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$^{18}O(p,t)^{16}O$ reaction and the coexistence model of ^{16}O

Donald G. Fleming
Department of Chemistry, University of British Columbia, Vancouver, Canada, V6T 1W5

Akito Arima
Department of Physics, State University of New York, Stony Brook, New York 11790 and University of Tokyo, Bunkyo-ku, Tokyo, Japan

H. W. Fulbright and Marshall Blann
Nuclear Structure Laboratory and Department of Physics and Chemistry, University of Rochester, Rochester, New York 14627
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The $^{18}O(p,t)^{16}O$ reaction has been studied at 20 MeV. Relative cross sections to the ground (0^+), 6.05 MeV (0^+), 6.92 MeV ($2^+, K=0$), and 9.85 MeV ($2^+, K=2$) states have been compared with standard distorted-wave Born-approximation (DWBA) calculations using SU_3 parentage factors and the nuclear coexistence model. The agreement is good for the 0^+ states but poor for the 2^+ states, given the uncertainties inherent in the DWBA calculation itself. The presence of competing second-order processes is indicated by the relatively strong population of the 8.88 MeV 2^- level.

[NUCLEAR REACTIONS, STRUCTURE $^{18}O(p,t)$ $E=20$ MeV; measured $d\sigma(\theta)$; resolution 25 keV. DWBA calculation of $d\sigma(\theta)$. Nuclear coexistence model.]

I. INTRODUCTION

In the nuclear coexistence model, spherical and deformed states "coexist" in the same nucleus. This model has enjoyed a considerable degree of success in the deformed rare-earth nuclei, where, among other things, it has been used to explain the observation of "shape transitions" in (t,p) and (p,t) reactions.[1,2] The model has also been considered in much lighter nuclei, notably around ^{16}O[3-8] and ^{40}Ca[9,10] and generally leads to a rotational type of structure in these nuclei, consistent with that predicted by calculations based on a deformed Hartree-Fock field.[11] In the past, evidence for support of the basic underlying concepts of the coexistence model in such nuclei has mainly been based on energy levels and electromagnetic transition probabilities,[3-7,9,11,12] although most of the theoretical calculations to date have made use of harmonic oscillator wave functions; more realistic Woods-Saxon calculations can be expected to make some changes.[8,13] The basic coexistence model has often been described in terms of an SU_3 representation for the rotational states, which has been fairly successful in predicting both energy spectra and transition rates,[3-7,14] as well as α-particle structure in light nuclei.[15] The SU_3 scheme can be expected to work well in the $1p$ shell and in the early part of the $(2s, 1d)$ shell.[16]

The calculations of Zuker, Buck, and McGrory (ZBM)[17] have taken a somewhat different approach; these authors have attempted to unify the spherical and deformed character of the coexistence model within the framework of an "exact" shell model calculation. While their calculations might be criticized on the grounds that they employ a rather restricted spherical shell model basis ($1p_{1/2}$, $2s_{1/2}$, and $1d_{5/2}$) in what is essentially a deformed core, their conclusions regarding low-lying states in ^{16}O and ^{18}O are probably correct. Indeed, for electromagnetic transition probabilities and one-nucleon-transfer reactions, their wave functions have generally provided good agreement with experiment,[18,19] although there are notable exceptions for particular transitions. Other shell model calculations have also been performed in this mass region[20] including further exact calculations in an enlarged model space.[21]

It is well known that one-nucleon-transfer reactions provide a rather insensitive test of different model wave functions; the two-nucleon-transfer reaction is a much more sensitive probe of nuclear structure effects. Accordingly, we would like to report herein on a 20 MeV study of the $^{18}O(p,t)^{16}O$ reaction; a preliminary report of which can be found in Ref. 22. This reaction has also been investigated by Lutz *et al.* at 18.2 MeV,[23] by Pignanelli *et al.* at 24.4 MeV,[24] by Adelburger *et al.* at 41.7 MeV,[25] by Cerny *et al.* at 43.7 MeV,[26] and by Sørensen at 50 MeV.[8] The inverse reaction, $^{16}O(t,p)^{18}O$, has also previously been reported,[27] and distorted-wave Born-approximation (DWBA) analyses have been carried out by Donau *et al.*[6] and Kolltveit, Muthukrishnan, and Trilling.[5] In the DWBA calculations of Ref. 6, reasonable agreement with the (t,p) experimental cross sections could only be obtained by the inclusion of deformed components in the parentage overlap; i.e., the (t,p) data were found to be in good agreement with the predictions of the coexistence model, in particular, with the Brown-Green wave functions for ^{16}O.[4]

On the other hand, more recent DWBA calculations of the (p,t) reaction are found to be sharply at variance with the predictions of the coexistence model. In the $^{18}O(p,t)^{16}O$ analysis of Adelburger *et al.*,[25] the 0^+ cross section ratio of the 6.05 MeV to ground state disagrees with experiment by a factor of 10. Similar results are given by these authors for the $^{42}Ca(p,t)^{40}Ca$ reaction, which agree with the conclusions of Erikson, Horsfjord, and Nilsson[10] in a later study of the same reaction. In the $^{18}O(p,t)^{16}O$ calculations of Ref. 25, the "microscopic" ZBM wave functions were found to give much better agreement with the experimental cross sections, a conclusion which is supported also by the calculations of Pignanelli *et al.*[24] Finally, calculations by Sørensen,[8] of a somewhat different nature, are also critical of the coexistence model, although in this case a coupled-channel treatment of the $^{18}O(p,t)^{16}O$ reaction cross sections was found to be necessary.

In the present study at 20 MeV, we have made use of the SU_3 coupling scheme within the framework of the coexistence model to calculate the appropriate two-nucleon parentage factors for the $^{18}O(p,t)^{16}O$ reaction, with the purpose in mind of providing a further test of the basic concepts of the model.

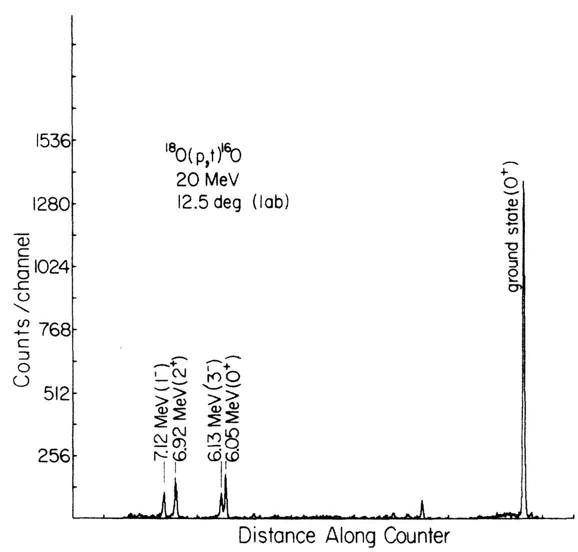

FIG. 1. Energy spectrum for the $^{18}O(p,t)^{16}O$ reaction at 20 MeV, 12.5° lab. The energy resolution is typically 25 keV. Levels beyond the 7.12 MeV (1^-) state are not shown.

II. EXPERIMENTAL RESULTS

These experiments were performed using the 20 MeV proton beam of the University of Rochester tandem accelerator. Outgoing tritons were detected by a sonic spark counter positioned in the focal plane of the Enge split-pole magnetic spectrometer. Details and the mode of operation of the data acquisition system based on this counter have been described elsewhere.[28] The target was $H_2{}^{18}O$ ice, obtained by spraying $H_2{}^{18}O$ water vapor onto a gold foil glued to a frame and supported by a liquid nitrogen cold finger. An energy spectrum taken at 12.5° (lab) is shown in Fig. 1; the energy resolution was typically 25 keV. The 6.05 MeV 0^+ state and the 6.13 MeV 3^- state are well separated; these two levels had not been resolved in previous studies of this reaction.[8,23-26] It was not possible to obtain any information on the states of ^{16}O beyond the 2^+ $(K=2)$ state at 9.85 MeV (not shown in Fig. 1) due to the appearance of the elastic line on the spark counter; indeed, the background in this region of the spectrum was high, causing some difficulty in the extraction of the cross sec-

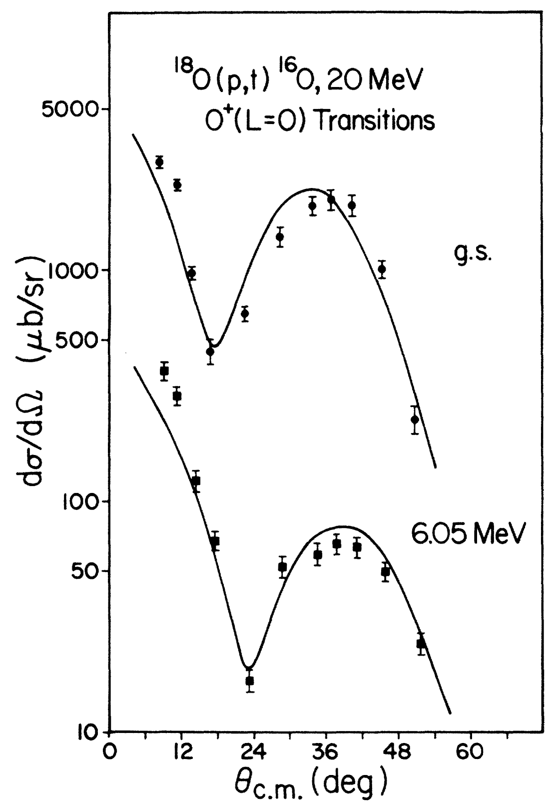

FIG. 2. Experimental angular distributions for the 0^+ ground state and 6.05 MeV states populated in the $^{18}O(p,t)^{16}O$ reaction at 20 MeV. The solid curves are DWBA calculations as discussed in the text. Each curve is individually and arbitrarily normalized to the data.

tion to the 9.85 and 8.88 MeV states. An angular distribution for the 8.88 MeV 2^- level was, in fact, not obtainable, but the state was observed at three angles (see later discussion) and its strength was entirely consistent with that reported elsewhere.[23,24] As such, it is relatively strongly populated (Table I) although, in first order, states of unnatural parity are forbidden for (p, t) reactions on a 0^+ target.

The experimental angular distributions are shown in Figs. 2–4; Fig. 2 presents the ground state and 6.05 MeV 0^+ states, Fig. 3 the 6.92 and 9.85 MeV 2^+ states, and Fig. 4 the 1^- and 3^- $(T=0)$ states at 7.12 and 6.13 MeV, respectively. The error bars shown in Figs. 2–4 are, in some cases, rather large and in these cases represent a combination of relatively poor statistics and uncertain background subtraction. The absolute cross sections are given in μb/sr and were determined by comparison of the yields of elastically scattered protons off protons in the water target (the p-p cross sections are well known; Ref. 29) with tritons from the $^{18}O(p,t)^{16}O$ ground state (g.s.) transition, for the same angle and beam current. There was some uncertainty inherent in this procedure due to the buildup of cracked hydrocarbons condensing on the target (which in later runs was minimized by surrounding the target in a large metal shroud); the absolute cross sections determined in this way are believed to be accurate to 30%. They, in fact, agree well with those reported in Ref. 23 at 18.2 MeV and in Ref. 24 at 24.4 MeV. The curves shown in Figs. 2–4 are DWBA fits, individually normalized to the data, and are referred to again in the discussion to follow.

Table I presents a compilation of the present data for the $^{18}O(p,t)^{16}O$ reaction at 20 MeV, showing the excitation energies, spins and parities,[30] the peak angle, and its differential cross section and the summed cross sections (10–50° c.m.), in 2° steps. These latter values were obtained by means of smooth curves drawn through the data points (not shown); some form of an integrated yield is important to have when comparing transitions of different multipolarities.

III. DWBA CALCULATIONS

As mentioned above, nuclear parentage factors have been calculated based on the SU_3 coupling

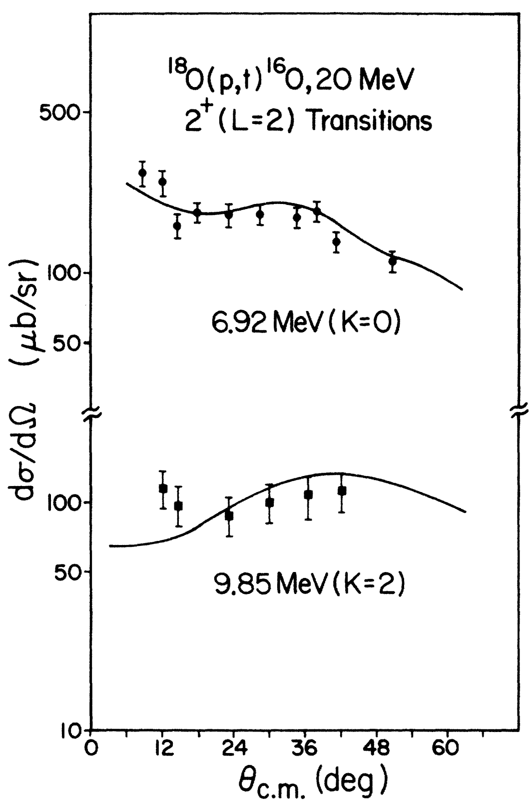

FIG. 3. Experimental angular distributions and DWBA fits to the 6.92 and 9.85 MeV 2^+ states populated in the $^{18}O(p,t)^{16}O$ reaction. See caption to Fig. 2.

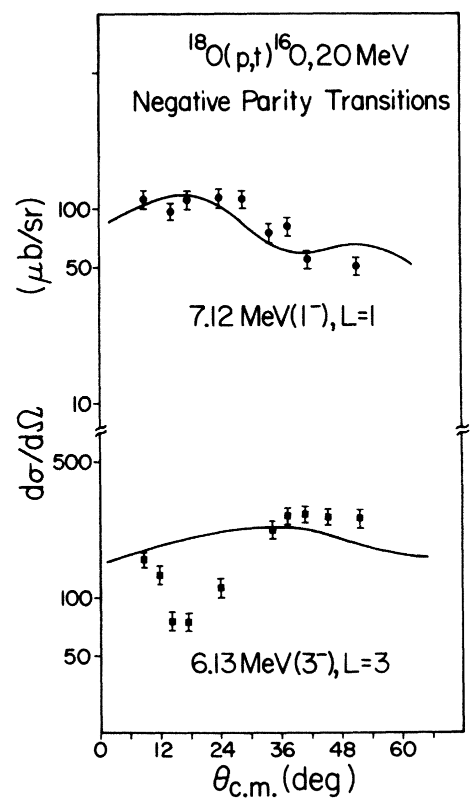

FIG. 4. Experimental angular distributions and DWBA fits to the 6.13 MeV (3^-) and 7.12 MeV (1^-) states populated in the $^{18}O(p,t)^{16}O$ reaction. See caption to Fig. 2.

scheme within the underlying framework of the coexistence model for ^{16}O and ^{18}O. The ^{16}O wave functions employed are given in Table II and, with the exception of a phase change, are identical with those of Brown and Green.[4] We assumed that their two particle-two hole wave functions belong to the (42) representation of the SU_3 group and their four particle-four hole wave functions to the (84) representation. The phase change was made because their phase convention differs from ours. The ^{18}O g.s. wave function is assumed to consist of $|2p\text{-}0h\rangle$ and $|4p\text{-}2h\rangle$ components

$$(1p)^{12}(2s,1d)^2[4^3,2](40)^{31}S$$

and

$$(1p)^{10}(2s,1d)^4[4^3,2](82)^{31}S\,.$$

The mixture of other states is neglected for the sake of simplicity, but the theoretical error caused by this truncation is probably not serious. Inclusion of a $|6p\text{-}4h\rangle$ component in a recent calculation of the $^{42}Ca(p,t)^{40}Ca$ reaction[10] caused no significant change in the calculated cross sections. The matrix element of the two-body interaction between the $|2p\text{-}0h(40)^{31}S\rangle$ state and the $|4p\text{-}2h(82)^{31}S\rangle$ state is proportional to that between the $|0p\text{-}0h\rangle$ state and the $|2p\text{-}2h\rangle$ state in ^{16}O;

$$\langle 2p|V|4p\text{-}2h\rangle = 0.176\langle 0p\text{-}0h|V|2p\text{-}2h\rangle\,.$$

Brown and Green[4] have estimated the matrix element $\langle 0p\text{-}0h|V|2p\text{-}2h\rangle$ to be −4.3 MeV, which gives

$$\langle 2p|V|4p\text{-}2h\rangle = -0.76\text{ MeV}.$$

A weak coupling model[31] predicts a $|4p\text{-}2h\rangle$ state around 3 MeV in ^{18}O, although the isospin structure of the $|np\text{-}mh\rangle$ state can affect its excitation energy.[32] We have assumed 4 MeV then as the energy difference between the $|4p\text{-}2h\rangle$ and $|2p\text{-}0h\rangle$ states in ^{18}O. These assumptions lead to the following ^{18}O g.s. wave function:

$$|^{18}O,\text{ g.s.}\rangle = 0.982|2p\text{-}0h\rangle + 0.189|4p\text{-}2h\rangle\,.$$

The 3.5% admixture of $|4p\text{-}2h\rangle$ is considerably less than that chosen by other authors and is referred to again in the later discussions. The SU_3 parentage factors based on these wave functions are given in Table III. These have been employed herein in 20 MeV DWBA calculations of the $^{18}O(p,t)^{16}O$ reaction.

The two-nucleon form factor was constructed in two well known ways: (1) A harmonic oscillator wave function matched to a Hankel-function tail at the nuclear surface (HO), as originally formulated by Glendenning,[33] and (2) a Woods-Saxon (WS) wave function with projected $\lambda=0$ relative motion, as originally formulated by Bayman and Kallio.[34] The HO treatment for the (p,t) reaction requires as input two-nucleon "structure factors," which incorporate the parentage factors of Table III, the wave function amplitudes of ^{18}O and ^{16}O (Table II), and the appropriate Moshinsky bracket for the $\lambda=0$ motion.[33] These factors (G_{NLJ}) are given in Table IV. The WS calculation requires only the appropriate pickup amplitudes, which incorporate the parentage factors of Table III and the wave function amplitudes. These pickup amplitudes (β) are given for the 0^+ states in Table V (in jj coupling), in order to facilitate the later discussion.

The DWBA calculations were carried out in zero range in both cases, utilizing the Colorado code DWUCK.[34] Finite range calculations of two-nucleon-transfer reactions have generally indicated good agreement both in angular distribution shapes and relative magnitudes when compared with the zero-range results[35] and we have made this assumption here. Specific calculations near ^{16}O tend to support this assumption.[24,36] The zero-range HO and WS form factors for two-nucleon transfer have previously been compared in the Ca nuclei[37] and in the Gd nuclei,[38] where the calculated angular distribution shapes were essentially identical and the relative cross sections were the same to within 30%, over 7 MeV of excitation.[37] However, in the present study of the $^{18}O(p,t)^{16}O$ reaction we find the same comparison

TABLE I. Experimental data, $^{18}O(p,t)^{16}O$ at 20 MeV.

Exc. (MeV)	J^π	$d\sigma$ (μb/sr)	$\sum_\theta$ (10–50°, c.m.) [a]
0.0	0^+	2100 (36°)	25 300
6.05	0^+	65 (38°)	1600
6.13	3^-	270 (40°)	3600
6.92	2^+ $(K=0)$	250 (12°)	3500
7.12	1^-	110 (24°)	1800
8.88	2^-	105 (34°)	Not obtained [b]
9.85	2^+ $(K=2)$	115 (12°)	2100

[a] The summed cross sections (in μb) to states shown are accurate to ±10% for the 0^+ states, ±20% for all other states, with the exception of the 9.85 MeV level, which is only accurate to ±30%.

[b] See Ref. 23 and/or Ref. 24, for an angular distribution.

TABLE II. The Brown-Green coexistence model wave functions for positive parity ^{16}O states.

State	J^π, K	$\lvert 0p\text{-}0h\rangle$	$\lvert 2p\text{-}2h\rangle$	$\lvert 4p\text{-}4h\rangle$
g.s.	$0^+,0$	0.874	0.469	−0.130
6.05	$0^+,0$	−0.262	0.229	−0.937
11.3	$0^+,0$	−0.410	0.853	0.323
6.92	$2^+,0$		0.377	−0.923
9.85	$2^+,2$		0.397	−0.918

TABLE III. SU_3 parentage factors for $^{18}O(p,t)^{16}O$.

^{18}O Configuration	(λ,μ)	$^{(2S+1)}L_J$	$(np-nh)$	^{16}O $(\lambda,\mu)J\ K$	Parentage factors $\beta(l)^2(^{2S+1}L_J)$
			0 0	(0,0)0 0	$0.667\ d^2(^1S_0)+0.745\ s^2(^1S_0)$
$[4^3,2](sd)^2$	(4,0)	$^1S_0$	2 2	(4,2)0 0	$-0.360\ p^2(^1S_0)$
			4 4	(8,4)0 0	0.0
			0 0	(0,0)0 0	0.0
$[4^3,2]p^{10}(sd)^4$	(8,2)	$^1S_0$	2 2	(4,2)0 0	$0.159\ d^2(^1S_0)+0.177\ s^2(^1S_0)$
			4 4	(8,4)0 0	$0.694\ p^2(^1S_0)$
			2 2	(4,2)2 0	$-0.326\ p^2(^1D_2)$
$[4^3,2](sd)^2$	(4,0)	$^1S_0$	2 2	(4,2)2 2	$-0.656\ p^2(^1D_2)$
			4 4	(8,4)2 K	0.0
			2 2	(4,2)2 0	$-0.314(sd)^2(^1D_2)$
$[4^3,2]p^{10}(sd)^4$	(8,2)	$^1S_0$	2 2	(4,2)2 2	0.0
			4 4	(8,4)2 0	$0.563\ p^2(^1D_2)$
			4 4	(8,4)2 2	$1.071\ p^2(^1D_2)$
			4 4	(8,4)2 4	0.0

to be much less satisfactory and far more sensitive to small parameter changes both in the bound-state and in the optical model parameters. Indeed, relative theoretical cross sections are no more reliable than a factor of 2 on this light nucleus, for DWBA calculations run at both 20 and 41.7 MeV (the beam energy in Ref. 25). Se also Refs. 24, 39, and 40. Nevertheless, even within this limitation, one should be able to make some definite statements with regard to the model at hand, a point which has also been emphasized elsewhere.[41]

Several different choices of optical model parameters were used in the calculations; representative ones are shown in Tables VI and VII. The values shown are as entered into the DWUCK code; the form of the optical potential can be found in Ref. 42. No attempt was made to fit any elastic scattering data, although the potentials shown were by and large determined from such data in similar mass and energy regions. An exception is the DX potential taken from Ref. 46, for 20 MeV (p,t) reactions on the tin isotopes, which generally gave poor fits to the present data. In some cases there have been slight changes in geometry, when compared with the original references, in order to improve the quality of the fits. No spin-orbit potential has been included in the triton potential [the 24.4 MeV $^{18}O(p,t)$ calculations of Ref. 24 show some improvement in back angle data when a spin-orbit term is included, but our data only extend to 60°]. The best over-all angular distribution shapes were obtained with either the AW or the CW potential combinations; the W triton potential had previously given good fits to (p,t) reactions on light nuclei,[39] although at higher exit triton energies. The DWBA fits shown in Figs. 2–4 were obtained with the CW potential and utilizing the HO form factor; similar curves could be obtained with the WS form factor. Each DWBA fit presented in Figs. 2–4 is individually and arbitrarily normalized to the data.

With regard to the WS calculation, there was a marked effect on the shapes (and the magnitudes) of the angular distributions (at both 20 and 41.7 MeV) for the 0^+ transitions, depending on whether $A=16$ or $A=17$ was used in the bound-state calculation. Indeed, satisfactory WS fits normalized to the data (which agreed well with the HO results shown in Figs. 2–4) could only be obtained with the $A=17$ case. This 6% change from $A=16$ to $A=17$ affects both the geometry of the WS well

TABLE IV. Two-nucleon structure factors (G_{NLJ}) for $^{18}O(p,t)^{16}O$.

Transition	(JK)	$G_N=1$	$G_N=2$	$G_N=3$
$0^+\to 0^+$, g.s.	0 0	+0.0091	−0.148	+0.538
$0^+\to 0^+$, 6.05	0 0	+0.0078	−0.138	−0.151
$0^+\to 0^+$, 11.3	0 0	+0.0112	−0.176	−0.223
$0^+\to 2^+$, 6.92	2 0	−0.155	−0.0121	
$0^+\to 2^+$, 9.85	2 2	−0.312		

TABLE V. Two-nucleon $(L=0)$ pickup amplitudes (β) for $^{18}O(p,t)^{16}O$.

Transition	$\beta(d_{5/2})^2$	$\beta(d_{3/2})^2$	$\beta(s_{1/2})^2$	$\beta(p_{3/2})^2$	$\beta(p_{1/2})^2$
$0^+\to 0^+$, g.s.	0.454	0.371	0.655	−0.149	−0.105
$0^+\to 0^+$, 6.05	0.128	0.104	0.184	0.166	0.118
$0^+\to 0^+$, 11.3	0.188	0.154	0.272	0.212	0.150

TABLE VI. Optical model parameters for the proton channel.

Label	V	r_v	a_v	W	r_w	a_w	W_D	r_D	a_D	V_{so}	r_{so}	a_{so}	r_c	Ref.
A	−46.5	1.15	0.60				54.0	1.16	0.43	−30.0	1.25	0.47	1.25	43
B	−51.5	1.04	0.67	−1.7	1.17	0.52	32.4	1.17	0.52	−24.8	1.01	0.75	1.17	44
C	−55.7	1.19	0.70				31.0	1.37	0.59	−5.80	1.07	0.13	1.10	45
D	−55.7	1.20	0.70				45.2	1.25	0.70	−12.0	1.10	0.70	1.20	46

and the zero-range integration in the cross-section calculation. While one can possibly mount an argument for binding one neutron in a well of $A = 16$ and the other in a well of $A = 17$, binding both with $A = 17$ clearly makes no physical sense. However, such a change does point out the sensitivity of the calculated cross section to small parameter changes; the result obtained was not anticipated and is an indication of the level of uncertainty inherent in (p, t) DWBA calculations on light nuclei. See also Refs. 24, 39, 40, and 49.

Moreover, the whole question of how to treat the binding energy of the two nucleons in a WS well in order to guarantee the proper behavior of the form factor is not at all understood.[50] Our assumption is the usual one[34,35] of assigning half the two-nucleon separation energy to each of the transferred particles, but this is surely incorrect for the transfer of particles of different spins. Systematic calculations of (p, t) reactions throughout the $1p$ shell by Kahana and Kurath,[51] however, have been rather successful in using this approach, although these authors do refer to the *ad hoc* nature of treating the two-nucleon bound state in this manner. In fact, the truncated HO (i.e., matched directly to a Hankel-function tail whose argument is characteristic of the separation energy) may well be the more pragmatic approach. However, it also suffers from considerable uncertainty, particularly in light nuclei,[39,40] depending on the chosen matching radius of the Hankel-function tail. In the $^{15}N(p, t)^{13}N$ analysis of Ref. 39, variations in relative cross sections of a factor of 2 over 10 MeV of excitation were reported depending on the position of matching. These effects are referred to again in the following discussion.

The relative cross sections obtained with the HO form factor for the four positive parity states observed are compared with the data in Table VIII. What is shown in the table are the results of the AW and CW optical model calculations (the CW normalized fits are shown in Figs. 2–4) as well as the results of an average calculation involving several combinations of optical model parameters (13 for the 0^+ and 10 for the 2^+ transitions), including the ones shown in Tables VI and VII. The errors shown for the average calculation are average deviations, the maximum deviation being a factor of 2. The values shown are, in all cases, *ratios* (relative to the g.s.) obtained from summed cross sections in the range 10–50° in 2° steps. The same comparison based on integrated cross sections (i.e., weighted by $\sin\theta$) gave very similar results.

IV. DISCUSSION

A. 0^+ states

The 0^+ states associated with the coexistence model in ^{16}O are the ground, 6.05, and 11.3 MeV states, which are predominantly $|0p\text{-}0h\rangle$, $|4p\text{-}4h\rangle$, and $|2p\text{-}2h\rangle$, respectively (Table II). The $|4p\text{-}4h\rangle$ state lies considerably lower in energy than the $|2p\text{-}2h\rangle$, which is usually explained in terms of its large shape deformation.[3,4,6,11] Some time ago Zamick[32] showed that the isospin structure of the $|4p\text{-}4h\rangle$ state also contributed to a considerable lowering in energy and Sørensen[8] has recently examined the structure of this state in terms of a superfluid phase transition in the ground state, thereby creating a shape isomeric state at an equilibrium deformation. In the present experiment at 20 MeV, the only 0^+ states ob-

TABLE VII. Optical model parameters for the triton channel.

Label	V	r_v	a_v	W	r_w	a_w	W_D	r_D	a_D	r_c	Ref.
W	−153.0	1.25	0.65				64.0	1.25	0.54	1.30	39
X	−176.0	1.14	0.72	−18.0	1.61	0.82				1.14	46
Y	−131.8	1.30	0.73				51.7	1.47	0.48	1.40	47
Z	−147.0	1.40	0.55	−18.4	1.40	0.55				1.40	48

served are the ground and 6.05 MeV states, the latter state being well resolved from the neighboring 6.13 MeV (3^-) level (Fig. 1). Strength to the 11.3 MeV and other 0^+ states in the $^{18}O(p,t)$-^{16}O reaction has been reported in Refs. 25 and 26, at higher bombarding energies. The experimental summed cross-section ratio of the 6.05 MeV to the ground state of ~6% (Table VIII) is essentially the same as that found at higher energies,[25] but our theoretical calculations and those of Adelburger *et al.*[25] for the same ratio differ considerably. As can be seen from Table VIII, with the possible exception of the CW calculation, our DWBA results employing the (SU_3) structure factors of Table IV agree well with the data. However, in Ref. 25, the same ratio disagrees with the experiment by a factor of 10 (the 6.05 MeV cross section being too small). Indeed, largely on this basis, Adelburger *et al.* (at 41.7 MeV) have concluded that the coexistence model fails completely to account for the observed (p,t) cross sections to 0^+ states in ^{16}O. Our result is essentially orthogonal to this. Some plausible explanations for the difference and further discussion of the over-all reliability of the calculated cross sections follow.

In the HO calculations shown in Table VIII, some sensitivity in calculated cross sections was experienced depending on the choice of optical model potential, particularly for the 0^+ states. The difference of a factor of 2 in relative cross sections shown between the CW and AW calculations was the maximum deviation observed. Although these particular choices gave the best (normalized) fits to the shapes of the angular distributions (the CW curves are displayed in Figs. 2–4), many of the other optical model combinations employed gave acceptable fits also, to both the g.s. and 6.05 MeV transitions. For this reason, we feel that the most reliable value for the cross-section ratio (calculated with the Ho form factor) is the average result of 5.5 ± 1.2 for 13 different calculations, compared with the experimental value of 6.3 ± 0.6. These numbers agree well, within their respective errors.

As mentioned previously, the calculated cross sections shown in Table VIII are based on our 3.5% |4p-2h⟩ amplitude in the ^{18}O g.s. and the Brown-Green wave functions for ^{16}O. In fact, there is considerable disagreement on the amount of "deformed" |4p-2h⟩ component in the ^{18}O g.s. wave function, ranging from about 5% in Ref. 7 to about 40% in Ref. 17. The latter value is consistent with an effective interaction calculation of Kolltveit.[5] Experimentally, nuclear reactions have yielded spectroscopic strengths consistent with about 10% hole component,[52] although the recent $^{18}O(p,d)$ analysis of Ref. 24 argues for something like 25%. While our value of 3.5% is probably too low, it is rather close to the ~5% admixture employed in Ref. 7, which, in conjunction with the SU_3 coupling scheme, has resulted in nice agreement in level positions and $B(E2)$ values in ^{18}O. Certainly it is unlikely that a factor of 10 increase could be justified. However, in order to examine the effect of a larger |4p-2h⟩ admixture, we have also carried out calculations using a 17% admixture, which represents an average of all the values tabulated in Ref. 24.

The (average) calculated cross-section ratio using this 17% |4p-2h⟩ amplitude is 4.3. Compared to the 3.5% result of 5.5 (Table VIII), this value of 4.3 is in somewhat poorer agreement with the experimental ratio of 6.3, but not significantly so. Our calculation for the 0^+ states agree well with the case II calculations of Ref. 6 (with ~10% |4p-2h⟩ amplitude), recalculated for the present 20 MeV bombarding energy. This latter calculation yields a value of 7.7 for the average cross-section ratio of 6.05 MeV to ground state. Since the calculations of Ref. 6 were done quite independently, the consistency obtained can be taken as some degree of success of the basic coexistence model, at least for these particular 0^+ states. It should be noted that even a 3.5% |4p-2h⟩ amplitude in the ^{18}O g.s. is important in a calculation of the (p,t) cross section. Indeed, the average calculated strength to the 6.05 MeV 0^+ level would be a factor of 2 greater if it were ignored, thus worsening the agreement with experiment by the same factor. This result is consistent with that which we reported much

TABLE VIII. Theoretical DWBA calculations, $^{18}O(p,t)^{16}O$, 20 MeV.

Exc. (MeV)	J^π, K	$(\sum_\theta, \text{ratio})_{\text{exp.}}$ [b]	$(\sum_\theta, \text{ratio})$ [a] AW	CW	$(\sum_\theta, \text{ratio})_{\text{avg.}}$ [c]
0.0	$0^+, 0$	100.0	100	100	100
6.05	$0^+, 0$	6.3 ± 0.6	6.5	3.5	5.5 ± 1.2
6.92	$2^+, 0$	13.8 ± 2.8	0.4	0.5	0.5 ± 0.1
9.85	$2^+, 2$	8.3 ± 2.5	0.8	1.2	1.0 ± 0.2

[a] Theoretical summed cross-section ratios obtained with the HO form factor and the AW and CW optical potentials.

[b] Experimental summed cross-section ratios, obtained from Table I, relative to a value of 100 for the g.s. transition. See footnote a, Table I.

[c] Theoretical summed cross-section ratios obtained with the HO form factor and an average result of several optical model combinations. The errors shown represent average deviations for these particular calculations, as discussed in the text.

earlier and based on much simpler wave functions.[22]

However, both the present calculations given in Table VIII and those in Ref. 6 were carried out with an HO form factor and, as referred to above, this may well lead to considerable errors in the determination of relative cross sections in light nuclei, depending on the matching radius chosen for the Hankel-function tail. For this reason the 0^+ (p,t) cross sections were also computed with the WS form factor, using the pickup amplitudes of Table V and with each neutron bound with half the total separation energy. On the average, this calculation yielded a cross-section ratio of 6.05 MeV to ground state which was reduced with respect to the HO result by a factor of 2. However, in view of the above mentioned uncertainty inherent in treating the binding energy of the transferred neutrons in the WS well,[50] it is difficult to know how to properly assess this result. The cross-section calculations are extremely sensitive to the parameters of the WS well; the previously referred to change from $A=16$ to $A=17$ markedly affects the shape of the calculated angular distributions, particularly for the 6.05 MeV transition. Indeed, the relative cross sections computed with $A=16$ in the bound states were essentially identical with those obtained with the HO, but the fits were definitely inferior. Probably a meaningful value for the theoretical ratio of $\sum_\theta(6.05)/\sum_\theta(\text{g.s.})$ at 20 MeV would be 4.0 ± 1.5, calculated on the basis of the present SU_3/coexistence model and taking into account the uncertainties just described. This is to be compared with the experimental ratio of 6.3 ± 0.6.

Using a WS form factor, Adelburger *et al.*[25] have reported a calculated value of 0.76 for the same ratio at 41.7 MeV proton energy, compared to an experimental ratio of 7.7. We have repeated their 0^+ calculations at 41.7 MeV, using the same nuclear parentage factors given in Table III that gave rise to the 20 MeV theoretical values of Table VIII. Again, there was a high degree of sensitivity in the DWBA calculations, depending on the choice of $A=16$ or $A=17$ in the bound-state wave function and/or on the choice of form factor itself, i.e., either HO (using the structure factors of Table IV) or WS (using the pickup amplitudes of Table V). We obtained, for the ratios $\sum_\theta(6.05)/\sum_\theta(\text{g.s.})$ and $\sum_\theta(11.3)/\sum_\theta(\text{g.s.})$ at 41.7 MeV, values of 7 ± 3 and $20\pm10\%$, respectively. The errors shown are meant to account for the degree of uncertainty just discussed (the 11.3 MeV level is not observed at 20 MeV). Similar calculations reported in Ref. 6 at 25 MeV with an HO form factor yielded 7 and 17%, respectively, with no degree of uncertainty quoted. The calculated values of Adelburger *et al.* at 41.7 MeV are 0.76 and 1.34 for the same two ratios, compared to the experimental results of 7.74 and 2.61, respectively.[25] Our calculated values at 41.7 MeV for both 0^+ states (6.05 and 11.3 MeV) are consistent with those reported earlier by Donau *et al.*,[6] but are a factor of 10 higher than those of Adelburger *et al.*[25] Indeed, as previously referred to, based largely on the 6.05 MeV ratio, Adelburger *et al.* conclude that the basic coexistence model is completely inadequate in describing the nuclear structure of 0^+ states in ^{16}O. We cannot agree with this conclusion, at least for the 6.05 MeV state, since, on the basis of our calculations, the model works as well as can be expected for a DWBA calculation on such a light nucleus, at both 20 and 41.7 MeV proton energies.

The main reason for the difference in calculated 0^+ cross sections between Adelburger *et al.* and ourselves presumably lies in the basis interpretation of the coëxistence states. Adelburger *et al.* have a very large amplitude for $(p_{1/2})^2$ transfer to the 6.05 MeV state which, as they point out, essentially cancels the contribution due to $(sd)^2$ pickup, thus leading to an overly small cross section. Their calculation for the $^{42}Ca(p,t)$-^{40}Ca reaction yields very similar results. On the other hand, their calculation using the ZBM wave functions[17] leads to a smaller $(p_{1/2})^2$ amplitude and one which constructively interferes with the $(sd)^2$ pickup thus enhancing the cross section to the 6.05 MeV level such that it is within a factor of 2 of the experimental result. A large destructive $(p_{1/2})^2$ transfer in the coexistence model would follow if the ^{16}O $|2p\text{-}2h\rangle$ state were assumed to be largely spherical. In our calculations, this state is highly deformed and the SU_3 projected overlaps yield relatively small pickup amplitudes for both $(p_{1/2})^2$ and $(p_{3/2})^2$ transfer (Table V). These do interfere destructively with the $(sd)^2$ amplitudes, but not to the disastrous extent found in Ref. 25. The destructive interference in the 6.05 (and 11.3 MeV) 0^+ transitions is also apparent in the HO structure factors given in Table IV; the form factor is only constructive at the nuclear surface when the different N components have alternating phases, such as for the g.s. transition.

The predictions of the coexistence model for 0^+ cross sections have also been recently investigated by Erikson *et al.*, in a study of the ^{42}Ca-$(p,t)^{40}Ca$ reaction.[10] The usual axial symmetry requirements of the model were found to give calculated cross sections in very poor agreement with experiment, consistent with the same conclusions reached in Ref. 25. However, it was found in Ref. 10 that deviations from axial symmetry caused a significant redistribution of hole strength, which largely affects the cross sections to excited 0^+ states. Deviations from axial symmetry in ^{16}O have also recently been suggested[53] to account for the over-all spectrum and in particular for the existence of a deformed shape minimum corresponding to a triaxial rotational band based on the 6.05 MeV state. These calculations, in fact, reaffirm the basic viability of the coexistence model. In the ^{42}Ca$(p, t)^{40}$Ca 0^+ analysis of Erikson *et al.*,[10] this redistribution of hole strength introduces single particle orbits of mixed k values, and SU_3 calculations carried out on this basis result in a significant improvement in calculated strengths, particularly for the low-lying states in ^{40}Ca. The DWBA calculated shapes also agreed much better with the experimental angular distributions when nuclear overlaps in ^{40}Ca were calculated on the basis of the SU_3 coupling model, thus supporting similar calculations we have carried out on the ^{18}O$(p, t)^{16}$O reaction. As these authors state, it is not time yet to assign a "death certificate" to the coexistence model.[10]

B. 2^+ states

In our 20 MeV analysis of the ^{18}O$(p, t)^{16}$O reaction, the major discrepancy between the present SU_3/coexistence model calculation and experiment comes not with the 0^+ states but the 2^+. The HO results of Table VIII suggest that the model as a whole fails to account for the magnitude of the 2^+ cross sections relative to the ground state. It is unlikely that the factor of 10–20 discrepancy evident in Table VIII can be explained in terms of the kinds of DWBA uncertainties just discussed. Rather, it points to the need for some obvious changes in our basic assumptions or to a complete breakdown in the model itself. We note that the calculated ratio of just the 2^+ cross sections, i.e., 6.92 ($K=0$) to 9.85 ($K=2$), agrees considerably better with the experimental ratio for these states than either 2^+ state to the g.s., thus suggesting that the total $L=2$ strength incorporated in the present calculations is not adequate.

In these calculations, the main contribution to the 2^+ strength, both $K=0$ and $K=2$ comes via a (p^2) $L=2$ pickup (Table III), populating both the $|2p\text{-}2h\rangle$ and $|4p\text{-}4h\rangle$ amplitudes in ^{16}O. For the 6.92 MeV transition, there is an additional contribution due to an $(sd)^2$ $L=2$ transfer, which although small, is important, since it contributes to a surface form factor (this is the only contribution to the $N=2$ structure factor of Table IV). By contrast, the 9.85 MeV $K=2$ transfer is pure $(p)^2$. For both transitions, the $|4p\text{-}2h\rangle$ to $|4p\text{-}4h\rangle$ overlap is important and the calculations of Table VIII were carried out with the previously mentioned 3.5% $|4p\text{-}2h\rangle$ amplitude in the ^{18}O ground state. We have already discussed the effect of increasing this amplitude to 17% (an average of the different values given in Ref. 24) for the 0^+ transitions—there was not a significant change in the calculated cross section ratio of 6.05 MeV to ground. Such is not the case for the 2^+ transitions. In particular, a 17% $|4p\text{-}2h\rangle$ amplitude in the ^{18}O g.s. wave function increases the 6.92 MeV $K=0$ cross section by a factor of ~6 and the 9.85 MeV $K=2$ cross section by a factor of ~2. This would mean, with reference to the calculated values in Table VIII, a 6.92 MeV relative cross section of ~3% vs an experimental ratio of ~14% and a 9.85 MeV relative cross section of ~2% vs an experimental ratio of 8%. Since a 17% amplitude of $|4p\text{-}2h\rangle$ may well be too large, our calculations at 20 MeV show that the 2^+ strength is underestimated in the ^{18}O(p, t) reaction by a factor of ~8, a serious discrepancy and one which is probably outside the inherent error expected for DWBA calculations on light nuclei. We note that the calculated 2^+ shapes based on the 3.5% $|4p\text{-}2h\rangle$ amplitude agree reasonably well with the data (Fig. 3), as do the 0^+ shapes (Fig. 2) previously discussed.

The 2^+ coexistence model wave functions of Brown and Green (Table II) are predominantly $|4p\text{-}4h\rangle$, and this overlaps rather poorly with the predominant $|2p\text{-}0h\rangle$ amplitude in the ^{18}O ground state. If the $|2p\text{-}2h\rangle$ amplitude in these states were larger, this would increase the calculated 2^+ cross sections. For example, if the $|2p\text{-}2h\rangle$ and $|4p\text{-}4h\rangle$ amplitudes in the $K=2$ state were equally mixed ($1/\sqrt{2}$ of each) and the ^{18}O g.s. contained 17% $|4p\text{-}2h\rangle$, then the calculated cross section of pure $(p)^2$ pickup to the 9.85 MeV level would be enhanced by a factor of 10. The effect would be even greater for the 6.92 MeV transition because of the $(sd)^2$ contribution. It is this drastic kind of change which is needed and probably could only be obtained at the expense of the general consistency in spectra and electromagnetic transition probabilities which already exists.[4,12,18]

It is tempting on this basis to argue for more configuration mixing, in the spirit of the ZBM calculations[17] and/or a more expanded configuration space. The ZBM wave functions have been employed in the ^{18}O(p, t) calculations of Adelburger *et al.*[25] and Pignanelli *et al.*[24] and in both cases, good agreement with experiment is claimed. As discussed in Ref. 25, since many more configurations are available in the ZBM wave functions, there is much less destructive interference between particle and hole amplitudes. However, as we have already pointed out, the level of agreement obtained by Adelburger *et al.* for the 0^+ states, using the ZBM wave functions, is not better than we have obtained using the SU_3 coupling scheme and the coexistence model. With regard to the 2^+ states, the ZBM calculations of Pignanelli *et al.*[24] are also in relatively good agreement with the experimental results. In fact, all their DWBA calculated cross sections appear to agree well with the experimental results, thus making a strong argument for the validity of the ZBM wave functions. (We assume that relative cross sections are being compared in Ref. 24, although this is not actually stated in the paper and no procedure is given for the normalization of the calculations to the data.) It will be interesting to have the results of similar calculations in the calcium region,[54] in order to see the level of agreement with experiment in the $^{42}Ca(p, t)^{40}Ca$ reaction.[10,25,55]

C. Negative parity levels

No nuclear structure information is presented for the negative parity states—the DWBA fits presented in Fig. 4 were calculated on the assumption of a pure $(d_{5/2}p_{1/2})$ transfer for the 6.13 MeV 3^- transition and a pure $(p_{1/2}s_{1/2})$ transfer for the 7.12 MeV 1^- transition. These are the leading amplitudes in shell model calculations for these levels which, in general, involve many amplitudes.[17] The calculated *shapes* of the (p, t) angular distributions are not expected to change appreciably if a full configuration mixed wave function were employed, particularly within the same major oscillator shell.

Only three negative parity states were populated in the present experiment, the 6.13 MeV (3^-), 7.12 MeV (1^-), and 8.88 MeV (2^-). These states are all $T=0$. It was not possible to observe levels any higher in excitation than the 9.85 MeV (2^+) level, for the reasons discussed earlier. Higher-lying negative parity levels (both $T=0$ and $T=1$) have been seen in a 43.7 MeV study of the $^{18}O(p, t)^{16}O$ reaction.[26] Despite the fact that a meaningful angular distribution for the 8.88 MeV (2^-) level was not obtained, it was observed at three angles in a separate series of measurements (20, 30, and 40°) and found to be relatively strongly populated, in all cases comparable to the 7.12 MeV (1^-) transition (see Table I). As previously mentioned, an angular distribution has been reported by Lutz *et al.*[23] at 18.2 MeV and by Pignanelli *et al.*[24] at 24.4 MeV; in both cases these are in good agreement with the relative cross sections that we have found at the above three mentioned angles. This level has also been observed at 43.7 MeV.[26] Since this 2^- state is one of unnatural parity, it can be populated in the $^{18}O(p, t)^{16}O$ reaction either by a breakdown in the usual selection rules and/or by accompanying second-order processes. A proper understanding then of the reason(s) for the population of this level is probably crucial to an understanding of the whole reaction mechanism, not only in this particular reaction, but for two-nucleon-transfer reactions on light nuclei in general.

In a direct two-neutron-transfer reaction, a $0^+ \rightarrow 2^-$ transition is forbidden under the usual assumption of a $\lambda=0$ relative motion of the two transferred neutrons.[33] The "few percent" ^4D state admixture in the triton g.s. wave function[56] could allow the direct population of such a 2^- state, as argued in Ref. 24, but its relatively strong population in the $^{18}O(p, t)^{16}O$ reaction may be hard to justify on this basis. More likely, this 2^- state is being populated through an accompanying inelastic excitation (coupled channels via the 3^- level in either ^{18}O or ^{16}O), or possibly by a sequential reaction process.[57] Based on analogy with coupled-channel (CC) calculations in rare-earth nuclei,[58] the 2^+ relative cross sections would be the ones most strongly affected, which is at least consistent with the lack of agreement discussed earlier. The suggestion that CC effects are probably present in the $^{18}O(p, t)^{16}O$ reaction is supported by the recent calculations of Sørensen[8] previously referred to, on the same reaction, and by recent calculations of the similar population of a 2^- state in the $^{22}Ne(p, t)^{20}Ne$ reaction.[59] In the CC treatment of Ref. 8, specific reference to the 2^- state populated in the $^{18}O(p, t)^{16}O$ reaction is not given, but Sørensen argues for such an approach in order to fit all transitions, in analogy with the situation in rare-earth nuclei.[58] In the $^{22}Ne(p, t)$ calculations of Ref. 59, it is in fact stated that such a CC approach is just as important as it is in the rare earths. Curiously enough, despite the well established presence of CC contributions to (p, t) reactions in rare-earth nuclei, there is little evidence for the population of states of unnatural parity in these nuclei.[1,38] Additional general evidence for the presence of CC effects in light nuclei may possibly be found in the "anomalous" $(p, t)/(p, {}^3He)$ cross-section ratios found in the $1p$ shell.[60]

Unlike the situation in the rare earths,[1,38,58,61] at least for the 2^+ levels, it appears that the forward angle behavior of the 2^+ angular distributions in the $^{18}O(p, t)^{16}O$ reaction can be reasonably well reproduced by an ordinary DWBA treatment (Fig. 3). See also Refs. 6 and 24. In similar fashion, the calculated angular distribution shape for the 7.12 MeV 1^- transition agrees well with the 20 MeV data (Fig. 3). The same statement can clearly not be made for the 3^- transition populating the 6.13 MeV level (Fig. 3). No combination of optical model parameters was able to reproduce the cross-section minimum observed at 18° (see also Ref. 23), which, in itself, is an indication of the presence of a coherent interference due to some competing process. Indeed (p, t) transitions populating 3^- final states are generally difficult to fit by DWBA calculations over a wide range of mass.[38,46,49] The excellent agreement found in the lead region[62] constitutes an exception to this statement. In general, 3^- levels contain many (p-h) amplitudes and it may be possible to obtain better DWBA fits with a highly configuration mixed wave function involving more than one major oscillator shell. This is consistent with the suggestion made earlier of the possible need for more configuration mixing in the 2^+ states of ^{16}O.

V. CONCLUSIONS

The ^{18}O$(p, t)^{16}$O reaction has been investigated at 20 MeV bombarding energy. The relative cross sections for the positive parity states have been calculated within the framework of the coexistence model using an SU_3 coupling scheme to determine the two-nucleon parentage factors. The normal first-order DWBA calculations are found to be overly sensitive to small parameter changes so that agreement between theory and experiment cannot be relied on to better than a factor of 2. Within this degree of uncertainty, the cross-section ratio of the 6.05 MeV 0^+ level to the g.s. agrees well with the experimental result, consistent with the earlier coexistence model calculations by Donau *et al.*[6] but inconsistent with the more recent ones of Adelburger *et al.*[25] See also Ref. 10.

The calculated strengths to the 2^+ states at 6.92 MeV ($K=0$) and 9.85 MeV ($K=2$) do not agree with the experimental results. The theoretical 2^+ results are sensitive to the degree of $|4\text{p-}2\text{h}\rangle$ amplitude in the ^{18}O ground state wave function. Calculations with a 3.5% amplitude severely underpredict (by a factor of ~15) the 2^+ strength; a 17% admixture gives improved but still unsatisfactory agreement. It may be also that a more expanded model space should be considered or that the $|n\text{p-}n\text{h}\rangle$ amplitudes in the coexistence model wave functions themselves are not correct. The possibility of contributing inelastic processes is also indicated, thus raising the question of whether the usual DWBA approach is valid at all in light nuclei.

Three negative parity states were observed; 6.13 (3^-), 7.12 (1^-), and 8.88 MeV (2^-); no detailed nuclear structure calculations for the negative parity states were carried out. Of these, the relatively strong population of the 8.88 MeV 2^- level (see also Refs. 23 and 24) argues in favor of the necessity of a complete coupled-channel calculation of the ^{18}O$(p, t)^{16}$O reaction.[8,59]

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