# Spectroscopy of <sup>29</sup>P via the <sup>28</sup>Si(p, $p'\gamma$ ) reaction\*

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Excitation functions have been measured for inelastic proton scattering from the 1.78 MeV,  $2^+$  first excited state of <sup>28</sup>Si between 3.1 and 5.7 MeV. Two previously unidentified resonances corresponding to states in <sup>29</sup>P with excitation energies of 6.577 and 7.272 MeV have been observed. Triple angular correlations in the Goldfarb-Seyler geometry have been measured as a function of energy over all the resonances observed in the inelastic excitation function. Information obtained from the angular correlation data has been combined with previous information concerning these states to allow  $J^{\pi}$  assignments for 9 of the 11 states observed.

 $\begin{bmatrix} \text{NUCLEAR REACTIONS} & {}^{28}\text{Si}(p, p_1), \text{ measured } \sigma(E), & E = 3.1-5.7 \text{ MeV}. & {}^{28}\text{Si}(p, p_1\gamma), \\ \text{measured } p - \gamma \text{ correlations, deduced } J^{\pi} \text{ of } {}^{29}\text{P levels.} \end{bmatrix}$ 

#### I. INTRODUCTION

There has recently been renewed interest in the theoretical study of the properties of the excited states of nuclei in the mass range of A = 27-29. The wealth of experimental information available on the properties of excited states in this region have served as a challenge to the various theoretical models used to describe these nuclei. Model analysis have proceeded from the simplest structures of a spherical <sup>28</sup>Si inert core plus a single nucleon to the most recent detailed calculations by deVoigt, Glaudemans, de Boer, and Wildenthal<sup>1</sup> and by Wildenthal and McGrory<sup>2</sup> using many particle shell model wave functions in a truncated configuration space.

Experimental results for the nucleus <sup>29</sup>P appear to show that some properties of the excited states are associated with collective motion of the nucleus, while other states appear to be single-particlelike in behavior. This led Ejiri<sup>3</sup> to attempt to analyze the structure of <sup>29</sup>P in terms of a unified model. In this model, he pictured the nucleus as a system of collective vibrational and rotational modes coupled to the independent motion of a few loosely bound nucleons and made his calculations both in the strong-coupling and the weak-coupling models. Both models give reasonable good agreement with the properties of some of the measured states. The nucleus <sup>29</sup>Si, the mirror nucleus of <sup>29</sup>P, has been studied in a modified intermediatecoupling model by Castel, Stewart, and Harvey<sup>4</sup> in which they treat individual particles as coupled to harmonic and anharmonic vibrations of the core nucleus <sup>28</sup>Si.

As the properties of the low-lying bound states of these nuclei become better understood the theoretical challenge comes in understanding the structure of the resonant states lying close to the proton separation energy. Most of the properties of the low-lying bound states in <sup>29</sup>P, such as spins, parities, decay modes, and lifetimes have been determined.<sup>5</sup> Properties of the unbound states in <sup>29</sup>P between  $E_x = 5.7$  and 7.7 MeV have previously been determined primarily by shape fitting to the elastic proton excitation curves and some spin ambiguities exist. We have recently begun a systematic study of the properties of resonant states in <sup>29</sup>P in this energy region which have appreciable yield in the inelastic proton decay channel leading to the first excited 2<sup>+</sup> state of <sup>28</sup>Si. This paper reports on measurements of the spins of these resonant states using the particle- $\gamma$  ray correlation geometry of Goldfarb and Seyler.<sup>6</sup>

#### **II. EXPERIMENTAL CONSIDERATION**

The Ohio State University's model CN Van de Graaff accelerator provided the particle beams used during the course of these experiments. The target material was natural SiO<sub>2</sub>. Although naturally occurring SiO<sub>2</sub> has an isotopic abundances of 92.2% <sup>28</sup>Si, the energy of the first excited state inelastic proton group for <sup>28</sup>Si is well separated from the proton groups from other target isotopes. The SiO<sub>2</sub> was evaporated onto a thin carbon backing for support using electron gun techniques. The thickness of the <sup>28</sup>Si was found to be 80  $\mu$ g/cm<sup>2</sup> by Rutherford scattering of protons at  $E_p = 1.8$  MeV.

The protons were detected using a silicon surface barrier detector with a solid angle of 11.3 msr. The  $(p, p' \gamma)$  angular correlations were taken using apparatus described previously.<sup>7</sup> Two  $10.2 \times 12.7$ cm NaI detectors were used to detect the  $\gamma$  rays in coincidence with the inelastic protons. Conventional fast-slow coincidence electronics, allowing



FIG. 1. The <sup>28</sup>Si(p, p')<sup>28</sup>Si\*(2<sup>+</sup>, 1.78 MeV) excitation function at  $\theta_p = 90^{\circ}$  c.m. All proton energies are in laboratory coordinates. The solid curve is not a theoretical fit but serves merely to guide the eye.

pile-up rejection<sup>8</sup> for both  $\gamma$  detectors and the proton detector, were used to gate the linear  $\gamma$  signals. An on-line IBM 1800 computer was used to collect a real-plus-accidental and accidental spectrum for each  $\gamma$  detector. Channel-by-channel subtraction of the two spectra then allowed corrections for the accidental events. The data were then corrected for proton and  $\gamma$ -ray pileup, with a typical correction of 10%.

#### III. RESULTS

# A. Inelastic scattering

The excitation function for inelastic proton scattering was measured at a proton angle of  $90^{\circ}$  c.m. over a bombarding energy of 3.1 to 5.7 MeV. Energy steps of 20 keV were taken between the resonances and 2.5–10 keV steps were taken over the resonances. Figure 1 shows the resulting excitation function.

The features of the measured inelastic excitation function agree with those seen by Vorona, Olness, Haeberli, and Lewis<sup>9</sup> (henceforth referred to as VOHL), and by Belote, Kashy, and Risser<sup>10</sup> (henceforth referred to as BKR), and by Brenner, Hoogenboom, and Kashy<sup>11</sup> (henceforth referred to as BHK), with the exception of two narrow resonances at  $E_p$  = 3.975 and = 4.690 MeV. The results of the analysis of these resonances are described in Sec. III C.

# B. Angular correlations

The Goldfarb-Seyler geometry enables one to make model-independent determinations of the

spins of resonant states. If the axis of quantization,  $\hat{z}$ , is taken along the momentum direction of the outgoing particle, the angular correlation function between these particles and the corresponding  $\gamma$  rays in a plane perpendicular to  $\hat{z}$  is given by

$$W(\theta_{\gamma} = \frac{1}{2}\pi, \phi_{\gamma}) = \sum_{\substack{\kappa = 0\\ \kappa = \text{ even}}}^{\kappa \max} A_{\kappa} \cos(\kappa \phi_{\gamma}), \qquad (1)$$

where

$$\kappa_{\max} \leq \min[2j_c, 2L_{\max}, 2(l_1)_{\max}, (2j_b - 1)_{\max}]$$

and  $j_c$  is the spin of the  $\gamma$ -emitting state,  $l_1$  is the orbital angular momentum of the incident particle,  $L_{\max}$  is the multipolarity of the emitted  $\gamma$  ray and  $j_b$  is the spin of the resonant state.

Since <sup>28</sup>Si is an even-even nucleus, the ground state has  $J^{\pi} = 0^+$  and the first excited state has  $J^{\pi} = 2^+$ . Therefore,  $j_c = 2$ ,  $L_{max} = 2$ , and  $l_1$ , which is now the orbital angular momentum of the resonant state, is a unique number. The relationship between the correlation complexity ( $\kappa_{max}$ ) and the spin of the resonant states is shown in Table I.

TABLE I. Correspondence between correlation complexity and the spin of the resonant state.

κ <sub>max</sub>	<sup>ј</sup> ь	
0	$\frac{1}{2}$	
2	$\frac{3}{2}$	
4	$\geq \frac{5}{2}$	

Goldfarb and Seyler warn that this formalism is strictly true only for an isolated resonance. With an isolated resonance, one needs only to measure one correlation at the resonance energy to deduce the spin of the states in question. In the presence of a background, one must be careful to obtain the behavior of the correlation coefficients off resonance as a function of energy to avoid erroneous assignments. Therefore triple angular correlations have been measured in the Goldfarb-Seyler goemetry as a function of proton bombarding energy in the range of  $E_p = 3.05 - 5.3$  MeV. The correlations were taken in 2.5 to 10.0 keV steps (depending upon the resonance width) over the resonances in the inelastic excitation function. The correlation data were then fitted by means of a leastsquares program to determine the coefficient  $A_{\kappa}$ . The resonating coefficient with the largest value of  $\kappa$ , used in conjunction with Table I, places a lower limit on the spin of the resonant state.

The reason for assigning a lower limit rather than a unique value to the spin of the resonant state involves the possibility of accidental cancellation of a coefficient with a larger  $\kappa$  value. Accidental cancellations can occur when the variation of a coefficient over a resonance is identically zero or smaller than the experimental uncertainty in



FIG. 2. Energy dependence of the angular correlation coefficients for  $E_p = 3.10 - 4.80$  MeV. Solid curve in the plot of  $A_0$  versus energy is the normalized inelastic proton excitation function.

that coefficient. Therefore, every assignment made has the inherent possibility of cancellation effects. In order to reduce the number of erroneous assignments due to accidental cancellations, the work of other<sup>9-11</sup> has been relied upon to complement as well as supplement our spin determination. Collectively, all of these data permit the logical assignments of spin and parity for these resonant states. The angular correlation coefficients as a function of energy are plotted Figs. 2 and 3 and are discussed below for each resonance. The results of the spin assignments are presented in Table II.

#### C. Analysis of the spins of the resonances

## 1. 3.10 MeV resonance

The angular coefficients determined over the 3.10 MeV resonance are shown in Fig. 2. Both the  $A_2$  and the  $A_4$  coefficient resonate indicating a spin of  $\frac{5}{2}$  or greater. This is consistant with the results of both VOHL and BKR, whose elastic scattering data give assignments of l=3,  $J=\frac{5}{2},\frac{7}{2}$ . Ejiri *et al*.<sup>12</sup> report a value of  $l_p=3$  for the <sup>28</sup>Si-(<sup>3</sup>He, d)<sup>29</sup>P reaction to this state.



FIG. 3. Energy dependence of the angular correlation coefficients for  $E_p$ =4.65-5.30 MeV. Solid curve in the plot  $A_0$  versus energy is the normalized inelastic proton excitation function.

Е <sub>р</sub> (MeV)	$E_x^{a}$ (±0.005) (MeV)	<sup>28</sup> Si l	(p,p <sub>0</sub> ) J	$\frac{28}{J}$ Si $(p, p'\gamma)$ J (current work)	Assignment $J^{\pi}$
3.10	5.737	3	$(\frac{5}{2})^{b,c}$	$\geq \frac{5}{2}$	$(\frac{5}{2}^{-}, \frac{7}{2}^{-})$
3.33	5.959	2	<u>3</u> b,c	32	$\frac{3}{2}$ +
3.57	6.191	1	3 b, c	<u>3</u> 2	$\frac{3}{2}$ -
3.71	6.326	1	$\left(\frac{1}{2}\right)^{c}$	$\frac{3}{2}$	$\frac{3}{2}$ +
		2	$\left(\frac{3}{2}\right)^{b}$		-
3.97	6.577	•••	•••		
4.23	6.828	2	$\frac{3}{2}c$	$\geq \frac{5}{2}$	$\frac{5}{2}$ +
4.43	7.021	1	$\frac{1}{2}^{c}$	3	$\frac{3}{2}$
4.69	7.272	•••	•••	• • •	
4.88	7.456	3	$\frac{5}{2}c, d$	$\geq \frac{5}{2}$	<u>5</u> -
4.95	7.523	2	$(\frac{3}{2}, \frac{5}{2})^d$	$\frac{1}{2}$	$(\frac{1}{2})$
5.19	7.755	2	$(\frac{3}{2}, \frac{5}{2})^d$	<u>3</u>	$(\frac{3}{2}^+)$

TABLE II. Summary of spins and parities for resonances in <sup>29</sup>P.

<sup>a</sup> Based upon  $Q(p, \gamma) = 2.744$  MeV.

<sup>b</sup> See Ref. 9.

<sup>c</sup> See Ref. 10. <sup>d</sup> See Ref. 11.

#### 2. 3.33 MeV resonance

By shape fitting analysis of the proton elastic scattering excitation curves, both VOHL and BKR make an assignment of l=2,  $J=\frac{3}{2}$  or  $\frac{5}{2}$  for the 3.33 MeV resonance. Because of the similarity of the  $\frac{3}{2}^+$  and  $\frac{5}{2}^+$  line shapes at all angles, a *J*-value uncertainty exists for l=2 assignments. Consequently, BKR measured the angular distribution of inelastically scattered protons leaving the residual nucleus <sup>28</sup>Si in its first excited  $2^+$  state. These measurements were made only on the peak of the resonance and were fitted with  $J^{\pi} = \frac{3}{2}^+$  for the 3.33 MeV resonance. The angular distribution was not symmetric about  $\theta_p = 90^\circ$  and indicated interference with neighboring levels of opposite parity.

The results of the angular correlation measurements in Fig. 2 show a strong resonance in both the  $A_0$  and  $A_2$  coefficients with no observable  $A_4$ coefficient above background. These results are consistent with a  $J = \frac{3}{2}$  spin assignment. Although the data are sparse, there appears to be a slight assymmetry in the shape of the  $A_2$  coefficient which could indicate that there is interference with a state or states of the same parity as well as with states of opposite parity as was noted by BKR.

#### 3. 3.57 and 3.71 MeV doublet

It is evident from the elastic scattering data of both VOHL and BKR that for these two resonances, the elastic partial width is only a small fraction of the total widths; the major strengths for these resonances being in the inelastic channel. Neither VOHL nor BKR were able to determine the spins and parities of these levels from the elastic data alone. However, both groups assigned spins and parities to the two levels by studying the inelastic scattering angular distribution in conjunction with the elastic scattering data. Although both groups concurred on a *p*-wave  $\frac{3}{2}$  assignment for the 3.57 MeV resonance, VOHL concluded a d-wave  $\frac{3}{2}^+$  assignment for the 3.71 MeV resonance, whereas BKR concluded a *p*-wave  $\frac{1}{2}$  assignment for this resonance. Additional information on the assignment for these two levels was reported later by Soroka and Pucherov<sup>13</sup> who measured the polarization of protons elastically scattered by  $^{\rm 28}{\rm Si}$  over the bombarding energy range of 2.7 to 3.9 MeV. Although they were unable to observe either of these two resonances in the polarization scattering, Soroka and Pucherov calculated a polarization from the resonance parameters reported by VOHL and BKR for the doublet. They concluded that their energy-averaged experimental data were in best agreement with their calculations of the polarization using the values of BKR and a p-wave  $\frac{1}{2}$  value for the 3.71 MeV level.

The results of our angular correlation measurements over these two resonances are shown in Fig. 2. Both  $A_0$  and  $A_2$  coefficients resonate across the doublet, whereas the  $A_4$  coefficient is zero within statistics. From the absence of an  $A_4$  coefficient across the resonances, we conclude that both states have spins of  $\frac{3}{2}$ . It is interesting to note that the  $A_2$  coefficient has a positive excursion over the 3.57 MeV resonance, whereas it has a negative excursion over the 3.71 MeV resonance.

It would be inviting to conclude from the nature of the resonating  $A_2$  coefficient that we have resolved the spin contradiction between VOHL and BKR for the 3.71 MeV level. Unfortunately, that is not necessarily true. At first glance it appears that the spins of both states are  $\frac{3}{2}$ . Indeed the  $A_2$ coefficient has the same general energy dependence as do the two resonances; i.e., the peaks of the  $A_2$  coefficient occur at the peaks of the resonances and the total widths of the resonances, as measured for the  $A_2$  coefficient, are  $85 \pm 10$  keV and  $50 \pm 10$  keV, respectively, which agree with the quoted widths by both VOHL and BKR. In addition the two peaks in the  $A_2$  coefficient do not appear to be due to interference, since the  $A_2$  coefficient goes to zero approximately half-way between the peaks. Since only states of the same parity will show interference in the even correlation coefficients, using the *l*-value assignments of VOHL we would conclude that the 3.57 MeV resonance is a *p*-wave  $\frac{3}{2}$  state and the 3.71 MeV resonance is a *d*-wave  $\frac{3}{2}^+$  state. On the other hand, since the

3.57 MeV resonance and the 3.71 MeV resonance overlap one another, one could possibly conclude that the energy dependence of the  $A_2$  coefficient represents an interference between states of the same parity. Although a  $\frac{1}{2}$  isolated resonance would have no  $A_2$  coefficient in the Goldfarb-Seyler goemetry, it is possible that the interference of a  $\frac{1}{2}$  level with another negative parity level of higher spin (say, the  $\frac{3}{2}$  - 3.57 MeV level) could induce an  $A_2$  coefficient in the vicinity of the  $\frac{1}{2}$  level. One might conclude from this assumption that the 3.57 MeV state and the 3.71 MeV state have the same parity. Hence using the *l*-wave assignments of BKR, this analysis would be consistent with the 3.57 MeV resonance being a *p*-wave  $\frac{3}{2}$  state and the 3.71 MeV resonance being a p-wave  $\frac{1}{2}$  state.

In order to examine further the possibility that the negative  $A_2$  coefficient associated with the 3.71 MeV resonance might be due to interference between the two overlapping resonances, we have calculated the terms associated with the interference between two Breit-Wigner type resonances. The energy-dependent terms for the real part of the product of the two elements of the *R* matrix associated with interference between the two reso-

nances can be expressed as

$$\frac{\Gamma_{1}\Gamma_{2}\Gamma_{1}'\Gamma_{2}'\left\{\left[\cos\phi(E-E_{0}')-\sin\phi(\frac{1}{2}\Gamma)\right](E-E_{0})+\cos\phi\left[\frac{1}{4}(\Gamma\Gamma')\right]+(\frac{1}{2}\Gamma)\sin\phi(E-E_{0}')\right\}}{\left[(E-E_{0})^{2}+(\frac{1}{4}\Gamma^{2})\right]\left[(E-E_{0}')^{2}+(\frac{1}{4}\Gamma')^{2}\right]},$$
(2)

where  $\Gamma_1 \Gamma'_1$  are the partial widths for formation of the primed and unprimed resonances,  $\Gamma_2 \Gamma_2'$  are the partial widths for the decay of the primed and unprimed resonances assuming only a single partial wave in the inelastic channel,  $E_0$  and  $E'_0$  are the respective resonance energies, the phase angle  $\phi$  is the over-all phase for the channels assumed, and  $\Gamma$  and  $\Gamma'$  are the total widths. The numerator of Eq. (2) is broken up into two terms: The first term is associated with that part of the interference which is asymmetric about the resonance energy  $E_0$ ; the second term is associated with that part of the interference which is symmetric about the resonance energy  $E_0$ . At the resonance energy  $E = E_0$ , the asymmetric term is zero and the only contribution to the interference part of the cross section is symmetric about  $E_0$  and is

$$\frac{1}{4}(\Gamma\Gamma')\cos\phi + (\frac{1}{2}\Gamma)\sin\phi(E_0 - E'_0). \tag{3}$$

Rearranging the terms into a part symmetric about  $E'_0$  and antisymmetric about  $E'_0$  [in the same form as (2)] we note that at the resonance energy  $E'_0$ , the asymmetric term about  $E'_0$  is again zero and the only contribution to the interference part of the cross section is symmetric about  $E'_0$ . This term is identical to Eq. (3). We conclude from

this that the part of the cross section due solely to interference between two resonances has the *same* sign at the peaks of both resonances.

For the Goldfarb-Seyler geometry used in this experiment, Eq. (1) contains only even terms in the expansion coefficient  $\kappa$ . For interference to occur in these even coefficients, the resonances must have been the same parity, as is suggested by the spin and parity assignments of BKR, whereas the spin and parity assignments of VOHL would indicate no interference in the  $A_2$  coefficients. If we now assume that the negative  $A_2$  coefficient at the 3.71 MeV resonances is due solely to interference from the 3.57 MeV resonances, then some part of the positive  $A_2$  coefficient at the 3.57 MeV resonance must be due to interference from the 3.72 MeV resonance and this part must also be a negative contribution. This would then imply that the self-resonant contribution of the 3.57 MeV resonance to its  $A_2$  coefficient would have to be a larger positive value than the measured  $A_2$  coefficient.

Recent calculations by Seyler<sup>14</sup> have shown that in the Goldfarb-Seyler geometry the ratio of  $A_2/A_0$ is sensitive to the partial wave through which the resonant state decays. The predictions for this

TABLE III. Ratio of the correlation coefficients, calculated assuming that only *one* indicated partial wave is involved in the decay of the spin  $\frac{3}{2}$  resonance,  $0^+ \rightarrow \frac{3}{2}$  $\rightarrow l_j \rightarrow 2^+$ .

l <sub>j</sub>	l <sub>j</sub>		
$p_{1/2}$	$s_{1/2}$	-0.33	
$P_{3/2}$	$d_{3/2}$	0.00	
$f_{5/2}$	$d_{5/2}$	+0.63	
f 7/2	87/2	-0.89	

ratio, with the assumption that only *one* partial wave is involved in the decay are shown in Table III.<sup>15</sup> Now we assume that the spin and parity of the 3.57 MeV resonance have been established as  $\frac{3}{2}$ <sup>-</sup>. Hence the self-resonant contribution of the 3.57 MeV state (assuming only a single decay mode) to its  $A_2$  coefficient are these of Table III.

The measured value of the ratio  $A_2/A_0$  for the 3.57 MeV resonance is  $+0.20 \pm 0.5$ , whereas the measured ratio of  $A_2/A_0$  for the 3.71 MeV resonance is  $-0.35 \pm 0.05$ . Assuming the spin and parity assignments of BKR, we find that the selfresonant contribution to the  $A_2/A_0$  ratio of the 3.57 MeV state would have to be as large as +0.55. For this to be true it appears that one would have to invoke almost a pure  $f_{5/2}$  decay (see Table III) to account for such a large positive  $A_2/A_0$  value and it is unlikely that there can be such a strong f-wave contribution at this energy. Of course, the measured  $A_2/A_0$  value would not rule out some  $p_{3/2}$  decay also, since the contribution of the  $p_{3/2}$ channel to the  $A_2$  coefficient is so small. In addition, the measured  $A_2/A_0$  ratio for the 3.71 MeV member of the doublet  $(-0.35 \pm 0.05)$  is consistent with a dominant  $s_{1/2}$  partial wave in the decay channel (-0.33). In conclusion, it appears that the 3.71 MeV resonance is a *d*-wave  $\frac{3}{2}^+$  state in agreement with the assignment of VOHL and not  $\frac{1}{2}$  state as suggested by BKR and by Soroka and Pucherov.

#### 4. 3.97 MeV resonance

The 3.97 MeV resonance ( $E_x = 6.577$  MeV) was not observed by VOHL or by BKR in either the elastic or inelastic excitation curves due probably to the very narrow width of the resonance. A very broad ( $\Gamma = 200 \text{ keV}$ ) spin  $-\frac{1}{2}$  resonance was observed in the elastic channel by BKR at a bombarding energy of 3.98 MeV, but presumably this is not the narrow resonance we observe at 3.97 MeV in the inelastic channel. A narrow resonance is reported at an excitation energy in <sup>29</sup>P of 6.54 MeV in the compilation of Endt and Van der Leun,<sup>5</sup> but this assignment was based upon a paper presented at a scientific meeting and has not been further reported in the literature. We have recently received a communication from Bair<sup>16</sup> who points out that the 3.97 MeV resonance ( $\Gamma \leq 3 \text{ keV}$ ) was observed in the <sup>28</sup>Si( $p, p'\gamma$ ) reaction at The Oak Ridge National Laboratory but was not reported in the literature,<sup>17</sup> since they were uncertain as to whether or not this resonance was due to a contaminant.

In order to verify that this resonance was in the compound nucleus <sup>29</sup>P and not due to a target contaminant, we measured the excitation curves for the  $\gamma$  rays emitted from the first excited 2<sup>+</sup> state of <sup>28</sup>Si at 1.78 MeV. The  $\gamma$  rays were detected with a 38 cm<sup>3</sup> Ge(Li) detector. The excitation curve for the  $\gamma$  ray are shown in Fig. 4 along with the inelastic proton excitation curve over the same energy range. The measured energy width in both channels is due primarily to the target thickness (~10 keV), the natural width being much smaller.

Figure 2 shows the angular correlation coefficients near the 3.97 MeV resonance. We are unable to draw any conclusions as to the spin of the state from the behavior of the correlation coefficients.

## 5. 4.23 MeV resonance

From the results of the elastic scattering shapes at the 4.23 MeV resonance, BKR report a *d*-wave  $\frac{3}{2}^+$  assignment to this resonance. The results of our angular correlation measurement are shown in Fig. 2. The  $A_0$ ,  $A_2$ , and  $A_4$  coefficients all show a resonance behavior over this energy range. On the basis of the resonating of the correlation coefficients and using the *l*-wave assignment of BKR our results are consistent only with a *d*-wave  $\frac{5}{2}^+$ spin assignment.

## 6. 4.43 MeV resonance

The 4.43 MeV resonance is a broad (~100 keV), strong, resonance in the inelastic proton channel



FIG. 4. Inelastic proton and  $\gamma$ -ray excitation functions for <sup>28</sup>Si(p,  $p'\gamma$ ) near  $E_p$ =3.97 MeV: (a) inelastic proton excitation function at  $\theta_p$ =90° c.m.; (b)  $\gamma$ -ray excitation function at  $\theta_{\gamma}$ =35° c.m.

but, as reported by BKR, it appears only weakly in the elastic channel. In the elastic channel, the 4.43 MeV level is near a broad (120 keV) *s*-wave resonance; the *s*-wave resonance appears to have little or no strength in the inelastic channel. BKR concluded from the shape fitting in the elastic channel, that the cross section is consistent only with a *p*-wave  $J = \frac{1}{2}^-$  or  $\frac{3}{2}^-$  value. They note that even with  $\Gamma_p/\Gamma$  as small as 0.15, the  $\frac{3}{2}^-$  assignment did not give agreement with the elastic scattering data. Their assignment for the 4.43 MeV resonance is a *p*-wave  $\frac{1}{2}^-$  state.

The results of the angular correlation measurements are shown in Fig. 2. Both the  $A_0$  and  $A_2$  coefficients show structure over the extent of the resonance, whereas the  $A_4$  coefficient is zero within statistic, indicating a spin of  $\frac{3}{2}$ .

Since the 4.43 MeV resonance is only very weakly excited in the elastic channel, it appears that an *l*-value assignment based upon the elastic scattering data is premature. Hence we conclude only that the state has a spin of  $\frac{3}{2}$  and could correspond either to a *p*-wave or a *d*-wave resonance.

## 7. 4.69 MeV resonance

The narrow resonance in the inelastic proton channel at a bombarding energy of 4.69 MeV  $(E_x = 7.272 \text{ MeV})$  has not previously been reported in the literature either in the elastic or the inelastic channel.<sup>16</sup> As with the other new narrow resonance we observed at 3.97 MeV, we measured the excitation curve for the  $\gamma$  ray emitted from the first excited 2<sup>+</sup> state of <sup>28</sup>Si. The excitation curves for the  $\gamma$ -ray yield is shown in Fig. 5 along with the inelastic proton yields over the same energy range. The measured energy width in both channels is due primarily to the target thickness (~10 keV), the natural width being much smaller.



FIG. 5. Inelastic proton and  $\gamma$ -ray excitation functions for <sup>28</sup>Si(p,  $p'\gamma$ ) near  $E_p$ =4.70 MeV: (a) inelastic proton excitation function at  $\theta_p$ =90° c.m.; (b)  $\gamma$ -ray excitation function at  $\theta_{\gamma}$ =35° c.m.

There is no evidence of the existence of either an  $A_2$  or an  $A_4$  coefficient above background over the resonance which would normally indicate a  $\frac{1}{2}$  spin assignment for the state. However, the state is sufficiently narrow compared to our target thickness that any resonating coefficients could be obscured by the background contributions. Hence we do not make a spin assignment for this state.

#### 8. 4.88 MeV resonance

The resonance in the elastic channel at 4.88 MeV is reported by both BKR and BHK as being an f-wave  $\frac{5}{2}$  state. Angular correlations were measured over this resonance solely as a check since we can only distinguish spins of *d*-wave, or less, resonances. The energy dependence of the correlation coefficients are shown in Fig. 3. All three coefficients show a definite resonance effect suggesting an assignment of  $\frac{5}{2}$  or greater. This is consistent with the assignments made from the elastic scattering data.

### 9. 4.95 MeV resonance

The 4.95 MeV resonance is only weakly excited in both the elastic and inelastic channels. BHK conclude that an assignment of a *d*-wave  $\frac{5}{2}^+$  resonance is most consistent with their elastic scatter-



FIG. 6. Energy dependence of the angular correlation coefficients near  $E_p$ =4.95 MeV. Solid curve in the plot of  $A_0$  versus energy is the normalized inelastic proton excitation function.

ing data although it is not clear that other assignments could not equally well fit the same data.

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The angular correlation measurements over this energy region are shown in Fig. 3 and an exploded energy scale depicted in Fig. 6. Only the  $A_0$  coefficient shows a resonance behavior; the coefficients  $A_2$  and  $A_4$  are constant over the extent of the resonance. Our results are therefore consistent only with a spin of  $\frac{1}{2}$  assignment.

### 10. 5.19 MeV resonance

The elastic scattering data of BHK suggests a d-wave,  $J = \frac{3}{2}$  or  $\frac{5}{2}$ , assignment to the 5.19 MeV resonance with the authors expressing a preference for the  $J = \frac{5}{2}$  assignment. The angular correlation coefficients as shown in Fig. 3 indicate  $J = \frac{3}{2}$ , since the  $A_4$  coefficient is constant and the  $A_0$  and  $A_2$  coefficients resonate. Hence we assign this resonance the spin and parity of  $\frac{3}{2}^+$ .

As previously noted, the possibility of accidental cancellations of coefficients always exists. Therefore, spin and parity assignments have been made using the angular correlation data in conjunction with elastic scattering data and angular distribution of inelastic protons and/or  $\gamma$  rays. Only in the case of four resonances  $(E_p = 3.33, 3.71, 4.95, and$ 5.19 MeV), did the angular correlation data indicate a spin which was the lower  $(j = l - \frac{1}{2})$  of the two spins determined from the elastic scattering data. For two of these resonances  $(E_{p} = 3.33 \text{ and } 3.71$ MeV), the angular distribution of inelastic protons and  $\gamma$  rays indicated a spin assignment consistent with the angular correlation data. Therefore, the spin assignments of only two resonances,  $E_{b} = 4.95$ and 5.19 MeV, contain an uncertainty and these assignments are made with this uncertainty so noted.

#### IV. DISCUSSION

One of the most important conclusions that we have drawn from our studies in the properties of resonant states in nuclei is that one must be careful in extracting spectroscopic information from any one experiment of a single type (particle angular distributions, elastic scattering shape fitting, particle- $\gamma$ -ray angular correlations, etc.) and that one must instead do a more complete correlation experiment or large number of different types of experiments in order to extract the parentage of the states of interest. The information obtained from experiments of these different types in general complement our knowledge of the resonance structures rather than merely supporting previous knowledge. As an example of how the results of VOHL and BKR on elastic scattering shape fitting

and our experiment on particle- $\gamma$ -ray correlation experiments can complement each other, we note that shape fitting in the elastic channel is most sensitive to the l value of the reaction through a particular resonance, whereas the Goldfarb-Seyler correlation experiments are most sensitive to the J value of the resonance. Hence the results of both analysis permit the assignment of *both* the spin *and* parity of the resonance, whereas each experiment alone may be ambiguous.

A second area where the results of both experiments complement one another is in the determination of decay-channel information. As an example, both VOHL and BKR make the reasonable assumption that the inelastic decay channels are dominated by the lowest l' value in the outgoing channel (in fact assumed to be the only l' value), since the penetrability factors increase by an order of magnitude with increasing l'. Consequently, their analysis of  $\frac{3}{2}^+$  resonant states are done on the assumption of only l' = 0 contributions to the decay. If this assumption is true (that only the l' = 0 and not say l' = 2 outgoing partial wave is present in the decay of the  $\frac{3^+}{2}$  resonant state to the  $2^+$  first excited state of <sup>28</sup>Si), then according to Table III the ratio of the  $A_2/A_0$  coefficient in the Goldfarb-Seyler geometry would be primarily -0.33 unless interference with other positive parity states dominated. We measure the value  $A_2/A_0 = -0.35 \pm 0.07$  for all four  $\frac{3}{2}$ <sup>+</sup> resonances which we studied and this value is consistent with a dominant l' = 0 outgoing partial wave in agreement with the original assumption. However, if we consider the 3.57 MeV  $\frac{3}{2}$  - state, we find that this assumption is not true. The lowest  $l^{\,\prime}$  value for the decay of the 3.57 MeV resonance is l' = 1,  $p_{1/2}$  or  $p_{3/2}$  wave. From Table III we see that a  $p_{1/2}$  wave would lead to an  $A_2/A_0$  value of -0.33 and a  $p_{3/2}$  wave would lead to no contribution. The first outgoing partial wave which could contribute to the measured positive  $A_2/A_0$  value of +0.20 is an  $f_{5/2}$  wave. The presence of the f wave does not rule out some contribution to the decay from *p* waves but shows that the assumption of only the lowest l' value contributing to the decay is not necessarily true. The presence of both p and fpartial waves in the outgoing channel points up the additional complication presented in the analysis of the properties of the resonances associated with interference between the various outgoing partial waves.

We have found that the results of these two sets of experiments, elastic scattering and Goldfarb-Seyler angular correlations do not of themselves contain enough experimental information to enable us to make unique parentage assignments to the outgoing decay channels. Consequently, we have begun a comprehensive set of other types of experiments, inelastic proton angular distributions and spin-flip angular correlations, over the same energy region as the resonances of interest in this paper in order to permit us to determine quantitatively all the resonance parameters.

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