PHYSICAL REVIEW C VOLUME 10, NUMBER 4 OCTOBER 1974

Linear relations between high-purity energy levels in $A = 12$–16 nuclei

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(Received 28 May 1974)

We have assumed the $T = 0$ and 1 negative-parity quartets of levels in $A = 14$ nuclei, as well as the $T = 1$ negative-parity quartet in $A = 16$ nuclei, to represent pure configurations. Linear relationships between the energies of these groups of levels produce results which are consistent with the relevant part of the single-particle energy spectrum in this region. From the given $A = 14$ energy values conclusions are made concerning the low positive-parity levels in $A = 15$. Due consideration is given to the necessary Coulomb corrections in all the calculations.

NUCLEAR STRUCTURE $A=12$–16; calculated Coulomb energies, single-particle energies. $A=16$ negative-parity states and $A=15$ positive-parity states; deduced level positions.

1. INTRODUCTION

The energy spectra of the $A = 14$ nuclei are characterized by the fact that the lowest $T = 0$ and $T = 1$ negative-parity levels form two distinctive quartets, each of which is well separated from other levels with the same isospin and parity. Numerous calculations and analyses of experimental data leave little doubt that these states are very predominantly, or even pure, closed ^{12}C shell plus $0p_{1/2}\,1s_{1/2}$ and $0p_{1/2}\,0d_{5/2}$ configurations.[1-7]

In the $A = 16$ spectra the $T = 1$ lowest negative-parity levels also form an isolated quartet, but not the corresponding $T = 0$ levels in ^{16}O. The first group can be described remarkably well as 1p-1h states with the $0p_{1/2}^{-1}\,0d_{5/2}$ and $0p_{1/2}^{-1}\,1s_{1/2}$ configurations highly predominant.[1,8-11] (See however Ref. 12.) The latter group have highly complex structures, requiring also appreciable 3p-3h mixing at least for most of the levels, in order to account especially for their low positions in the ^{16}O spectrum.[13-17]

The purpose of this article is to investigate whether the $A = 14$ quartets can be successfully related to the $T = 1$ quartet in $A = 16$. Several other aspects associated with this will be dealt with. Basically we are guided by the work of Goldstein and Talmi[18] and Pandya,[19] who related the energies of the low-lying quartet of negative-parity states of ^{38}Cl and ^{40}K through a, by now well-known, exact linear relationship between particle-particle and particle-hole interaction energies. Later work on these and other nuclei[20-22] confirmed the general success of this method, but at the same time, its limitations were accentuated. Energy states described as pure shell-model configurations are actually never just that, but small admixtures can often be "swept under the carpet," so to speak, by incorporating them in the phenomenological effective two-particle interaction. This might work when energy level positions alone are being evaluated, but very likely not in calculations on properties such as electromagnetic moments and transitions, which are usually very sensitive to small changes in wave functions. At present we consider energy values only and the levels concerned are generally regarded as presenting nearly pure states. Under these circumstances our assumption of exact purity seems in order. Our matrix elements for the effective two-particle interaction will then be simply related to those of Talmi and Unna,[1,23] although slight numerical differences will occur.

All the energy level data in this paper are from Ajzenberg-Selove,[24,25] supplemented where necessary by the binding energy data of Wapstra and Gove.[26] In ^{14}N the $(0,1)^-$ level at 4.91 MeV is assumed to be 0^-; further assumptions of this kind are indicated in our diagrams.

2. COULOMB ENERGY CORRECTIONS

Coulomb energy contributions to the energy levels of the $A = 12$–17 nuclei can be calculated straightforwardly if the average nuclear potential is assumed to be that of a harmonic oscillator and if the relevant energy levels represent pure configurations.[27-29] Let C_j^A represent the Coulomb interaction of a j proton with a closed-shell core with A nucleons, and $\Delta E_c(j_1 j_2, J)$ that between a j_1 and a j_2 proton. The expressions for these two quantities are given in Ref. 29, in units of $K = e^2(\nu/\pi)^{1/2}$, where ν is the oscillator strength parameter. Since we are concerned only with a small A interval, K can be assumed to be constant. We shall need only the following numerical values ($K = 0.345$ MeV):

$$C_p^{12} = 2.99 \quad C_d^{12} = 2.67 \quad C_s^{12} = 2.84$$

$$C_d^{16} = C_{d'}^{16} = 3.57 \quad C_s^{16} = 3.75$$

$$\Delta E_c(p^2, 0) = 0.52$$

$$\Delta E_c(pd, J) = \Delta E_c(ps, J) = 0.45 . \qquad (1)$$

Here, and hereafter, all numerical values for energies are given in units of MeV; further, the symbols p, s, d, d' are abbreviations for the $0p_{1/2}$, $1s_{1/2}$, $0d_{5/2}$, $0d_{3/2}$ orbits, respectively. The value in the last equation is merely an average one, the errors thus made (<3%) being of negligible consequence hereafter. From the values given we obtain the Coulomb energy of each level of present interest, relative to the ^{12}C or ^{16}O ground state, the value in the latter case being 6.50 MeV less than in the first.

The above method is based on the assumption of an infinite potential well, whereas a finite well would be more realistic. Such sacrifice for the sake of simplicity is most disastrous for levels close to nucleon emission threshold, which have proton configurations involving low orbital angular momentum. However, as is suggested by Fig. 1, a single type of correction to the calculated Cou-

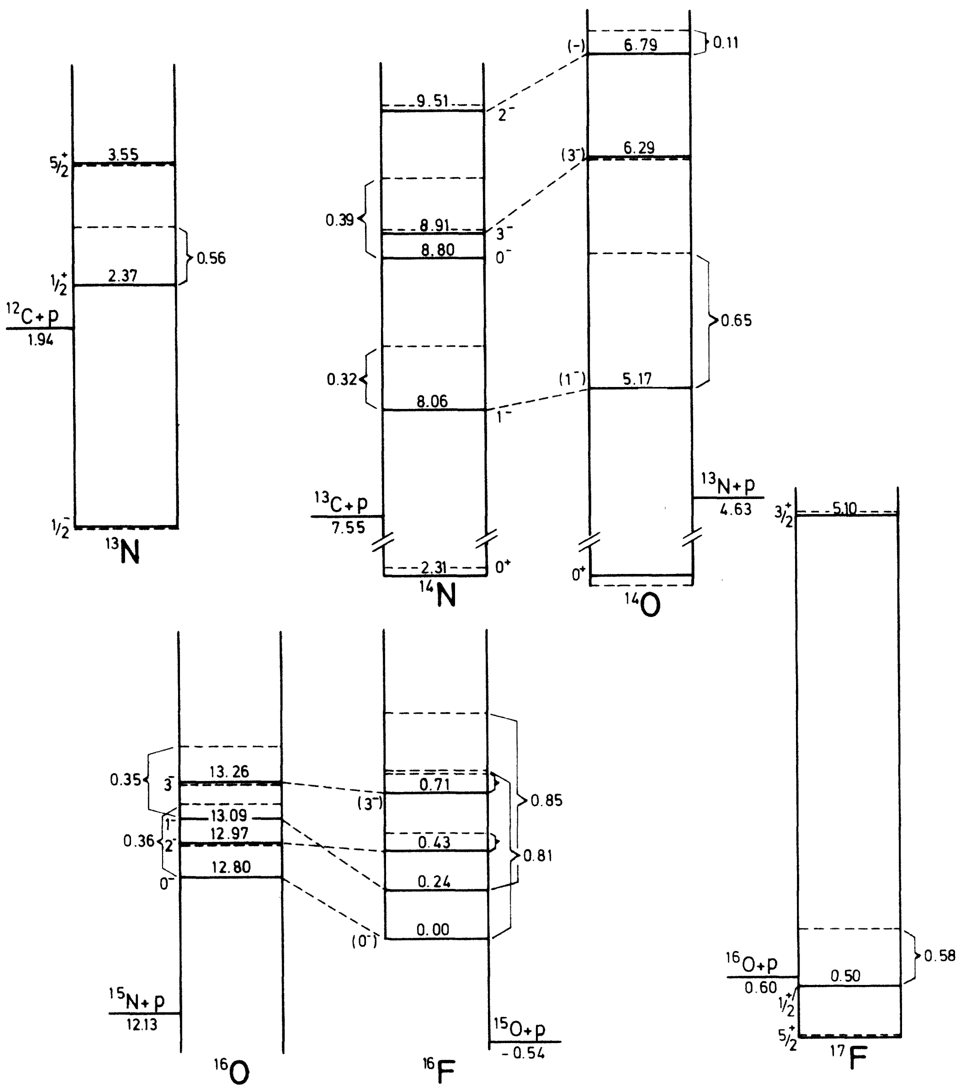

FIG. 1. Solid lines are experimental levels assumed to represent pure p^n and $p^n j$ configurations, $j = s,\ d,\ d'$; dashed lines give the positions derived, through Eq. (1), from the corresponding experimental levels of respectively ^{13}C, ^{14}C, ^{16}N, and ^{17}O.

lomb energies seems adequate in our region of interest. Practically all the energy levels shown lie more or less close to proton emission threshold, but Eq. (1) produces significant discrepancies (Thomas-Ehrman shifts) only for levels with $p^n s$ configurations. For these levels the shifts are rather close to an average of approximately 0.70 MeV, when the proton occupancy of the s level is 1, and 0.35 MeV when the occupancy is $\frac{1}{2}$ (as in ^{14}N and ^{16}O).

Thus we may regard the present method as essentially a two-parameter one; besides K there occurs only one further parameter, the TE shift, which is nonzero only whenever s levels close to proton emission threshold are involved, in which case it can be regarded as constant in the limited region under consideration. Hereafter we assume this constant to be equal to 0.70 MeV, or some definite fraction of it, depending on proton occupancy. Thus, with this extreme simplification as starting point, we can calculate the Coulomb displacement energies, which are as obtained from Eq. (1) minus the necessary TE shifts. The values thus obtained for the levels in Fig. 1 compare with experiment at least as favorably as in general the computer-calculated values of De Meijer, van Royen, and Brussaard.[30] Although our procedure is more empirical than theirs, their calculations involve more parameters, even if their electromagnetic shift estimate is not counted. Of course our method is equally adaptable to a wider range of nuclei than considered here.

As suggested by the small numerical differences between comparable quantities in Eq. (1), the Coulomb energy for a pure state may differ only slightly from that of one in which the relevant configuration is merely predominant, provided the admixtures are of a reasonable nature. Hence the excellent agreements in Fig. 1 are not necessarily proof of the exact, or even very high, purity of the levels concerned. It merely indicates that the assumption of exact purity will give reliable results for Coulomb energies at least. Figure 2 gives the Coulomb-corrected energy values for the presumably pure p^2 (or p^{-2}), $p^{\pm 1}s$, $p^{\pm 1}d$ states in $A=14$ and 16, the $T=1$ level positions being each

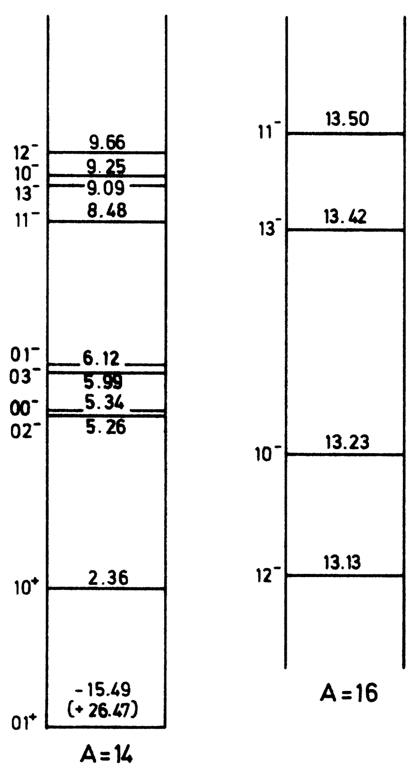

FIG. 2. Average Coulomb-corrected positions, relative to the ^{12}C(^{16}O) ground state, for p^2, ps, pd (p^{-2}, $p^{-3}s$, $p^{-3}d$) states in $A=14$, and the same, relative to the ^{16}O ground state, for $p^{-1}s$ and $p^{-1}d$, $T=1$, states in $A=16$. (Proton particle charges outside the reference core switched off, proton hole charges left on.)

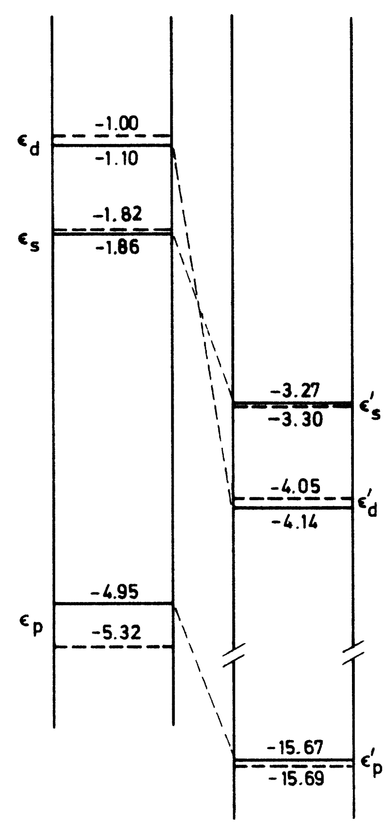

FIG. 3. Single-particle spectra for the ^{12}C and ^{16}O closed shells, respectively. Solid lines are experimental levels, dashed lines are levels as calculated from Fig. 2; $e'_p = -\epsilon'_p$.

the average of three values close together. The Coulomb energy for each $T=0$ level in $A=14$ is assumed to be the same as for its $T=1$ counterpart.[23] This may be questionable; especially the 00^- level of ^{14}N at 4.91 MeV is well bound (by more than 2.5 MeV), and its *TE* shift may not be significant. However, our end results will be affected only slightly if we do take the *TE* shift to be zero for, say, all the $T=0$ levels.

3. CALCULATIONS ON RELATIONS BETWEEN $A=14$ AND $A=16$ LEVELS

The energy values for the odd-parity levels in Fig. 2 are given by

$$E(pjTJ)=\epsilon_p+\epsilon_j+V(pjTJ)\,. \tag{2}$$

$$\begin{aligned}E'(p^{-1}jTJ)&=e'_p+\epsilon'_j+V(p^{-1}jTJ)\\&=e'_p+\epsilon'_j+\sum\alpha(pjTJ,T'J')V(pjT'J')\,,\end{aligned} \tag{3}$$

where

$$\alpha=-(2T'+1)(2J'+1)\begin{Bmatrix}\frac{1}{2}&\frac{1}{2}&T\\\frac{1}{2}&\frac{1}{2}&T'\end{Bmatrix}\begin{Bmatrix}\frac{1}{2}&j&J\\\frac{1}{2}&j&J'\end{Bmatrix}\,. \tag{4}$$

$$V(abTJ)=\langle abTJ|V_{12}|abTJ\rangle \tag{5}$$

$p=0p_{1/2}$, $j=0d_{5/2}$ or $1s_{1/2}$.

E is the value relative to the ^{12}C ground state, E' that relative to the ^{16}O ground state. The symbols ϵ and e signify neutron single-particle and single-hole energies, respectively; unaccented ones refer to energies in a field produced by the ^{12}C ground state core, accented ones refer likewise to the ^{16}O ground state core. Experimental values for these quantities, as provided by the neutron separation energies for the relevant levels in ^{13}C, ^{16}O, and ^{17}O, are given in Fig. 3. Given these values and the $A=14$ spectrum in Fig. 2, we calculate the level positions for the $p^{-1}j$ states in $A=16$ by combining Eqs. (2) and (3). Figure 4 shows the Coulomb-corrected results, the assumed 0.35 MeV *TE* shift for all $p^{-1}s$ levels of ^{16}O being

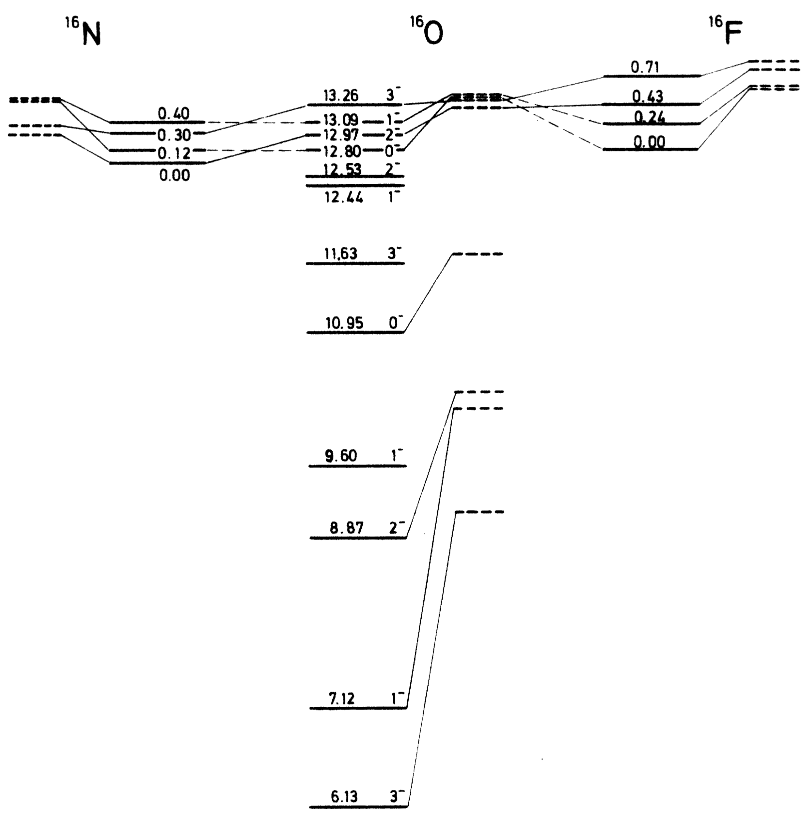

FIG. 4. Calculated $p^{-1}j$ level positions (dashed lines) compared to the experimental positions; the $T=0$ levels are connected to those with presumably dominant $p^{-1}j$ configurations. (Refs. 9, 17, 31). The three experimental 10^- levels are arbitrarily positioned at the same energy.

probably wrong, but of little relevance anyway, in the case of the 01^- level. The downward displacement, as the proton number increases, of the $p^{-1}s$ levels within the $T=1$ quartet is correctly reproduced, being slightly less than expected from the *TE* shift. The compressed nature of this quartet is also reproduced and hence this property can be regarded as due to the reversal of the order of the s and d levels in the single-particle spectrum, as the p shell is being filled, and to the reversed order of each pair of pj levels in the two quartets of $A=14$.

The calculated levels of the $T=1$ quartet lie slightly too high on the average, the determining quantities being

$$\Sigma_j = e_p' + \epsilon_j' + \epsilon_p + \epsilon_j \,. \tag{6}$$

Even a better average would relate to this sum only and not to the single-particle and hole energies individually. For consistency it is therefore appropriate to consider the following converse and practicable procedure, namely to calculate each single-particle and hole energy by relating the experimental energies for the pj quartets in $A=14$ to those of the $T=1$, $p^{-1}j$ quartet in $A=16$. We shall need only averages of these energies. Equations (2) and (3) give, for the weighted average over J,

$$\bar{E}'(p^{-1}j\,T=1) = \Sigma_j - \tfrac{1}{2}\bar{E}(pj\,T=0) - \tfrac{1}{2}\bar{E}(pj\,T=1)\,. \tag{7}$$

From the two spectra in Fig. 2 we obtain

$$\Sigma_s = 5.25, \quad \Sigma_d = 5.32\,. \tag{8}$$

Instead of Eq. (3), the $A=16$ level values are also given by

$$E(p^3j\,TJ) = 3\epsilon_p + \epsilon_j + 3\bar{V} + 3\sum \beta(pj\,TJ, T'J')V(pj\,T'J')\,, \tag{9}$$

where

$$\beta = (2J'+1)\begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ j & J & J' \end{Bmatrix}^2 \delta_{TT'} + (2T'+1)\begin{Bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & T & T' \end{Bmatrix}^2 \delta_{JJ'}, \tag{10}$$

$$\bar{V} = \langle p^2 | V_{12} | p^2 \rangle_{\text{av}} = \tfrac{1}{2}[V(p^2 01) + V(p^2 10)] = \langle p^{-2} | V_{12} | p^{-2} \rangle_{\text{av}}\,, \tag{11}$$

$$E(p^3jTJ) = E'(p^{-1}jTJ) + \mathrm{BE}(^{12}\mathrm{C}) - \mathrm{BE}(^{16}\mathrm{O}) - 6.50 = E'(p^{-1}jTJ) - 41.96\,. \tag{12}$$

As before, the average obtained from Eqs. (2), (9), and (12)

$$\bar{E}'(p^{-1}jT=1) - 41.96 = 3\bar{V} - 2\epsilon_j + \tfrac{1}{2}\bar{E}(pjT=0) + \tfrac{5}{2}\bar{E}(pjT=1) \tag{13}$$

yields

$$3\bar{V} - 2\epsilon_s = -6.70, \quad 3\bar{V} - 2\epsilon_d = -8.35\,. \tag{14}$$

Equations (8) and (14) give

$$\epsilon_d - \epsilon_s = 0.82, \quad \epsilon_d' - \epsilon_s' = -0.75\,. \tag{15}$$

This clearly reproduces the well-known reversal in the order of ϵ_s and ϵ_d as the p shell is being filled up; also the spacings between these two levels agree approximately with the experimental values.

These results are independent of the values for ϵ_p, e_p', and $\bar{V}$. The most probable values for these quantities are obtained by considering all the states which are usually regarded as approximately, if not pure, closed-shell core plus $p^{\pm n}$ configurations. Binding energies, Coulomb corrected according to Eq. (1), give the following relations under the assumption of exact purity:

$$\begin{array}{lll} A=12 & & 4e_p' + 6\bar{V} = 41.96 \\ A=13 & \epsilon_p = -4.94 & 3e_p' + 3\bar{V} = 37.02 \\ A=14 & 2\epsilon_p + \bar{V} = -14.31 & 2e_p' + \bar{V} = 27.65 \\ A=15 & 3\epsilon_p + 3\bar{V} = -26.31 & e_p' = 15.65 \\ A=16 & 4\epsilon_p + 6\bar{V} = -41.96\,. & \end{array} \tag{16}$$

The numerical values for $A=13, 15$ are averages. For $A=14$ the $T=0$ and 1 level values (given in Fig. 2) have been averaged over. The least-squares solutions (in agreement with Talmi and Unna[1]) are:

$$\epsilon_p = -5.32 \quad \bar{V} = -3.45 \quad e_p' = 15.69 \tag{17}$$

with standard errors of the order of 0.1.

The combination of Eqs. (8), (14), and (17) yields the "theoretical" single-particle energy spectra shown in Fig. 3, indicating clearly the lowering of the levels as the p shell is being filled up. The only discrepancy worth mentioning is that for ϵ_p. In this connection it may be mentioned that the ^{16}O ground state is a major closed shell and the three relevant levels in the adjacent nuclei should therefore be well described as pure single-particle and hole configurations. This is clearly confirmed by experimental evidence, as summarized by Bohr and Mottelson.[32] Equation (17) is therefore reliable in the sense that they are almost exactly

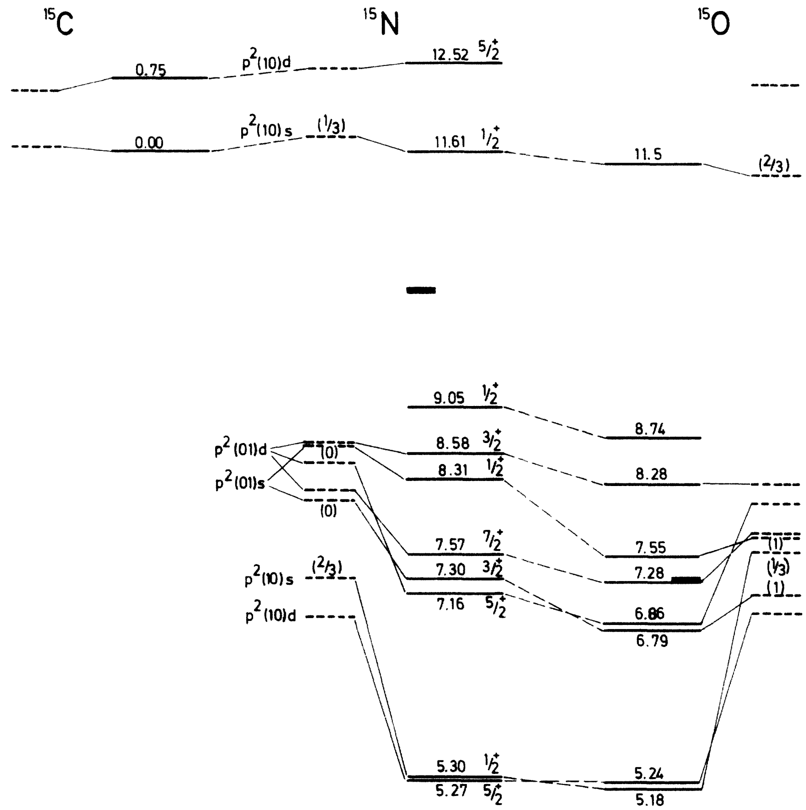

FIG. 5. Calculated p^2j level positions (dashed lines), the correspondence between these and the experimental levels indicating the accepted dominant configurations for the latter. (Refs. 33–35). The s orbit proton occupancy for each calculated p^2s level, needed to estimate the TE shift, is given in brackets. The two thick lines indicate the thresholds for proton emission. The $\frac{3}{2}\ \frac{1}{2}^+$ experimental levels of ^{15}C and ^{15}N are arbitrarily positioned at the same energy.

the solutions of the last three equations in Eq. (16) alone. By the same measure, however, one should not necessarily expect the three relevant levels of ^{13}C all to be so pure as to give equally reliable single-particle energies. The lower ϵ_p value of Eq. (17) is just about right for obtaining good agreement between calculated and experimental averages of the $T=1$ quartets in Fig. 4.

4. LOW-LYING POSITIVE-PARITY STATES IN $A = 15$ NUCLEI

Pure p^2s and p^2d state energy values in $A=15$ nuclei are given by

$$E[p^2(T_0J_0)j,\,TJ]=2\epsilon_p+\epsilon_j+V(p^2T_0J_0)+2(2T_0+1)(2J_0+1)\sum(2T'+1)(2J'+1)\begin{Bmatrix}\frac{1}{2}&\frac{1}{2}&T_0\\ \frac{1}{2}&T&T'\end{Bmatrix}^2\begin{Bmatrix}\frac{1}{2}&\frac{1}{2}&J_0\\ j&J&J'\end{Bmatrix}^2 V(pjT'J') \tag{18}$$

(or the equivalent expression for $p^{-2}j$ relative to the ^{16}O core); to this must be added the necessary Coulomb energy corrections. We replace $V(pjT'J')$ in terms of $E(pjT'J')$, according to Eq. (2), and $V(p^2T_0J_0)=V(p^{-2}T_0J_0)$, according to:

$$E'(p^{-2}T_0J_0)=2e_p'+V(p^2T_0J_0)=E(p^2T_0J_0)+41.96\,. \tag{19}$$

In this manner we obtain the energy spectra for $A=15$, shown in Fig. 5, from the experimentally based energy values of Figs. 2 ($A=14$) and 3, but without having to use the somewhat problematical ϵ_p value.

As in $A=16$, the discrepancies between theoretical and experimental level positions increase as we get down to the lower levels. Any mixing of the p^2j configurations among themselves will not improve matters for the thus affected level pairs $T=\frac{1}{2}$, $J^\pi=\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$, since such mixing alone will not alter the already too high mean position of each pair.

Our picture, for the six lowest levels shown, is generally consistent with that of Lie, Engeland, and Dahll,[36] insofar as their results indicate roughly increasing mixtures of (relative to the ^{16}O core) 3p-4h configurations with decreasing energy. Thereby the four lowest level positions are approximately correctly given. However, their configuration assignments for the other levels are not accompanied by a similar agreement between theoretical and experimental level positions, and further comparison with our results is perhaps not sensible.

Saayman and De Kock[35] have done several types of calculations from which they conclude that 3p-4h configurations are not essential for a better understanding of the level structure of Fig. 5. Their calculations III and IV do include such configurations, and our results seem to be most consistent with the latter, which is the least restrictive. The two lowest levels contain the largest 3p-4h mixtures, which is consistent with their pronounced depression relative to our theoretical level positions. The levels that come closest to being described as pure *and* having their energies correctly fitted are the levels at 8.31/7.55, 8.58/8.28, and 11.61/11.5 MeV. Our results are in agreement with this.

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