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Pseudo-resonance behavior of partial waves in elastic pion scattering

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Glauber theory employing observed πN phases is used to compute πd differential cross sections for $E \lesssim 600$ MeV. Lacking experimentally extracted phase shifts, we used the model to construct model phases, which have been plotted in Argand diagrams. These display for many J^π values a pseudo-resonance behavior. It is restressed that these do not correspond to resonances in a $B=2$ system but simply reflect a N^* in the weakly bound deuteron.

[NUCLEAR REACTIONS Calculated $\sigma_{el}^{\pi d}(E,\theta)$; theoretical phase shift analysis for $E_\pi \lesssim 600$ MeV. Interpretation of phase behavior displayed in Argand plots.]

The scattering of projectiles on composite systems in the energy region around a resonance of the projectile and an individual constituent has been discussed in a number of recent publications. Evidence from pion scattering on light nuclei shows the existence of a peak in total cross sections[1] which is substantially broadened and slightly displaced in comparison with the peak in the elementary πN cross section. Much of the relevant information necessary to explain shift and broadening of these peaks should be contained in differential cross sections, or more precisely in phase shifts. But in spite of accumulating information, notably on pion scattering[2,3] the data are neither accurate enough nor sufficiently dense in energy to allow a reliable extraction of phase shifts.

Attempts have nevertheless been made for π-^{12}C[4] and in particular for π-^{4}He scattering.[5] Argand plots of partial wave amplitudes then show resonancelike behavior for several partial waves around the position of the elementary resonance.[5] A discussion of such behavior is the topic of this note.

Lacking sufficient data, one has to rely on models for scattering on composite targets to extract full and partial wave amplitudes. As an example we mention π-^{12}C scattering in the Δ region, which has been analyzed using an impulse approximation for the (Fourier transform of the) optical potential[6]:

$$-(4\pi^2\mu)^{-1}V^{\pi\,^{12}\mathrm{C}}(E,q)=\tfrac{11}{12}f^{\pi\,^{12}\mathrm{C}}(E,q)|^{\mathrm{Born}}$$
$$=\tfrac{11}{12}F(q)f^{\pi N}(E,q)\,. \qquad (1)$$

We forego details in the derivation of (1) and focus on the dominant resonating p wave in the scattering from every individual nucleon. When viewed from the c.m. of the π-C system these elementary p waves will have components in several partial waves for π-^{12}C scattering. Differently stated, the ground state form factor $F(q)$ smears the elementary resonances over several waves, and one easily estimates for $E\sim E_r^{\pi N}$

$$f^{\pi\,^{12}\mathrm{C}}(E)|^{\mathrm{Born}}\sim(2l+1)^{-1}\frac{3}{k}\left(\frac{\Gamma}{E-E_r+\frac{1}{2}i\Gamma}\right)\times\int_{-1}^{+1}F(2k\sin\tfrac{1}{2}\theta)\cos\theta\,P_l(\cos\theta)d\cos\theta\,. \qquad (2)$$

All partial wave Born amplitudes (2) clearly resemble the elementary resonance amplitude with an apparent l and E dependent elastic width determined by the integral in (2).

Explicit calculation of the complete amplitude generated by $V^{\pi C}$, Eq. (1), has shown that these features typical of the *Born approximation*, Eqs. (1) and (2) are approximately preserved after multiple collisions.[6]

The reflection of the Δ in nuclear matter should be most conspicuous if (i) the "smearing" by $F(q)$ is weak, and (ii) the Born approximation (1) dominates.

Condition (i) is obviously met by light nuclei, ideally ^{2}H. No general criterion holds for (ii) but also here one expects the importance of multiple collisions to decrease with decreasing mass number. A host of theories has been applied to describe $\pi, K\ldots$ scattering on ^{2}H in particular for the resonance region.[7-9] By far the easiest is the Glauber prescription, which ascribes the scattering to single and double collisions. The theory accounts surprisingly well for elastic scattering down to $E_\pi \sim 150$ MeV and often out to large angles.[7,10]

Without understanding the reason for the rather unexpected agreement, we took the model for granted and investigated $(d\sigma/d\Omega)^{\pi d}$ for $600 \gtrsim E$ (MeV) $\gtrsim 140$. Published phase shifts[10] have been used to construct elementary amplitudes which have been fed into the π-d Glauber amplitude.[7,11]

Figure 1 demonstrates the measure of agreement between experimental π-d distributions for relatively low energies ($E_\pi \lesssim 250$ MeV) and cross sections computed as indicated above. We also extended these calculations to $E = 600$ MeV and some predicted angular distributions are shown in Fig. 2. Next the noted agreement for cross sections is assumed to imply that the underlying Glauber amplitude is at least qualitatively correct. The latter has then been subjected to a *theoretical* phase shift analysis.

The results for Argand plots of some unique unnatural parity $[\pi = (-)^{J+1}]$ and some natural parity $[\pi = (-)^J]$ eigenamplitudes are displayed in Fig. 3. Pronounced resonance-like loops appear for low-J amplitudes. As expected these loops are indeed qualitatively described by partial wave projections of the Born amplitude, Eq. (2). Its approximate sufficiency is understood from the computational observation that the double scattering term contributes (depending on energy and angle) 10–40% to the differential cross section.[12] Equation (2) could thus be used to readily estimate the apparent "inelasticity" as the ratio of the radii of resonance and unitary circles as well as the shift in the resonance position for each J and in the summed cross section.

However, it is abundantly clear that the loops in the Argand plots are pseudo-resonances which essentially reflect the smearing of the elementary π-N resonance over several partial waves in the π-d system. The same phenomenon should be, and has been, observed in competing channels like $\pi^+ + d \to p + p$.[13] Yet the apparent inelasticity of the elastic amplitudes is totally unrelated to the coupling of competing channels to the elastic one. There is in particular no meaning to an extraction of the usual resonanance and background parameter from Argand plots, as has been done for π-^{4}He scattering.[5]

By the same token one ascribes broadening and shift of peaks in pion total cross sections to a distribution of the elementary resonance over several partial waves which, indeed, is hardly an interesting dynamical phenomenon. The reasoning should also hold for pion-scattering on somewhat heavier targets. The argument of course breaks down for medium weight targets, but we also ex-

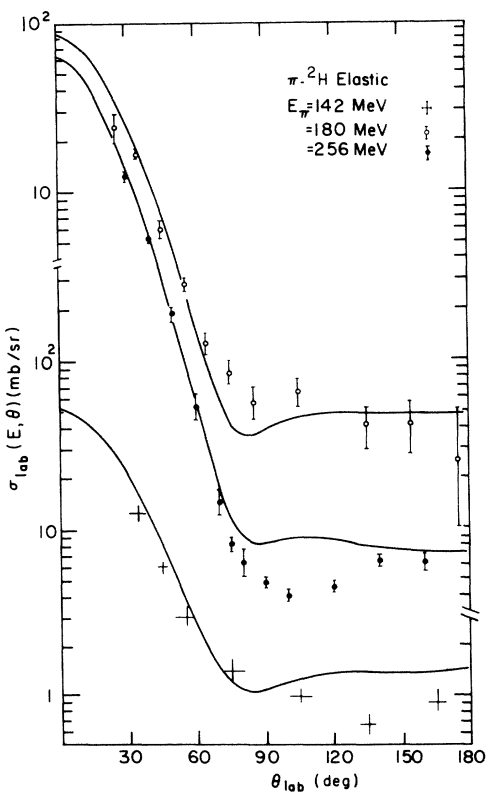

FIG. 1. Some measured angular distribution of elastically scattered pions from ^{2}H and their Glauber predictions ($E_\pi = 142$, 180, 256 MeV).

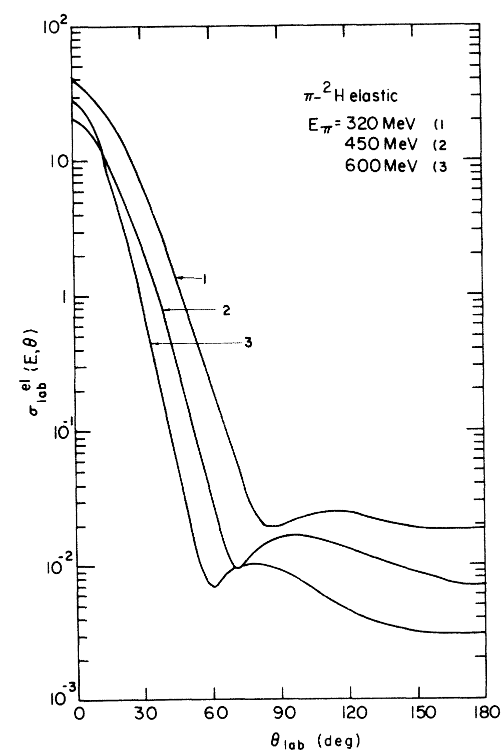

FIG. 2. Glauber predictions for π-d distributions; $E_\pi = 320$, 450, 600 MeV.

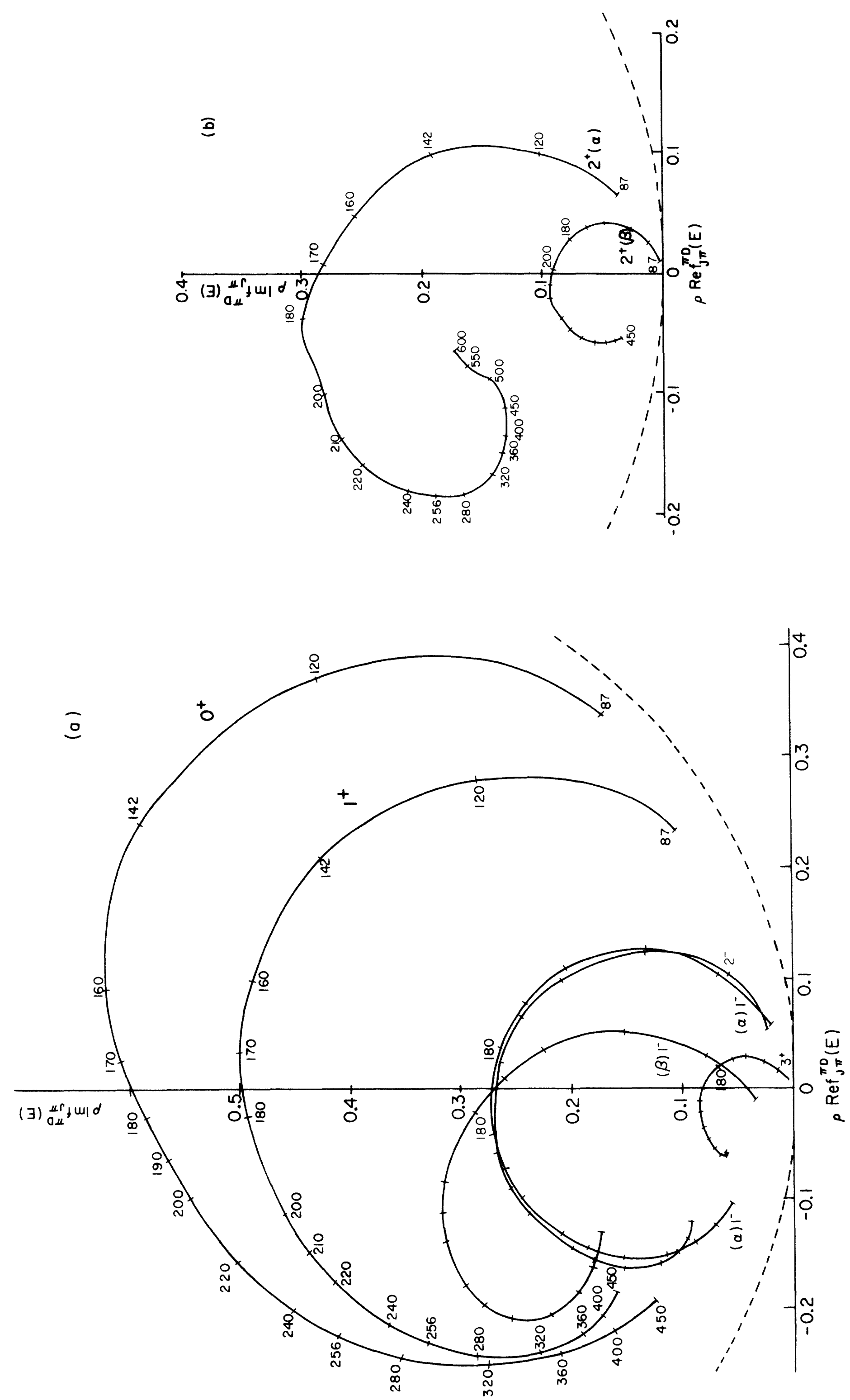

FIG. 3. Argand plots for some unique unnatural parity $[\pi=(-)^{J+1}]$ π-d partial wave/amplitudes and natural parity $[\pi=(-)^{J}]$ eigenamplitudes. (ρ is the phase space factor). The marks on the curves correspond to E_π = 87, 120, 142, 160, 170, 180, 200, 210, 220, 240, 256, 280, 320, 360, 400, 450, (500, 550, 600) MeV. The dashed curve is part of the unitary circle.

pect the peak to become unnoticable for increasing mass number.

One may ask whether a similar phenomenon occurs in the scattering of other entities: At first sight elastic ^{4}He-d scattering seems to be a possible candidate but there is not the slightest indication as to the existence of pseudo-resonances in the reliably extracted phase shifts.[14] There may be several reasons for their absence, like the proximity of the otherwise pronounced ^{4}He-N resonance(s) to threshold and, maybe, the importance of multiple ^{4}He-N collisions in the elastic ^{4}He-d scattering.[15]

In conclusion we stress that the discussed reactions caution against uncritical interpretations of energy loops in an Argand diagram.

Note added in proof: Dr. J. Schiffer has brought our attention to a publication[16] where a resonance in the ^{28}Si compound system that was seen in ^{12}C + ^{16}O elastic reactions persists in ^{13}C(^{16}O, ^{17}O)^{12}C. Due to the weak binding of the last neutron in both ^{13}C and ^{17}O the latter reaction may be an example of a pseudo-resonance. It would also be of interest to study the elastic ^{16}O + ^{13}C reaction.

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