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Nuclear structure of ^{46}K: Studies with ^{48}Ca(d, $\alpha\gamma$)^{46}K and deuteron-transfer reactions*

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The reactions ^{48}Ca(d, α)^{46}K and ^{48}Ca(d, $\alpha\gamma$) were studied at $E_d = 17$ and 7 MeV, respectively. Angular distributions for the ^{48}Ca(d, α) reaction were obtained in 5° steps from $\theta_{lab} = 10°$ to $\theta_{lab} = 145°$. Data for $\theta \leq 60°$ were taken with a split-pole spectrograph with resolutions of 13–25 keV. Only the levels at 0, 0.587, 0.691, 0.886, 1.370, 1.738, and 1.941 MeV were seen significantly above background. Excitation functions for $16 \leq E_d \leq 17$ MeV at $\theta = 90$ and 100° showed energy dependent cross section variations of $\lesssim \pm 15\%$ for the ground state and 0.587 MeV level, larger ones for the 1.738 MeV state and negligible variations for the 0.691, 0.886, and 1.941 MeV levels. The level energies listed were also deduced from the fast α-γ coincidence spectra which also served to set independent spin limits. A simultaneous microscopic distorted wave Born approximation analysis for the (d, α) data and the ^{48}Ca(p, ^{3}He) and ^{48}Ca(p, t)^{46}Ca ($T = 4$) cross sections previously studied at $E_p = 42$ MeV was performed. Under the assumption that ^{48}Ca is doubly magic theoretical and empirical wave functions for the six low-lying negative parity states of ^{46}K were deduced and used to derive empirical $(s_{1/2}f_{7/2})^2$, $(d_{3/2}f_{7/2})^2$, and $(s_{1/2}f_{7/2}, d_{3/2}f_{7/2})$ residual n-p interaction matrix elements. The ^{48}Ca(d, α) reaction at 17 MeV, as well as the ^{48}Ca(p, ^{3}He) and the ^{48}Ca(p, t) transitions at $E_p = 42$ MeV show some features that cannot be explained by one-step direct two-nucleon transfer. The excitation of unnatural parity states in ^{48}Ca(p, t)^{46}Ca ($T = 4$) and the importance of $L > j - 1$ transitions for the corresponding states in ^{46}K point to significant two-step contributions. Similarly, the triplet ($\Delta S = 1$), ($\Delta T = 0$) transfers in (p, ^{3}He) are underpredicted with respect to singlet transfers.

NUCLEAR REACTIONS Measured ^{48}Ca(d, $\alpha\gamma$)^{46}K fast α-γ coincidence spectra, $E_d = 7$ MeV, resolution 4–8 keV. Deduced ^{46}K level energies, J limits. Measured ^{48}Ca(d, α)^{46}K, $\sigma(\theta, E)$, $E_d = 17.0$ MeV, resolution 13–30 keV. Microscopic DWBA analysis for ^{48}Ca(d,α), ^{48}Ca(p,^{3}He), and ^{48}Ca(p, t)^{46}Ca ($T = 4$). Deduced: J^π values, ^{46}K wave functions, residual interaction matrix elements.

I. INTRODUCTION

The expected simplicity of ^{46}K in the shell model interpretation has motivated various studies of this nucleus in spite of its limited accessibility. In the earliest interpretations the lowest states of ^{46}K were viewed as members of a pure $\pi d_{3/2}^{-1}\nu f_{7/2}^{-1}$ multiplet.[1] This model implied that other configurations, such as $s_{1/2}^{-1}f_{7/2}^{-1}$ or $d_{5/2}^{-1}f_{7/2}^{-1}$, etc., lie considerably higher in energy and produce negligibly small admixtures to the ground state and lower excited levels of ^{46}K; however, information gathered since the initial discovery of this isotope by Marinov and Erskine[2] gives reason to doubt the applicability of this very simple picture.[3-5] Contrary to expectations from the ^{38}Cl($d_{3/2}f_{7/2}$) particle-particle spectrum it was found that the first excited state of ^{46}K (at 0.587 MeV) is not a 5^- level, but probably a 3^- state, while the expected 5^- level seems to lie at 0.886 MeV, i.e., 216 keV higher than the corresponding 5^- level in ^{38}Cl. Recent publications[6,7] confirm these initially unexpected spin assignments and produce strong evidence that at least $\pi s_{1/2}^{-1}\nu f_{7/2}^{-1}$ admixtures to the $\pi d_{3/2}^{-1}\nu f_{7/2}^{-1}$ multiplet must be considered. The present paper aims to justify previous J^π suggestions for the unnatural parity states[6] and presents an attempt to obtain quantitative information on the sf-df mixing in ^{46}K. To the extent that reliable and consistent empirical wave functions for ^{46}K can be found the effective two-nucleon $(d_{3/2}f_{7/2}, d_{3/2}f_{7/2})$, $(s_{1/2}f_{7/2}, s_{1/2}f_{7/2})$ and $(s_{1/2}f_{7/2}d_{3/2}f_{7/2})$ matrix elements in the $s_{1/2}$-$d_{3/2}$-$f_{7/2}$ shell model space can be deduced. It is of interest to compare such empirical values for ^{46}K with experimental and theoretical matrix elements obtained for lighter elements.

II. ^{48}Ca(d,$\alpha\gamma$)^{46}K EXPERIMENT

A. Procedure

A 0.9 mg/cm² self-supporting Ca target, enriched to 96% in ^{48}Ca, was bombarded by a well focused deuteron beam with $E_d = 7$ MeV. α particles from the ^{48}Ca(d, $\alpha\gamma$) reaction were detected at $\theta \approx \pm 135°$ in the laboratory system by two large heavy-ion surface-barrier counters with 50 μm depletion depth. The choice of these high-field counters with a shallow and well defined depletion depth permitted the detection of the α particles of interest with full energy ($\leq$8 MeV), whereas the numerous deuterons, protons, and tritons entering the counter lost only a fraction ($\leq$3 MeV) of their energy. Only pulses corresponding to α energies of 4 MeV or higher were accepted. α resolution was limited by the thick target and by kinematic broadening to about 800 keV. However, this proved adequate to distinguish α groups populating the 0.587, 0.691, and 0.886 MeV levels from those populating higher levels, in particular the 1.738 and 1.941 MeV states. The deuteron beam was stopped by an insulated collector about 5 m behind the target.

γ rays from the ^{48}Ca(d, $\alpha\gamma$) reaction were detected by a 40 cm^3 Ge(Li) counter placed at 90° with respect to the beam direction. Since the great majority of γ counts did not result from the (d, α) reaction, chance coincidence rates had to be kept small. Standard fast-slow coincidence techniques were employed at relatively low counting rates. The experimental coincidence time peak was 15 nsec full width at half-maximum (FWHM) and was centered in the accepted time window of 40 nsec. The estimated true to chance coincidence ratio was ≈50. The data were accumulated on magnetic tape in a two-parameter event-by-event mode with 1024 channel resolution for γ rays and 256 channel resolution for α particles. Coarse resolution monitoring of the coincident events was carried out with a 4096 channel two-dimensional analyzer (ND-160). The experiment was repeated at different counting rates and with a different set of α detectors. In each run a coincidence γ spectrum as shown in Fig. 1 was obtained. The high neutron background from ^{48}Ca + d set a practical limit for γ statistics.

B. Experimental results

With typical (total) γ counting rates of about 4000 counts per sec γ resolutions of 4–5 keV were obtained for low energy γ rays. Resolution for high energy γ rays (~1.9 MeV) was consistently worse, although tests with radioactive sources (such as ^{56}Co) at similar counting rates still produced resolutions of approximately 6–8 keV for γ rays near 2 MeV for the Ge(Li) counter used. It is of interest that the 1941 γ ray in all ^{48}Ca(d, $\alpha\gamma$) runs was consistently much broader, i.e., ~15 keV. (See Fig. 1.) It cannot be ruled out that much of this width is due to amplifier-analyzer gain drifts during the long runs needed for reasonable statistics; however a more probable explanation would be to assume that the main contribution to the peak width comes from Doppler shifts which go in opposite directions for γ rays in coincidence with the (summed) α particles measured at θ = +135° and at −135°, respectively. If most of the 1941 keV line width is due to Doppler effects the full shift is seen, in agreement with the suggestion

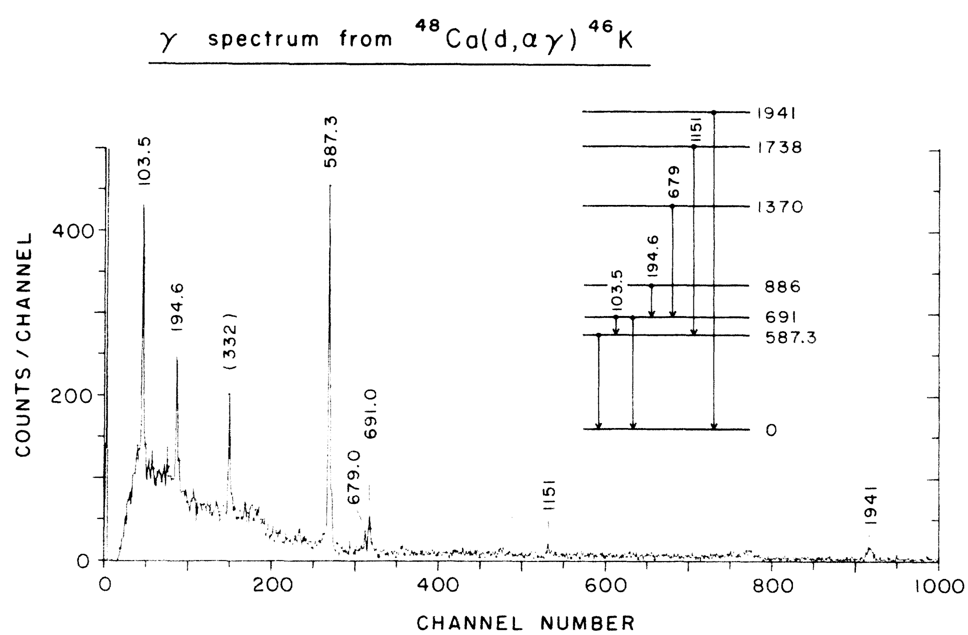

FIG. 1. Spectrum of all γ rays in fast coincidence with 4–8 MeV α's from the ^{48}Ca(d,$\alpha\gamma$)^{46}K reaction at E_d = 7 MeV. The insert shows the γ decay spectrum deduced from energy balance considerations and stripping of the two-dimensional E_α-E_γ data set. The 332 keV transition stems from the ^{40}Ca target impurity (see text).

below that the transition has $E1$ multipolarity. The symmetry of the apparatus assures that the centroids of the peaks are not Doppler-shifted. It should be mentioned that a known ~1945 keV γ ray from the $^{40}Ca(d, \alpha\gamma)^{38}K$ impurity reaction might intrude and broaden the 1941 keV ^{46}K line; however, intensity considerations based on the known $^{40}Ca(d, \alpha)^{38}K$ spectrum[8] and ^{38}K γ decay[9] make such an explanation very unlikely. Data stripping with respect to α energies yielded the decay scheme shown in the insert of Fig. 1.

The 1941 and 1151 keV γ peaks do not stand out very sharply in Fig. 1 because of the limited resolution and detector efficiency at higher γ energies, but integrated peak counts of about 120 ± 15 and 50 ± 20, respectively, (after background subtraction) were measured in each of three separate coincidence runs.

The distinct 332 keV transition does not fit into the ^{46}K level scheme found for the $^{48}Ca(d, \alpha)$ reaction, although it definitely is in fast coincidence with α particles in the 5–7 MeV windows. The explanation offered is that it results from the decay of the 460 keV (1^+) level which is very strongly excited in the $^{40}Ca(d, \alpha)^{38}K$ impurity reaction (^{40}Ca is present in the target with 4% abundance). This 332 keV line was found to be by far the dominant line in the $^{40}Ca(d, \alpha\gamma)^{38}K$ γ spectrum measured at 4.4 MeV.[9] The 460 keV ^{38}K state was also found to be extremely strongly excited in $E_d = 7.7$ MeV[8] and in our own $^{40}Ca(d, \alpha)^{38}K$ spectra taken at 17 MeV.[10] We measure the excitation energy of this strong ^{38}K level as 460 ± 5 keV in slight disagreement with the value given in Ref. 8. But our energy is in very good agreement with a recent compilation[11] which lists the excitation of the $T = 0, J^\pi = 1^+$ state as 461 ± 2 keV and gives the first excited state ($T = 1, J^\pi = 0^+$) as 131 ± 2 keV. The difference is 330 ± 3 keV, in good agreement with the observed 332 ± 1 keV γ line. In retrospect it is not hard to explain why the coincident α particles were not seen at their higher energy, expected from the more positive $^{40}Ca(d, \alpha)$ Q values ($Q_0 = +4.650$ MeV): As mentioned above the α detectors were run at 50 μm depletion depth, hence they would not stop α particles above 8 MeV. A 10 MeV α particle would, therefore, lose only about 5.7 MeV in the depleted region.

The measured ^{46}K γ energies together with their relative intensities corrected for the Ge(Li) detector efficiency are listed in *Table I*. All transitions were seen in fast coincidence so that their half-lives are $t_{1/2} \ll 10^{-7}$ sec. Hence all observed transitions should have $E1$, $M1$, or $E2$ character.

III. $^{48}Ca(d, \alpha)^{46}K$ ANGULAR DISTRIBUTIONS AT $E_d = 17$ MeV

A. Experimental procedure

Angular distributions for the $^{48}Ca(d, \alpha)$ reaction were measured in 5° steps for $10° \leq \theta_{lab} < 145°$ in our 46 cm scattering chamber with three cooled, fully depleted (Si) transmission detectors of 200–300 μm thickness (a thicker, fourth, detector served as a monitor). In addition, data for $\theta \leq 60°$ were retaken with the Enge split-pole spectrograph with position sensitive surface barrier counters in the focal plane. The procedure for the latter mode of high-resolution measurements was described previously.[12] Spectrograph analysis permitted the elimination of pile-up background from elastically scattered deuterons and enabled us to obtain high resolution for a number of spectra.

Resolution for the spectrograph runs was determined by target thickness, target condition, and nonuniformity. Spectrograph spectra typically had a resolution of 20–25 keV FWHM. Resolution for

TABLE I. Energies and relative intensities of γ decays in fast coincidence with α particles from the $^{48}Ca(d, \alpha)^{46}K$ reaction, corrected for detector efficiency.

γ energy (keV)	Initial → Final state state (keV)	Intensity[a] (relative to the 587.3 → 0 transition)	Branching
103.5 ± 0.5	691 → 587.3	31 ± 7	→72%
194.6 ± 0.5	886 → 691	6.3 ± 1	
587.3 ± 0.5	587.3 → g.s.	100	
679.0 ± 0.5	1370 → 691	5.0 ± 1.3	
691.0 ± 0.5	691 → g.s.	11.9 ± 2.2	→28%
1151 ± 2	1738 → 587.3	6.6 ± 2.3	
1941 ± 2	1941 → g.s.	29 ± 12	

[a] The errors given for the relative intensities reflect counting statistics (dominant for the weak transitions) and uncertainties in the detector efficiency (important for the strong transitions). Corrections for $(d, \alpha\gamma)$ angular dependence were not made.

the thin-detector runs in the scattering chamber ranged from 30 to 50 keV depending on scattering angle. It is apparent from Fig. 2 that even with 50 keV resolution all ^{46}K states investigated would be well resolved, although background peaks from the ^{40}Ca and ^{16}O impurities in the target would interfere more severly with the analysis. Nevertheless, the unexpected angular distributions obtained for the g.s. and 691 keV groups prompted us to make several attempts at very good resolution. Using a new, very thin ^{48}Ca target two spectra with 13 keV resolution were taken. No indication for a doublet nature of any of the strong peaks was found.

Targets used for the ^{48}Ca(d, α)^{46}K angular distributions consisted of 60 to 100 μg/cm^2 thick CaO layers on 20 μg/cm^2 carbon backings. Calcium was 96% isotopically enriched in ^{48}Ca. Target thicknesses were found by elastic deuteron scattering and comparison to known cross sections.

Target condition and beam current were monitored at all times by the simultaneous use of an (elastic)scattering monitor detector and charge integration. Good agreement of both monitoring methods indicated that the targets did not deteriorate during the runs.

The Q value for the ^{48}Ca(d, α)^{46}K reaction is $Q_0 = +1.915$ MeV,[2] and particle type selection in the scattering chamber runs could be accomplished by range discrimination. α particles from the ^{48}Ca(d, α) reaction had a range of up to 200 μm for 17 MeV deuteron energy and lost their full energy in the 200–300 μm detectors used. In the same detectors tritons would at most lose 9.5 MeV. Thus, potentially, α groups from ^{46}K levels up to 9 MeV excitation might have been observable since the Q value for ^{48}Ca(d, ^{3}He) is -10.31 MeV. However, copious α groups from the carbon backing [Q_0 for ^{12}C(d, α) = -1.341 MeV] and ^{16}O obscured the spectra at energies corresponding to $E^* > 4$ MeV excitation in ^{46}K. This problem, of course, was also present for the spectrograph data.

B. Experimental errors

Uncertainties in the target thickness account for the dominant scale error of ±15% in the absolute cross sections for the ^{48}Ca(d, α) data. Other systematic errors could be kept well below 5%. Random errors from statistics and background subtraction (mostly impurity peaks) varied greatly and are indicated by error bars in the figures.

Excitation energies in the (d, α) spectra could be measured to an accuracy of ±3 keV for levels below and to ±5 keV for levels above 1 MeV. The values obtained agreed well within these limits with the more accurate (<±2 keV) energies deduced from our (d, $\alpha\gamma$) study. Hence only the latter are listed in our figures and tables.

C. Experimental differential cross sections for ^{48}Ca(d,α)^{46}K

The 17 MeV ^{48}Ca(d, α) spectra permitted extraction of angular distributions for seven groups be-

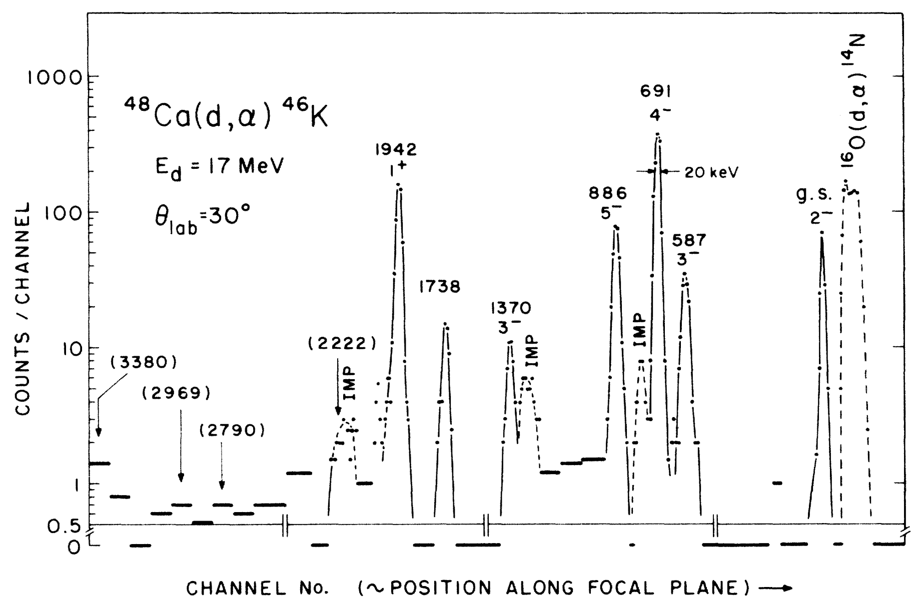

FIG. 2. Typical semilog spectrum for ^{48}Ca(d,α) observed with (4) position sensitive detectors in the focal plane of the split pole spectrograph. Horizontal bars denote averages over regions with statistically insignificant counts per channel. Impurities (indicated by dotted lines) were ^{16}O, ^{40}Ca, and ^{12}C in order of importance.

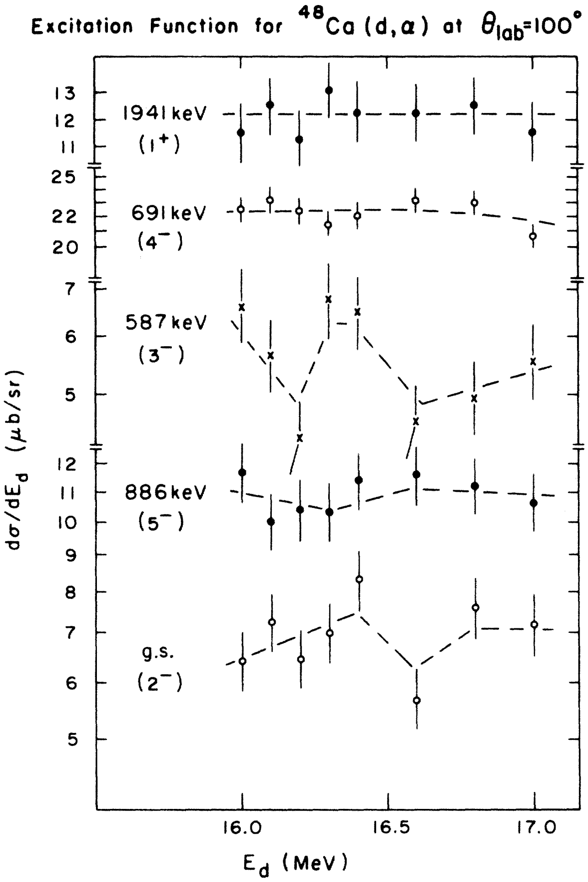

FIG. 3. Semilog plot of $d\sigma/dE_d$ for ^{48}Ca(d,α) as a function of deuteron energy. The angle of observation was $\theta_{lab} = 100°$. A similar behavior was seen at $\theta_{lab} = 90°$. Fluctuations were significantly smaller at forward angles (i.e., inside statistics), except for the weak 1.738 MeV level (not shown).

low 2 MeV excitation, i.e., for transitions leading to the ground state and the 587, 691, 886, 1370, 1738, and 1941 keV levels. ^{46}K excitation energies for these groups were determined for most angles between 20 and 90° and were found to be constant, with an accuracy that ruled out the possibility that they derived from the ^{40}Ca(d, α) reaction. The same constancy was not found for the other weak groups in Fig. 2 labeled "impurity." It is, of course, possible that beneath or near these impurity peaks other very weak transitions to ^{46}K are hidden, for instance near 2222, 2790, 2969, and 3383 keV excitation, where ^{46}K states have been seen in the ^{48}Ca$(p, {}^3$He) experiment,[6] but these peaks would be about 2 orders of magnitude weaker than the strongest (691 keV) group seen in ^{48}Ca(d, α). Only at 3620 (±12) keV did we find a statistically significant group of higher excitation ($\sigma_{max} \approx 5$ μb/sr) at several angles.

Comparison with a spectrum taken at 12 MeV and inspection of the angular distributions suggested that some of the weaker levels, in particular the 1738 and 587 keV states may have nondirect contributions at back angles. In order to check the importance of compound-nuclear contributions, excitation functions with ~6 keV energy resolution were measured at two back angles, $\theta_{lab} = 90$ and 100°. The 100° data are presented in Fig. 3. It is seen that the stronger transitions show no significant energy dependence whereas the 587 keV and ground state transition seems to exhibit ±15% fluctuations. Detailed excitation curves at $\theta = 100°$ for the very weak transitions to the 1370 and 1738 keV states could not be obtained; however, the 1738 keV transition was found to fluctuate strongly with energy even at forward angles, being enhanced by about a factor of 5 as the bombarding energy is lowered to 12 MeV. The 1370 keV level showed no fluctuations outside the rather large (±25%) statistical errors.

D. DWBA calculations

Calculated and measured angular distributions for the seven strongest (d, α) transitions at 17 MeV are shown in Fig. 4. The solid curves represent conventional direct, one-step transfer calculations in the distorted wave Born approximation (DWBA) performed with code DWUCK. In order to compute transfers of two-particle clusters of different spin S and isospin (T, N) the output of DWUCK II was multiplied by isospin coupling coefficients and the well known statistical factors.[13] For the pickup reactions $A(a, b)B$ we used

$$\sigma_{\text{DWBA}} = Na^2(S,T)\langle T_B N_B TN \mid T_A N_A\rangle^2 \times \frac{2S_b+1}{2S_a+1}\,\frac{1}{2J+1}\,\sigma_{\text{DWUCK}},$$

where N remains a reaction dependent over-all normalization. The factor $a^2(S, T)$ is discussed below. J is the total angular momentum of the transfered cluster, and here equal to that of the final state. DWUCK II computes the microscopic form factor according to the method of Bayman and Kallio.[13] Form factors with $d_{3/2}f_{7/2}$ and $s_{1/2}f_{7/2}$ or $d_{3/2}{}^2$, $s_{1/2}{}^2$, and $s_{1/2}d_{3/2}$ configurations were used for the negative or positive parity states, respectively.

The shape of the angular distributions obtained from microscopic deuteron transfer calculations was not sensitive to the mixture of the configurations considered, neither is there a pronounced Q dependence; but the predicted angular distributions are sensitive to the optical model parameters used. Nevertheless, as long as the parameters are realistic, e.g. derived from optical model fits to existing data, such as ^{48}Ca(d, d) and ^{48}Ca(α, α)

or ^{48}Ti(α, α), and one does not switch to a completely different parameter family, i.e., other discrete ambiguities,[14] the position of the various *dominant* maxima and minima is changed little and remains in fair agreement with experiment. One does, however, obtain significant changes of the relative height of the main and secondary maxima for different choices of optical model parameters. The use of finite-range and nonlocality corrections in the local energy approximation, which reduce contributions from the nuclear interior, merely affects the slope of the angular distributions. In the calculations shown in Fig. 4 standard nonlocality parameters of $\beta_i = 0.54$ for deuterons and $\beta_0 = 0.2$ for α particles were used. The finite-range parameter R (as used in code DWUCK[13]) was set to $R = 0.4$ for consistency with our earlier (d, α) calculations[14] and kept at zero for the (p, t) and (p, ^{3}He) calculations. It had no pronounced effect on the angular distributions beyond increasing their falloff by about 10% over the region shown, compared to zero-range calculations.

The procedure of careful "well matching" recently suggested for the elimination of parameter ambiguities[14] was difficult to implement for ^{48}Ca(d, α) at $E_d = 17$ MeV. ^{48}Ca(d, d) scattering potentials do not follow typical (d, d) systematics at this energy. Restriction of r_0 and a to $r_0 = 1.20$ and $a = 0.75$ fm leads to an unusually shallow real well ($V_0 = 74.1$ MeV).[15] On the other hand, relatively poor ^{48}Ca(α, α) fits to 30 MeV ^{48}Ca(α, α) data were found if r_0 was restricted to $r_0 < 1.2$ fm. It must, furthermore, be noted that ^{48}Ca(α, α) potentials may differ substantially from the potentials for ^{46}K(α, α) actually

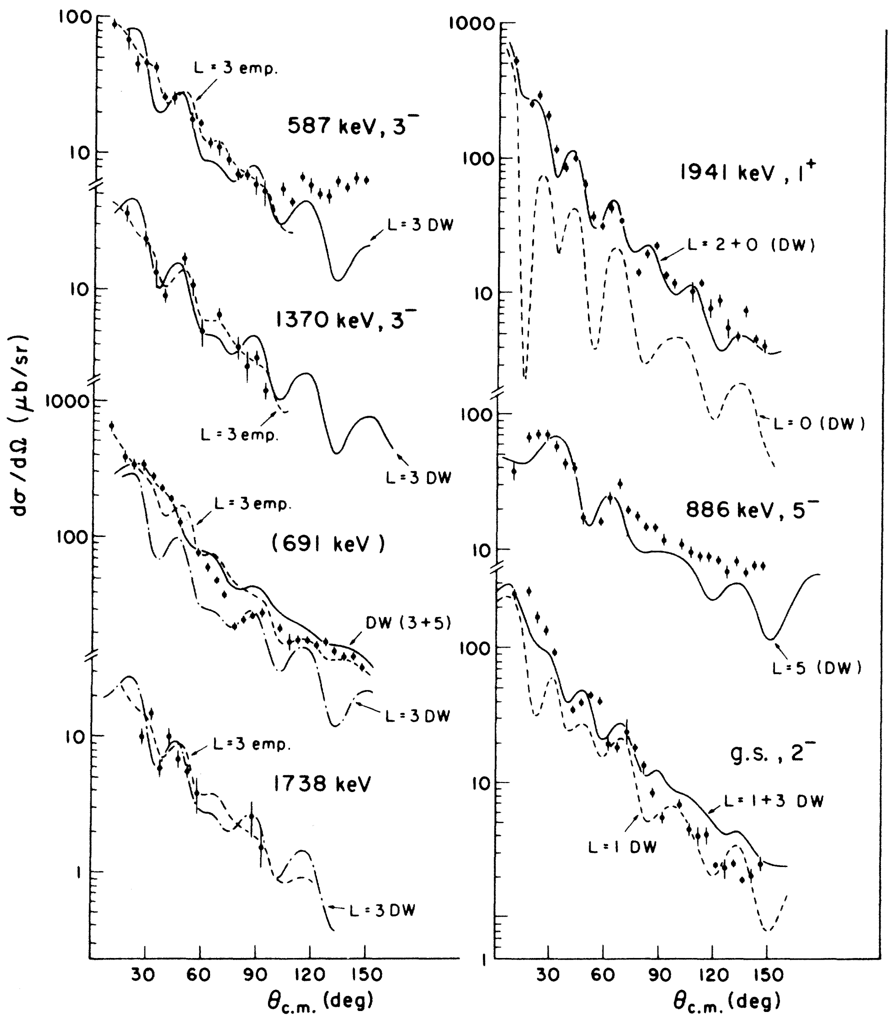

FIG. 4. Experimental ^{48}Ca(d, α)^{46}K angular distributions at $E_d = 17$ MeV compared with DWBA calculations and empirical $L = 3$ curves. For DWBA curves which are the sum of two L transfers the dominant contributor is also shown separately. Experimental error bars include statistics, random monitoring errors, and estimated errors due to background subtraction (in quadrature).

TABLE II. Optical model parameters used for the DWBA calculations. The notation is the same as that of Becchetti and Greenlees (a).

Particle type	E (MeV)	V	r_0	a	W	$4W_D$	r_I	a_I	r_c	λ_{so}	Source
p	41.65	46.7	1.17	0.75	6.53	13.2	1.32	0.62	1.25	⋯	a
d	17.0	96.38	1.0	0.9137	⋯	77.55	1.476	0.5208	1.3	⋯	b
t, ^{3}He	(20)	166.0	1.16	0.75	16.7	⋯	1.498	0.817	1.25	⋯	c
α	(30)	213.2	1.33	0.622	23.05	⋯	1.33	0.622	1.3	⋯	d
Single particle bound state well, e		f	1.25	0.75	⋯	⋯	⋯	⋯	1.3	25	

[a] F. D. Becchetti and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).
[b] See Ref. 15.
[c] E. R. Flynn, D. D. Armstrong, J. G. Beery, and A. G. Blair, Phys. Rev. 182, 1113 (1969).
[d] A. Bernstein, private communication.
[e] Binding energy computed as $\frac{1}{2}[Q(\gamma,d)-E^*]$ for triplet transfer and as $\frac{1}{2}[Q(\gamma,np)-E^*]$ for singlet transfer.
[f] Adjusted to give the correct separation energy.

needed in the calculations. Hence the potentials available (e.g. the deuteron and α parameters shown in Table II) are deficient in some respects, and our less than satisfactory fits may partly be explained in this way.

However, the main difficulty is likely to be found elsewhere: It is easily verified that DWBA calculations with reasonable wave functions for ^{46}K, for instance those computed by Bertsch[16] and presented in Table III, predict pure or practically pure $L=3$ transitions to the four 3^- and 4^- states, and a practically pure $L=1$ transition for the 2^- ground states. On the other hand, it is apparent that of the six transitions which lead to likely members of the mixed $s_{1/2}f_{7/2}$ and $d_{3/2}f_{7/2}$ multiplets only two (instead of four) agree with each other, i.e., with the empirical $L=3$ curve and resemble the theoretical $L=3$ angular distribution (see Fig. 4). The strong 691 keV (4^-) transition, however, seems to have a very substantial higher L (=5?) admixture. Similarly, the (2^-) ground state transition has a large higher L (presumably $L=3$) admixture. In ^{48}Ca(d, α) such higher L admixtures are allowed by selection rules in principle, but their observed magnitude can only be explained if there is a noticeable two-step transfer, for instance of the type ^{48}Ca(d, t)(t, α)^{46}K.

The presence of a two-step transfer contribution would also explain why a combination of empirical $L=3$ and $L=5$ angular distributions does not produce better agreement with the observed transition to the 4^- state than the addition of the presumably imperfect DWBA cross sections: If two-step and single-step transitions compete, the resultant cross section would not in general be reproducible as the simple sum of $L=3$ and $L=5$ cross sections. The importance of two-step "stripping" contributions to direct transfers has been demonstrated for (^{3}He, t) reactions.[17] More recently the observation of spin forbidden ^{48}Ca(p, t) transitions to the isobaric analogs of the unnatural parity ^{46}K states under discussion[6] has provided additional evidence that two-step processes must be considered even where transitions through the inelastic channel are not likely.

In the subsequent discussion we shall assume that two-step processes are primarily responsible for deviations from pure L shapes without affecting *drastically* the cross section at the *forward stripping peaks*. The latter also have been found rather insensitive to the choice of optical model parameters—apart from an over-all normalization variation of ±50%. Subject to the uncertainties in this hypothesis it is possible to use the ^{48}Ca(d, α), ^{48}Ca(p, ^{3}He), and ^{48}Ca(p, t) data to check shell model wave functions for ^{46}K and propose new wave functions that would lead to better agreement with the experimental transfer data.

IV. DISCUSSION OF ^{46}K TWO-HOLE STATES

A. J^π assignments of ^{46}K

In interpreting the level scheme of ^{46}K we start with the observation that: (a) the levels shown in Fig. 1 (insert) have now been seen independently (i) in various laboratories, (ii) by various reactions, i.e. (d, α) (p, ^{3}He), and (d, $\alpha\gamma$); and (b) that there are no indications that (deuteron) pickup populates any levels below 2 MeV apart from those shown.

Published ^{48}Ca(p, ^{3}He)^{46}K work[6] and studies of ^{48}Ca(p, t) transitions to the analogs of low-lying

^{46}K levels[6,7] at 42 MeV bombarding energy have established with a reasonable degree of certainty the assignments 587 keV (3^-), 886 keV (5^-), and 1370 keV (3^-). It also seems certain that the states at 0.00, 691, and 1941 keV are levels of unnatural parity, since (p, t) transitions to their analog states are suppressed by about an order of magnitude.[6] The early assignment of 2^- to the ^{46}K ground state[18] has been found consistent with all more recent experimental results. Finally, it is clear on the basis of the (d, α) excitation functions shown in Fig. 3 that at least the ground state, 587, 691, 886, and 1941 keV levels are excited by predominantly direct (d, α) transfer and hence have $T = T_3 = 4$ isospin and $J^\pi \neq 0^+$.

The γ transitions shown in Fig. 1 are fully consistent with and confirm the results listed above. Their observation in fast coincidence demonstrates that these transitions are of multipolarity 1 or 2. On the basis of typical lifetimes, it should be safe to exclude $M2$ except, perhaps, for the decay of the 1941 keV level. The observed dominant cascade 886 (5^-) → 691 → 587 (3^-) → g.s. (2^-) strongly suggests that the 691 keV level has $J = 4$. Although energy favored by a factor >300, the branch to the ground state is moderately weak (~28%), therefore the 691–587 keV transition should be of multipolarity 1, but even if $E2$ is admitted one finds $J^\pi \leq 3^\pm$, 4^- because of competing transitions to the 2^- and 3^- levels. Because of feeding of this state by the 5^- level [which could decay by $E2$ to the lower 587 (3^-) state] we also have the restriction $3^-, 4^\pm \leq J^\pi$. Hence the observed γ lines restrict J^π to 4^- or 3^-. The fact that the 691 keV state has been found to have unnatural parity[6,7] eliminates 3^-. Thus we assign $J^\pi = 4^-$ to the 691 keV level, subject only to the reasonable assumption that none of the low-lying levels consists of very close, unresolved doublets of which different members are populated in the $(p, {}^3\text{He})$ and $(d, \alpha\gamma)$ reactions.

It is interesting to emphasize that the 886 keV level (5^-) has no visible transitions to either the ground state (2^-) or the 587 (3^-) level, both of which would be energy favored over that to the 691 (4^-) keV state. This is in good agreement with multipolarities expected from the present spin assignments. However, if the spin sequence 2^-, 4^-, 5^-, 3^- tentatively proposed by Paul *et al.*[19] were correct, the missing transitions could go by $M1$ whereas the observed 194.6 keV transition would have to go by $E2$. An even stronger argument against the spins suggested in Ref. 19 is the *observed* fast ground state branch of the 691 keV level which would have to go by $M3$ (in competition with an $M1$ branch).

Compared to the strong arguments given above the ^{48}Ca(d, α) angular distributions at 17 MeV provide more limited if independent proof for the adopted J^π assignments. Only the (d, α) transition to the 886 keV (5^-) state shows a very characteristic angular distribution. It is fit best by $L = 5$ DWBA curves. Together with the knowledge that this state has natural parity (i.e., only one L value may contribute) one concludes $J^\pi = 5^-$. This spin has also been deduced on the basis of (d, α) transfer work alone,[3] but this assignment was somewhat model dependent in as much as it is assumed (a) that (d, α) two-step contributions do not significantly change the angular distribution for this transition and (b) that on the basis of shell model arguments a 4^+ state is unlikely in ^{46}K near this energy (for $L = 4$ might still be acceptable on the basis of an only moderately inferior DWBA fit).

For the deduction of other J^π values *on the basis of (d, α) data alone* one must rely on shell model expectations: On the basis of single-hole energies it is reasonable to assume that the lowest (two-hole) states in ^{46}K have negative parity and the dominant configurations $(\pi d_{3/2} \nu f_{7/2})^{-1}$ and $(\pi s_{1/2} \nu f_{7/2})^{-1}$. Within this space one has one 2^-, two 3^-, two 4^-, and one 5^- levels. Hence the transition dominated by $L = 1$ should lead to the $(d_{3/2} f_{7/2})_{2^-}$ state. The ground state transition is the only one where a strong $L = 1$ contribution is seen (Fig. 4). It is also apparent from Fig. 4 that four transitions to low-lying states (587, 691, 1370, and 1738 keV) are *consistent* with dominant $L = 3$ transfer (compare with empirical $L = 3$ curve). For the 587 and 1370 keV levels we have previous evidence (see above) for their 3^- nature. The (d, α) data are consistent with these assignments, and hence suggest $J^\pi = 4^-$ for the remaining (691 and—possibly—1738 keV) levels. As mentioned above there is strong evidence for nondirect contributions to the 1738 keV transition. However, the energy and weakness of this level agree well with shell model expectations for the upper 4^- state. The wave functions of Table III predict almost complete cancellation for the (d, α) and $(p, {}^3\text{He})$ $L = 3$ transition amplitude. Hence the weak cross section and even the fluctuations in the (d, α) cross sections as a function of energy would be understandable. There is no evidence known to us that would speak against a 4^- value for this level, and there are several arguments in favor of a $J^\pi = 4^-$ assignment, although they are fairly weak: (a) If the 1738 keV level has $J^\pi = 4^-$ it should and does undergo γ decay to the lowest state that can be reached by $M1$ [here the 587 (3^-) level]; (b) it should (and does seem to) show an essentially $L = 5$ angular distribution at high energy where competing reactions are negligible. [The unassigned, fragmentary $(p, {}^3\text{He})$ angular distribu-

TABLE III. Experimental assignments for ^{46}K together with empirical and theoretical wave functions for the lowest two-hole states in ^{46}K (phases are in shell model convention with single particle wave functions positive near origin).

Experimental Energy (keV)	$L_{d,\alpha}$	J^π	Configuration $d_{3/2}^{-1}f_{7/2}^{-1}$	$s_{1/2}^{-1}f_{7/2}^{-1}$	$d_{5/2}^{-1}f_{7/2}^{-1}$	Origin of matrix elem. or wave function
0	(1+3)	2^-	1.0(−0.02)	⋯	⋯	Empirical =a
			0.990	⋯	0.139	Kuo G bare =b
			0.993	⋯	0.114	Kuo + core polar. = c
			0.979	⋯	0.203	Bertsch G bare =d
587	3	3^-	−0.81 ± 0.02	0.586 ± 0.03	⋯	a (empirical)
			−0.790	0.611	0.041	b
			−0.711	0.701	0.050	c
			−0.766	0.639	0.067	d
691	(3+5)	4^-	0.49 ± 0.04	0.87 ∓ 0.02	⋯	a (empirical)
			0.452	0.882	0.130	b
			0.372	0.923	0.096	c
			0.448	0.875	0.184	d
886	5	5^-	−1.0(−0.02)	⋯	⋯	a (empirical)
			−0.999	⋯	−0.042	b
			−0.999	⋯	−0.040	c
			−0.998	⋯	0.064	d
1370	3	3^-	−0.586 ± 0.03	−0.81 ∓ 0.02	⋯	a (empirical)
			−0.612	−0.791	−0.007	b
			−0.703	−0.709	−0.057	c
			−0.642	−0.765	−0.041	d
(1738)	(?)	4^-	−0.87 ± 0.02	0.49 ± 0.04	⋯	a (empirical)
			−0.887	0.461	−0.044	b
			−0.926	0.376	−0.029	c
			−0.883	0.465	−0.060	d
1941	(0+2)	1^+				

[a] From experiment, this work.
[b] Matrix elements from Ref. 26.
[c] Matrix elements from Ref. 26.
[d] See Ref. 16.

tion[6] for the 1738 level is consistent with $L=5$.] There is also some evidence in the 80 MeV (d, α) data taken by Frascaria[20] for a relatively strong excitation of this level. (c) There is no other known candidate for the 4^- level expected close to 2 MeV.

The highest state (1941 keV) strongly excited in ^{48}Ca$(d, \alpha)^{46}$K at $E_d=17$ MeV does not belong to the negative parity multiplets. The observed γ decay eliminates $J \geq 5$. For any intermediate spin, i.e., $2 \leq J \leq 5$ there are three possible transitions of multipolarity 1 or 2. However, only the transition to the (2^-) ground state is observed, in spite of the fact that experimentally it would be the hardest one to detect (Fig. 1). Adjusted for detector efficiency (Table I) this 1941 keV γ ray is second in intensity only to that depopulating the first excited state. Hence a reasonable, although not fully compelling conclusion on the basis of γ decay alone would be $J<2$.

Previously reported work[6,7] established that the 1941 keV level has unnatural parity. Independently, 0^+ is ruled out by the strong (d, α) cross section. The J^π possibilities $(0^-, 1^+, 2^-, 3^+, 4^-)$ which are not excluded by strong selection rules can be reduced by the two-particle transfer angular distributions of Refs. 6 and 20, which disagree with DWBA curves dominated by $L=1$ or $L=3$. On the other hand, in each case and in the present study good fits are obtained with mixtures of $L=2$ and $L=0$ contributions. For the present data and for the $(p, {}^3\text{He})$ experiment $L=0$ contributions are essential to fit the small angle structure and the steep falloff with θ. Hence only $J^\pi=1^+$ is consistent with all known data.

B. Low-lying ^{46}K states as mixed $(d_{3/2}f_{7/2})^{-1}$ and $(s_{1/2}f_{7/2})^{-1}$ multiplets. Empirical matrix elements for the residual interaction

There is good evidence that ^{47}Ca and ^{47}K may be viewed as "one-hole" nuclei with an inert ^{48}Ca core. The first excited state (2^+) of ^{48}Ca is found at 3.833 MeV. The first excited state ($\frac{3}{2}^-$) for ^{47}Ca lies at 2.016 MeV,[21] and the ground state seems to be well described as pure $\nu f_{7/2}{}^{-1}$ ($C^2S_{7/2}=6.6$). For ^{47}K the ground state is essentially $\pi s_{1/2}{}^{-1}$, to be followed by the $\pi d_{3/2}{}^{-1}$ excited state at 0.359 MeV. No other low-lying states are known.[22] In ^{45}K ($\frac{3}{2}^+$) the position of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ levels is almost exactly reversed; hence in ^{46}K the $s_{1/2}{}^{-1}$ and $d_{3/2}{}^{-1}$ single particle energies should be nearly degenerate, and for low-lying states of ^{46}K proton $2s_{1/2}$ and $1d_{3/2}$ and neutron $1f_{7/2}$ orbits have to be considered, as a minimum. The additional consideration of $\pi d_{5/2}{}^{-1}$ orbits (Table III) produces very small corrections to the ^{46}K wave functions, which should probably be neglected as long as core excitations in ^{48}Ca are not explicitly included in the configuration space and target wave function.

The results of our ^{48}Ca$(d,\alpha)^{46}$K study give good evidence for the importance of the $(s_{1/2}f_{7/2})^{-1}$ configuration; first, through the observation of a second low-lying 3^- state, significantly excited in ^{48}Ca(d,α) as well as in $(p,{}^3\text{He})$, and secondly through constructive and destructive coherence effects in the direct deuteron transfer amplitudes to the 3^- and 4^- states.

Although 4^- states may be excited by $L=3$ and $L=5$ deuteron transfer, at 17 MeV DWBA predictions for $L=3$ cross sections are more than an order of magnitude larger for $(d_{3/2}f_{7/2})_{4^-}$ transfer than $L=5$ cross sections. Hence the strong 4^- state (at 691 keV) should show constructive coherence for $L=3$. This is also predicted from the wave functions of Table III. [There is no significant coherence effect expected for $L=5$ in this case as the only important competing configuration $(s_{1/2}f_{7/2})_{4^-}$ cannot couple to $L=5$.] As a consequence the weak transition to the orthogonal 4^- state must show destructive $L=3$ interference, and its cross section will be small and may also begin to show $L=5$ features. The largest candidate for the second 4^- transition seen in (d,α) is that to the 1738 keV state. In $(p,{}^3\text{He})$ a possible 4^- candidate was seen at 2.969 MeV, but this level is extremely weak in (d,α). (See Fig. 2.)

The ratio of the experimental (d,α) cross sections at forward angles for the two $(df, sf)_4$-states, which should only depend on the form factors[23] and not on the other DWBA details, is $R_{4^-}(d,\alpha)\gtrsim 18$ where the > sign is required because part of the 1738 keV cross section is known to come from nondirect contributions and any other 4^- candidates are still weaker. The ratio for the pair of 4^- levels in ^{48}Ca$(p,{}^3\text{He})$ is $R_4(p,{}^3\text{He})\approx 20\pm 5$ for either of the two 4^- candidates.[6] A small, but statistically significant and partly resolved peak at about 1738 keV is also seen in (d,α) work at 80 MeV[20] where $L=5$ would be dynamically favored. Although somewhat imprecise, these large ratios reflect very strong $L=3$ cancellations for the higher level and effectively limit any possible 4^- wave functions in the $(s_{1/2}f_{7/2}, d_{3/2}f_{7/2})$ space to either

$$\psi_s \approx [(0.87\pm 0.02)s_{1/2}f_{7/2}+(0.49\mp 0.04)d_{3/2}f_{7/2}]_{4^-}$$

or

$$\psi_s' \approx [(0.99\pm 0.01)s_{1/2}f_{7/2}+(0.14\mp 0.1)d_{3/2}f_{7/2}]_{4^-}$$

(shell model phases) for the strong (691 keV) level. A change of ±0.1 in the $d_{3/2}f_{7/2}$ term, i.e., a nearly negligible change in the $s_{1/2}f_{7/2}$ amplitude would accommodate the range of $10<R_{4^-}<80$ in the experimental ratio for direct one-step deuteron transfer for either (d,α) or $(p,{}^3\text{He})$. ψ_s and its orthogonal 4^- wave function

$$\psi_{\text{weak}}=[0.49s_{1/2}f_{7/2}-0.87d_{3/2}f_{7/2}]_{4^-}$$

are very close to some theoretical 4^- shell model predictions shown in Table III.

In the absence of significant two-step or compound contributions (d,α) transitions to 3^- states must go by pure $L=3$, $\Delta T=0$, $\Delta S=1$. Experimentally, the observed angular distributions to the 3^- states at 587 and 1370 keV are quite similar, their magnitudes and Q values comparable. Hence experimental 3^- cross section ratios are easily deduced. We find $R_{3^-}(d,\alpha)=2.2\pm 0.4$ at $E_d=17$ MeV. However, at $E_d=80$ MeV the 1.370 MeV level is very much weakened relative to the other four low-lying levels in ^{46}K. Since the 0.587 MeV level is not resolved in the high energy data available[20] a definite 3^- ratio at $E_d=80$ MeV cannot be extracted, but comparison with the resolved 2^- ground state or the sum of the 3^- and 4^- levels at both energies strongly suggests R_{3^-} (80 MeV) > 4. This difference between 17 and 80 MeV (d,α) results points to a potentially serious limitation of our ratio method as it can be taken as evidence that the weak transitions are not dominated by one-step processes.

The ratios for ^{48}Ca$(p,{}^3\text{He})^{46}$K (3^-) and ^{48}Ca-(p,t) (3^- analog) transitions were observed as $R_{s+t}=2.9\pm 10\%$ (which becomes $R'_{s+t}=2.43$ if Q value effects are divided out) and $R_s=3.0\pm 20\%$ (corresponding to $R_s^1=2.7$), respectively.[6] *Equality* of of the various ratios is not expected, because the transitions are governed by different interactions and coupling rules. The $0^+\to 3^-$ transition for (d,α) should go by $\Delta T=0$, $\Delta S=1$, the (p,t) transition to the 3^- analog states by $\Delta T=1$, $\Delta S=0$, and the $(p,{}^3\text{He})$ transition by a roughly predictable[24,25] mixture of the two processes. For one particular

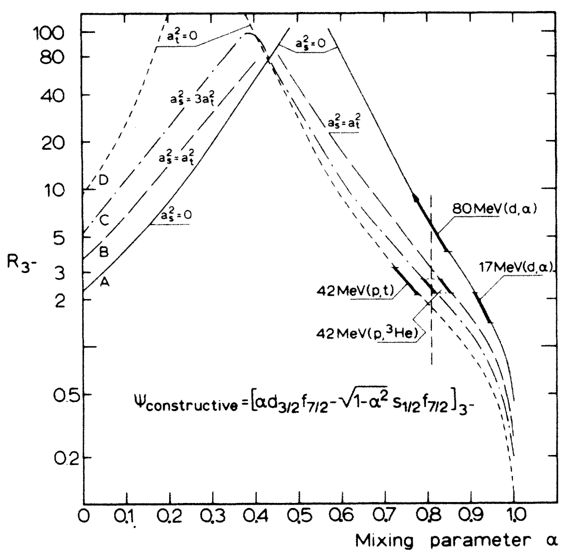

FIG. 5. DWBA predictions for

$$R_{3^-} = \frac{\sigma(L=3,\ \text{constructive})}{\sigma(L=3,\ \text{destructive})},$$

as a function of the df-sf mixing parameter α. The ratios plotted are those for the $L=3$ maxima near 22.5°, computed for ^{48}Ca(d, α), $E_p=17.0$ MeV, $E^*=1$ MeV (curve A, pure triplet), ^{48}Ca(p, ^{3}He), $E_p=41.65$ MeV, $E^*=1$ MeV (curve B using $a_{\text{singlet}}{}^2=a_{\text{triplet}}{}^2$, curve C using $a_{\text{singlet}}{}^2=3\ a_{\text{triplet}}{}^2$) and for ^{48}Ca($p$, t)^{46}Ca ($T=4$) $E_p=41.65$ MeV, $E^*=14.5$ MeV (curve D, pure singlet). The thickened portions on the curves indicate the ranges consistent with the corresponding (Q-value corrected) experimental ratios, assuming that *all* of the experimental cross section is interpretable as direct, single-step two-nucleon transfer. It is seen that the 17 MeV (d, α) range is outside the range $0.77<\alpha<0.85$ consistent with the higher energy (p, He^3) and (d, α) data. Weighing the data according to the expected likelihood of two-step contributions a preferred mixing parameter $\alpha=0.81\pm0.02$ is chosen. (For the 4^- ratios different ($L=3$) + ($L=5$) triplet curves are applicable and all experimental ratios are consistent.)

transfer the observed ratio again can be reproduced by two sets of wave functions as noted for the 4^- case. However, if all three ratios are to be reproduced simultaneously only the solution with strong mixing of $s_{1/2}f_{7/2}$ and $d_{3/2}f_{7/2}$ components seems possible. In Fig. 5 we show the predicted singlet and triplet two-nucleon transfer cross section ratios as a function of the mixing parameter α. Also indicated are the measured (p, t) (singlet), (d, α) (triplet), and (p, ^{3}He) ($s=0$, $s=1$ mixture) cross section ratios (taken at $\theta=22.5°$ and corrected for Q dependence). It can be seen that the high energy (p, ^{3}He) and (d, α) select $\alpha\approx0.81\pm0.02$. This value is hardly affected if $a_{T=1}{}^2/a_{T=0}{}^2$ is varied within the acceptable range. The 17 MeV (d, α) ratio ($R'_3=1.8\pm0.4$, corrected for Q dependence), however, is only qualitatively consistent with that of the higher energy data. It is *smaller* than expected by a factor of 3. The (p,t) data give a R'_{3-} ratio too *large* by a factor of 1.5. Both types of transfers are probably affected by higher order contributions.

The empirical wave functions obtained by this method of matching cross section ratios are compared in Table III with shell model wave functions obtained from Kuo-Brown matrix elements.[26] Apart from our inability to deduce the very small $d_{5/2}f_{7/2}$ admixtures from experiment rather good agreement for the important terms is seen. The empirical wave functions of Table III were used to predict 17 MeV (d, α) and 42 MeV (p, ^{3}He) and (p, t) cross sections for all negative parity levels. As illustrated in Fig. 6 agreement to ±30% with all three experiments is found.

Although the differences between theoretical and empirical wave functions seem minor there are fairly large differences in the corresponding energy levels, which are compared in Fig. 7. Consequently, there are differences between the theoretical matrix elements and the empirical ones which are deduced from the relation

$$V_{ij}(J^\pi)+\epsilon_{ij}\delta_{ij}=\sum_k \langle\phi_i\,|\,\phi_k\rangle E_k(J^\pi)\langle\phi_k\,|\,\phi_j\rangle,$$

where here

$$\phi_1=d_{3/2}f_{7/2},\quad \phi_2=s_{1/2}f_{7/2},\quad \text{and } k=1,2.$$

Before such empirical matrix elements can be compared with V_{ij} values computed elsewhere it must be recalled that for these ^{46}K configurations the terms ϕ_i have $T=0$ or $T=1$ terms with equal probability. Hence, existing $T=0$ and $T=1$ matrix elements were averaged for the purpose of comparison. For the zero order shell model energies we took $\epsilon(d_{3/2}f_{7/2})=-1.952$ MeV and $\epsilon(s_{1/2}f_{7/2})=-1.59$ MeV. Table IV shows a comparison of our empirical matrix elements with those used by Bertsch and those obtained by other authors. Some differences might be expected as the present values refer to a ^{48}Ca core, whereas all previous ones refer to ^{40}Ca or even lighter cores. Nevertheless, the deduced values for V_{ij} are found in the middle of the range of values previously quoted in the literature. For the $(d_{3/2}f_{7/2})^2$ diagonal matrix elements a comparison with previous experimental data is possible and shown in Table IV. Our ^{46}K values are systematically *more* attractive by about 100 keV (300 keV for $J^\pi=2^-$) than other experimental values. They are generally 50 to 700 keV *less* attractive than theoretical values derived from the bare G matrix. It is possible to deduce semiempirical matrix elements by using theoretical wave functions ϕ_k and experimental ener-

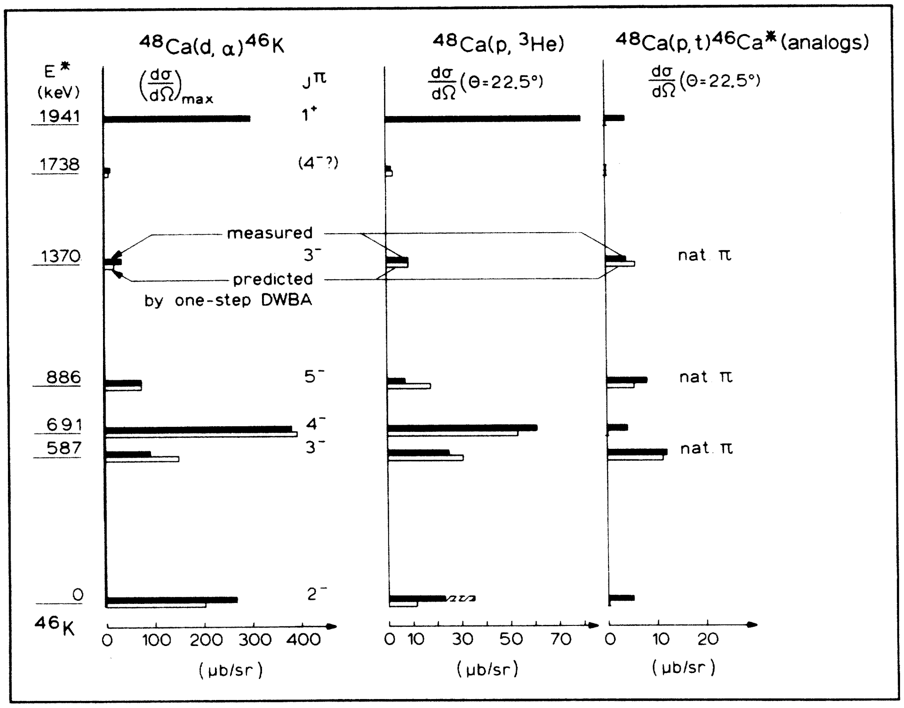

FIG. 6. Measured small angle cross sections for the three two-nucleon transfer reactions and one-step, zero-range DWBA calculations with the empirical wave functions listed in Table III. Note that the observed spin forbidden 1^+, 2^-, 4^- (p, t) cross sections are fairly strong although significantly reduced compared to the analogous $(p, {}^3\mathrm{He})$ transitions. For the DWBA $(p, {}^3\mathrm{He})$ cross sections $a_s^2 = a_t^2$ was assumed. The absolute cross section scales for the three reactions were individually normalized.

gies E_k. If Set d in Table III is used average deviations of ±60 keV from the empirical values listed in Table IV are found. Whether our empirical or semiempirical matrix elements are taken, all fall somewhere between the Kuo-Brown values[26] for the bare matrix and their values with three-hole-one-particle corrections. This comparison suggests the importance of the largely repulsive contributions obtained by Kuo and Brown from three-hole-one-particle core polarization corrections.

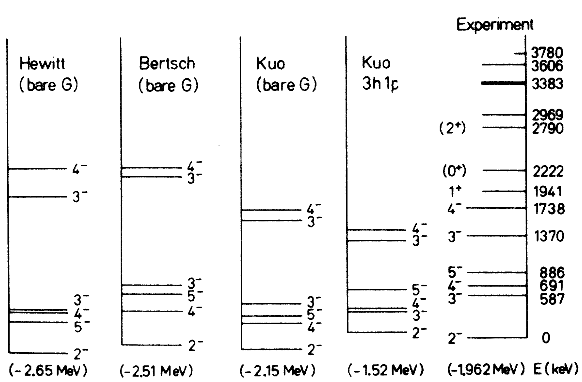

FIG. 7. The empirical spectrum of ^{46}K compared with shell model calculations for negative parity states, based on different sets of matrix elements listed in Table IV. The calculation with Kuo-Brown matrix elements with core polarization corrections (Ref. 26) yields the correct level order and the relatively best level spacing.

V. SUMMARY AND CONCLUSIONS

As ^{46}K can only be reached by two-nucleon transfer reactions it has been found most helpful to consider previously published ^{48}Ca$(p, {}^3\mathrm{He})$ and ^{48}Ca$(p, t)^{46}$Ca$(T=4)$ transfer data[6] in conjunction with the ^{48}Ca$(d, \alpha\gamma)$ and ^{48}Ca$(d, \alpha)^{46}$K data presented in this paper. The simultaneous interpretation of these four reactions yields ^{46}K J^π assignments which appear reasonably model independent.

In this respect the ^{48}Ca$(d, \alpha\gamma)^{46}$K fast α-γ coincidence spectra proved particularly useful by eliminating certain J^π choices left open in the transfer experiments, and by providing a sensitive test for the empirical ^{46}K wave functions proposed (Table III). γ-decay estimates based on these wave functions show that all observed transitions should be strong and that no unfavored transitions are seen. Only one transition expected to be relatively strong, the decay of the weakly excited 1370 keV (3^-) state to the (2^-) ground state, is not seen

TABLE IV. Proton-neutron matrix elements derived from empirical ^{46}K wave functions compared with $T=0$ and $T=1$ averages derived from theoretical or systematic studies for lighter nuclei. (Values in MeV.)

Configurations	J^π	This work $V_{ij}+\epsilon$	This work V_{ij} [d]	Kuo and Brown [a] G_{bare}	Kuo and Brown [a] G_{b+3p1h}	Hewitt [b] G_{bare}	Bertsch [c] G_{bare}	Dieper. [e]	Ernè [f]	Mairle [g]	Erskine [h]
$(s_{1/2}f_{7/2}, s_{1/2}f_{7/2})$	3^-	1.100	−0.503	−0.472	−0.370	−0.725	−0.43				
$(s_{1/2}f_{7/2}, s_{1/2}f_{7/2})$	4^-	0.945	−0.663	−1.048	−0.648	−1.315	−1.12				
$(s_{1/2}f_{7/2}, d_{3/2}f_{7/2})$	3^-	0.382	0.382	0.525	0.476	0.735	0.69				
$(s_{1/2}f_{7/2}, d_{3/2}f_{7/2})$	4^-	−0.449	−0.449	−0.542	−0.332	−0.690	−0.66				
$(d_{3/2}f_{7/2}, d_{3/2}f_{7/2})$	2^-	0	−1.962	−1.988	−1.416	−2.645	−2.22	−2.24	−1.64	−1.625	−1.70
$(d_{3/2}f_{7/2}, d_{3/2}f_{7/2})$	3^-	0.856	−1.106	−1.108	−0.776	−1.405	−1.08	−0.73	−0.93	−0.875	−0.93
$(d_{3/2}f_{7/2}, d_{3/2}f_{7/2})$	4^-	1.484	−0.481	−0.545	−0.268	−0.645	−0.41	−0.08	−0.43	−0.300	−0.39
$(d_{3/2}f_{7/2}, d_{3/2}f_{7/2})$	5^-	0.886	−1.076	−1.662	−0.942	−2.165	−1.78	−1.30	−1.23	−0.935	−1.00

[a] See Ref. 26.
[b] Hewitt and Comins (for $\omega=65$) (private communication through B. Barrett); see also B. Barrett *et al.* Phys. Rev. C 3, 1137 (1973).
[c] See Ref. 16.
[d] Referred to ^{48}Ca core. Here the valence nucleon interaction energy is taken as

$$\epsilon_{1/2^+} = \mathrm{BE}(^{46}\mathrm{K}) + \mathrm{BE}(^{48}\mathrm{Ca}) - \mathrm{BE}(^{47}\mathrm{Ca}) - \mathrm{BE}(^{47}\mathrm{K}) = 1.603\ \mathrm{MeV};\quad \epsilon_{3/2^+} = 1.962\ \mathrm{MeV}.$$

[e] See Ref. 1.
[f] F. C. Ernè, Nucl. Phys. 84, 91 (1966).
[g] G. Mairle, Habilitationsschrift, MPI Heidelberg V28, 1972 (unpublished).
[h] J. R. Erskine *et al.*, Phys. Rev. C 3, 1976 (1971).

and must be at least a factor of 2–3 smaller than the observed decay to the 691 keV (4^-) state.

Although the excitation of the 1738 keV level at $E_d=17$ MeV seems to be largely nondirect, the consistent observation of this state in the high energy $(p, {}^3\mathrm{He})$ and (d, α) data, its γ decay, and shell model systematics suggest that it may be the missing upper 4^- state of the $s_{1/2}f_{7/2}$-$d_{3/2}f_{7/2}$ multiplets. This tentative assignment has little effect on the empirical wave functions of Table III, but if this (4^-) excitation energy is found in error one would obtain changes in the empirical 4^- matrix elements shown in Table IV.

As a final step in the analysis the experimental ^{46}K spectrum was compared with shell model calculations with various effective matrix elements derived from the approach of Kuo and Brown[26] (Fig. 7). It was found that calculations with matrix elements from the bare G matrix were inferior to those including core polarization corrections. Exploratory calculations for positive parity states suggest the level order $1^+, 3^+, 0^+, 1^+, 2^+, 0^+$ (spread over 3 MeV). However, the uncertainty in the $\nu d_{3/2}$ and $\nu s_{1/2}$ single particle energies does not allow conclusions beyond the observation that the 1^+ state always appears as the lowest positive parity state, in good agreement with the empirical spectrum of ^{46}K.

As in the ^{48}Ca(p, t) study,[6] deviations from the direct one-step two-nucleon transfer process are seen in ^{48}Ca(d, α). Nevertheless, the general consistency of spectroscopic deductions for the more strongly excited ^{46}K levels from ^{48}Ca$+p$ reactions at $E_p=42$ MeV and ^{48}Ca$+d$ at $E_d=17$ and 80 MeV[20] leads us to expect that two-step transfer contributions, while present and important for detailed predictions have not obscured the basic information sought. Further quantitative studies of the $(d, t)(t, \alpha)$ and $(p, d)(d, t)$ contributions would be of great importance as we see no other way to explain the unaccountably large contribution of $L>J-1$ components for all transfers to unnatural parity states. In our ^{48}Ca(d, α) DWBA analysis (Fig. 4) these $L_>$ contributions have been taken as $L=J+1$ in conformity with one-step transfer selection rules; however, it is quite conceivable that parity forbidden (i.e., $L_>=J$) amplitudes contribute strongly in the two-step process.[27] Such contributions could give a significant improvement in the fits of Fig. 4 and may even reduce the ~30% scatter of expected versus observed two-nucleon transfer cross sections shown in Fig. 6; for this scatter is mainly caused by a systematic underprediction of transfer cross sections to unnatural parity states. The relative overprediction for transitions to natural parity states is particularly severe for ^{48}Ca$(p, {}^3\mathrm{He})$ where $S=0$ and $S=1$ amplitudes contribute simultaneously. If the singlet interaction strength a_s^2 ($S=0$, $T=1$) is taken as about 3 times the triplet strength a_t^2 as suggested by Fleming, Hardy, and Cerny[25] large discrepancies are found for the summed 4^- vs 3^- and 5^- strengths. In Fig. 6 $a_s^2=a_t^2$ is used, i.e., the limit of a spin independent interaction, and reasonable agreement of data and predictions is maintained. Other authors have encountered similar problems in (p, ^{3}He) singlet vs triplet normalization[28,29] and this interesting question should be the subject of further study.

With the caveat that two-step effects may have larger effects on our two-nucleon transfer cross sections than seems likely at this point and the suggestion that the transfer experiments may be profitably repeated at even higher energies we present empirical wave functions and empirical $(s_{1/2}f_{7/2}, s_{1/2}f_{7/2})$, $(s_{1/2}f_{7/2}, d_{3/2}f_{7/2})$, and $(d_{3/2}f_{7/2}, d_{3/2}f_{7/2})$ matrix elements that correlate the existing information on ^{46}K and are, in part, overdetermined by the data. The $(d_{3/2}f_{7/2})^2$ matrix elements compare well with those previously measured in lighter nuclei. The $(s_{1/2}f_{7/2})^2_{3^-,4^-}$, and in particular the off-diagonal $(s_{1/2}f_{7/2}, d_{3/2}f_{7/2})_{3^-,4^-}$ matrix elements appear to be the first data of this kind obtained for a simple nucleus and are, perhaps, especially interesting. In each case these eight matrix elements, which are generally close to those computed by Kuo and Brown, are smaller (slightly less attractive) than the bare G-matrix results. The core polarization corrections always seem to go in the right direction and represent a definite improvement, although more often than not they are larger than needed for optimum agreement with the empirical values.

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