = |\Phi_2(r_1 \cdots r_A)\rangle\langle\Phi_2(r_1 \cdots r_A)| \tag{9}$$

which project onto the P_1 and P_2 spaces, or the initial and the final nuclear states, respectively. We next define the Q-space operator Q which projects onto the isobar-doorway states $|\Phi_{n\alpha}\rangle$ as

$$Q = \sum_{n,\alpha} |\Phi_{n\alpha}(r_1 \cdots r_{A-1}, \Delta_\alpha)\rangle\langle\Phi_{n\alpha}(r_1 \cdots r_{A-1}, \Delta_\alpha)| . \tag{10}$$

We finally define the q-space operator q which projects onto the rest of the Hilbert space (the compound inelastic states) defined by the system Hamiltonian, i.e.;

$$q = 1 - P - Q . \tag{11}$$

It is easy to see that the operators P, Q, and q satisfy the projection operator and the orthogonality conditions.

The π-nucleus interaction V_0 and V_R may now be shown to satisfy the following conditions: (1) The nonresonant interaction V_0 does not connect P and Q spaces:

$$Q V_0 P_i = 0, \quad i = 1, 2 \tag{12}$$

and

$$P_i V_0 P_j \neq 0, \quad i, j = 1, 2 ; \tag{13}$$

and similarly (2) the resonant part V_R satisfies

$$P_i V_R Q \neq 0, \quad i = 1, 2 \tag{14}$$

and

$$P_i V_R P_j = Q V_R Q = 0, \quad i, j = 1, 2 . \tag{15}$$

We have now properly specified our projection operators. We may proceed to solve the scattering problem.

The complete wave function of the system $|\Psi\rangle$ is described by the Schrödinger equation

$$(E - H)|\Psi\rangle = 0 , \tag{16}$$

where E is the energy of the system. This equation may be rewritten as the following coupled equations, by using the projection-operator techniques[19];

$$(E - H_{11})P_1|\Psi\rangle = H_{12}P_2|\Psi\rangle + H_{1Q}Q|\Psi\rangle , \tag{17a}$$

$$(E - H_{22})P_2|\Psi\rangle = H_{21}P_1|\Psi\rangle + H_{2Q}Q|\Psi\rangle , \tag{17b}$$

$$(E - H_{QQ})Q|\Psi\rangle = H_{Q1}P_1|\Psi\rangle + H_{Q2}P_2|\Psi\rangle + H_{Qq}q|\Psi\rangle , \tag{17c}$$

$$(E - H_{qq})q|\Psi\rangle = H_{qQ}Q|\Psi\rangle , \tag{17d}$$

where we have used the usual notations: $H_{ij} = P_i H P_j$, $H_{iQ} = P_i H Q$, and $H_{QQ} = QHQ$, etc. In Eq. (17), we have already made the doorway-state hypothesis: There is no coupling between P and q spaces, or

$$P_i V_0 q = P_i V_R q = 0, \quad i = 1, 2 . \tag{18}$$

We have assumed that the elastic scattering states in both the incident and final channels are not *directly* coupled to the compound inelastic states. It is clear that the assumption on the reaction mechanisms in the preceding discussion is identical to the one used in the elastic scattering.[16]

To simplify Eq. (17), let us introduce an effective Q-space Hamiltonian by eliminating the q space, i.e.,

$$\tilde{H}_{QQ} = H_{QQ} + H_{Qq}(E - H_{qq})^{-1}H_{qQ} . \tag{19}$$

We obtain an effective Hamiltonian for the P space as

$$\mathcal{H} = H + H(E - \tilde{H}_{QQ})^{-1}H , \tag{20}$$

and write the coupled-channel equations for the charge exchange reactions as

$$(E - \mathcal{H}_{11})P_1|\Psi\rangle = \mathcal{H}_{12}P_2|\Psi\rangle , \tag{21a}$$

$$(E - \mathcal{H}_{22})P_2|\Psi\rangle = \mathcal{H}_{21}P_1|\Psi\rangle , \tag{21b}$$

where $\mathcal{H}_{ij} = P_i \mathcal{H} P_j$. Equation (21) may be solved for the desired T matrix. These coupled equations are, however, rather complicated. In order to exhibit the important dynamical effects of the isobar resonance, we choose to introduce a distorted-wave Born-approximation (DWBA) for the T matrix. The DWBA SCX amplitude is defined as

$$T = \langle \chi_2^{(-)} | \mathcal{H}_{21} | \chi_1^{(+)} \rangle , \tag{22}$$

where the distorted-wave functions $|\chi_i^{(\pm)}\rangle$ are the homogeneous solutions of Eq. (21), i.e., we have

$$(E - \mathcal{H}_{ii})|\chi_i^{(\pm)}\rangle = 0, \quad i = 1, 2 . \tag{23}$$

Equations (22) and (23) are the basic starting point of our discussions.

Formally our results so far are rather similar to the (p, n) reactions. However, for the case of pion-nucleus charge exchange reactions, both the wave functions $|\chi^{(\pm)}\rangle$ and the interaction $\mathcal{H}_{21}$ have strong energy dependence due to the (3, 3) reso-

nance. The fact that the π-nucleus interaction is strong and resonating indicates important initial- and final-state interactions. To show these effects explicitly, we have to study the behavior of the continuum wave functions (the distorted waves). This may be done conveniently by using the isobar-doorway model, where the resonant and the nonresonant parts of the pion wave functions are treated separately.

We write Eq. (23) explicitly as

$$(E - H_{ii} + H_{iQ}(E - \tilde{H}_{QQ})^{-1}H_{Qi})\,|\,\chi_i^{(\pm)}\rangle = 0\,. \tag{24}$$

The distorted wave $|\,\chi_i^{(\pm)}\rangle$ may be formally obtained in terms of the nonresonant and resonant parts as

$$|\,\chi_i^{(\pm)}\rangle = |\,\phi_i^{(\pm)}\rangle + |\,\phi_i^{R(\pm)}\rangle\,, \tag{25}$$

where the nonresonant wave function $|\,\phi_i^{(\pm)}\rangle$ is the solution of the homogeneous equation,

$$(E - H_{ii})\,|\,\phi_i^{(\pm)}\rangle = 0\,, \tag{26}$$

and the resonant wave function $|\,\phi_i^{R(\pm)}\rangle$ is related to the nonresonant part by

$$|\,\phi_i^{R(\pm)}\rangle = \frac{1}{E - H_{ii} \pm i\eta}\,P_i\,V_R\,Q \times \frac{1}{E - \tilde{H}_{QQ} + H_{Qi}(E - H_{ii} \pm i\eta)^{-1}H_{iQ}} \times Q\,V_R P_i\,|\,\phi_i^{(\pm)}\rangle\,. \tag{27}$$

In order to simplify Eq. (27), let us introduce the isolated doorway approximation[19] and represent the inverse operators by the eigenstates of the operator H_{QQ}, as defined in Eq. (10). We may write Eq. (27) as

$$|\,\phi_i^{R(\pm)}(\vec{k})\rangle = \int \frac{d\vec{k}\,'}{E - E_{k'} \pm i\eta}\,|\,\phi_i^{(\pm)}(\vec{k}\,')\rangle \times T_i^{(\pm)}(\vec{k}\,', \vec{k}; E)\,, \tag{28}$$

where we have denoted k as a continuum energy variable in $|\,\phi_i^{(\pm)}(\vec{k})\rangle$, the resonant T matrix $T_i^{(\pm)}$ is defined as

$$T_i^{(\pm)}(\vec{k}\,', \vec{k}; E) = \sum_{n,\alpha} \frac{\langle\,\phi_i^{(\pm)}(\vec{k}\,')\,|\,V_R\,|\,\Phi_{n\alpha}\rangle\langle\,\Phi_{n\alpha}\,|\,V_R\,|\,\phi_i^{(\pm)}(\vec{k})\rangle}{E - (E_n + \epsilon_\alpha + \Delta_{n\alpha i}^{\uparrow} + \Delta_{n\alpha}^{\downarrow}) + \frac{1}{2}i(\Gamma_{n\alpha i}^{\uparrow} + \Gamma_{n\alpha}^{\downarrow})}\,, \tag{29}$$

where the continuum width $\Gamma_{n\alpha i}^{\uparrow}$ and shift $\Delta_{n\alpha i}^{\uparrow}$ to the ith channel are defined as

$$\Delta_{n\alpha i}^{\uparrow} - \frac{i\,\Gamma_{n\alpha i}^{\uparrow}}{2} = \langle\,\Phi_{n\alpha}\,|\,V_R(E - H_{ii} + i\eta)^{-1}V_R\,|\,\Phi_{n\alpha}\rangle\,. \tag{30}$$

The compound width $\Gamma_{n\alpha}^{\downarrow}$ and shift $\Delta_{n\alpha}^{\downarrow}$ are defined as

$$\Delta_{n\alpha}^{\downarrow} - \frac{i\,\Gamma_{n\alpha}^{\downarrow}}{2} = \sum_q \frac{|\langle\,\phi_q\,|\,V_R\,|\,\Phi_{n\alpha}\rangle|^2}{E - \epsilon_q + i\,\Gamma_q/2}\,, \tag{31}$$

where we have used an energy averaging procedure by assigning a width parameter Γ_q to each q state.[19] In Eq. (31), ϵ_q and $|\,\phi_q\rangle$ are the eigenenergies and the eigenstates of the q space:

$$(\epsilon_q - H_{qq})\,|\,\phi_q\rangle = 0\,, \tag{32}$$

which may contain inelastic continuum states. The resonant T matrix $T_i^{(\pm)}$ as defined in Eq. (29) is very close to the T matrix for the elastic scattering in channel i.[16] This fact may enable us to use the results from the elastic scattering calculation to estimate the matrix elements $T_i^{(\pm)}$. We shall later return to such approximation. The quantity $T_i^{(\pm)}(\vec{k}\,', \vec{k}; E)$ will become one of the most important factors in our model for charge exchange reactions.

Substituting Eq. (25) into the DWBA T matrix, we obtain the results in terms of a nonresonant

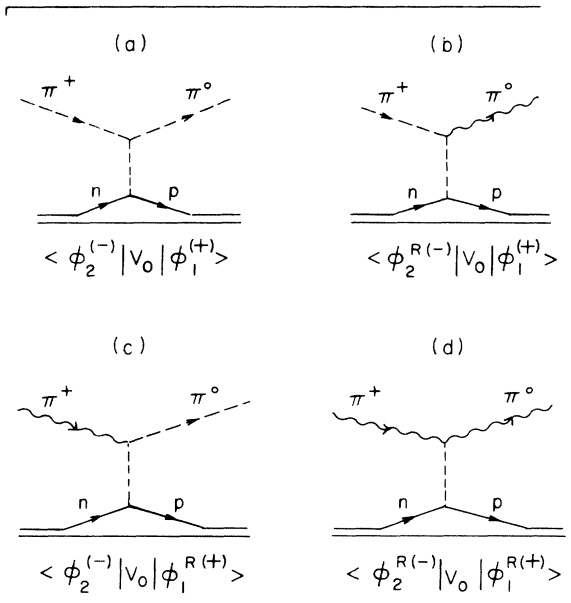

FIG. 1. Nonresonant charge-exchange reactions. Diagram (a) is the (NR-NR-NR) term, where both initial and final pion states are nonresonant; diagrams (b) and (c) are the (NR-NR-R) and (R-NR-NR) terms where one of the pion state is resonating; diagram (d) is the (R-NR-R) term with both pion states resonating. Here we identify each term by ⟨initial pion state–interaction–final pion state⟩. The nonresonant pion states are shown as the dashed lines (----) and the resonating pion states by wavy lines (∿∿∿).

SCX T_{SCX}^{NR} and a resonant SCX amplitude T_{SCX}^{R} as

$$T_{SCX} = T_{SCX}^{NR} + T_{SCX}^{R}, \tag{33}$$

where the nonresonant SCX amplitude is given by

$$T_{SCX}^{NR} = \langle \phi_2^{(-)} \mid V_0^{21} \mid \phi_1^{(+)} \rangle + \langle \phi_2^{R(-)} \mid V_0^{21} \mid \phi_1^{(+)} \rangle + \langle \phi_2^{(-)} \mid V_0^{21} \mid \phi_1^{R(+)} \rangle + \langle \phi_2^{R(-)} \mid V_0^{21} \mid \phi_1^{R(+)} \rangle, \tag{34}$$

where $V_0^{21} = P_2 V_0 P_1$ is the charge-exchange part of the π-nucleus interaction V_0, and the resonant SCX amplitude T_{SCX}^{R} is

$$\begin{aligned} T_{SCX}^{R} = {} & \langle \phi_2^{(-)} \mid H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1} \mid \phi_1^{(+)} \rangle + \langle \phi_2^{(-)} \mid H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1} \mid \phi_1^{R(+)} \rangle \\ & + \langle \phi_2^{R(-)} \mid H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1} \mid \phi_1^{(+)} \rangle + \langle \phi_2^{R(-)} \mid H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1} \mid \phi_1^{R(+)} \rangle . \end{aligned} \tag{35}$$

All these terms are represented in Figs. 1 and 2, where, for simplicity, we have denoted the resonant charge exchange operator as

$$X_{ij} = H_{iQ}(E - \tilde{H}_{QQ})^{-1} H_{Qj} . \tag{36}$$

Equations (34) and (35) give a complete DWBA description of the single charge exchange reactions. The four terms in T_{SCX}^{NR} or T_{SCX}^{R} are related. To show this property, we now take the Born amplitudes, i.e., the first terms in Eqs. (34) and (35), which are close to the amplitudes in the plane-wave Born approximation (PWBA). We define the nonresonant Born amplitude as

$$U_1^{NR}(\vec{k}', \vec{k}) = \langle \phi_2^{(-)}(\vec{k}') \mid V_0^{21} \mid \phi_1^{(+)}(\vec{k}) \rangle \tag{37}$$

then we may show that

$$\begin{aligned} U_2^{NR}(\vec{k}', \vec{k}) &= \langle \phi_2^{R(-)}(\vec{k}') \mid V_0^{21} \mid \phi_1^{(+)}(\vec{k}) \rangle \\ &= \int \frac{d\vec{q}}{E - E_q + i\eta} T_2^{(-)}(\vec{q}, \vec{k}'; E) U_1^{NR}(\vec{q}, \vec{k}), \end{aligned} \tag{38}$$

$$\begin{aligned} U_3^{NR}(\vec{k}', \vec{k}) &= \langle \phi_2^{(-)}(\vec{k}') \mid V_0^{21} \mid \phi_1^{R(+)}(\vec{k}) \rangle \\ &= \int \frac{d\vec{q}}{E - E_q + i\eta} T_1^{(+)}(\vec{q}, \vec{k}; E) U_1^{NR}(\vec{k}', \vec{q}), \end{aligned} \tag{39}$$

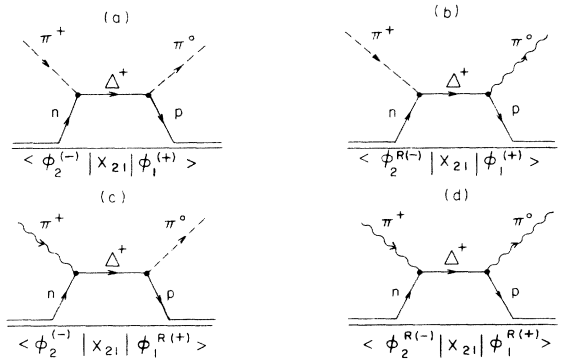

FIG. 2. Resonant charge exchange through an isobar-doorway state. The order of the diagrams follows Fig. 1: (a) the (NR-R-NR) term, (b) the (NR-R-R) term, (c) the (R-R-NR) term, and (d) the (R-R-R) term.

$$\begin{aligned} U_4^{NR}(\vec{k}', \vec{k}) &= \langle \phi_2^{R(-)}(\vec{k}') \mid V_0^{21} \mid \phi_1^{R(+)}(\vec{k}) \rangle \\ &= \int\int \frac{d\vec{q}\, d\vec{q}'}{(E - E_q + i\eta)(E - E_q' + i\eta)} T_2^{(-)}(\vec{q}, \vec{k}'; E) \\ &\quad \times T_1^{(+)}(\vec{q}', \vec{k}; E) U_1^{NR}(\vec{q}, \vec{q}'), \end{aligned} \tag{40}$$

where $T_i^{(-)}$ are also defined by Eq. (29). The nonresonant SCX amplitude is then

$$T_{SCX}^{NR} = \sum_{i=1}^{4} U_i^{NR}(\vec{k}', \vec{k}) . \tag{41}$$

It is clear from Eqs. (37) through (40) that once the Born amplitude $U_1^{NR}(\vec{k}', \vec{k})$ is known on and off the energy shell, we may determine the complete nonresonant amplitude in terms of the T matrix $T_i^{(\pm)}(\vec{k}', \vec{k}; E)$ defined by Eq. (29). We may follow exactly the same argument for the resonant exchange interaction. Let us define the Born amplitude:

$$\begin{aligned} & U_1^{R}(\vec{k}', \vec{k}; E) \\ & \quad = \langle \phi_2^{(-)}(\vec{k}') \mid H_{2Q}(E - \tilde{H}_{QQ})^{-1} H_{Q1} \mid \phi_1^{(+)}(\vec{k}) \rangle , \end{aligned} \tag{42}$$

where we have the explicit energy dependence. Then the other three terms in Eq. (35) are related to $U_1^{R}(\vec{k}', \vec{k}; E)$ by exactly the same relations as given by Eq. (38) through (40) with V_0^{21} changed to X_{ij}, and $U_1^{NR}(\vec{q}, \vec{q}')$ changed to $U_1^{R}(\vec{q}, \vec{q}'; E)$; the connection is also made by the use of the T matrix $T_i^{(\pm)}$. The complete resonant SCX amplitude is then given as

$$T_{SCX}^{R} = \sum_{i=1}^{4} U_i^{R}(\vec{k}', \vec{k}; E) . \tag{43}$$

We have now shown that the complete DWBA amplitude for single charge exchange reactions is determined by $U_1^{NR}(\vec{k}', \vec{k})$, $U_1^{R}(\vec{k}', \vec{k}; E)$, and $T_i^{(\pm)}(\vec{k}', \vec{k}; E)$. The usefulness of the method may be viewed as follows. The energy dependence due to the resonance is completely explicit in these basic quantities. The basic Born amplitudes may be evaluated with minimum uncertainty in the pion wave functions, since they are expressed only in terms of the nonresonant parts of the wave func-

tions, which may be reliably evaluated from a first-order optical model calculation. Near the resonance where the nonresonant distortion may be neglected, we may be able to use plane waves for $|\phi_i^{\pm}\rangle$.

Before we make approximations on these basic quantities, we would first like to extend the same formalism to DCX reactions in a second-order distorted-wave approximation. For simplicity, we shall discuss only the resonant charge exchange interaction. We first extend Eqs. (23) through (29) to include the doubly-charge-exchanged channel, hereafter denoted as $i=3$ channel in these equations. The resonant DCX reactions proceed through two successive interactions of X_{ij}. The simplest term with nonresonant *internal* and external pion wave functions is shown in Fig. 3. We shall call this simplest term the nonresonant Born term and denote it as $W_1(\vec{k}', \vec{k}; E)$; it may be related to $U_1^R(\vec{k}', \vec{k}; E)$ of Eq. (41) by

$$W_1(\vec{k}', \vec{k}; E) = \int \frac{d\vec{q}}{E - E_q + i\eta} \times U_1^R(\vec{k}', \vec{q}; E) U_1^R(\vec{q}, \vec{k}; E). \quad (44)$$

There are three other terms with nonresonant external pion wave functions. We may write these terms as (see Fig. 4)

$$W_2(\vec{k}', \vec{k}; E) = \int \frac{d\vec{q}}{E - E_q + i\eta} \times U_1^R(\vec{k}', \vec{q}; E) U_2^R(\vec{q}, \vec{k}; E), \quad (45)$$

$$W_3(\vec{k}', \vec{k}; E) = \int \frac{d\vec{q}}{E - E_q + i\eta} \times U_2^R(\vec{k}', \vec{q}; E) U_1^R(\vec{q}, \vec{k}; E), \quad (46)$$

$$W_4(\vec{k}', \vec{k}; E) = \int \frac{d\vec{q}}{E - E_q + i\eta} \times U_3^R(\vec{k}', \vec{q}; E) U_2^R(\vec{q}, \vec{k}; E). \quad (47)$$

It is readily shown that the Born term with nonresonant external pion wave functions is given as

$$W_1^B(\vec{k}', \vec{k}; E) = \langle \phi_3^{(-)}(\vec{k}') | X_{32} | \frac{1}{E - \mathcal{H}_{22} + i\eta} | X_{21} | \phi_1^{(+)}(\vec{k}) \rangle = \sum_{n=1}^{4} W_n(\vec{k}', \vec{k}; E). \quad (48)$$

In Eqs. (45) through (47), we have also used $U_2^R(\vec{k}', \vec{k}; E)$ and $U_3^R(\vec{k}', \vec{k}; E)$, as defined in Eqs. (38) and (39), for simplification in notations. The diagrams corresponding to $W_n (n = 2, 3, 4)$ are shown in Fig. 4. It is important to note that the Born term $W_1^B(\vec{k}', \vec{k}; E)$ is also completely determined by the basic quantities in the theory: $U_1^R(\vec{k}', \vec{k}; E)$ and the T matrix $T_i^{(\pm)}(\vec{k}', \vec{k}; E)$. We do not need any extra parameters.

After we have obtained the Born term, it is straightforward to generalize our consideration to the cases with *resonant* external pion states. The results are given similar to Eqs. (38), (39), and

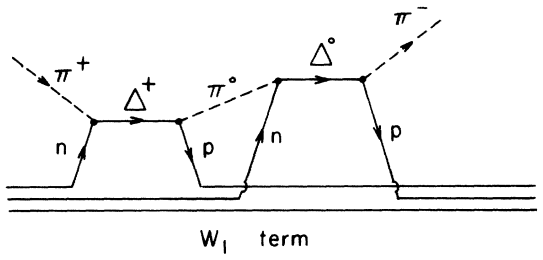

FIG. 3. Double charge exchange (DCX) interaction through two successive isobar formations. In this diagram, all the pion states are nonresonant. For DCX interaction diagrams (Figs. 3 and 4), we denote the diagram by ⟨initial pion state–intermediate pion state–final pion state⟩; Fig. 3 is therefore ⟨NR-NR-NR⟩.

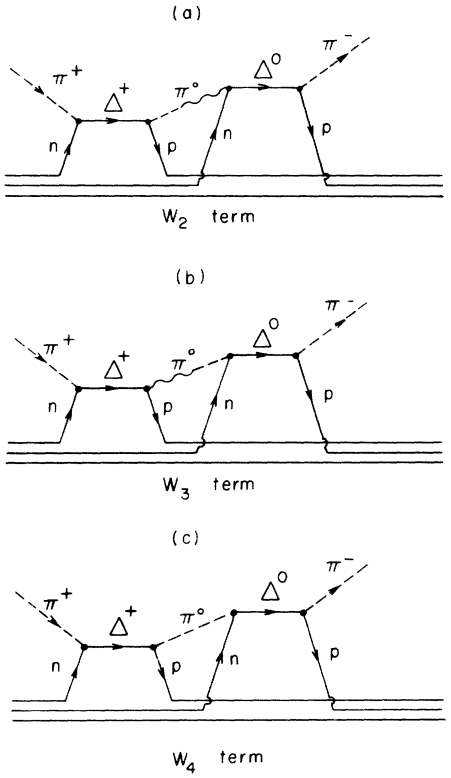

FIG. 4. Double-resonance DCX diagrams with nonresonant external pion states. Diagrams (a) and (b) are ⟨NR-(NR, R) – NR⟩ and ⟨NR – (R, NR) – NR⟩, respectively, where the intermediate states contain one resonant component of the pion wave function, their values being given by Eqs. (45) and (46). Diagram (c) is the ⟨NR – R – NR⟩ term given by Eq. (47).

(40), with the following replacements

$$U_i^{\mathrm{NR}}(\vec{k}', \vec{k}) \to W_i^{\mathrm{B}}(\vec{k}', \vec{k}; E), \quad i = 1, 2, 3, 4$$

and

$$T_2^{(\pm)}(\vec{k}', \vec{k}; E) \to T_3^{(\pm)}(\vec{k}', \vec{k}; E)$$

[with $i = 3$ in Eq. (29)], where we have defined

$$W_2^{\mathrm{B}}(\vec{k}', \vec{k}; E) = \langle \phi_3^{R(-)}(\vec{k}') \mid X_{32}(E - \mathcal{H}_{22} + i\eta)^{-1} X_{21} \mid \phi_1^{(+)}(\vec{k}) \rangle , \tag{49}$$

$$W_3^{\mathrm{B}}(\vec{k}', \vec{k}; E) = \langle \phi_3^{(-)}(\vec{k}') \mid X_{32}(E - \mathcal{H}_{22} + i\eta)^{-1} X_{21} \mid \phi_1^{R(+)}(\vec{k}) \rangle , \tag{50}$$

and

$$W_4^{\mathrm{B}}(\vec{k}', \vec{k}; E) = \langle \phi_3^{R(-)}(\vec{k}') \mid X_{32}(E - \mathcal{H}_{22} + i\eta)^{-1} X_{21} \mid \phi_1^{R(+)}(\vec{k}) \rangle . \tag{51}$$

The DCX amplitude in second-order distorted-wave approximation is therefore given as

$$T_{\mathrm{DCX}}^{\mathrm{R}} = \sum_{i=1}^{4} W_i^{\mathrm{B}}(\vec{k}', \vec{k}; E) . \tag{52}$$

We have now completed our formulation of the SCX and DCX reactions in the isobar-doorway model. Both SCX and DCX amplitudes are shown to depend on two basic quantities: $T_i^{(\pm)}(\vec{k}', \vec{k}; E)$ of Eq. (29) and $U_1^{\mathrm{R}}(\vec{k}', \vec{k}; E)$ of Eq. (42), if we neglect the small nonresonant contributions from $U_1^{\mathrm{NR}}(\vec{k}', \vec{k}; E)$ (which also appears in the nonresonant DCX amplitudes which have not been discussed here). These two quantities have clear dynamical meanings in the theory. In the next section, we shall use a phenomenological model for these quantities, where they are related to the basic π-nucleon interactions.

III. SIMPLE MODEL

In this section we shall show a model calculation of our basic quantities $T_i^{(\pm)}(\vec{k}', \vec{k}; E)$ and $U_1^{\mathrm{R}}(\vec{k}', \vec{k}; E)$ with the same approximations used in Ref. 16. We first rewrite Eqs. (29) and (42) as

$$T_i^{(\pm)}(\vec{k}', \vec{k}; E) = \sum_{n\alpha} \frac{\langle \phi_i^{(\pm)}(\vec{k}') \mid V_R \mid \Phi_{n\alpha} \rangle \langle \Phi_{n\alpha} \mid V_R \mid \phi_i^{(\pm)}(\vec{k}) \rangle}{E - \tilde{E}_{n\alpha i} + \frac{1}{2} i \tilde{\Gamma}_{n\alpha i}} \tag{53}$$

and

$$U_1^{\mathrm{R}}(\vec{k}', \vec{k}; E) = \sum_{n\alpha} \frac{\langle \phi_2^{(-)}(\vec{k}') \mid V_R \mid \Phi_{n\alpha} \rangle \langle \Phi_{n\alpha} \mid V_R \mid \phi_1^{(+)}(\vec{k}) \rangle}{E - E_{n\alpha} + \frac{1}{2} i \Gamma_{n\alpha}^{\downarrow}} , \tag{54}$$

where we have introduced the following definitions:
(1) The resonance energy

$$\tilde{E}_{n\alpha i} = E_n + \epsilon_\alpha + \Delta_{n\alpha i}^{\uparrow} + \Delta_{n\alpha}^{\downarrow} ; \tag{55}$$

(2) the modified isobar resonance energy

$$E_{n\alpha} = E_n + \epsilon_\alpha + \Delta_{n\alpha}^{\downarrow} ; \tag{56}$$

(3) the quasitotal width

$$\tilde{\Gamma}_{n\alpha i} = \Gamma_{n\alpha i}^{\uparrow} + \Gamma_{n\alpha}^{\downarrow} . \tag{57}$$

In the static model for the interaction V_R,[17] we may write (ignoring a phase factor)

$$\langle \phi_j^{(0)}(\vec{k}') \mid V_R \mid \phi_{n\alpha} \rangle \langle \Phi_{n\alpha} \mid V_R \mid \phi_i^{(0)}(\vec{k}) \rangle \simeq [\Gamma_{\pi N}^{\alpha}(k') \Gamma_{\pi N}^{\alpha}(k)]^{1/2} \mathfrak{A}_{ji} \frac{\hat{K}' \cdot \hat{K}}{2\pi} F_j^{\eta}(\vec{k}') F_i^{\eta}(\vec{k}) \tag{58}$$

where $\mid \phi_i^{(0)}(\vec{k}) \rangle$ is related to $\mid \phi_i^{(\pm)}(\vec{k}) \rangle$ by a phase, $\hat{K}$'s refer to unit vectors of the pion momenta in the π-nucleon c.m. system. For the static model, we may use $\vec{K} = \vec{k}$. To simplify our discussions, we assume plane wave solutions for $\mid \phi_i^{(\pm)}(\vec{k}) \rangle$ so that we may drop the designation of $(\pm)$ in our notations.[20] The factor $\mathfrak{A}_{ji}$ depends on the isospin states of the transition $i \to j$. We have also defined the width

$$\Gamma_{\pi N}^{\alpha}(k) = 2\pi \mid \langle \vec{k} \mid V_R \mid \Delta_\alpha \rangle \mid^2 , \tag{59}$$

where $\mid \vec{k} \rangle$ is a state of the pion nuclear system, and $\mid \Delta_\alpha \rangle$ is the single particle state of Δ in the nucleus. The form factors are defined

$$F_i^{\eta}(\vec{k}) = \sum_{j=1}^{A} \langle \Phi_n(\vec{r}_1 \cdots \vec{r}_{j-1} \vec{r}_{j+1} \cdots \vec{r}_A) \mid e^{i\vec{k}\cdot\vec{r}_j} \mid \Phi_i(\vec{r}_1 \cdots \vec{r}_j \cdots \vec{r}_A) \rangle = \int d\vec{r}\, e^{i\vec{k}\cdot\vec{r}} \rho_{n,i}(\vec{r}) . \tag{60}$$

$\rho_{ni}(\vec{r})$ is a product of a single particle wave function and a spectroscopic factor which depends upon the parentage of the state of the A nucleon system in terms of the state Φ_n of the $(A-1)$ nucleon system. If we assume that the scattering is not far off shell, i.e., $|\vec{k}| \approx |\vec{k}'|$, we may approximate

$$\Gamma^{\alpha}_{\pi N}(k') \approx \Gamma^{\alpha}_{\pi N}(k), \tag{61}$$

since $\Gamma^{\alpha}_{\pi N}(k)$ is not as critically energy-dependent as the energy denominator in Eqs. (53) and (54). We may rewrite Eqs. (53) and (54) as

$$T_i(\vec{k}', \vec{k}; E) = \frac{\mathfrak{N}_{ii}}{2\pi} \sum_{n\alpha} \frac{\Gamma^{\alpha}_{\pi N}(k) \hat{K}' \cdot \hat{K} F^n_i(\vec{k}') F^n_i(\vec{k})}{E - \tilde{E}_{n\alpha i} + \frac{1}{2} i \tilde{\Gamma}_{n\alpha i}} \tag{62}$$

and

$$U^{R}_1(\vec{k}', \vec{k}; E) = \frac{\mathfrak{N}_{21}}{2\pi} \sum_{n\alpha} \frac{\Gamma^{\alpha}_{\pi N}(k) \hat{K}' \cdot \hat{K} F^n_2(\vec{k}') F^n_1(\vec{k})}{E - E_{n\alpha} + \frac{1}{2} i \Gamma^{\downarrow}_{n\alpha}} \tag{63}$$

We now introduce the closure property of the isobar-doorway states and sum over $n\alpha$. We obtain[16]

$$T_i(\vec{k}', \vec{k}; E) = \frac{\mathfrak{N}_{ii}}{2\pi} \frac{\Gamma_{\pi N}(E) \hat{K}' \cdot \hat{K} F_{ii}(\vec{k}', \vec{k})}{E - M(1236) - \delta E + \frac{1}{2} i (\Gamma^{i}_{el} + \Gamma_{in})} \tag{64}$$

and

$$U^{R}_1(\vec{k}', \vec{k}; E) = \frac{\mathfrak{N}_{21}}{2\pi} \frac{\Gamma_{\pi N}(E) \hat{K}' \cdot \hat{K} F_{21}(\vec{k}', \vec{k})}{E - M(1236) - \delta E^{\downarrow} + \frac{1}{2} i \Gamma_{in}}, \tag{65}$$

where we have approximated the average width $\langle \Gamma^{\alpha}_{\pi N} \rangle$ by the free $\Delta(1231)$ decay width $\Gamma_{\pi N}(E)$, and the average resonance energy $\langle \tilde{E}_{n\alpha i} \rangle$ by

$$\langle \tilde{E}_{n\alpha i} \rangle \approx M(1231) + \delta E, \tag{66}$$

where δE is the total energy shift of the isobar energy in the nuclear medium.[16] We have also denoted the averaged quasitotal width for the ith channel as

$$\langle \tilde{\Gamma}_{n\alpha i} \rangle = \Gamma^{i}_{el} + \Gamma_{in}, \tag{67}$$

where

$$\Gamma^{i}_{el} = \langle \Gamma^{\downarrow}_{n\alpha i} \rangle \quad \text{and} \quad \Gamma = \langle \Gamma^{\downarrow}_{n\alpha} \rangle. \tag{68}$$

The form factors are defined as

$$F_{ij}(\vec{k}', \vec{k}) = \sum_n F^n_i(\vec{k}') F^n_j(\vec{k}). \tag{69}$$

We note that the difference between the total width Γ^{i}_{tot} and the quasitotal width is the decay width to charge-exchange channel.[21] Since the charge-exchange width is very small compared to the total width, we may take

$$\Gamma^{i}_{el} + \Gamma_{in} \approx \Gamma^{i}_{tot}. \tag{70}$$

In Eq. (65) we also have an average energy shift

$$\delta E^{\downarrow} = \langle \Delta^{\downarrow}_{n\alpha} \rangle. \tag{71}$$

Since the energy shifts are probably small, the two energy shifts δE and $\delta E^{\downarrow}$ may be taken to be equal.

We have now reduced these basic quantities to their simplest forms. It is interesting to make the connection of charge exchange-reactions to the elastic scattering in the isobar-doorway model. Compared to the results we have in Ref. 16, the $T_i(\vec{k}', \vec{k}; E)$ is nothing but the off-shell extension of the *elastic* T matrix in the isobar-doorway model [with Eq. (70)]. We also find that $U^{R}_1(\vec{k}', \vec{k}; E)$ is closely related to the modified π-nucleon t matrix,[16]

$$\alpha(E) \tilde{t}_{33}(\vec{k}', \vec{k}; E) \equiv \frac{\frac{1}{2}\Gamma_{\pi N}(E)}{E - M(1236) - \delta E + \frac{1}{2} i \Gamma_{in}} \hat{K}' \cdot \hat{K}, \tag{72}$$

by the off-diagonal form factor $F_{21}(\vec{k}', \vec{k})$ as

$$U^{R}_1(\vec{k}', \vec{k}; E) = \frac{\mathfrak{N}_{21}}{2\pi} \alpha(E) \tilde{t}_{33}(\vec{k}', \vec{k}; E) F_{21}(\vec{k}', \vec{k}). \tag{73}$$

In analogy to $U^{R}_1(\vec{k}', \vec{k}; E)$, we may write

$$T_i(\vec{k}', \vec{k}; E) = \frac{\mathfrak{N}_{ii}}{2\pi} \tilde{T}^{i}_{33}(\vec{k}', \vec{k}; E) F_{ii}(\vec{k}', \vec{k}), \tag{74}$$

where

$$\tilde{T}^{i}_{33}(\vec{k}', \vec{k}; E) = \frac{\frac{1}{2}\Gamma_{\pi N}(E)}{E - M(1236) - \delta E + \frac{1}{2} i (\Gamma^{i}_{el} + \Gamma_{in})} \hat{K}' \cdot \hat{K}. \tag{75}$$

As we have shown in Ref. 16, the total width (i.e., $\approx \Gamma^{i}_{el} + \Gamma_{in}$ may be determined by a fit to the total cross section as a function of energy.[22] By the same procedure, we may also estimate the inelastic width Γ_{in} by the ratio of the total reaction cross section $\sigma_r(E)$ to the total elastic cross section $\sigma_{el}(E)$. Therefore, all the necessary parameters in the model may be checked with the measurements of the total cross sections alone. The angular distributions of the SCX and DCX reaction cross section and of the elastic scattering may then be predicted within the same approximations.

The model is therefore shown to be capable of making direct and simple connections between the SCX and DCX reactions and the elastic scattering. Furthermore, the model is particularly useful in its separation of the resonant factors from the nonresonant parts. As examples, we shall explicitly show this property in the DWBA amplitudes. We first consider the nonresonant terms.

Since the treatment of the completely nonresonant interaction $U^{NR}_1(\vec{k}', \vec{k})$ of Eq. (37) is not in the model, we shall assume that it is known from an ap-

propriate optical-model calculation.[23] This term should have smooth energy dependence. The resonant terms of Eqs. (38) and (39) may be combined to the following form

$$U_2^{\mathrm{NR}}(\vec{k}',\vec{k})+U_3^{\mathrm{NR}}(\vec{k}',\vec{k})=\frac{1}{2\pi}[a(\vec{k}',\vec{k})|\tilde{T}^2_{33}(E)| + b(\vec{k}',\vec{k})|\tilde{T}^1_{33}(E)|], \tag{76}$$

where we have defined

$$\tilde{T}^i_{33}(\vec{k}',\vec{k};E)\equiv|\tilde{T}^i_{33}(E)|\hat{K}'\cdot\hat{K}, \tag{77}$$

$$a(\vec{k}',\vec{k})=\mathfrak{N}_{22}\int d\vec{q}(E-E_q+i\eta)^{-1}(\hat{k}'\cdot\hat{q}) \times F_{22}(\vec{k}',\vec{q})U_1^{\mathrm{NR}}(\vec{q},\vec{k}), \tag{78}$$

$$b(\vec{k}',\vec{k})=\mathfrak{N}_{11}\int d\vec{q}(E-E_q+i\eta)^{-1}(\hat{q}\cdot\hat{k}) \times U_1^{\mathrm{NR}}(\vec{k}',\vec{q})F_{11}(\vec{q}',\vec{k}). \tag{79}$$

We note that the energy dependence of Eq. (76) appears mainly in the factor $|\tilde{T}^i_{33}|$, since $a(\vec{k}',\vec{k})$ and $b(\vec{k}',\vec{k})$ are smooth functions of energy. The most resonating term in the nonresonant charge exchange reaction is then

$$U_4^{\mathrm{NR}}(\vec{k}',\vec{k})=\frac{\mathfrak{N}_{11}\mathfrak{N}_{22}}{4\pi^2} \times|\tilde{T}^2_{33}(E)|c(\vec{k}',\vec{k})|\tilde{T}^1_{33}(E)|, \tag{80}$$

where the smoothly energy-dependent factor $c(\vec{k}',\vec{k})$ is given as

$$c(\vec{k}',\vec{k})=\int\int d\vec{q}\,d\vec{q}'(E-E_q+i\eta)^{-1} \times(E-E_{q'}+i\eta)^{-1}(\hat{k}'\cdot\hat{q}')(\hat{k}\cdot\hat{q}) \times F_{22}(\vec{k}',\vec{q}')U_1^{\mathrm{NR}}(\vec{q}',\vec{q})F_{11}(\vec{q},\vec{k}). \tag{81}$$

We next consider the resonant terms. The basic term is given in Eq. (73) with main energy dependence in $|\tilde{t}_{33}(E)|$ defined by

$$\alpha(E)\tilde{t}_{33}(\vec{k}',\vec{k};E)\equiv|\tilde{t}_{33}(E)|\hat{K}'\cdot\hat{K}. \tag{82}$$

We may write

$$U_2^{\mathrm{R}}(\vec{k}',\vec{k};E)+U_3^{\mathrm{R}}(\vec{k}',\vec{k};E) =\frac{\mathfrak{N}_{21}}{4\pi^2}|\tilde{t}_{33}(E)|[d(\vec{k}',\vec{k})|\tilde{T}^2_{33}(E)|+e(\vec{k}',\vec{k})|\tilde{T}^1_{33}(E)|], \tag{83}$$

where

$$d(\vec{k}',\vec{k})=\mathfrak{N}_{22}\int\frac{d\vec{q}(\hat{k}'\cdot\hat{q})(\hat{q}\cdot\hat{k})}{E-E_q+i\eta} \times F_{22}(\vec{k}',\vec{q})F_{21}(\vec{q},\vec{k}) \tag{84}$$

and

$$e(\vec{k}',\vec{k})=\mathfrak{N}_{11}\int\frac{d\vec{q}(\hat{k}'\cdot\hat{q})(\hat{q}\cdot\hat{k})}{E-E_q+i\eta} \times F_{21}(\vec{k}',\vec{q})F_{11}(\vec{q},\vec{k}). \tag{85}$$

Finally, we have

$$U_4^{\mathrm{R}}(\vec{k}',\vec{k};E) =\frac{\mathfrak{N}_{22}\mathfrak{N}_{21}\mathfrak{N}_{11}}{8\pi^3}|\tilde{t}^2_{33}(E)||\tilde{T}^2_{33}(E)|f(\vec{k}',\vec{k})|\tilde{T}^1_{33}(E)|, \tag{86}$$

where

$$f(\vec{k}',\vec{k})=\int\int\frac{d\vec{q}\,d\vec{q}'(\hat{k}'\cdot\hat{q}')(\hat{q}'\cdot\hat{q})(\hat{q}\cdot\hat{k})}{(E-E_q+i\eta)(E-E'_q+i\eta)} \times F_{22}(\vec{k}',\vec{q}')F_{21}(\vec{q}',\vec{q})F_{11}(\vec{q}\cdot\vec{k}). \tag{87}$$

As noted before, the factors $d(\vec{k}',\vec{k})$, $e(\vec{k}',\vec{k})$, and $f(\vec{k}',\vec{k})$ have relatively smooth energy dependence, so that the strong energy dependence in all the resonant terms is completely separated out in our final expressions. Furthermore, the energy dependence is in turn directly related to the energy dependence in the elastic scattering. We finally note that the main nuclear structure effects are contained in the form factors which are separated out from the main effects of the resonance energy dependence.

The same discussion may be extended to the DCX reactions, where the main energy dependence will also be contained in $|\tilde{t}_{33}(E)|$ and $|\tilde{T}^i_{33}(E)|$ factors. The extension is straightforward and will not be discussed here.

IV. CONCLUDING REMARKS

We have extended the isobar-doorway model for pion-nucleus scattering to the charge-exchange reactions. We have shown that energy dependence of the SCX and DCX reaction amplitude in DWBA may be conveniently separated from the parts which contain nuclear structure information. The energy dependence depends on the pion optical potential, or the self-energy effects in the nuclear medium; it is therefore shown to be closely related to the elastic scattering amplitude.

Within the model, the SCX and DCX reactions may be treated on the same footing as the elastic scattering with common factors depending only on the energy and therefore may be consistently described by a simple parametrization from the energy dependence of the total cross sections. These factors may eventually be evaluated by a more detailed interaction model, such as a microscopic theory where the motion of the isobar is ex-

plicitly taken into account. The model we present here is general enough to allow variations in the detailed assumptions of the pion-nucleon interaction in the medium.

The same procedure as described in this work may also be applied to separate the resonant and nonresonant components in an optical-model calculation. This separation of the optical potential, of course, is an extension of the usual DWBA calculation; for the case of pion-nucleus scattering, this approach may be useful in order to gain more insight into the roles of the (3, 3) resonance in the optical potential. However, this type of optical-model approach will be formally equivalent to our formulation if the optical-model wave functions used are obtained from formally exact optical potentials, since our wave functions formally contain all orders of multiple scattering.

Finally, we would like to point out that our model has the distinctive feature of displaying explicitly the roles of the (3, 3) resonance in the reaction dynamics, including the initial and final state interactions. The effect of the nonresonant background interactions is also properly retained in the formalism. The validity of the model can only be tested by experimental data. We propose to apply the model near the (3, 3) resonance region (i.e., 150 MeV $\lesssim T_\pi \lesssim$ 300 MeV, $T_\pi \equiv$ pion laboratory kinetic energy). From Eqs. (70) through (87), a rough observation suggests that the SCX and DCX reaction cross sections be resonating across the (3, 3) region, following the energy dependence of the elastic scattering modulated by smoothly energy-dependent factors. This resonant behavior in the total elastic charge-exchange cross sections indicates a quite different result as from the first-order optical model calculations and the multiplet-scattering theory.[10,14,15]

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[20]We note that, if we keep only the resonant π-nucleon interactions, $\phi_i^{(\pm)}(\vec{k})$ will be plane waves, so this is a good approximation near the resonance energy.

[21]We note that the decay width to the charge-exchanged channel (from ith to jth channels) is of the order of $[(N-Z)/A]^2$ of the elastic scattering width in the ith channel. This estimate is based on the isospin factors associated with the resonance part of the charge-exchanging interaction [N= neutron number, Z = proton number, $A = N + Z$].

[22]It is understood that the total cross section for a neutral pion (π^0) may be difficult to obtain, so our parameter Γ_{tot} for π^0 channel is not easily determined. However, the total widths for all the channels (in the Born approximation) are equal, to the order of $[(N-Z)/A]$. Therefore, for $(N-Z) \ll A$, we may neglect even the i dependence of the total width.

[23]An appropriate choice of the nonresonant optical potential for $|\varphi^{(\pm)}>$ was discussed in Ref. 16. The nonresonant charge exchange interaction is contained in H_{21}. By definition, this nonresonant charge-exchange interaction contains no effects of the isobar resonance and is rather weak, so a reliable estimate of $U_1^{\text{NR}}(\vec{k}', \vec{k})$ may be obtained in the usual multiple scattering formalism or in a first-order optical potential calculation where only the nonresonant π-nucleon interactions are considered.