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Recoil corrections in distorted-wave Born approximation calculations of heavy-ion transfer reactions*

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Various distorted-wave Born approximation (DWBA) theories which take only approximate account of recoil are assessed by comparing numerically their predictions to exact DWBA calculations of cross sections for heavy ion trnasfer. Of these the most successful for single nucleon transfer appears to be that of Baltz which treats recoil exactly through second order in the mass of the transferred particle. Also rather accurate are the first order theories of Baltz and Kahana and of Reisdorf. The theory of Buttle and Goldfarb which incorporates their approximate correction for recoil is useful near and below the Coulomb barrier, especially when there is good "Q matching." Theories which completely neglect recoil are distinctly less reliable. None of the approximate theories here considered appear adequate for an example of α particle transfer.

[NUCLEAR REACTIONS ^{88}Sr(^{16}O, ^{15}N), ^{30}Si(^{16}O, ^{15}N), ^{40}Ca(^{16}O, ^{12}C). Calculated $\sigma(\theta)$, comparison of exact and approximate finite range DWBA.]

I. INTRODUCTION

A bewildering variety of calculations employing the distorted wave Born approximation (DWBA) have been made recently for the analysis of transfer reactions induced by heavy ions. Since exact DWBA calculations can be time consuming and costly, it is worthwhile to inquire how reliable are those calculations which introduce additional approximations. The leading difference between the exact and approximate calculations concerns the treatment of so-called "recoil effects"; it already has been established that in some instances the cross sections predicted when the recoil terms are neglected differ substantially from those which result when recoil is treated exactly.[1]

The purpose of this paper is to compare numerically exact DWBA calculations to several other more approximate DWBA calculations in order to discover their limits of accuracy and regions of applicability. Specifically, we will consider that the calculations of DeVries,[2] which treat finite range and recoil effects exactly, define the standard. Following are the other treatments considered here: (1) the finite range but no recoil procedure of Sawaguri and Tobocman,[3] as programmed by Schmittroth, Tobocman, and Golestaneh[4] (referred to hereafter as STSG); (2) the exact treatment of so-called first order recoil effects by Baltz and Kahana[5] (BK), building upon STSG as the exact zeroth order theory; (3) the extension of this to second order by Baltz[6] (B); (4) the method of Buttle and Goldfarb,[7,8] with and without their recoil corrections (referred to as BG); (5) an extension of this method by Reisdorf[9] (R) which includes exactly first order recoil corrections within the BG framework; and (6) an analytic approximation[8] to the Buttle-Goldfarb expressions, relevant only to sub-Coulomb reactions, which will be termed the Ter-Martirosyan-Sommerfeld (TMS) method. In addition some comments will be added about calculations, presumably equivalent to those of DeVries, recently reported by Tamura and Low.[10] In Sec. II we briefly summarize the key features of the various procedures listed above.

Theories which also take some account of recoil have been developed by Nagarajan[11] and by Braun-Munzinger and Harney.[12] These are more approximate than those of BK, B, and R in that they use a "local momentum" approximation when introducing recoil corrections into the distorted waves; on the other hand, the procedure of Ref. 12 incorporates recoil corrections to arbitrarily high order in the mass of the transferred particle. We have made no comparisons to the predictions of these theories.

As test cases we have considered the following reactions: The proton transfer reactions ^{88}Sr-(^{16}O, ^{15}N)^{89}Y(g.s.) and ^{88}Sr(^{16}O, ^{15}N)^{89}Y(0.91 MeV) are meant to typify single nucleon transfer to a moderately heavy target in situations where the values of Q are favorable. The transfer to the ground state ($\frac{1}{2}^-$) involves only small angular momentum quantum numbers while those for transfer to the first excited state at 0.91 MeV ($\frac{9}{2}^+$) are fairly large. Further, the ground state is a $j_<$ state for which the total angular momentum of the transferred proton around the ^{88}Sr target ($\frac{1}{2}$) is a half unit less than the orbital angular momentum l, while the 0.91 MeV first excited state is a $j_>$ state with corresponding angular momenta $\frac{9}{2}$ and 4, respectively. The proton transfer reactions ^{30}Si-(^{16}O, ^{15}N)^{31}P(g.s.) and ^{30}Si(^{16}O, ^{15}N)^{31}P(1.26 MeV) are intended to typify reactions on lighter targets in situations where the values of Q are less favorable. Here the ground state is a $j_>$ state ($s_{1/2}$) while the excited state is a $j_<$ state ($d_{3/2}$). As a final example we consider transfer of an α particle in the reaction ^{40}Ca(^{16}O, ^{12}C)^{44}Ti(g.s.); because the mass transferred is large, we here expect the recoil effects to be very important.

For the first two sets of reactions, cross sections will be calculated for a range of bombarding energies extending from below the Coulomb barrier to well above that barrier. Experimental excitation functions[13] and angular distributions[14,15] are available in this range but the objective of this paper will not be a critique of the confrontation between calculations and these data.

II. SUMMARY OF DWBA THEORIES

A. "Exact" DWBA

A DWBA theory which includes finite range and recoil effects exactly has been formulated by Austern *et al.*[16] The program LOLA uses this formulation and is discussed in detail in Ref. 2. The DWBA amplitude in the post representation for a transfer reaction $A(a, b)B$ where $B = A + x$ and $a = b + x$ is given by

$$T = \int d\vec{r}_b \int d\vec{r}_a \chi_b^{*(-)}(\vec{r}_b) F(\vec{r}_b, \vec{r}_a) \chi_a^{(+)}(\vec{r}_a), \tag{1}$$

where the form factor $F(\vec{r}_b, \vec{r}_a)$ is defined by

$$F(\vec{r}_a, \vec{r}_b) = \langle \psi_B(\vec{r}_{Ax}) | V_a(\vec{r}_{bx}) | \psi_a(\vec{r}_{bx}) \rangle . \tag{2}$$

Here χ represents a distorted wave and ψ a bound state wave function. The spatial vectors involved are shown in Fig. 1. In this expression the only approximations made beyond the DWBA itself are that ψ_a and ψ_B are taken to be single particle wave functions in a spherically symmetric potential and that the full perturbing potential ΔV has been replaced by $V_a(\vec{r}_{bx})$, the shell model nuclear potential of x relative to the core b.

In order to simplify the six-dimensional integral demanded in the above equation, approximations have been developed as discussed below.

B. Method of Sawaguri, Tobocman, Schmittroth, and Golestaneh

In this method the form factor is evaluated exactly as a function of $\vec{r}_a$ by means of the Sawaguri-Tobocman modified harmonic oscillator expansion[3] but a no-recoil approximation is made for the distorted wave function $\chi_b^{(-)*}(\vec{r}_b)$: considered as a function of $\vec{r}_a$ and $\vec{r}_{bx}$, the coordinate $\vec{r}_b$ is identically

$$\vec{r}_b = \beta \vec{r}_a + \gamma \vec{r}_{bx}, \tag{3}$$

where $\beta \equiv (m_A/m_B)$ and $\gamma \equiv m_x(m_B + m_b)/(m_B m_a)$; in the no-recoil approximation the argument of the distorted wave $\vec{r}_b$ is replaced by $\beta \vec{r}_a$. The computer code RDRC is used to evaluate[4] the cross sections.

C. First order recoil correction of Baltz and Kahana

One may improve upon the above no-recoil approximation by expanding $\chi_b^{(-)*}(\vec{r}_b)$ about $\beta \vec{r}_a$ in powers of γ:

$$\chi_b^{(-)*}(\vec{r}_b) = \chi_b^{(-)*}(\beta \vec{r}_a) + \frac{\gamma}{\beta} \vec{r}_{bx} \cdot \vec{\nabla}_{r_a} \chi_a^{(-)*}(\beta \vec{r}_a) + \cdots . \tag{4}$$

The method of Baltz and Kahana[5] consists of retaining only the zeroth and first order terms above. In cases where second and higher order terms become small, this method then approaches the exact result.

One may perform a check on the convergence of the first order result by writing

$$\chi_b^{(-)*}(\vec{r}_b) \simeq \chi_b^{(-)*}(\alpha \vec{r}_a) + \left[\frac{(\beta - \alpha)}{\alpha} \vec{r}_a + \frac{\gamma}{\alpha} \vec{r}_{bx} \right] \times \vec{\nabla}_{r_a} \chi_b^{(-)*}(\alpha \vec{r}_a) \tag{5}$$

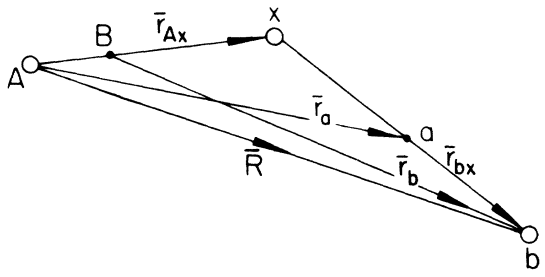

FIG. 1. Vector diagram for coordinates which enter the DWBA integral for the transfer reaction $A(a, b)B$ where $a \equiv b + x$ and $B = A + x$.

and allowing α to take some value other than β [such as the Buttle-Goldfarb prescription for longitudinal recoil as used in this paper, Eq. (9)]. If the cross section is relatively independent of the expansion parameter α, then the indication is that the calculation is relatively well converged. In this paper, however, tabulated cross sections are for the conventional choice $\alpha=\beta$. The computer code generated to perform these calculations is an extension of RDRC.

D. Second order recoil correction of Baltz

In a straightforward manner the method of BK may be extended to treat recoil through second order in γ exactly.[6] One keeps the next term in the Taylor expansion of $\chi_b^{(-)*}(\vec{r}_b)$, namely

$$\frac{1}{2}\left(\frac{\gamma}{\beta}\right)^2(\vec{r}_{bx}\cdot\vec{\nabla}_{r_a})^2\chi_a^{(-)*}(\beta\vec{r}_a)\cdots, \tag{6}$$

and adds this to the zeroth and first order terms of Eq. (4). Again convergence may be checked by expanding about some $\alpha\vec{r}_a$ where $\alpha\neq\beta$.

E. Method of Buttle and Goldfarb

This method rests on two approximations: (a) The radial part of the bound state wave function ψ_B is represented by a spherical Hankel function of the first kind. When the transferred particle is a neutron this replacement is exact in the asymptotic region; when the transferred particle is a proton, as it is for the present test cases, this replacement can still be rather accurate.[13,17] (b) Concerning recoil, either of two recipes can be followed. The exact expressions for $\vec{r}_a$ and $\vec{r}_b$ in terms of $\vec{R}$ and $\vec{r}_{bx}$ are

$$\vec{r}_a=\vec{R}-\frac{m_x}{m_a}\vec{r}_{bx},\quad \vec{r}_b=\frac{m_A}{m_B}\vec{R}+\frac{m_x}{m_B}\vec{r}_{bx}. \tag{7}$$

For the no-recoil approximation in the post representation one sets[7]

$$\vec{r}_a\simeq\vec{R},\quad \vec{r}_b\simeq\frac{m_A}{m_B}\vec{R}. \tag{8}$$

On the other hand, some account can be taken of recoil through the prescription[8]

$$\vec{r}_a\simeq\left(1-\frac{m_x}{m_a}\frac{R_a}{d}\right)\vec{R},\quad \vec{r}_b\simeq\left(\frac{m_A}{m_B}+\frac{m_x}{m_B}\frac{R_a}{d}\right)\vec{R}\cdots. \tag{9}$$

Here R_a is taken to be radius of the potential V_a. For incident energies below the Coulomb barrier, d is set equal to R_{cl}, the average of the classical distances of closest approach in the entrance and exit channels for head-on Coulomb collisions. For energies above the Coulomb barrier, d is taken to be R_{cl} at the Coulomb barrier.

Approximation (a) permits use of a powerful addition theorem for the wave function $\psi_B(\vec{r}_{Ax})$. Combining this theorem with either the no-recoil approximation or the approximate recoil correction, one can reduce the original six-dimensional integral to products of two three-dimensional integrals, one of which, denoted as $T_{l\lambda}$, has the familiar form of the conventional zero-range approximation.

To carry through this procedure a code entitled TRANSFER[13] optimizes the fit between the radial part of ψ_B and a spherical Hankel function in a region around a "match center" radius defined as $[A_1^{1/3}/(A_1^{1/3}+A_2^{1/3})]d$ and also calculates an overlap integral, denoted as A_{l_1} by Buttle and Goldfarb. The integral $T_{l\lambda}$ is evaluated through a modified version of the computer code DWUCK.[18]

F. First order recoil correction of Reisdorf

Here one retains the spherical Hankel function approximation of BG but gives an exact first order treatment of recoil. Specifically one adopts the approximations[9]

$$\chi_a^{(+)}(\vec{r}_a)\simeq\left(1-\frac{m_x}{m_a}\vec{r}_{bx}\cdot\vec{\nabla}_R\right)\chi_a^{(+)}(\vec{R}) \tag{10a}$$

and

$$\chi^{(-)*}(\vec{r}_b)\simeq\left(1+\frac{m_x}{m_B}\vec{r}_{bx}\cdot\nabla_{R'}\right)\chi_a^{(-)*}(\vec{R}'), \tag{10b}$$

where $\vec{R}'\equiv(m_A/m_B)\vec{R}$. This approximation for recoil is nearly, but not quite, equivalent to that of BK, since one is expanding both the incident and final distorted waves about $\vec{R}$ and $\vec{R}'$, respectively.

G. Ter-Martirosyan-Sommerfeld method

For incident energies below the Coulomb barrier an approximate analytic expression for the Buttle-Goldfarb cross section has been derived by those authors[8] using relations given by Ter-Martirosyan[19] and Sommerfeld.[20] This method rests on three further assumptions: (1) The radial part of the form factor entering the integral $T_{l\lambda}$, which is a spherical Hankel function of order l, may be replaced by the product of a constant times a spherical Hankel function of order zero. (2) The angular-dependent term in the form factor $Y_{l\lambda}^*(\hat{R})$ may be replaced by $Y_{l\lambda}^*(\hat{R}_{rec})$, where $\hat{R}_{rec}$ is the recoil direction along which the contribution to the integral is expected to be a maximum. (3) The Coulomb parameters in the incident and final channels, η_i and η_f, are much larger than unity. The resulting cross section is given in Eqs. (2.11), (2.13), (2.15), and (2.16) of Ref. 8. For the no-recoil approximation, the wave numbers involved in the post form of the cross section are k_i and $k_f'=(m_A/m_B)k_f$; for the approximate recoil correction of Buttle and Goldfarb, k_i is replaced by $[1-(m_x/m_a)(R_a/d)]k_i$ and $k_f'=[m_A/m_B+(m_x/m_B)(R_a/d)]k_f$.

Given a plot of the bound state radial wave functions, one may evaluate the above cross section, if need be, using graph paper and a desk calculator. In the present instance, though, the code TRANSFER[13] carries out the necessary matching of radial functions and evaluates the cross section.

III. PARAMETERS AND DETAILS OF THE CALCULATIONS

Potentials of the Woods-Saxon form are used to generate both bound states and distorted waves. For the ^{88}S(^{16}O, ^{15}N)^{89}Y reactions, we have used the parameters chosen by Anantaraman, Katori, and Körner[14] although, for energies near the Coulomb barrier, these do not give the best fit to elastic scattering cross sections. The proton separation energy in ^{16}O is 12.130 MeV; that of ^{89}Y is 7.073 MeV. The central potential for ^{16}O has radius equal to 2.9595 fm and diffuseness 0.6 fm; that for ^{89}Y has radius 5.338 fm and diffuseness 0.65 fm. The charge radii are taken equal to those of the central potentials. Spin orbit potentials of the standard form were adopted with radii and diffusenesses equal to those of the central potentials and depth equal to 7 MeV. For the distorted waves we use $V=-100$ MeV, $W=-25$ MeV, $a=0.50$ fm, $R_{\rm inc}=8.431$ fm, and $R_{\rm fin}=8.456$ fm.

The bound state parameters for the reaction ^{30}Si-(^{16}O, ^{15}N)^{31}P are for ^{16}O: $R=3.0828$ and $a=0.65$ fm, and for ^{31}P: $R=3.8840$ and $a=0.65$ fm. The proton separation energy in ^{31}P is 7.29 MeV. The spin orbit potentials were set equal to zero. Here the distorted waves have the parameters $V=-100$ MeV, $W=-40$ MeV, $R_{\rm inc}=6.8651$ fm, $a_{\rm inc}=0.49$ fm, $R_{\rm fin}=6.8413$, and $a_{\rm fin}=0.60$ fm.

For the reaction ^{40}Ca(^{16}O, ^{12}C)^{44}Ti, the parameters chosen are those used previously in an exact DWBA calculation[21] which gave an adequate fit to the observed angular distribution at a bombarding energy of 42 MeV. The separation energy of the $3s$ α-particle orbital in ^{16}O is 7.160 MeV, while the separation energy of the $7s$ α-particle orbital in ^{44}Ti is 5.235 MeV. The radius parameter r_0 for the central potentials is taken to be 1.25 fm, while for the Coulomb potential it is 1.0 fm. [Here $R\equiv r_0(4^{1/3}+A_{\rm core}{}^{1/3})$ so that, for example, the radius of the potential in ^{16}O is $1.25(4^{1/3}+12^{1/3})=4.846$ fm.] The diffuseness parameter is 0.65 fm. The parameters for the optical potentials are $V=-100$ MeV, $W=-40$ MeV, $r_0=1.22$ fm, $a_0=0.8$ fm, and $a_I=0.7$ fm.

In all calculations, masses were set equal to mass numbers in units of amu. The spectroscopic factor for the proton in ^{16}O was taken to be 2.0 while those for the states in ^{89}Y and in ^{31}P are chosen to be 1.0. For the α-particle transfer reaction, both spectroscopic factors are set equal to 1.0.

For the Sr and Si calculations the step size of numerical integrations for the code LOLA was chosen to be 0.10 fm; reducing this to 0.075 fm led to changes of less than a percent in the peak cross sections. The BG calculations used a step size of 0.05 fm; doubling this produced changes of about 1% in the cross sections. The integrations in the BK calculations were carried out with step sizes such that on doubling the step sizes the peak cross sections changed by less than 0.2%.

In calculations which approximate the bound state wave function with a spherical Hankel function (BG, R, and TMS) there is inevitably some uncertainty accruing to the precise manner in which the match is made. In the present calculations of proton transfer these uncertainties do not appear to exceed 10% except at the higher energies for the reaction ^{30}Si(^{16}O, ^{15}N)^{31}P (1.26 MeV), a case of poor Q match, where the uncertainty appears to be about 20%. Further, cross sections calculated for bombarding energies above the Coulomb barrier with

TABLE I. Peak cross sections for the reaction ^{88}Sr(^{16}O, ^{15}N)^{89}Y(g.s.). For all entries but LOLA, the table shows the ratio of the calculated cross section to that of LOLA. By convention, at energies below the Coulomb barrier we have given the cross sections at 170° rather than 180°, the true peak location.

$E_{\rm lab}$ (MeV)	θ Peak (deg)	LOLA $\sigma(\theta)$ (mb)	B	BK	STSG No Rec.	R	BG With Rec.	BG No Rec.	TMS With Rec.	TMS No Rec.
42.5	170	0.0493	1.010	1.010	0.682	1.01	0.99	0.85	1.06	0.86
44	170	0.1017	1.009	1.013	0.70		1.01	0.87	1.05	0.87
46	170	0.222	1.014	1.018	0.72	1.00	1.01	0.92	1.14	0.97
48	135	0.311	1.019	1.026	0.72		1.04	0.92		
50	115	0.395	1.018	1.027	0.71		1.05	0.92		
52	100	0.475	1.021	1.036	0.71	0.98	1.06	0.91		
56	80	0.621	1.021	1.043	0.69		1.07	0.94		
59	70	0.717	1.017	1.049	0.68	0.99	1.09	0.88		

the BG method become increasingly inaccurate if d, which enters the definition of the "match center" radius, is taken to be the Coulomb distance of closest approach corresponding to these bombarding energies. We believe the procedure stated in Sec. II E and followed in this work, that d be equated to R_{cl} at the energy corresponding to the Coulomb barrier, is much more reliable since we find that the over-all quality of the wave function fit rapidly deteriorates as d is decreased below this value.

IV. RESULTS AND DISCUSSION

A. ^{88}Sr(^{16}O, ^{15}N)^{89}Y

Angular distributions have been calculated for this reaction from energies well below the Coulomb barrier to energies well above it. (In the entrance channel, the barrier formed by the Coulomb potential and the real part of the optical potential is classically surmounted at an energy of 47.5 MeV in the laboratory frame.) The predicted shapes of the different theoretical angular distributions are in very good agreement with each other and with the experimental data[14]; therefore, we have not shown them here. The magnitudes of the different theoretical predictions at the peak angle are given in Tables I and II relative to the exact recoil (LOLA) predictions. At low energies, where the angular distributions peak at 180°, we actually quote the cross sections at 170°, the largest angle where data have been obtained. Only the "normal" l-transfer value cross sections are shown; the cross sections for "nonnormal" l transfer are very small except for transfer to the $p_{1/2}$ state at the highest energies. The TMS results are shown only at low energies where there is some hope that a sub-Coulomb theory is relevant.

It is clear that, of all the approximate calculations, those that use the no-recoil approximation are the least satisfactory. On the other hand, all the calculations which take some account of recoil are in fairly good accord with the exact DWBA calculations; only in one instance do the deviations of the BG cross sections from LOLA exceed 10% and, for the BK and R calculations, they never exceed 7%. The Baltz second order recoil calculations are practically equivalent to exact finite-range calculations for this case; here the maximum deviation from LOLA is 2.1%. TMS approximation with the inclusion of recoil is useful at the two lowest energies but by 46 MeV it is starting to break away from the exact calculations; not surprisingly, the TMS approximation appears more accurate for the ground state transition, where the angular momentum transfer is only 1, than for the excited state transition, where it is 5.

The energy dependence of the ratio of approximate to exact peak cross sections is worth special comment. For the ground state transition, the STSG and BG no-recoil calculations show only a small energy variation while, for the transition to the $g_{9/2}$ excited state, the ratio increases by over 35% between 42.5 and 59 MeV. (Parenthetically we note that there is a sizable discrepancy between the magnitude of the STSG and BG no-recoil cross sections for the ground state transition; we must remember that the recoil approximations of these two theories are not the same even though both bear the same appellation, "no recoil." Also STSG specifies a unique "exact no-recoil" calculation while all methods based on BG use further the Hankel function approximation, which involves nonunique matching prescriptions.) In previous papers[13,14] the hope was expressed that some observed energy variations of spectroscopic factors for the ground state transition, deduced by fitting experiment to the no-recoil calculations, could be explained by exact DWBA calculations; but since the energy dependence of the exact and the no-recoil ground state cross sections are so similar, it appears that other explanations must be sought for any apparent energy dependence of the spectroscopic factors.

TABLE II. Peak cross sections for the reaction ^{88}Sr(^{16}O, ^{15}N)^{89}Y(0.91 MeV). The format is similar to that of Table I.

E_{lab} (MeV)	θ Peak (deg)	LOLA $\sigma(\theta)$ (mb)	B	BK	STSG No Rec.	R	BG With Rec.	BG No Rec.	TMS With Rec.	TMS No Rec.
42.5	170	0.0713	0.993	0.97	0.64	1.07	1.06	0.81	1.15	0.86
44	170	0.1724	0.998	0.98	0.66		1.06	0.83	1.14	0.87
46	170	0.477	1.008	0.99	0.69	1.03	1.06	0.86	1.18	0.92
48	155	0.811	1.011	1.00	0.71		1.08	0.91		
50	120	1.061	1.011	1.02	0.76		1.08	0.98		
52	105	1.292	1.012	1.03	0.80	0.98	1.08	1.02		
56	85	1.631	1.021	1.06	0.85		1.14	1.12		
59	75	1.854	1.013	1.07	0.88	0.93	1.08	1.09		

For the ground state transition, the energy dependence of the ratio is slight for the two calculations which include recoil exactly to first order; for BG with recoil it increases by 10% in the energy span between 44 and 59 MeV. For the excited state transition, there are differences: for BK there is an increase of 10%, for BG with recoil the increase is not as great, while for R the ratio decreases by 13%. The ratio is practically energy-independent when second order recoil is included; there is an increase of 1% for the ground state and 2% for the excited state.

Calculations which make no approximation for recoil have been performed by Tamura and Low[10] and applied to the data of Anantaraman *et al.*[14] We were disturbed to find that their average spectroscopic factors differed substantially from those made using the exact DWBA cross sections listed in Tables I and II. The authors of Ref. 10 have recently informed us, however, that their parameters differ from those given in Ref. 14; consequently the comparison between the two calculations is not meaningful.

B. ^{30}Si(^{16}O, ^{15}N)^{31}P

This reaction differs from the preceding one in that the target is much lighter and, more important, the Q-matching conditions are not as well satisfied. The barrier in the entrance channel, formed by the Coulomb and the real part of the nuclear potentials, is classically surmounted at an energy of 16.69 MeV in the center of mass system, corresponding to an incident energy of 25.59 MeV in the laboratory, while in the exit channel the barrier height equals 14.75 MeV. The difference in barrier heights, −1.94 MeV, is nearly 3 MeV larger than the Q value for the ground state transition, −4.84 MeV, and consequently the reaction is inhibited. This is in contrast to the preceding ^{88}Sr(^{16}O, ^{15}N)^{89}Y(g.s.) reaction for which the entrance barrier height equals 40.19 MeV and the exit barrier equals 35.80 MeV. There, the Q value, −5.06 MeV, nearly matches the difference in barrier heights, −4.39 MeV, and thus that reaction is favored.

Angular distributions for the reaction on ^{30}Si have been computed at four energies, the lowest of which (25 MeV) lies just below the classical barrier height. For the transition to the $s_{1/2}$ ground state only the normal $l=1$ angular momentum transfer is permitted. For the transition to the excited $d_{3/2}$ state at 1.26 MeV, the cross sections for non-normal $l=2$ transfer at the higher bombarding energies are not negligible compared to those for normal $l=1$ transfer. Consequently, we consider separately the cross sections for the normal and nonnormal l transfer leading to the $d_{3/2}$ state. Again the predicted shapes of the different theoretical angular distributions are essentially indistinguishable for the normal l transfers and are in good agreement with experimental data.[15] At the two higher energies the normal cross sections develop a forward peak as well as the usual peak corresponding to a grazing collision. The peak cross sections for the nonnormal l transfer and the cross sections for the normal l transfer at the "grazing collision peak" are listed in Table III.

It will be seen that the calculations which neglect recoil give cross sections which are far too small at almost all energies. The TMS calculations are in poor agreement with the BG calculations they are meant to approximate. The BG calculations which correct for recoil are tolerable for the ground state transition near the barrier but rapidly

TABLE III. Peak cross sections for the reaction ^{30}Si(^{16}O, ^{15}N)^{31}P. The format is similar to that of Table I.

E_{lab} (MeV)	θ Peak (deg)	LOLA $\sigma(\theta)$ (mb)	B	BK	STSG No Rec.	R	BG With Rec.	BG No Rec.	TMS With Rec.	
25	170	0.0082	1.01	0.94	0.50	0.97	1.07	0.61	1.37	
30	105	0.1345	0.99	0.90	0.54	0.92	1.14	0.69		g.s.
42	45	1.256	1.00	1.01	0.67	0.99	1.48	0.85		$s_{1/2}$
60	25	3.125	1.03	1.09	0.74	1.07	1.83	0.95		$L=1$
25	170	$3.80(10^{-5})$	0.97	0.84	0.32	0.84	0.78	0.41	1.06	
30	115	0.004 00	0.94	0.83	0.31	0.81	0.67	0.38		$d_{3/2}$
42	45	0.2216	1.02	0.88	0.28	0.89	0.64	0.35		$L=1$
60	25	1.447	1.05	0.88	0.22	0.85	0.59	0.29		Normal
25	120	$2.60(10^{-7})$	1.01	0.63	···	0.85	···	···	···	$d_{3/2}$
30	90	$1.002(10^{-4})$	0.85	0.57	···	0.75	···	···		$L=2$
42	45	0.018 73	1.03	0.69	···	0.84	···	···		Non-normal
60	20	0.2746	1.15	0.71	···	0.83	···	···		

get worse as the energy is increased. Of course, neither the BG nor STSG method is capable of calculating the nonnormal component which, according to the exact recoil calculations, has a peak cross section at 60 MeV that is about $\frac{1}{5}$ the peak cross section of the normal component.

In contrast to this, the BK and R calculations for the normal component are not too distant from the corresponding exact calculations at all four energies. The BK calculations of the cross sections for nonnormal l transfer, however, appear to be not as reliable while the R calculations for nonnormal and normal l transfer are of comparable quality. The B second order calculations of nonnormal l transfer agree rather well with LOLA except at 30 and 60 MeV. More important, second order normal l-transfer cross sections are quite well converged at all energies.

C. ^{40}Ca(^{16}O, ^{12}C)^{44}Ti

For this example of a multinucleon transfer, none of the approximate methods yield cross sections whose magnitudes at an incident energy of 42 MeV agree with those of the exact calculation. The BG method fails because a fit of the highly oscillatory 7s orbital in ^{44}Ti to a spherical Hankel function cannot be made at those values of r_{Ax} which contribute most to the reaction amplitude. A satisfactory match can be made at rather large values of r_{Ax}; the resulting angular distributions then approximate those of the exact calculation but the cross sections are a factor of 20 larger in magnitude.

When the cross sections are computed with the BK or B methods, they are still much larger than those of the exact calculation, as is shown in Fig. 2. Although the problem of fitting a bound state wave function to a spherical Hankel function does not arise, the BK method too is unsatisfactory for this case of large mass transfer since it discards terms beyond first order in the mass of the transferred particle. But even when the second order terms are included, method B, the large discrepancy yet remains.

Calculations have also been performed with less extended α-particle orbitals; specifically, the radii of the central potentials of ^{16}O and ^{44}Ti were changed to $1.25(12^{1/3}) = 2.862$ fm and $1.25(40^{1/3}) = 4.275$ fm, respectively. The deviations between the approximate and exact calculations are then diminished but the agreement is still quite unsatisfactory. The second order calculations of B and the exact calculations again yield very similar angular distributions but at 40°, where both calculations give a grazing peak, the second order calculations are $2\frac{1}{2}$ times larger than the exact calculations; at smaller angles this factor is somewhat reduced. Similarly, the BG calculations are now only twice as large at the grazing peak as are the exact calculations but this grazing peak is displaced, occurring at an angle slightly larger than 50°, and further, the forward angle structure is much more damped than is the case for the exact calculations.

V. CONCLUSIONS

Since DWBA calculations which exactly include recoil are frequently expensive, a number of approximate recoil methods have been developed. We have compared several approximate DWBA codes with exact recoil calculations to determine the regions of applicability of the approximate methods.

For single nucleon transfer reactions we find that the calculated angular distributions are all nearly identical, but sizable variations are often found in the predicted magnitudes, particularly as a function of incident energy. We conclude:
(a) DWBA calculations which completely neglect recoil are distinctly less reliable than those that

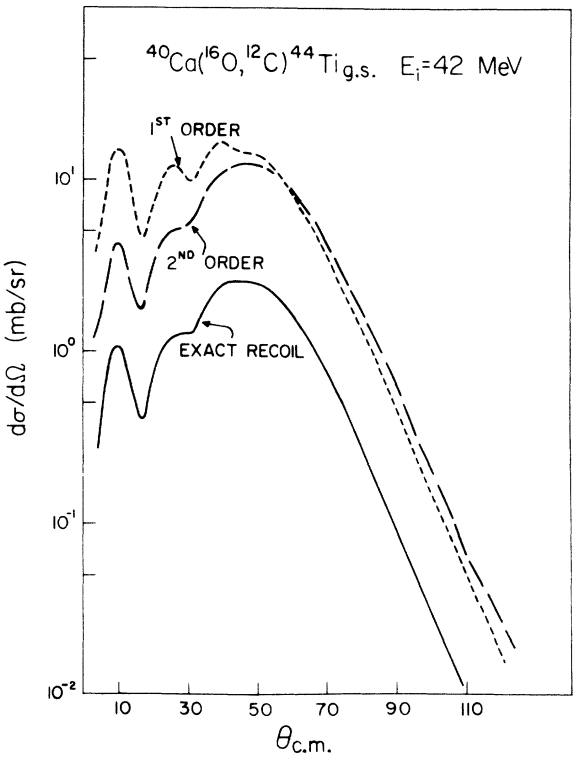

FIG. 2. Calculated differential cross sections for the reaction ^{40}Ca(^{16}O, ^{12}C)^{44}Ti$_{g.s.}$ at an incident energy of 42 MeV as given by the exact calculation (LOLA) and the exact first order and second order theories of BK and B, respectively.

do take some account of recoil. When the Q-matching conditions are well satisfied, some of the "no-recoil" cross sections are close to the exact values but others show discrepancies well over 10%. When the Q-matching conditions are poorly satisfied, the cross sections tend to be very much less than the exact results.

(b) The semiclassical TMS method with the BG type recoil correction is only safely applied when the incident energy is well below the Coulomb barrier and when there are good Q-matching conditions.

(c) The BG method incorporating their approximate correction for recoil is useful below and near the Coulomb barrier, particularly for reactions with good Q-matching conditions.

(d) The methods of BK and R, which treat recoil exactly to first order in the mass of the transferred particle, may be applied over a broad range of single nucleon transfer reactions provided that the ratio of nonnormal to normal components is small. The near equivalence of the BK and R calculations argues that even for energies well above the barrier the spherical Hankel function approximation of BG is fairly accurate when care is taken in matching that function to the radial part of the bound state wave function ψ_B.

(e) For cases of reasonably well Q-matched single nucleon transfer which we have examined the second order method of B deviates from the exact DWBA calculations by only a percent or two. The deviations for both the normal and nonnormal components are somewhat larger in a situation with poorer Q matching but these deviations are distinctly less than is the case for the first order method of BK.

We do add the caveat that for the examples here considered the bombarding energies are near the Coulomb barrier or exceed it by only a few tens of MeV. We have seen indications that the reliability of the approximate calculations worsens for higher bombarding energies.

For an α-particle transfer reaction, none of these approximate methods reliably predicted the magnitude of the cross section.

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