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Coriolis-coupling model applied to odd-neutron nuclear spectra of the $1g_{9/2}$ shell

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A Coriolis-coupling model extended to include a residual pairing type interaction is applied to investigate the energy levels of low lying states of the odd nuclei of the $1g_{9/2}$ shell ($73 < A < 87$). This deformed rotational treatment includes spectra of both parities originating from overlapping states of the $N = 4$ and $N = 3$ positive and negative parity shells. Single particle energy levels and wave functions throughout a shell are computed using spin-orbit and well-flattening parameters $C = -0.26\hbar\omega_0$ and $D_{N=4,3} = -0.040\hbar\omega_0$, $-0.053\hbar\omega_0$, respectively, the latter values being consistent with $d_{5/2}$ parentage states of ^{79}Se (positive parity) and $f_{5/2}$ states of ^{83}Kr (negative parity). Inclusion of an extracore pairing interaction primarily introduces a multiplicative effective overlap which serves as a quenching factor to the strength of off-diagonal Coriolis coupling terms. All parameters in the analysis are taken from available experimental considerations. The rotational constant A and deformation β, in particular, are derived from the location of the first excited 2^+ state of the neighboring even-even nucleus and its observed $E2$ transition rate to the ground state. For each set of parameters both a single particle and a Coriolis coupling diagonalization are performed over an entire shell of 10 or 15 Nilsson states to obtain final negative or positive parity spectra and wave functions. The results of this essentially fixed parameter model are presented and compared with experimental determinations with which they are in substantial agreement. In general, the model favors a prolate description for the nuclei in this region. Energy excitation spectra are all in general agreement, and in particular: (a) ground state spins and lowest lying opposite parity spins are always correctly predicted; (b) many otherwise anomalous low lying and often ground state $7/2^+$, $5/2^+$ spins are well predicted. The systematic reproduction of experimental spectra provided by the model points to the validity of a statically deformed interpretation of the odd neutron nuclei of the $1g_{9/2}$ shell, in contrast to the purely spherical and vibrational approaches to which the region has been restricted in the past.

[NUCLEAR STRUCTURE ^{73}Ge, ^{75,77,79,81}Se, ^{83,85}Kr, ^{87}Sr; calculated levels, I, π. Deformed Coriolis coupling treatment with pairing. Both parities.]

I. INTRODUCTION

The Coriolis coupling model[1-4] has been applied by Malik and Scholz (MS) with considerable success to nuclei in the $1f_{7/2}$ shell and later in the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ shells. Until recently, little use of this model was made in the $1g_{9/2}$ shell since the nuclei in this mass region were generally considered to display spherical-vibrational characteristics. Investigations involving suitable residual interactions between nucleons in $(1g_{9/2})^n$ configurations[5,6] as well as the coupling of quasiparticles to quadrupole surface phonons (QPC)[7,8] have failed to describe the spectra of nuclei in the ($A = 73-121$) region. Extensions of the QPC model to include quasihole-phonon coupling[9] and quadrupole moment effects[10] still did not remove many of the energy spectral difficulties.

In particular, the anomalous presence of many low lying $\frac{5}{2}^+$, $\frac{7}{2}^+$ states, which often lie below the $\frac{9}{2}^+$, are unaccounted for in the above models. Moreover, equivalent pairs, i.e., nuclei having the same number of particles and holes, are predicted by the spherical shell model[7] to possess essentially similar excitation spectra, whereas there is apparent disagreement, for example, between the low lying spectra of (^{77}Se-^{81}Se) and (^{75}Se-^{85}Kr or ^{83}Se). ^{81}Se lacks a very low lying $\frac{5}{2}^+$ state while the second set possesses ground states different from one another.

Finally, only one of these treatments[7] has attempted to describe the negative parity spectrum always present in nuclei of this region which in some cases represents much of the known spectrum and is often the parity of the ground state.

With more complete experimental data and with more extensive theoretical investigations[11-16], interest has recently been revived in the applicability of the rotational model and has broadened considerably the realm of nuclei which lend themselves to a deformed model interpretation. The work of MS reflected the importance of the often ignored Coriolis or rotational-particle coupling terms in destroying band structure and thereby disguising the rotational appearance of nuclei in the middle mass region. Other current workers[17,18] reported positive results using deformation treatments for $43 < A < 110$ nuclei. Specifically, Sanderson[19] used a Coriolis model to interpret his (d, t) reaction studies on ^{75}Se.

Further, a reinvestigation of the even-even nuclei in the $1g_{9/2}$ neutron shell equivocates their presumed characteristics. For example: (a) in ^{72}Ge the first excited 0^+ state is below the first 2^+ level; (b) in ^{80}Se, ^{86}Se, ^{82}Kr, ^{92}Zr, and ^{94}Zr the two phonon triplet is not well separated from other higher or lower lying states; and (c) the important E_4/E_2 ratio in this region averages 2.5, a value which is intermediate between the vibrational and strong coupling ideals. In comparison to the $1f_{7/2}$ shell, an even stronger case can be made for a rotational analysis for the $1g_{9/2}$ shell since: (a) the even-even nuclei possess greater E_4/E_2 ratios throughout; (b) the general pattern of rotational constants as illustrated in Fig. 1 and Ref. 2 is more suggestive of deformation; and (c) there is a greater availability of extracore particles (both neutrons and protons) which is generally considered as the cause of permanent deformation.

In view of the emerging pattern of recent research as well as of the shell model discrepancies discussed above, there is reason to expect that odd-neutron nuclei of the $1g_{9/2}$ neutron region could be understood in terms of an alternate rotational approach, and sufficient evidence exists for attempting a detailed investigation along these lines.

To this end a model[20] is developed using a deformed core, an effective pairing-type residual interaction, and rotational-particle coupling, and applied to describe both the positive and negative parity levels of the nuclei in this region. This treatment is able to: (a) predict the ground state spins and lowest lying opposite parity spins for all the odd-neutron nuclei throughout this shell; and (b) substantially reproduce the low lying spectra for both positive and negative parity states. In a subsequent paper, the computed wave functions will be used to calculate the electromagnetic properties of these nuclei (without the imposition of an effective charge) to add further support to these conclusions.

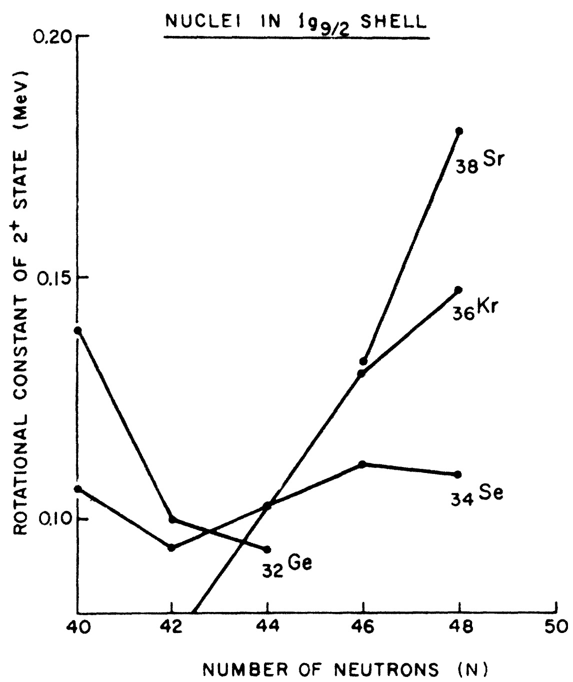

FIG. 1. Rotational constant versus neutron number for odd-neutron nuclei in $1g_{9/2}$ shell.

II. THE MODEL

Based upon the strongly coupled collective treatment first introduced by Bohr and Mottelson,[21] the relevant terms of the Hamiltonian of the present model are

$$H = H_R + H_{p-c} + H_P \tag{1}$$

for a single unpaired nucleon coupled to a deformed many body core. The three terms represent, respectively: the rotational motion of the core; the intrinsic Hamiltonian, i.e., the residual pair-correlation interaction of the core nucleons beyond the closed shell; and the single particle motion of the last nucleon with respect to the core.

The assumption of a permanently deformed system symmetric with respect to the body-fixed axis, together with the strong coupling relationship $\vec{R} = \vec{I} - \vec{j}$ leads to $\mathcal{G}_x = \mathcal{G}_y = \mathcal{G}$ and $K = \Omega$, where $\mathcal{G}_x$ and $\mathcal{G}_y$ are Cartesian components of the core moment of inertia and K and Ω are the z components of $\vec{I}$ and $\vec{j}$, respectively.

In the following sections, the individual terms of the Hamiltonian and their interactions will be developed in detail.

A. Single particle aspect

The single particle contribution to the Hamiltonian is

$$H_P = \frac{-\hbar^2}{2\mu}\Delta + \frac{\mu}{2}(\omega_x^2 x^2 + \omega_x^2 y^2 + \omega_z^2 z^2) + C\vec{l}\cdot\vec{s} + D\vec{l}^2 \tag{2}$$

for an anisotropic harmonic oscillator potential with rotational symmetry about the z axis, a spin-orbit coupling and a Nilsson[22] l^2 term. μ is the reduced mass of the last odd nucleon.

The particle Hamiltonian is then diagonalized in terms of isotropic harmonic oscillator wave functions $|j\Omega\rangle$, leading to eigenfunctions

$$\chi_{\Omega,\nu} = \sum_j c_{j,\Omega,\nu} |j\Omega\rangle;$$
$$\chi_{-\Omega,\nu} = \sum_j (-)^{j-1/2} c_{j,\Omega,\nu} |j-\Omega\rangle, \tag{3}$$

with eigenvalues $h_{\Omega,\nu}$ and expansion coefficients $c_{j,\Omega,\nu}$ as functions of the deformation parameter β. The index ν is included to distinguish single particle levels of common Ω and anticipates a Coriolis interaction applied to an entire shell.

Recognizing the two-particle nature of the nuclear potential, the corrected single particle energies are given by

$$\epsilon_{\Omega,\nu} = \tfrac{3}{4}h_{\Omega,\nu} - \tfrac{1}{4}\langle C\vec{1}\cdot\vec{s} + D\vec{1}^2\rangle , \tag{4}$$

where $\langle C\vec{1}\cdot\vec{s} + D\vec{1}^2\rangle$ indicates the expectation value of the enclosed operator in terms of the deformed wave function $\chi_{\Omega,\nu}$. The band head or total system energies for a given nucleus are formed by $E=\sum_i \epsilon_{\Omega,\nu}$ summing over all occupied states beyond the closed shell.

In order to formulate an allowable basis, excited particle states are designated by the position of the last elevated unpaired particle; core excited or "hole" states are uniquely constructed by lifting a core particle and pairing it with the last odd nucleon. Both types of states, particle and hole, complement one another and generate a complete basis of states equal to the number of Nilsson states within a shell and are both derivable from the same Hamiltonian given in Eq. (2). Accordingly, other core excited states formed by locating a core particle at any other state, other than that occupied by the odd nucleon, or by relocating more than one core particle, are not considered.

B. Residual interaction aspect (pairing)

In Eq. (1) H_{p-c}, the intrinsic Hamiltonian of the paired core nucleons beyond the closed shell, may be represented essentially by the simple intranuclear residual interaction, i.e. the pairing force. Employing the well known BCS approach[7,23,24] leads to a core overlap integral of the form

$$(\phi_C^{(\mu)}, \phi_C^{(\mu')}) = (u_\mu^{(\mu')}u_\mu^{(\mu)} - v_\mu^{(\mu')}v_\mu^{(\mu)}) \times \prod_{i\neq\mu,\mu'} (u_i^{(\mu')}u_i^{(\mu)} + v_i^{(\mu')}v_i^{(\mu)}) \tag{5}$$

which becomes significant in regulating the strength of the Coriolis coupling terms to be discussed in the following section. The v_μ and u_μ are normalized variational parameters whose approximate solutions are well known[25] and which represent the probability of occupancy and nonoccupancy of a pair in state μ. The second factor of Eq. (5), since it is always summed over all interactions, approaches unity, and can be ignored, whereas the first factor defined $\xi_{\mu-\mu'} = (u_\mu^{(\mu')}u_\mu^{(\mu)} - v_\mu^{(\mu')}v_\mu^{(\mu)})$ may vary from 0 to ± 1.[26] $\xi_{\mu-\mu'}\,(\mu\neq\mu')$ then plays the role of an "effective overlap parameter" which in this analysis is treated as constant over each of the following three classes of interactions in which, for purposes of explanation, a sharp Fermi surface is assumed:

(1) p-p $u_\mu = u_{\mu'} = 1$ $v_\mu = v_{\mu'} = 0$ $\xi = 1$

(2) h-h $u_\mu = u_{\mu'} = 0$ $v_\mu = v_{\mu'} = 1$ $\xi = -1$

(3) p_e-h $u_\mu = 0, u_{\mu'} = 1$ $v_\mu = 0, v_{\mu'} = 1$ $\xi = 0.$

p-p, h-h, p_e-h represent interactions between two-particle, two-hole (core excited) and excited particle-hole states. The ground state may be regarded as either a particle or a hole state. A strictly enforced selection rule is imposed by the third case, p_e-h, which corresponds physically to forbidding interactions which cannot be obtained by the simultaneous destruction of one particle in state μ and creation of one particle in state μ'. The first two interactions are used to determine the sign of $\xi_{\mu-\mu'}$ whose magnitude is allowed to vary from 1 as the condition of a sharp Fermi level is relaxed. Calculations are always performed in accordance with the properly applied selection rules discussed above. However, only the positive value of $\xi[\xi_{P-P'}\,(\mu\neq\mu')]$ will be the one referred to in the discussion. For all diagonal interactions, i.e. $\mu=\mu'$, the overlap factor is *always* taken as unity.

Since the renormalized expectation values $\langle H_C^{(\mu)}\rangle$ remain relatively constant throughout middle mass nuclear shells,[26] $\langle H_C^{(\mu)}\rangle$ may be taken in the limiting case for which it reduces to Eq. (4); hence the energy of the residual interaction adds a constant contribution, and no additional adjustment is made to the rotational band head energies.

C. Core rotation and particle coupling aspect

The rotational aspect of the Hamiltonian is generally regarded in terms of the diagonal and nondiagonal contributions of

$$H_R = A[\vec{I}^2 + \vec{j}^2 - 2(\vec{I}\cdot\vec{j})], \tag{6}$$

where $A=\hbar^2/2\mathcal{J}$ is the rotational constant for an axially symmetric system. In particular the $\vec{I}\cdot\vec{j}$, or Coriolis term, contains terms of the form $I^+j^- + I^-j^+$. This interaction was not considered by Nilsson with the exception of the $K=\Omega=\pm\frac{1}{2}$ case which contributes directly towards decoupling the energies.

In the absence of these nondiagonal contributions, an adequate wave function for the entire system may be stated as a properly symmetrized product of the D functions, $|IKM\nu\rangle$; the deformed single particle functions $\chi_{\Omega,\nu}$; and the intrinsic core functions $\phi_C^{(\Omega,\nu)}$:

$$|IMK\nu\rangle_s=\left[\frac{2I+1}{16\pi^2}\right]^{1/2}\times[D^I_{MK}(\theta_i)\chi_{\Omega,\nu}+(-)^{I-1/2}D^I_{M-K}(\theta_i)\chi_{-\Omega,\nu}]\,\phi_C^{(\Omega,\nu)}, \tag{7}$$

where $I_z=K$ is taken equal to $j_z=\Omega$ in order to satisfy the symmetry requirements, and θ_i represents the three Eulerian angles. This case leads to a rotational band sequence $I=K, K+1, K+2, \ldots,$ structured on each particle state ϵ_K in accordance with $\Delta(E_I)=A[I(I+1)-K(K+1)]$ with respect to the ground state $I=K\neq\frac{1}{2}$. For $K=\frac{1}{2}$ bands, decoupling considerations must be included.

In order to properly incorporate the entire Coriolis interaction, K must be sacrificed as a constant of the motion and the following expansion adopted as the total wave function of the entire coupled system:

$$|IM\rangle=\sum_{K,\Omega>0}C_{K,\nu}|IMK\nu\rangle_s. \tag{8}$$

Thus I and M are retained as the only good quantum numbers of the system while $|IM\rangle$ is recognized in terms of two sets of coefficients, $c_{j,\Omega,\nu}$ and $C_{K,\nu}$, both of which depend upon the nuclear deformation β and both of which are determined by successive exact diagonalization procedures for each set of parameters.

Since the summation in Eq. (8), unlike other conventions,[1,27] is taken only over the positive values of K in integral steps, the interaction connects states as follows:

$$|\Delta K|=1 \text{ and } \Delta K=0 \text{ for } K=\tfrac{1}{2} \text{ bands} \tag{9}$$

provided $\xi\neq\xi_{P_e-H}=0$ for nondiagonal matrix elements.

According to this representation the diagonal and nondiagonal matrix elements of the total model Hamiltonian inclusive of pairing, decoupling, and nondiagonal Coriolis elements may be written

$$\langle IMK\nu|H|IMK\nu\rangle=E_{K=\Omega,\nu}+A\left[I(I+1)-2K^2+\sum_j c_{j,\Omega,\nu}{}^2 j(j+1)+\delta(\Omega,\tfrac{1}{2})(-)^{I+1/2}(I+\tfrac{1}{2})a_{\nu,\nu}\right], \tag{10a}$$

$$\langle IMK\nu|H|IMK'\nu'\rangle=-A\xi\left\{\delta(K,K'\pm1)[(I\mp K)(I\pm K+1)]^{1/2}\sum_j c_{j,\Omega,\nu}c_{j,\Omega\pm1,\nu'}[(j\mp\Omega)(j\pm\Omega+1)]^{1/2}-\delta(\Omega,\Omega')\left[\sum_j c_{j,\Omega,\nu}c_{j,\Omega',\nu'}j(j+1)+\delta(K,\tfrac{1}{2})(-)^{I+1/2}(I+\tfrac{1}{2})a_{\nu,\nu'}\right]\right\}, \tag{10b}$$

where

$$a_{\nu,\nu'}\equiv-\sum_j(-)^{j+1/2}(j+\tfrac{1}{2})c_{j,1/2,\nu}c_{j,1/2,\nu'} \tag{10c}$$

may be designated the generalized decoupling parameter as compared to the a parameter often used in discussing partial or sub-shells.

Thus energy spectra and wave functions dependent upon β, ξ, C, D, and $\hbar\omega_0$ are generated in accordance with the above prescription. Ground state spins are uniquely assigned from the value of I corresponding to the lowest eigenvalue of Eq. (1).

III. DETAILS AND SCOPE OF ANALYSIS

Nuclear spectra and wave functions are calculated according to the model using the following sequence: Single particle energies $h_{\Omega,\nu}$ and corresponding wave function coefficients $c_{j,\Omega,\nu}$ are generated as a function of deformation β by exact diagonalizations over all $\frac{1}{2}(N+1)(N+2)$ states in an entire appropriate shell. The two-body corrected energies $\epsilon_{K,\nu}$ of Eq. (4) are then calculated. From $\epsilon_{K,\nu}$ and the individual structure of given nuclei in the $1g_{9/2}$ shell, the unmixed band head energies E are calculated and added diagonally to the Coriolis coupling matrix. No renormalization adjustments are made to these band head energies.

A second diagonalization of the matrix described by Eqs. (10a), and (10b) for each value of total spin I produces the final spectra and coefficients, $C_{K,\nu}$ as functions of the rotational constant A, overlap parameter ξ, and deformation β.

Standard values of oscillator quanta $\hbar\omega_0=41/A^{1/3}$ MeV, nuclear radius $R=1.2A^{1/3}$ fm, as well as a spin-orbit strength $C=-0.26\hbar\omega_0$ appropriate to $N=3$ and $N=4$ shells are used.

β, A, ξ, and the well-flattening parameter D_N are taken from the following experimental observations:

(a) β is always taken to within ±0.05 from the value of the neighboring nucleus' $E2$ transition probability from the first excited 2^+ to the ground state. The strong coupling approximation is given by

$$B(E2;i\text{-}f)=\left(\frac{5}{16\pi}\right)Q_0{}^2(I_i2K0|I_fK), \tag{11}$$

where Q_0, the intrinsic quadrupole moment, is proportional to $(\beta+0.16\beta^2)$. The exact value of β in this small range is chosen in each instance by optimization of the model's predictions.

(b) A is taken from the neighboring even-even 2^+ energy state assuming a rotational character for this state (refer to Fig. 1): $\Delta E_{I=2} = A[I(I+1)] = 6A$.

(c) $D_{N=4,3} = -0.040\hbar\,\omega_0$, $-0.053\hbar\,\omega_0$ is fixed for all nuclei throughout a shell, each shell being determined by comparison of the location of higher lying $d_{5/2}$ parentage states of ^{79}Se (positive parity $N=4$) and $f_{5/2}$ parentage states of ^{83}Kr (negative parity $N=3$).

(d) ξ is estimated according to the BCS variational solutions for the parameters u_μ, v_μ applied to interaction of levels near the ground state and is found always to be between $0.6 \leq \xi \leq 0.8$ for nuclei in this region.

All these values fall within ranges disclosed by other workers.[13,17,28] The adoption of these mutually consistent assumptions of rotational character serves to restrict the freedom of the calculation by removing most latitude from the abovementioned parameters.

The presence of the overlap parameter ξ does not directly influence calculations other than as a suppressant to the Coriolis coupling between states nearest the Fermi level. Its role appears essentially to lessen the value of β necessary for reproduction of experimental results; in this way β remains within the range expected for moderately deformed nuclei.

From the standpoint of an extreme single particle Coriolis coupling model all the nuclei of the $1g_{9/2}$ shell may be synthesized into five classes based upon the number of odd nucleons occupying the five available levels beyond the 40-neutron closed shell. For example, ^{73}Ge and ^{75}Se, each possessing 41 neutrons, fall in class I. ^{77}Se with 43 neutrons is in class II, etc. All nuclei of a given class are expected to display similar behavior with respect to deformation and core overlap. It should be emphasized that even though the spectra of different nuclei in the same class might ostensibly appear different (e.g. the different ground state spins of ^{73}Ge and ^{75}Se, both of class I), small variations in deformation easily account for otherwise inexplicable low lying doublets, triplets, and crossovers.

A detailed calculation of at least one characteristic nucleus from each class is presented, with somewhat greater emphasis given to the middle-shell nuclei where deformation and Coriolis coupling effects are significant and where most vibrational model anomalies occur.

An explanation of the prevalent negative parity spectra, and in particular, the ubiquitous $\frac{1}{2}^-$ present either as a low lying isomer or in some cases as a ground state, is a requirement of any comprehensive theoretical treatment of this region.

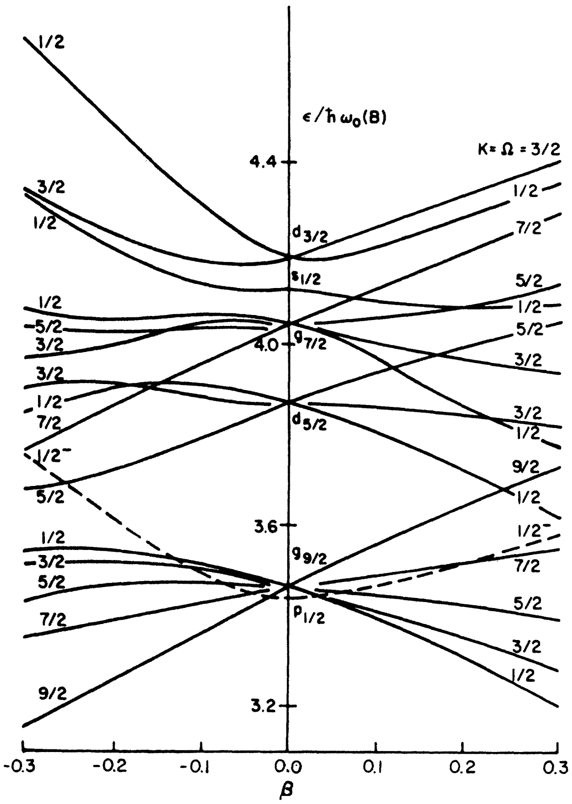

FIG. 2. Nilsson diagram for $N=4$ shell; energy in units of $\epsilon/\hbar\omega_0(B)$ for $-0.3<\beta<0.3$. Dotted curve represents $N=3$, $2p_{1/2}$ negative parity state.

In this analysis the intrusion of the $2p_{1/2}$ single particle state (on which the last odd nucleon might reside) from the underlying $N=3$ negative parity shell into the $1g_{9/2}$ sub-shell is assumed for both prolate and oblate deformations. (See Fig. 2.)

Although Coriolis coupling does not mix states of opposite parity, i.e. states of adjacent shells, separate diagonalizations involving the 15 states of the $N=4$ *or* the 10 states of the $N=3$ shell are performed. Then a suitable adjustment of the two sets of spectra is made to conform with experimental determinations.

Hence, at predicted deformations the negative parity spectra of all odd nuclei may be treated by a single Coriolis coupling calculation applied to the entire $N=3$ shell. The results of this calculation are presented summarily, with the exception of a specific treatment of ^{81}Se. The validity of such a general approach as representative of all negative parity nuclei is suggested by the similarity of the "opposite" parity spectra of nearly all nuclei in the region considered.

In the following presentation of results, nuclear spins are to be taken as of positive parity unless otherwise designated. In the accompanying dia-

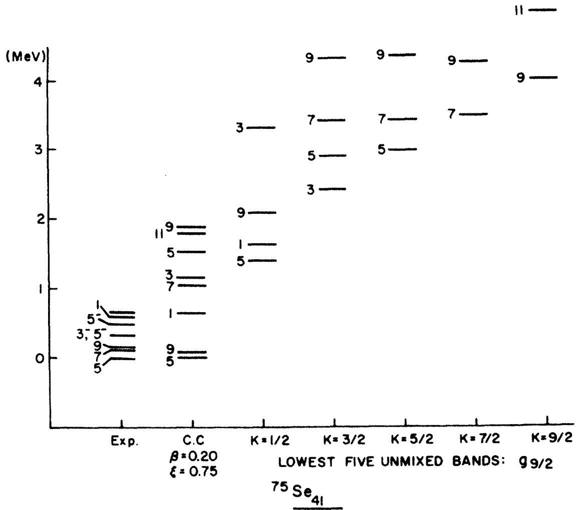

FIG. 3. Energy level diagram for $^{75}Se_{41}$ depicting experimental (Ref. 19), Coriolis coupled (C. C.) spectra, and the unperturbed rotational structure of the lowest five bands for $\beta=0.20$.

grams all spins are noted as $2I$, and all excitation spectra are plotted with respect to the known lowest positive parity state at 0 MeV.

IV. PRESENTATION AND DISCUSSION OF RESULTS

A. General summary of energy spectra and wave functions

The two most important consequences of this model with respect to excitation spectra are the correct predictions of (a) the ground and lowest lying opposite parity spins for all nuclei in the $1g_{9/2}$ shell, and (b) the anomalous low lying and

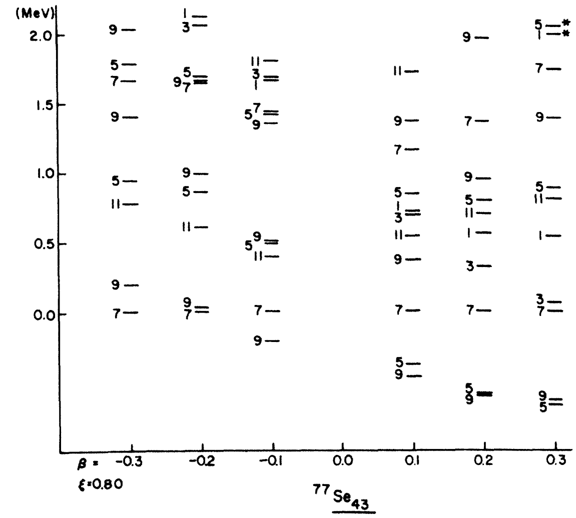

FIG. 5. Theoretical energy level diagram for $^{77}Se_{43}$ depicting the Coriolis coupled spectra for positive and negative deformation.

often ground state $\frac{7}{2}^+$, $\frac{5}{2}^+$ spin states as a natural outcome of the Coriolis mixing of the $K=\frac{5}{2},\frac{7}{2},\frac{9}{2}$ bands originating in the $1g_{9/2}$ sub-shell.

In general, predicted deformations throughout the region favor a prolate description. The prolate determined spectrum of every nucleus treated is presented in Figs. 3–10, the most representative being ^{79}Se. In all nuclei a low lying $\frac{11}{2}^+$ level of less than 2 MeV is predicted although, due to the difficulty of measuring higher spin states, as yet none has been detected. These conclusions are not possible using only the deformed Nilsson model, but require the Coriolis interaction to destroy the simple band structure. This effect on some of the lower lying unperturbed bands of ^{79}Se is illus-

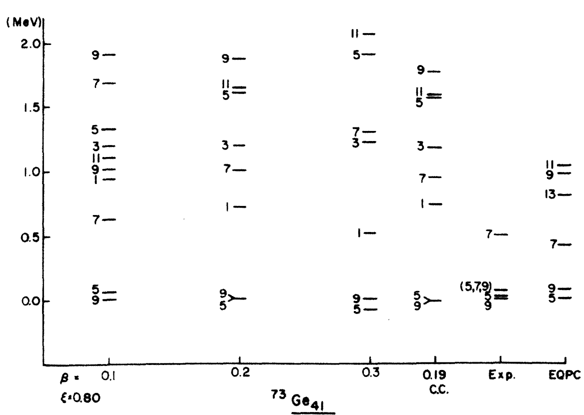

FIG. 4. Energy level diagram for $^{73}Ge_{41}$ depicting experimental (Ref. 29), EQPC (Ref. 34), and Coriolis coupled spectra for positive and optimum ($\beta=0.19$) deformation.

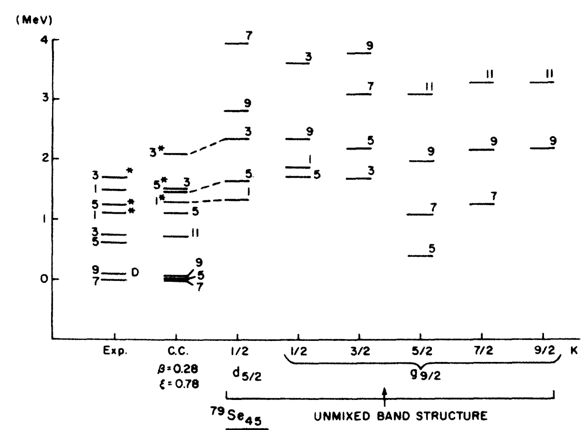

FIG. 6. Energy level diagram for $^{79}Se_{45}$ depicting experimental (Refs. 32, 33), Coriolis coupled spectra, and the unperturbed rotational structure of the lowest six bands for $\beta=0.28$.

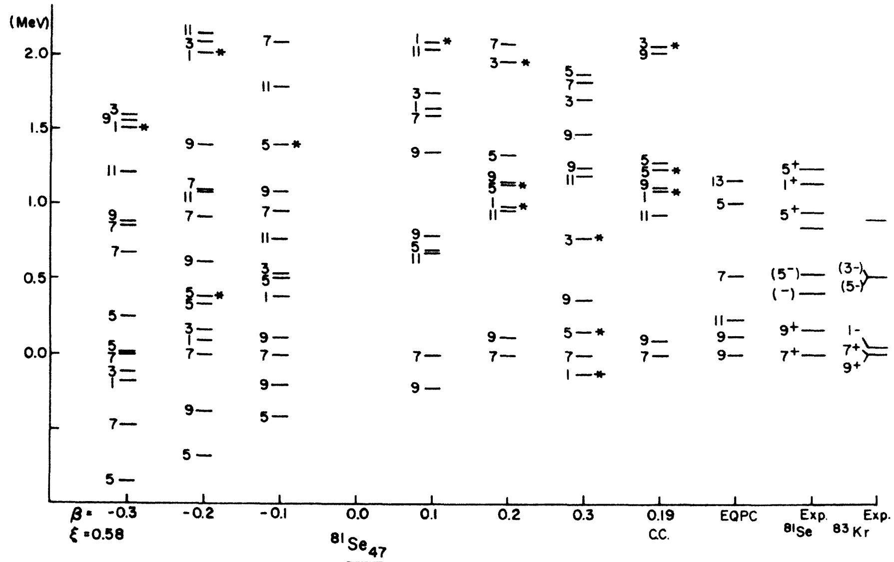

FIG. 7. Energy level diagram for $^{81}Se_{47}$ depicting experimental (Ref. 35), EQPC (Ref. 34), and Coriolis coupled spectra for positive, negative, and optimum deformation.

trated in Fig. 6.

In order to evaluate the total band head structure for a given nucleus upon which the Coriolis interaction is built, it is necessary to perform the single particle anisotropy perturbation and then correct each perturbed state in accordance with Eq. (4). Figure 2 is a Nilsson diagram corrected for a two-body potential. For all 15 states in the $N=4$ shell, the energy $\epsilon(\beta)/\hbar\,\omega_0(\beta)$ variation is plotted with respect to positive and negative deformation. Also included is the dotted $2p_{1/2}$ state whose behavior is calculated from a similar treatment of the $N=3$ shell.

The general pattern of energy spectra for all classes of nuclei due to the Coriolis mixing can be summarized as follows: For prolate deformation class I nuclei, whose lowest band is based on the $K=\frac{1}{2}$ Nilsson level, experience considerable decoupling effects. For example, in Fig. 3, the $K=\frac{1}{2}$ unmixed band has a decoupling coefficient $a_{\nu,\nu'}=4.9$ which leads to a reordering of its spin structure, $I=\frac{5}{2}^+, \frac{1}{2}^+, \frac{9}{2}^+, \frac{3}{2}^+, \cdots$. In this way the Nilsson model alone could account for a low lying $\frac{5}{2}$ spin state, but the entire Coriolis matrix is necessary to depress the $\frac{9}{2}$ spin states. The nuclei in this class, then, are expected to possess a low lying $\frac{5}{2}^+, \frac{9}{2}^+$ doublet (for $\beta>0$). Also predicted at small deformation is the presence of a $\frac{7}{2}^+$ state at approximately $\frac{1}{2}$ MeV.

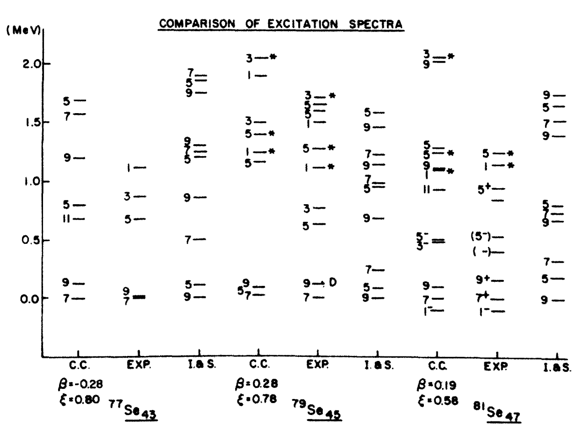

FIG. 8. Optimum excitation spectra for $^{77}Se_{43}$ (Ref. 36), $^{79}Se_{45}$, $^{81}Se_{47}$ compared with experiment and the predictions of Ikegami and Sano (Ref. 8).

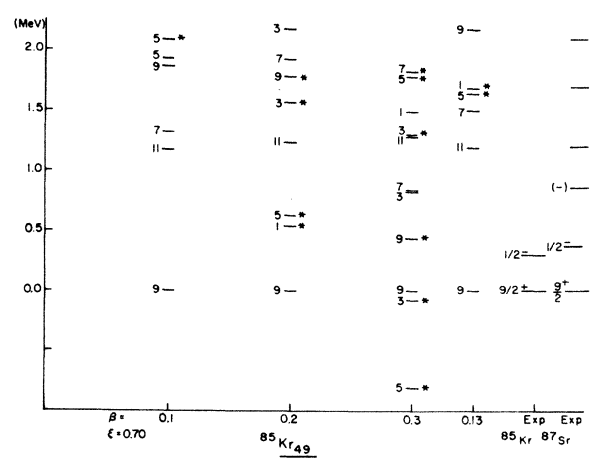

FIG. 9. Energy level diagram for $^{85}Kr_{49}$ depicting positive and optimum ($\beta=0.13$) deformation and the experimental spectra for $^{85}Kr_{49}$ and $^{87}Sr_{49}$ (Ref. 35).

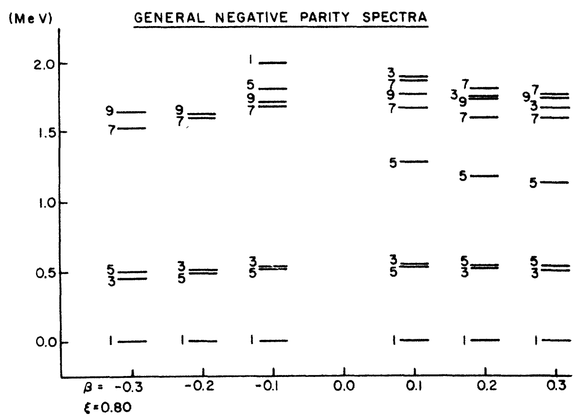

FIG. 10. General negative parity spectra presented for both positive and negative deformation.

Class II and III nuclei possessing lowest unmixed band head energies on $K=\frac{3}{2}$ and $\frac{5}{2}$ bands, respectively, are expected to exhibit a number of low lying states of $I \geq \frac{5}{2}$ due to the proximity of the higher spin states in the unmixed band head spectrum. An example of this is shown in Fig. 6 for ^{79}Se (class III) for which the unperturbed low energy structure contains available $\frac{7}{2}, \frac{9}{2}$ states, which when subject to the rotational particle coupling result in the tightly grouped $\frac{7}{2}$, $\frac{5}{2}, \frac{9}{2}$ triplet followed by an $\frac{11}{2}$ state. Again the unperturbed or Nilsson interpretation yields an excitation spectrum of $\frac{5}{2}, \frac{7}{2}, \frac{7}{2}$, etc., which does not account for the depressed $\frac{9}{2}$ level.

For class IV nuclei based on a lowered $K=\frac{7}{2}$ band, a $\frac{7}{2}, \frac{9}{2}$ doublet is anticipated with the $\frac{7}{2}$ level descending with increasing positive deformation due to divergence of the Nilsson $|\frac{7}{2}, \frac{9}{2}\rangle$ and $|\frac{9}{2}, \frac{9}{2}\rangle$ states. (See Fig. 2.)

Since the unperturbed class V prolate structure already contains an $|IK\nu\rangle = |\frac{9}{2}, \frac{9}{2}, \frac{9}{2}\rangle$ ground state, the Coriolis interaction pushes this state further down, isolating it from other $g_{9/2}$ parentage levels. However, the crossover of the $|\Omega, \nu\rangle = |\frac{1}{2}, \frac{5}{2}\rangle$ single particle $2d_{5/2}$ bands which occur at about $\beta = 0.25$ (Fig. 2), causes the rapid descent of the $I=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ states of this parentage. This is evident in Fig. 9.

For oblate deformation the over-all situation is somewhat related, with the exception that the band head pattern is reversed, there being a general first order similarity between nuclei of classes I and V, II and IV for opposite deformation.

In the results most levels are classified according to their shell model parentage in both theoretical and experimental energy spectra [whenever (d,p) reactions permit such identification]. The general location of $2d_{5/2}$ (indicated with an asterisk) with respect to $1g_{9/2}$ parentage states depends primarily on the well-flattening constant D_4. The relative locations of these levels is always in good agreement with experiment, especially considering that no attempt to adjust D_4 throughout the $1g_{9/2}$ shell has been made.

No inclusion of higher lying excitation spectra is attempted, since these states are usually a consequence of multiparticle excitations and their explanation falls outside the structure of a single particle model.

Negative parity spectra are also generally well explained including the occasional crossover of the first $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states as, for example, in ^{83}Kr. Figure 10 illustrates a representative negative parity Coriolis calculation for positive and negative β. Explicit calculations for a specific nucleus are included only for ^{81}Se since this model regards the negative parity states of all nuclei treated as being within the same class.

In some cases levels are predicted which have not been reported in the analysis of experimental data; however, inspection of the primary experimental evidence indicates that they may be there.

The positive parity wave function expansion coefficients of all the $1g_{9/2}$ nuclei reveal contributions almost exclusively of $1g_{9/2}$ parentage ($\nu=\frac{9}{2}$) for ground and first excited states. It is only for higher lying states that admixtures from $2d_{5/2}$ ($\nu=\frac{5}{2}$) and $1g_{7/2}$ ($\nu=\frac{7}{2}$) parentage enter into the wave functions and justify using the entire shell as a basis for the theoretical interaction.

The admixtures of the important ground and first excited states fed by $\nu=\frac{9}{2}$ derive mostly from bands adjacent to the lowest band for each class of nuclei, subject, of course, to the restriction $I \geq K$. For example, the $I=\frac{7}{2}^+$ level of ^{79}Se, a class III nucleus based on the lowest $K=\frac{5}{2}$ band (see Fig. 6) receives contributions of $|C_K|^2 = 0.138$, 0.609, 0.243 for $K=\frac{3}{2}, \frac{5}{2}, \frac{7}{2}$, but absolutely no contributions from the forbidden $K=\frac{9}{2}$ band.

The negative parity wave functions, typically for ^{77}Se, are also calculated for positive and negative deformation and for spins $I=\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$. Since they are based on the $2p_{1/2}$ shell model, $K=\frac{1}{2}$ band, dominant contributions are from this parentage ($K, \nu = \frac{1}{2}, \frac{1}{2}$) in all cases with only slight mixing occurring for the larger spins. This is due to the relatively large value of $D = -0.053\hbar\omega_0$ and relatively small $\xi = 0.6$ which effectively isolates the $2p_{1/2}$ band. For these conditions, the Coriolis interaction is exerted primarily through decoupling with a decoupling coefficient $a_{\nu,\nu'}$ of about 1, which is responsible for the creation of the first $\frac{3}{2}^-, \frac{5}{2}^-$ characteristic doublet. Since the decoupled terms are primarily diagonal, their role is only indirectly manifested in the wave function coefficients. The situation remains essentially the same for a large range of β of either positive or negative deformation.

B. Detailed discussion of nuclear spectra

1. *Class I: ^{75}Se, ^{73}Ge, ^{71}Zn*

For class I nuclei the odd nucleon falls on the $K=\frac{1}{2}^+$ or $K=\frac{9}{2}^+$ single particle state for positive or negative deformation, respectively. For the first two nuclei a $\frac{9}{2}^+, \frac{5}{2}^+$ doublet is predicted for positive β with an interchange of ground state spins due to a slight difference in β. Actually a triplet has been observed[19,29] for both ^{73}Ge and ^{75}Se which confirms the theoretical $\frac{9}{2}^+, \frac{5}{2}^+$ states. The third member of the triplet (a favored $\frac{7}{2}^+$ state) appears to indicate a smaller deformation than values obtained from neighboring even-even transition rates. This is commensurate with anticipated behavior of class I nuclei which are adjacent to shell closure. The 285 KeV state in ^{75}Se is generally considered to be a negative parity $\frac{3}{2}^-, \frac{5}{2}^-$ doublet[30,31] as predicted by the model for all negative parity nuclei. (See Fig. 10.)

2. *Class II: ^{77}Se*

The model, for class II nuclei, places the odd particle on either the $K=\frac{3}{2}^+$ or $\frac{7}{2}^+$ single particle state depending on deformation. (See Fig. 2.)

Referring to Figs. 5 and 8, the low-lying $\frac{9}{2}^+, \frac{7}{2}^+$, and $\frac{5}{2}^+$ states are reasonably well predicted for β positive or negative. A prolate interpretation for ^{77}Se is at least as satisfactory as the refined QPC model of Ikegami and Sano[8] which incorporates nine major shells in the pairing calculations and represents the best spherical model effort to date. Furthermore, the $\frac{7}{2}^+$ ground state could be achieved with positive deformation if the band head energies were renormalized in specific accordance with the pairing interaction, in which case the entire shell could be described in terms of prolate deformation.

However, with $\beta<0$, the excellent agreement obtained for the three lowest lying levels favors a negative deformation. This is consistent with the oblate description for middle shell nuclei in the MS^2 analysis of the $1f_{7/2}$ shell.

3. *Class III: ^{79}Se*

Spectral agreement for this nucleus is particularly good with eight well correlated positive parity levels at excitation energies less than 2 MeV. The model calls for an additional $\frac{5}{2}^+$ spin between the $\frac{7}{2}^+$, $\frac{9}{2}^+$ pair. Experimental studies[32,33] again indicate a possible doublet associated with the $\frac{9}{2}^+$ level with values of $l_n=1$ and 4. Since there exist no very low lying $\frac{1}{2}^+$ or $\frac{3}{2}^+$ levels throughout the shell, and considering the similarity between $l_n=1$ and $l_n=2$ (d,p) displays, the presence of the missing $\frac{5}{2}^+$ could be speculated from a possible $l_n=2$ transition to this level.

The locations of the $2d_{5/2}$ states denoted with asterisks are in excellent agreement. The states of this parentage appear to descend as the odd nucleon count increases, which is in keeping with the model prediction. The inversion of the $\frac{5}{2}$* and $\frac{3}{2}$* is a direct consequence of the decoupling of the $K=\frac{1}{2}$, $2d_{5/2}$ band as shown in Fig. 6.

Figure 6 also offers a partial picture of the unperturbed band structure of ^{79}Se and emphasizes the importance of the Coriolis coupling in achieving the $\frac{7}{2}^+$ ground state. The expansion coefficients of this state indicate a strong mixture of $K=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ bands. The Nilsson model alone cannot account for such a spin or so dense a low lying level structure.

4. *Class IV: ^{81}Se, ^{83}Kr*

The nuclei in this class are observed to possess a low lying positive parity $\frac{7}{2}, \frac{9}{2}$ doublet without any evidence of a close $\frac{5}{2}^+$ spin. The model accurately predicts this and accounts for the spin interchange for ^{81}Se, ^{83}Kr by means of a slight variation in deformation. ($\beta=0.19$ to 0.18) Other levels of ^{81}Se are in good agreement with known measurements. As a representative nucleus, Fig. 7 depicts the Coriolis coupling excitation spectrum of ^{81}Se for both positive and negative deformation.

Little information is known beyond the ground state doublet for ^{83}Kr. However, fitting these two levels in conjunction with what is observed in ^{81}Se is sufficient to generate the static and dynamic properties of the doublet states; these will be discussed in a subsequent paper. Also included for comparison is a recent calculation by Goswami *et al.*[34] in which they used the EQPC model with the addition of a static dipole-dipole interaction term.

5. *Class V: ^{85}Kr, ^{87}Sr*

Since only the $\frac{9}{2}^+$ ground state has been detected for ^{85}Kr, the exact β assignment is tentatively chosen at $\beta=0.13$. Figure 9 compares the theoretical spectrum to both ^{85}Kr and ^{87}Sr, which according to the present model should possess essentially similar characteristics. For this class, the odd nucleon resides on the $K=\frac{9}{2}^+$ single particle level for $\beta>0$. Except for the ground state, low level positive parity levels are not predicted for this class, although the $2d_{5/2}$ parentage states descend rapidly with increasing deformation, and their relative location awaits further experimental investigation of these nuclei.

C. Summary and conclusions

Nuclear distortions appear uniformly prolate with the exception of ^{77}Se which permits an acceptable explanation for either deformation, the oblate being preferred because of a correctly predicted ground state spin. The magnitude of β, smallest near both shell closures, increases for classes of nuclei more centrally located within the $1g_{9/2}$ subshell.

In view of the systematic reproduction of existing data provided by the present model, the general conclusion of this investigation is that the odd neutron nuclei in the $1g_{9/2}$ shell may be interpreted in terms of a rotational statically deformed approach. This conclusion is reinforced by the fact that all parameters are established by physical observations which permit a unified set of consistent assumptions to produce a model with essentially fixed parameters. Moreover, low energy excitations, including most spectral anomalies, and all ground state spins can be well accounted for.

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