Regions of the two-nucleon force

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Using an approximate Bethe-Salpeter equation approximate values were obtained for the partial wave contributions of the two and three pion exchange ladder diagrams. With these results the region hypothesis of the two-nucleon force was tested for this model. No definite region structure was observed for the S and part of the P and D waves ($^3P_0$, $^3P_1$, $^3P_2$, $^3D_1$, $^3D_3$). A definite region structure was observed for other waves, but it was impossible to locate common (to all partial waves) radii for the outer and intermediate regions of this model of the two-nucleon interaction.

I. INTRODUCTION

Present day ideas of the nucleon-nucleon interaction are based on the relationship between range and the number and kind of mesons exchanged. The more and the heavier mesons are exchanged between the two nucleons, the shorter range the resulting force is. According to Taketani *et al.*[1] the two-nucleon interaction can be divided into three regions. The outer region ($r > 2$ fm) is dominated by the one-pion exchange (OPE). In the intermediate region (1 fm $< r <$ 2 fm), in addition to one-pion exchange effects, the two-pion exchange (TPE) is also important. In the inner region ($r < 1$ fm) many-pion and heavy meson exchanges are important. This region classification was the basis of many theoretical calculations.[2] At present, the exploration of the two-pion exchange potentials seems to be a realistic and achievable goal.[3] Since the work of Taketani *et al.*[1] accurate phase shift analyses were performed[4] which lead to stricter requirements on the multipion regions. According to Breit and Haracz[5] the outer region of OPE dominance is for $r > 2.9$ fm. In a recent work[6] an estimate on the multipion regions is given according to which the outer region of OPE is for $r > 3.2$ fm and the intermediate region of TPE is for 1.6 fm $< r <$ 3.2 fm, with the exception of the attractive P-state phase shifts.

The aim of this work is to test the above ideas on a model where the relative strength of the multipion exchanges can be evaluated. We have chosen the relativistic approximate Bethe-Salpeter equation for the two-nucleon system as presented by Thompson, Gersten, and Green.[7] The successive approximations of this equation are obtained from which the approximate values of the ladder diagrams of the TPE and the three-pion exchange (3PE) are obtained. On the basis of this work estimates can be obtained on the contributions of the ladder diagram of the 3PE which (with the exception of the $^1S_0$ and $^3P_0$ states calculated by Gammel, Menzel, and Wortman) were not calculated till now.

II. APPROXIMATE BETHE-SALPETER EQUATION

We start our considerations from the approximate Bethe-Salpeter equation presented in Ref. 7. The partial wave equations in the momentum representation for the K matrix are given in the following form:

$$\langle p_f l_f | K^{JS} | p_i l_i \rangle = \langle p_f l_f | W^{JS} | p_i l_i \rangle + \sum_l \int_0^\infty \langle p_f l_f | W^{JS} | k l \rangle g(k, p_i) \langle k l | K^{JS} | p_i l_i \rangle k^2 dk, \tag{1}$$

where p_i is the c.m.s. momentum of the incoming nucleon, p_f the c.m.s. momentum of the outgoing nucleon, and k an intermediate momentum (p_f and k are off the energy shell, while p_i is on the energy shell), l_i l_f, and l are their respective angular momenta, W is the pseudopotential, g the Green's function, J the total angular momentum, and S the total spin. The Green's function is equal to

$$g(k, q) = \frac{4M^2\pi^3}{E^2(k)[E(k) - E(q) - i\epsilon]} = \frac{4M^2\pi^3[E(k) + E(q)]}{E^2(k)[k^2 - q^2 - i\epsilon]}, \tag{2}$$

where M is the nucleon mass and $E(q)=(M^2+q^2)^{1/2}$. In an operatorial notation Eq. (1) has the form

$$K=W+WgK. \tag{3}$$

The successive approximations are built according to

$$\begin{aligned} &K_0=W,\\ &K_n=WgK_{n-1},\\ &K=K_0+K_1+\cdots, \end{aligned} \tag{4}$$

where for W the one-pion exchange potential[7] is taken.

The integrands of the integrals of Eq. (1) or Eq. (4) have poles at $k=p_i$. In order to perform the numerical integration, the integral

$$\alpha\int_0^\infty \frac{dk}{k^2-p_i^2}=0$$

is subtracted from the integrals of Eq. (1) or Eq. (4). The number α is chosen in such a way that the singularity is removed. The second of Eqs. (4) is now

$$\langle p_f l_f K_n^{JS}|p_i l_i\rangle=\sum_l\int_0^\infty\bigg[\langle p_f l_f|W^{JS}|kl\rangle g(k,p_i)\langle kl|K_{n-1}^{JS}|p_i l_i\rangle k^2 - \langle p_f l_f|W^{JS}|p_i l\rangle\frac{8M^2\pi^3}{E(p_i)(k^2-p_i^2)}\langle p_i l|K_{n-1}^{JS}|p_i l_i\rangle p_i^2\bigg]dk \tag{5}$$

and the singularity at $k=p_i$ is removed. Now the numerical integration of the integrals of Eq. (5) is straightforward. The infinite range of integration is transformed to a finite one by changing the variable of integration to

$$x=\frac{k-k_0}{k+k_0}, \tag{6}$$

where k_0 is a suitably chosen momentum.

For the new variable x the range of integration is from -1 to 1 and the Gaussian integration method can be employed.

III. THE RESULTS

The results presented in this work were obtained using the following values: $g^2/4\pi=15.0$, $k_0=1000$ MeV, average pion mass $\mu=138.7$ MeV, average nucleon mass $M=938.5$ MeV, 12 point Gaussian integration, total angular momentum $J\leq 4$, and laboratory kinetic energy up to 500 MeV. In Table I we compare our results with the numerical results of Ref. 8, where the Bethe-Salpeter equation of the ladder diagrams is treated without approximate reductions. As one can see, the results obtained by using the approximate Bethe-Salpeter equation[7] are similar to these obtained in Ref. 8; the differences are smaller than 25%. Therefore this work gives a good first estimate for the ladder diagram contributions of the TPE and the 3PE. The results are presented in Fig. 1, where the successive approximations of Eqs. (4) and (5) K_0, K_1, K_2 for $g^2/4\pi=15$ are depicted by the three lines. From the results one may have not only estimates of the contributions of the ladder diagrams of the two- and three-pion exchanges but also a test of the regions of the two-nucleon interaction.

Our evaluations will be based on the impact parameter considerations as in Refs. 5 and 6 and other works.[2] Following Ref. 5 we will write the condition for a predominant contribution of up to n-pion exchanges for a partial wave of angular momentum L and for a laboratory kinetic energy

$$T<(83\text{ MeV fm}^2)n^2L(L+1)/R^2, \tag{7}$$

where R is the radius of the OPE region in fm. Thus one can compare the results presented in Fig. 1 with Eq. (7). Below we summarize our findings:

S waves: No region structure is observed, in accordance with Eq. (7).

P waves:

$^3P_0$, $^3P_1$: No region structure observed.

$^3P_2$: No OPE region observed, but for T below about 50 MeV ($R\approx 3.6$ fm) there is a predominant contribution of OPE + TPE.

TABLE I. The expansion coefficients k_n of the K-matrix elements at 100 MeV, $tg\delta=\sum k_n(g^2/4\pi)^n$.

	$^1S_0$ coefficients		$^3P_0$ coefficients	
n	Ref. 8	This work	Ref. 8	This work
1	−0.0426	−0.0425	0.0426	0.0425
2	0.0113	0.0088	0.00242	0.00279
3	−0.00223	−0.00194	0.00032	0.00038

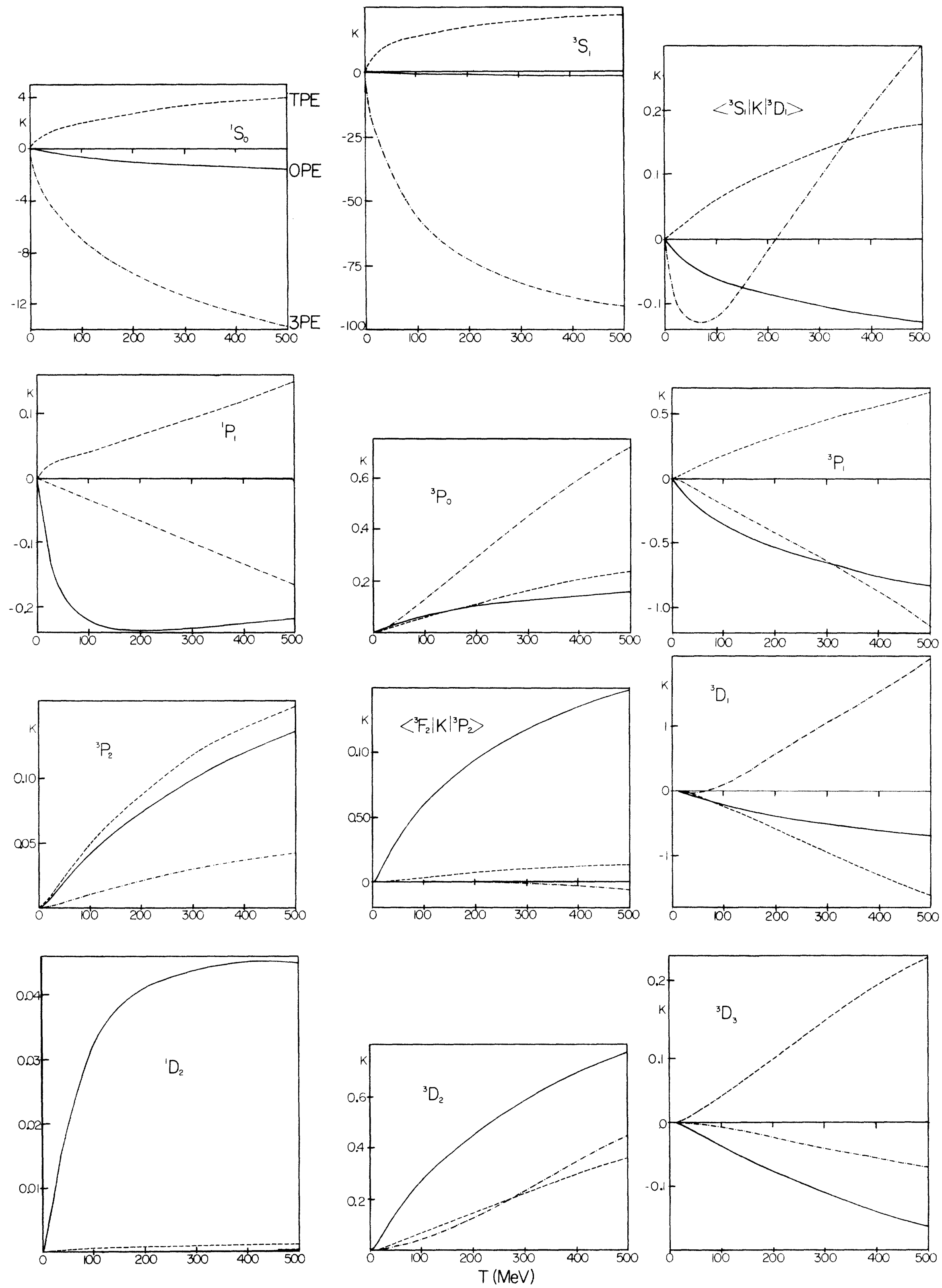

FIG. 1. The contributions to the K-matrix elements. The solid line corresponds to the second order contributions (OPE), the dashed line to the fourth order, and the dashed-dotted line to the sixth order contributions.

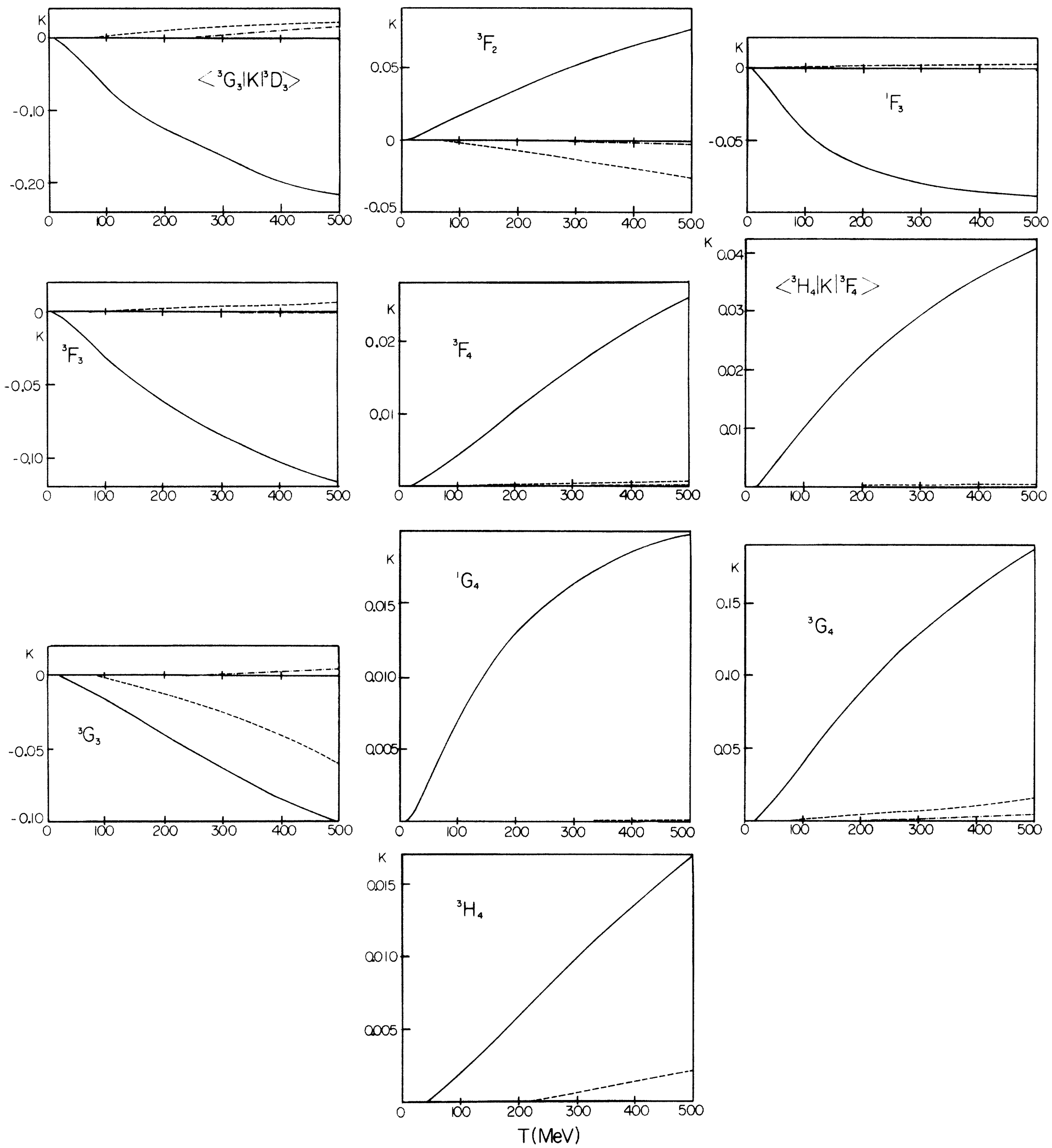

FIG. 1 *(Continued)*

${}^1P_1$: OPE region observed for T below 5 MeV ($R \approx 5.8$ fm); below about 12 MeV ($R \approx 7.4$ fm), there is a predominant contribution of OPE + TPE.

D waves:

${}^1D_2$: In the range of our calculations ($T < 500$ MeV) only the OPE region is observed ($R < 1$ fm).

${}^3D_1$: No regions are observed, this state is coupled to the ${}^3S_1$ state.

${}^3D_2$: OPE region observed for $T < 30$ MeV ($R \approx 4.1$ fm), OPE + TPE region for about $T < 50$ MeV ($R \approx 6.3$ fm).

${}^3D_3$: No OPE region observed; there are predominant OPE + TPE contributions for about $T < 100$ MeV ($R \approx 4.5$ fm).

F waves:

${}^1F_3$, ${}^3F_3$, ${}^3F_4$: Only the OPE region is observed

($R < 1.4$ fm).

$^3F_2$: This state is coupled to the $^3P_2$ state; the OPE region is observed for about $T < 150$ MeV ($R \approx 2.6$ fm); above that energy there is a predominance of OPE + TPE ($R < 2.8$ fm).

G waves:

$^1G_4$: Predominantly OPE ($R < 1.8$ fm).

$^3G_4$: Predominantly OPE for about $T < 350$ MeV ($R \approx 2.2$ fm).

$^3G_3$: Predominantly OPE for about $T < 100$ MeV ($R \approx 4.1$ fm); above that energy OPE + TPE prevails ($R < 3.7$ fm). This state is coupled to the $^3D_3$ state.

H waves:

$^3H_4$: Predominantly OPE for about $T < 300$ MeV ($R \approx 2.9$ fm); this state is coupled to the $^3F_4$ state.

IV. SUMMARY AND DISCUSSION

With the help of an approximate Bethe-Salpeter equation approximate values were obtained for the partial wave contributions of the two- and three-pion exchange ladder diagrams to the K-matrix elements. For the $^1S_0$ and $^3P_0$ states the accuracy is within 25%, where the comparison was made with the work of Gammel *et al.*[8]

With the approximate two- and three-pion exchange contributions the region hypothesis of the two-nucleon interaction is tested. Some region structure is observed but not in the simple fashion as the estimates described in Sec. I. There is no region structure for the S waves, as expected. Also the P waves show no definite region structure, with the exception of the $^1P_1$ wave, but for this wave the radius of the outer region is very large ($R \approx 5.8$ fm). The region structure depends strongly on the total angular momentum J. The $L = J + 1$ waves are affected strongly by their $L = J - 1$ counterparts. No definite region structure was found for the $^3D_1$ and $^3D_3$ waves. A definite region structure was observed for the $^1D_2$ and $^3D_2$ waves and for waves with $L > 2$, but it is impossible to give definite values for the radii of the OPE and TPE regions.

Our results must be interpreted with some caution. The contributions of the ladder diagrams are quite large, often much larger than the experimentally observed K-matrix elements. Also, experimentally there is only a weak coupling between states of the same J (rather small mixing angles); therefore, it is possible that the large contributions of the ladder diagrams should be suppressed. Thus in reality the radius of the outer region might be smaller than the one evaluated in Sec. III. On the other hand, some nonladder diagrams might have relatively large contributions for some partial waves. For instance the contribution of the crossed box diagram exceeds the contributions of the box diagram for the $^1D_2$ and $^1G_4$ states.[9]

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