Derivation of a coupling potential for Coulomb-nuclear interference effects

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A collective coupling potential for surface vibrations is calculated from an extended liquid drop model. It describes Coulomb and nuclear excitations. Applying the potential to the reaction $^{58}Ni(^{16}O, ^{16}O')^{58}Ni^*$ we obtain a good agreement with recent experimental data.

[NUCLEAR REACTIONS $^{58}Ni(^{16}O, ^{16}O)^{58}Ni^*$, $E = 34–58$ MeV; derivation of coupling potentials; calculated $\sigma(E)$.]

1. INTRODUCTION

Coulomb-nuclear interference measurements[1-3] are very sensitive on the heavy-ion potential in the region of the Coulomb barrier. Until now theoretical calculations[3-5] on this effect were based upon phenomenological optical potentials, whose parameters are fitted to the experimental data for each considered nuclear system. In this work we present a method to calculate the heavy-ion elastic and coupling potentials from an extended liquid drop model. In Sec. 2 the collective ion-ion potentials are derived, which are used in Sec. 3 for a semiclassical description of the collision process. The results obtained from the theory for the elastic and inelastic scattering of ^{16}O on ^{58}Ni are discussed in Sec. 4.

2. COLLECTIVE HAMILTONIAN AND THE ION-ION POTENTIAL

The Hamiltonian describing the scattering of two nuclei has the following general structure

$$\hat{H} = \frac{\hat{p}_r^{\,2}}{2m} + \hat{H}_1(\xi_1) + \hat{H}_2(\xi_2) + \hat{V}(\vec{r}, \xi_1, \xi_2)\,. \tag{1}$$

It consists of the kinetic energy of the relative motion, the intrinsic energies $\hat{H}_i(\xi_i)$ of the two nuclei, and their interaction potential $\hat{V}$, the latter depending on the relative distance $\vec{r}$ and the intrinsic coordinates ξ_i. In this paper we want to deal with the excitation of collective modes during the heavy-ion collision. The most dominant collective states in spherical even-even nuclei are the surface vibrations, and hence it seems suitable to use the collective vibrator model[6] for the intrinsic Hamiltonian

$$\hat{H}(\xi) \equiv \hat{H}_{\text{vib}}(\alpha^{[l]}) = \sum_l (-1)^l (2l+1)^{1/2} \left\{ \frac{1}{2B_l} [\pi^{[l]} \otimes \pi^{[l]}]^{[0]} + \tfrac{1}{2} C_l [\alpha^{[l]} \otimes \alpha^{[l]}]^{[0]} \right\}. \tag{2}$$

In the following we calculate the real part of the ion-ion potential $\hat{V}(\vec{r}, \alpha^{(1)}_{lm}, \alpha^{(2)}_{lm})$ by means of the extended liquid drop model.[7-9] In this model the binding energy E_B is assumed to be a functional of the nuclear density $\rho(\vec{r})$:

$$E_B[\rho(\vec{r})] = W_0 A + \frac{C}{2\rho_0} \int [\rho(\vec{r}) - \rho_0]^2 d\tau + \frac{V_0}{8\pi} \iint \rho(\vec{r}) \frac{e^{-|\vec{r}-\vec{r}\,'|/\mu}}{|\vec{r} - \vec{r}\,'|} [\rho(\vec{r}\,') - \rho(\vec{r})] d\tau d\tau'$$
$$+ \frac{1}{2}\left(\frac{eZ}{A}\right)^2 \iint \rho(\vec{r}) \frac{1}{|\vec{r} - \vec{r}\,'|} \rho(\vec{r}\,') d\tau d\tau' + \frac{G}{2\rho_0}\left(2\frac{Z}{A} - 1\right)^2 \int \rho(\vec{r})^2 d\tau\,. \tag{3}$$

The first contribution to E_B is proportional to the number of nucleons. The second term in Eq. (3) describes compression effects, since it lowers the binding energy if the nuclear density $\rho(\vec{r})$ differs from the saturation density ρ_0 in infinite nuclear matter. The Yukawa energy has essentially surface character. It is followed by the Coulomb and the symmetry energy. No pairing term is included since we restrict ourselves to even-even nuclei.

From the ansatz (3) we obtain for the ion-ion potential[7]

$$V = E_B[\rho] - E_B[\rho_1] - E_B[\rho_2], \tag{4}$$

where ρ denotes the density distribution of the system consisting of the two ions with the relative distance $|\vec{r}|$; ρ_1 and ρ_2 are the densities of the individual nuclei. For simplicity we assume the density of the system as the sum of the single densities ("sudden approximation"), i.e.

$$\rho = \rho_1(\vec{r}_1) + \rho_2(\vec{r}_2). \tag{5}$$

ρ_1 and ρ_2 are measured from the centers of the nuclei. Inserting Eq. (5) into Eq. (4) and using the expression (3) for the binding energy E_B, the ion-ion potential can be derived. It is of the form

$$V = V_{\text{Coul}} + V_{\text{Yuk}} + V_{\text{comp}} + V_{\text{sym}}, \tag{6a}$$

where

$$V_{\text{Coul}} = \frac{4\pi Z_1 Z_2 e^2}{A_1 A_2} \lim_{\nu\to\infty} Y(\vec{r}, \nu, \rho_1, \rho_2),$$

$$V_{\text{Yuk}} = V_0\left[Y(\vec{r}, \mu, \rho_1, \rho_2) - \mu^2 \lim_{\nu\to 0}\frac{1}{\nu^2} Y(\vec{r}, \nu, \rho_1, \rho_2)\right], \tag{6b}$$

$$V_{\text{comp}} = \frac{C}{\rho_0} \lim_{\nu\to 0}\frac{1}{\nu^2}\left[Y(\vec{r}, \nu, \rho_1, \rho_2) - \tfrac{1}{2} Y(\vec{r}, \nu, \rho_0, \rho_0)\right],$$

$$V_{\text{sym}} = \frac{G}{\rho_0}\left(2\frac{Z_1}{A_1} - 1\right)\left(2\frac{Z_2}{A_2} - 1\right)\lim_{\nu\to 0}\frac{1}{\nu^2} Y(\vec{r}, \nu, \rho_1, \rho_2).$$

Here we have introduced the basic Yukawa integral

$$Y(\vec{r}, \mu, \rho_1, \rho_2) = \frac{1}{4\pi}\iint d\tau_1 d\tau_2\, \rho_1(\vec{r}_1)\frac{e^{-|\vec{r}_1 - \vec{r}_2 + \vec{r}|/\mu}}{|\vec{r}_1 - \vec{r}_2 + \vec{r}|}\rho_2(\vec{r}_2) \tag{6c}$$

which can be solved after a Fourier transformation by the same method as described in Ref. 9. If we assume a sharp nuclear surface given by

$$R_i(\Omega_i) = R_{0i}\left[1 + \sum_{l,m}\alpha_{lm}^{(i)} Y_{lm}^*(\Omega_i)\right], \quad i = 1, 2 \tag{7}$$

and a homogeneous density distribution

$$\rho_i(\vec{r}_i) = \rho_{0i} = \frac{3A_i}{4\pi R_{0i}^3}, \quad i = 1, 2 \tag{8}$$

we obtain up to terms linear in the surface variables $\alpha^{[i]}$

$$Y(\vec{r}, \mu, \rho_{01}, \rho_{02}) = 4\rho_{01}\rho_{02}R_{01}^2 R_{02}^2\Big[F(\mu, r, R_{01}, R_{02}) + R_{01}\sum_{l,m}(-1)^l \alpha_{lm}^{(1)} Y_{lm}^*(\Omega) G_l(\mu, r, R_{01}, R_{02}) + R_{02}\sum_{l,m}\alpha_{lm}^{(2)} Y_{lm}^*(\Omega) G_l(\mu, r, R_{02}, R_{01})\Big]. \tag{9a}$$

In Eq. (9a) the following abbreviations have been used

$$F(\mu, x, y, z) = \int_{-\infty}^{+\infty} dk \frac{1}{k^2 + 1/\mu^2} j_0(kx) j_1(ky) j_1(kz), \tag{9b}$$

$$G_l(\mu, x, y, z) = \int_{-\infty}^{+\infty} dk \frac{k}{k^2 + 1/\mu^2} j_l(kx) j_l(ky) j_1(kz),$$

where j_l denotes the spherical Bessel function of order l. The integration over the momentum space in Eq. (9b) can be done in a rather lengthy, but straightforward way by means of the theory of residues.[10] Combining Eqs. (6a), (6b), and (9a) we write the ion-ion potential in the form

$$V(\vec{r}, \alpha_{lm}^{(1)}, \alpha_{lm}^{(2)}) = U(r) + \sum_{l,m}\left[S_l(r)(-1)^l \alpha_{lm}^{(1)} + T_l(r)\alpha_{lm}^{(2)}\right] Y_{lm}^*(\Omega). \tag{10}$$

The elastic potential $U(r)$ as well as the vibrational coupling potentials for projectile and target nucleus, $S_l(r)$ and $T_l(r)$, consist of a Coulomb, Yukawa, compression, and symmetry part. For $U(r)$ these various parts are given by:

$$U(r)_{\text{Coul}} = \frac{9}{\pi}\frac{Z_1 Z_2 e^2}{R_{01}R_{02}}\lim_{\nu\to\infty} F(\nu, r, R_{01}, R_{02}),$$

$$U(r)_{\text{Yuk}} = V_0\frac{9}{4\pi^2}\frac{A_1 A_2}{R_{01}R_{02}} \times\left[F(\mu, r, R_{01}, R_{02}) - \mu^2\lim_{\nu\to 0}\frac{1}{\nu^2}F(\nu, r, R_{01}, R_{02})\right], \tag{11}$$

$$U(r)_{\text{comp}} = \frac{4R_{01}^2 R_{02}^2 C}{\rho_0}\left(\rho_{01}\rho_{02} - \tfrac{1}{2}\rho_0^2\right) \times \lim_{\nu\to 0}\frac{1}{\nu^2}F(\nu, r, R_{01}, R_{02}),$$

$$U(r)_{\text{sym}} = \frac{G}{\rho_0}\left(2\frac{Z_1}{A_1} - 1\right)\left(2\frac{Z_2}{A_2} - 1\right)4\rho_{01}\rho_{02}R_{01}^2 R_{02}^2 \times\lim_{\nu\to 0}\frac{1}{\nu^2}F(\nu, r, R_{01}, R_{02}).$$

The expressions for $S_l(r)$ and $T_l(r)$ are very similar to those in Eq. (11).

As mentioned above we apply the theory to the scattering of ^{16}O on ^{58}Ni. Since the ^{16}O nucleus is an extremely stiff vibrator it will be hardly excited during the collision, i.e. $\alpha^{(1)}_{lm} \approx 0$. Furthermore it is known from Coulomb excitation[11] that the target nucleus is predominantly excited by the quadrupole part of the interaction. We can therefore simplify the collective potential, Eq. (10), to be

$$V(\vec{r}, \alpha^{(2)}_{2m}) = U(r) + T_2(r) \sum_{m=-2}^{+2} \alpha^{(2)}_{2m} Y^*_{2m}(\Omega) . \qquad (12)$$

Figure 1(a) shows the various contributions to the elastic potential for the system ^{16}O-^{58}Ni. From Eq. (11) it is immediately clear that the symmetry potential $U(r)_{\rm sym}$ vanishes in this case since for the ^{16}O nucleus $Z_1 = \frac{1}{2}A_1$. The entire potential $U(r) = U(r)_{\rm Coul} + U(r)_{\rm Yuk} + U(r)_{\rm comp}$ resembles a molecular one; it is drawn in Fig. 1(b). If the two nuclei are far apart only the Coulomb forces are acting. At smaller distances the short-range Yukawa forces counteract them and cause a maximum in the potential $U(r)$ at about 9 fm. The height of this maximum corresponds to the interaction barrier. For projectile energies above the barrier the nuclei begin to overlap. In the overlap region the density of the nuclear matter is heightened [see Eq. (5)] and hence a repulsive compression potential arises. The resulting increase of the elastic potential is even enhanced by the Yukawa part $U(r)_{\rm Yuk}$ which also becomes repulsive for small values of r.

The Coulomb-nuclear interference effect becomes quite transparent by looking at the corresponding coupling potentials, Figs. 2(a) and 2(b). When the nuclei reach a distance less than 12 fm during the scattering process, the Coulomb excitation—caused by $T_2(r)_{\rm Coul}$—is diminished by the Yukawa part $T_2(r)_{\rm Yuk}$. At $R_{\rm cr} = 10.4$ fm, Coulomb and nuclear excitation totally interfere: $T_2(R_{\rm cr}) = 0$. It is important to realize that the interference takes place even before the nuclei touch ($R_{01} + R_{02} = 8.3$ fm). If $r < 9$ fm, the Yukawa and compression part of the coupling potential are clearly dominant.

At this point we should mention how the various constants contained in the binding energy formula (3) are determined. Some of these are known from nuclear matter calculations[12]: $W_0 = -15.85$ MeV, $\rho_0 = 0.2$ fm^{-3}. The remaining constants V_0, C, and G can be calculated as a function of the range μ of the Yukawa force by the method outlined in Refs. 8 and 13. Assuming $\mu = 1.0$ fm we obtain for the system ^{16}O-^{58}Ni: $V_0 = -259.9$ MeV fm, $C = 25.8$ MeV, $G = 109.6$ MeV.

3. SEMICLASSICAL THEORY OF HEAVY-ION SCATTERING

Because of the large masses of the considered nuclei a semiclassical description of the scattering process seems to be suitable. We first calculate the classical trajectory $\vec{r}(t)$ in the elastic potential $U(r)$. Therefore the classical equations of motion for the Hamiltonian function

$$\mathcal{H} = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + U(r) \qquad (13)$$

are solved numerically by the Runge-Kutta method. Here, the reduced mass is denoted by $m = A_1A_2/(A_1 + A_2)M$. Figure 3 shows that slightly above the Coulomb barrier the orbital path differs consider-

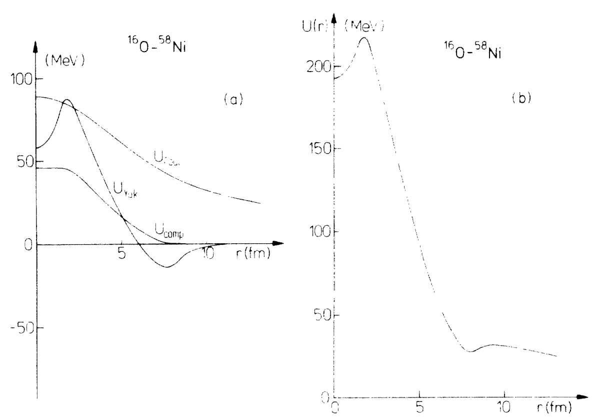

FIG. 1. (a) The elastic potential $U(r)$ for the system ^{16}O-^{58}Ni consists of a Coulomb, Yukawa, and compression part. The symmetry term vanishes in this system. (b) The total potential $U(r)$ has quasimolecular structure.

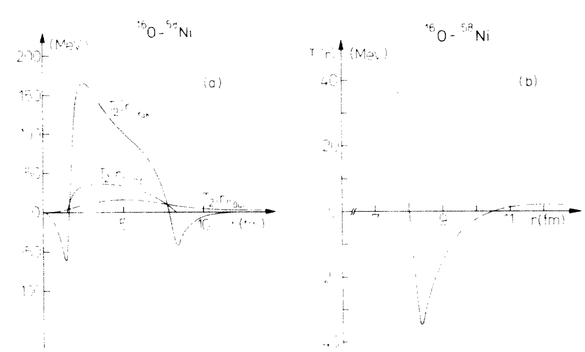

FIG. 2. (a) The various contributions to the quadrupole part $T_2(r)$ of the coupling potential for surface vibrations. (b) The coupling potential $T_2(r)$. The zero in T_2 at $R_{\rm cr} = 10.4$ fm causes the Coulomb-nuclear interference minimum in the inelastic excitation function.

ably from the Rutherford hyperbola. At $\vartheta_{c.m.}=90°$ the potential $U(r)$ gives an unique connection between impact parameters and scattering angle.

The excitation cross section of the target nucleus can then be obtained from the time dependent Schrödinger equation

$$[\hat{H}_0+\hat{H}'(t)]\,|\psi\rangle = i\hbar\frac{\partial}{\partial t}\,|\psi\rangle\,, \tag{14a}$$

where

$$\hat{H}_0=\hat{H}_{\text{vib}} = \frac{\sqrt{5}}{2B_2}[\pi^{[2]}\otimes\pi^{[2]}]^{[0]}+\tfrac{1}{2}\sqrt{5}\,C_2[\alpha^{[2]}\otimes\alpha^{[2]}]^{[0]} \tag{14b}$$

and

$$\hat{H}'(t)=T_2[r(t)]\sum_{\mu=-2}^{+2}\alpha^{(2)}_{2\mu}Y^*_{2\mu}[\Omega(t)]\,. \tag{14c}$$

If we expand the wave function $|\psi\rangle$ in terms of the eigenstates $\varphi_s(\xi,t)$ of the unperturbed Hamiltonian $\hat{H}_0$

$$|\psi(\xi,t)\rangle=\sum_s a_s(t)\,|\varphi_s(\xi,t)\rangle \tag{15}$$

we get the following set of linear coupled differential equations for the excitation amplitudes $a_r(t)$:

$$\dot{a}_r(t)=\frac{1}{i\hbar}\sum_s a_s(t)\,\langle r|\hat{H}'(t)|s\rangle\exp\left[\frac{i}{\hbar}(E_r-E_s)t\right]. \tag{16}$$

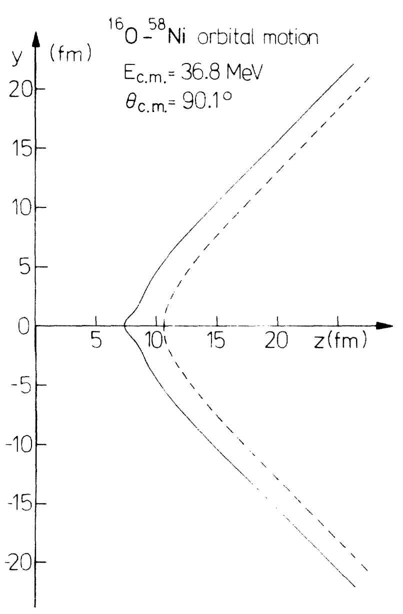

FIG. 3. Classical trajectory in the elastic potential $U(r)$. For an incident energy near the Coulomb barrier the orbital path (solid line) differs considerably from the Rutherford hyperbola (dashed line).

In order to solve Eq. (16) numerically, the Winther–de Boer program[11] has been extended. Since the excitation probability for the state $|r\rangle$ is given by

$$P_r(t)=|a_r(t)|^2\,, \tag{17}$$

we finally obtain for the corresponding cross section

$$\left(\frac{d\sigma_r}{d\Omega}\right)_{\text{c.m.}}=P_r(t\to+\infty)\frac{\rho(\vartheta_{\text{c.m.}})}{\sin\vartheta_{\text{c.m.}}}\left|\frac{d\rho(\vartheta_{\text{c.m.}})}{d\vartheta_{\text{c.m.}}}\right|. \tag{18}$$

Here ρ denotes the impact parameter and $\vartheta_{c.m.}$ the scattering angle in the center-of-mass system.

4. RESULTS

Within the theory developed above we have calculated the excitation functions for the system ^{16}O-^{58}Ni. In Fig. 4(a) the ratio between the elastic and Rutherford cross section is plotted for $\vartheta_{\text{lab}}=75°$; Fig. 4(b) shows the corresponding excitation function for the first 2^+ state (1.453 MeV) in ^{58}Ni. The experimental data are taken from Refs. 1 and 3.

The elastic cross section behaves as usual: If the projectile energies are sufficiently below the Coulomb barrier E_{Coul} ($\approx$46 MeV) one has essentially Rutherford scattering, while for $E>E_{\text{Coul}}$ the cross section drops exponentially because of the strong absorption in the elastic channel. This absorption is taken into account by adding an imaginary potential $iW(r)$ in Eq. (14c), which has been constructed in the following way:

$$W(r)=\begin{cases}0\,, & r>R_{01}+R_{02}\\ -W_{\text{im}}(R_{01}+R_{02}-r)^n\,, & r\le R_{01}+R_{02}\,.\end{cases} \tag{19}$$

The best fit to the scattering data was obtained with the parameter set $W_{\text{im}}=4.5$ MeV, $n=10$.

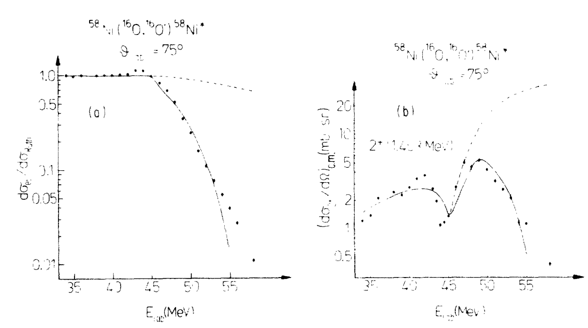

FIG. 4. (a) Cross section for elastic scattering of ^{16}O on ^{58}Ni at 75° lab. The solid curve has been calculated with, the dashed curve without an imaginary potential. Experimental data are taken from Refs. 1 and 3. (b) The excitation function of the 1.453 MeV 2^+ state in ^{58}Ni. Curves and data as in Fig. 4(a).

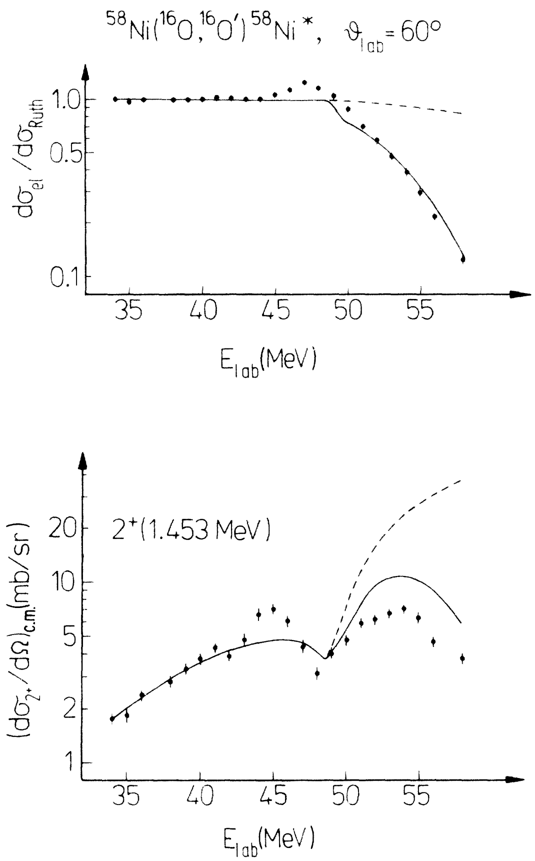

FIG. 5. Same as Fig. 4, but for 60° lab.

The inelastic cross section first rises with increasing energy as expected from Coulomb excitation. Then it drops due to the Coulomb-nuclear interference to the minimum at about 44 MeV. If the beam energy is increased further, $d\sigma_{2^+}/d\Omega$ rises again since now the nuclear forces are prevalent ("Yukawa excitation"). For still higher energies the cross section falls monotonically because of the dominance of absorption processes. At other scattering angles the excitation functions are quite similar (Fig. 5).

Although our semiclassical results show good agreement with experimental data, it seems that a quantum mechanical description is more appropriate, if the projectile energies exceed the interaction barrier.[3] The main reason is that the imaginary potential is only used in the coupled equations (16) for the excitation amplitudes, but neglected in the classical calculation of the orbital motion.

In the present work a model has been developed to calculate the real part of the heavy-ion coupling potentials. Until now the model only could be tested in the region of the Coulomb barrier. It would be very interesting to see whether the transitions predicted by the compression and symmetry part of our coupling potential also agree with experiment or not. A decision about this can be made by extending the heavy-ion inelastic scattering experiments to higher energies.

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