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Comment on the possible measurement of vacuum polarization in heavy ion scattering*

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Contrary to a claim by Rafelski and Klein, one cannot use nonrelativistic elastic heavy ion collisions to probe the large q^2 behavior of the photon propagator.

In a recent paper, Rafelski and Klein[1] claim that one can measure the high q^2 behavior of the photon propagator in nonrelativistic heavy ion scattering. For illustration, they consider ^{16}O on ^{208}Pb, with a 60 MeV ^{16}O beam, which is not sufficiently high to allow the nuclear densities to overlap. The amplitude is written [their Eq. (10)] as

$$\propto \frac{1}{\vec{q}^2}\left\{1+\int_1^\infty \sigma(t)\frac{\vec{q}^2}{(\vec{q}^2+4m_e^2t^2)} \times\left[1+i\eta\ln\frac{(1+\eta)^2}{1+(\nu t^2/2\sin^2\frac{1}{2}\theta)^{-1}}\right]dt\right\}, \quad (1)$$

where $\sigma(t)$ is the spectral function for the photon propagator, t is "mass" of the virtual photon measured in units of $2m_e$, m_e is the electron mass, η is the Coulomb factor $=Z_1Z_2\alpha c/V_\infty$ (≈ 50 for their example), and $\nu=(2m_ec/p)^2$ where p is the incident momentum of the ^{16}O ion, and θ the scattering angle.

If this claim were true it would be very exciting. But it violates simple physical intuition. For example, at $\vec{q}^2$ of $(1\ \text{GeV}/c)^2$, Eq. (1) indicates that we are probing virtual photon masses of $\approx 1\ \text{GeV}/c^2$. But that means probing distances of $\frac{1}{5}$ fm, whereas the original particles stay more than 12 fm apart. Aside from the question of form factors, how can one use this process to probe such small distances, since the Coulomb wave functions are so small inside the classical turning point?

The point is that their approximation in going from their exact Eq. (7) to their Eq. (10) is entirely invalid for large virtual photon masses. Suppose the correct version of (1), their Eq. (7), is written as

$$\frac{1}{\vec{q}^2}\left[1+\int_1^\infty \sigma(t)\frac{\vec{q}^2}{\vec{q}^2+4m_e^2t^2}W(t)dt\right],$$

where

$$W(t)=[(1+\rho)z]^{i\eta}\exp(-2\eta\tan^{-1}\rho^{1/2}) \times|\Gamma(1+i\eta)|^2, \quad {}_2F_1(-i\eta,\ 1+i\eta,\ 1,\ z)$$

where

$$z=(1+\rho/\sin^2\tfrac{1}{2}\theta)^{-1},$$

$$\rho=\nu t^2/2.$$

I have calculated $W(t)$ numerically for the case $\theta=30°$, i.e., $q^2=1(\text{GeV}/c)^2$ and the results are tabulated in Table I. Looking at the real parts, for example, we see that for virtual photon masses of only 10 MeV, the correct coefficient is suppressed relative to theirs by a factor of 40, and beyond 20 MeV the coefficient drops by about a factor of 10 for every increase of 10 MeV. This agrees with one's naive expectation, since the Coulomb repulsion suppresses the wave function inside 15 fm, which corresponds to photon masses larger than 13 MeV.

Consequently, such heavy ion techniques cannot probe the large q^2 behavior of the photon propagator.

TABLE I. $W(t)$ for $q^2=1\ (\text{GeV}/c)^2$.

$\lambda=2m_et$ (MeV)	Re$W(t)$	Im$W(t)$
1	0.866	−172
2	0.693	−102
5	0.283	−31.4
10	0.027	−6.05
20	0.010	−0.3
50	-0.2×10^{-4}	-0.6×10^{-4}
100	-0.9×10^{-10}	-0.1×10^{-10}
200	-0.6×10^{-21}	-0.8×10^{-21}

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[1] J. Rafelski and A. Klein, Phys. Rev. C 9, 1756 (1974).