

## Alpha-particle molecules in low-energy heavy-ion reactions\*

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From a survey of the heavy-ion systems measured until now two conditions can be deduced empirically which must be fulfilled in order to find correlated resonances of the type measured in the low-energy excitation functions of the  $^{12}\text{C} + ^{12}\text{C}$  reaction. With both conditions one is able to explain simultaneously the lack and the existence of correlated resonances in all measured reactions. The conditions are: (1) the level density of the corresponding compound nuclei and the number of open channels must be small, and (2) both nuclei in the entrance channel must be  $\alpha$ -particle nuclei. The latter condition seems reasonable because experimental evidence exists for the assumption that these resonances are due to  $\alpha$ -particle doorway states.

NUCLEAR REACTIONS Several heavy-ion reactions, energies in the vicinity of the Coulomb barrier, measured  $\sigma(E, \theta)$ , deduced explanation for the occurrence of correlated resonances.

### I. INTRODUCTION

One of the oldest questions in heavy-ion physics is whether nuclear molecules exist or not. This question has been discussed very enthusiastically since the early days of heavy-ion physics and has been the subject of many contrary opinions. A nuclear molecule is by definition a nuclear structure which is composed of two single nuclei held together by nuclear force. The lifetime of the nuclear molecule must be long compared with the collision time.

The first experimental evidence for the existence of nuclear molecules was obtained in 1960 by Almqvist, Bromley, and Kuehner.<sup>1</sup> These authors found unexpected correlated structure among all exit channels in the excitation functions of the  $^{12}\text{C} + ^{12}\text{C}$  reaction measured at energies just below the Coulomb barrier which could not be explained in the framework of existing reaction models. In contrast to this experimental result, measurements with similar heavy ions in the entrance channel failed to give any correlated structure.<sup>1</sup> Later on these experiments were extended to much higher projectile energies. Pronounced structure has been found in some of the measured excitation functions and was attributed to the formation of nuclear molecules.<sup>2</sup> However, these high-energy experiments will not be the subject of this paper because there are several important features which distinguish these experiments from the low-energy experiments (high level density of the compound nucleus, high  $l$  values) and may therefore result in a completely different interpretation.

The aim of this paper is (1) to summarize the results of those heavy-ion experiments which have been performed in order to see if resonances exist in other systems besides the  $^{12}\text{C} + ^{12}\text{C}$  system and (2) to give an explanation for the fact that correlated structure could be found only in a few systems and is completely absent in most of the systems investigated.

Section II deals with the  $^{12}\text{C} + ^{12}\text{C}$  experiment and some early interpretations in terms of nuclear molecules. In Sec. III the experimental results following the  $^{12}\text{C} + ^{12}\text{C}$  experiment are summarized and in Sec. IV conclusions are drawn from the existence of resonances in only 3 cases and the absence in all other cases. Experimental evidence for these conclusions are given in Sec. V.

### II. $^{12}\text{C} + ^{12}\text{C}$ EXPERIMENT

As mentioned above, the possible existence of nuclear molecules was deduced for the first time from the results of the  $^{12}\text{C} + ^{12}\text{C}$  experiment carried out by Almqvist, Bromley, and Kuehner (ABK) in 1960 at Chalk River.<sup>1</sup> These authors have measured excitation functions for the  $\alpha$ -particle, proton, and neutron exit channels, and also for the total  $\gamma$  yield at projectile energies below the Coulomb barrier ( $E_{CB} \approx 6.7$  MeV). Figure 1 (a) shows the experimental results of ABK and Fig. 1 (b) shows the measurements which have been performed at the Erlangen tandem. The excitation functions exhibit pronounced resonances which are correlated among all exit channels measured. These resonances lie just below

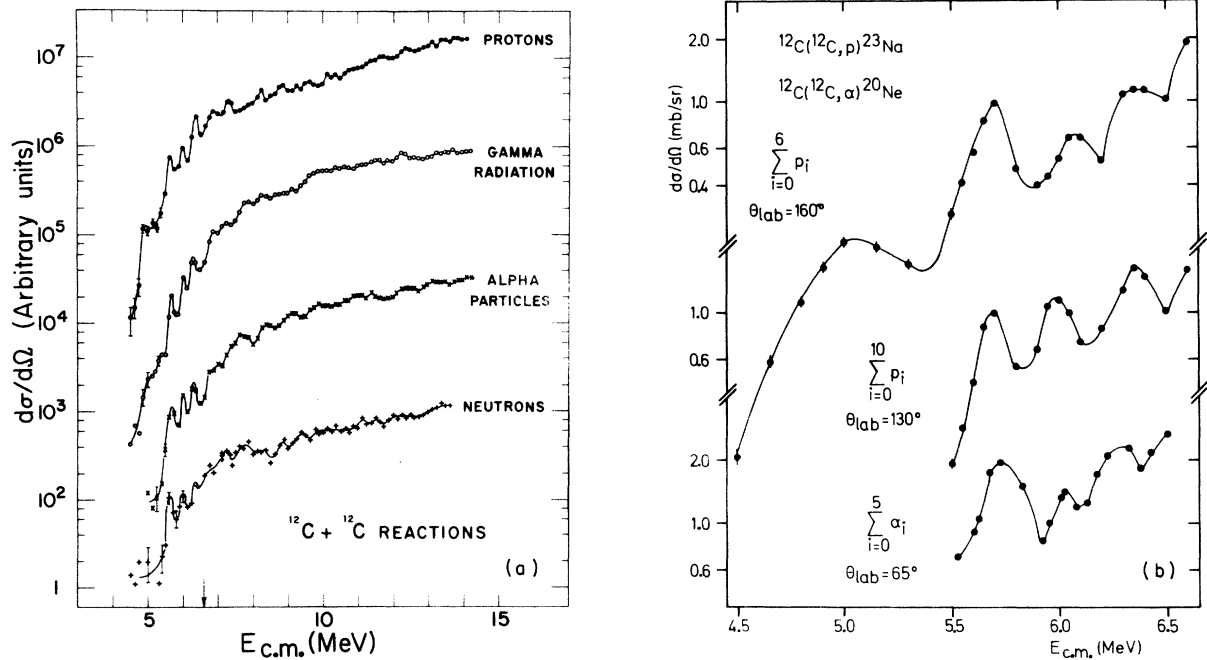


FIG. 1. Excitation functions for the reaction  $^{12}\text{C} + ^{12}\text{C}$ . (a) Shows the results of ABK (Ref. 1) and (b) measurements performed in Erlangen. The summation indices in (b) give the number of transitions to different excited states which have been summed together. The arrow in (a) marks the position of the Coulomb barrier.

the top of the Coulomb barrier [the arrow in (a) marks the position of the barrier]. The widths of the resonances are about 120 keV and they are separated by about 400 keV. The  $J^\pi$  assignments of the resonances at  $E_{c.m.} = 5.6$  and 6.0 MeV have been determined in Ref. 3 to be  $2^+$  and  $4^+$ , respectively. These resonances also exist in excitation functions measured at different angles. Figure 2 shows the  $\theta_{c.m.} = 90^\circ$  excitation function for the elastic channel measured at Erlangen. Here the maxima are anticorrelated with the maxima in the excitation functions of the reaction channels.

The experimental result of the  $^{12}\text{C} + ^{12}\text{C}$  experiment was very surprising. In the  $^{12}\text{C} + ^{12}\text{C}$  reaction the  $^{24}\text{Mg}$  compound nucleus is populated at an excitation energy  $E_x \approx 20$  MeV. For this high excitation energy one expects broad overlapping states<sup>4</sup> in  $^{24}\text{Mg}$  and therefore smoothly varying excitation functions. That means one cannot explain the measured resonances in terms of compound nuclear resonances.

The alternative explanation in terms of statistical fluctuations ( $\Gamma/D \approx 7$ ) fails too, because of the strong correlations which have been measured among all exit channels and different angles. Additionally the excitation functions shown in Fig. 1 contain many transitions to different states in the residual nuclei resulting in a strong damping of statistical effects. For instance, a damping coefficient  $N=52$  is valid for the  $\theta_{lab} = 130^\circ$  proton

excitation function of Fig. 1(b) which makes deviations from a mean cross section by a factor of 2, as found in the  $^{12}\text{C} + ^{12}\text{C}$  experiment, highly improbable

From the analysis of the measured resonance widths a large value for the ratio of the reduced carbon width  $\gamma_C$  to the single-particle width  $\gamma_{s.p.}$  could be deduced ( $\gamma_C/\gamma_{s.p.} = 14\%$  for the 5.6 MeV resonance, calculated in the Wigner limit).<sup>3</sup> This large ratio gave rise to the belief that  $^{12}\text{C} + ^{12}\text{C}$  single-particle states exist at the resonance energies. The lifetime of these states is  $4.10^{-21}$  sec

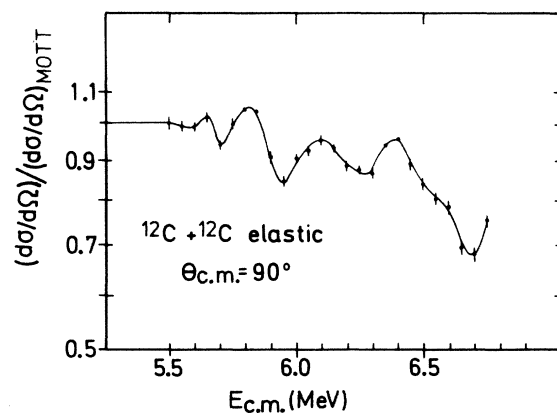


FIG. 2. Excitation function for  $^{12}\text{C} + ^{12}\text{C}$  elastic scattering.

which is long compared to a collision time of about  $7.10^{-22}$ . Therefore these states could be called nuclear molecules according to the definition given in Sec. I. One possible decay mode of these nuclear molecules is the amalgamation of both nuclei into the  $^{24}\text{Mg}$  compound nucleus which in turn decays after some time. The resonance-like creation of the nuclear molecules then results in cross section resonances for all particles evaporated from the compound nucleus. These resonances are therefore due to an entrance channel effect. This interpretation is supported by the fact that the excitation functions of the reaction  $^{23}\text{Na}(p, \alpha)^{20}\text{Ne}$  are almost smooth for projectile energies resulting in  $E_x \approx 20$  MeV in  $^{24}\text{Mg}$ . Figure 3 shows the excitation functions of the reaction  $^{23}\text{Na}(p, \alpha)^{20}\text{Ne}$  together with an excitation function of the  $\alpha$ -particle exit channel of the  $^{12}\text{C} + ^{12}\text{C}$  reaction.

ABK have also measured excitation functions for the  $\alpha$ -particle exit channel and the total  $\gamma$  yield of the  $^{16}\text{O} + ^{16}\text{O}$  system at projectile energies below the Coulomb barrier in order to see if there exist correlated resonances in the  $^{16}\text{O} + ^{16}\text{O}$  sys-

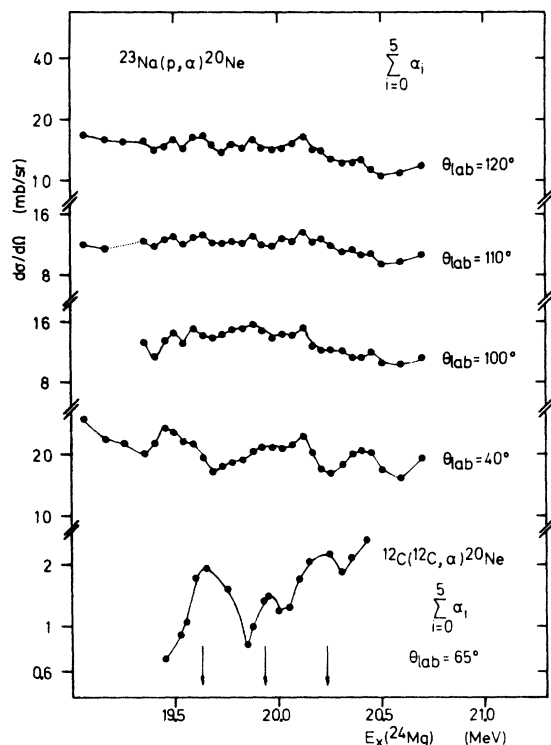


FIG. 3. Excitation functions for the reaction  $^{23}\text{Na}(p, \alpha)^{20}\text{Ne}$  for projectile energies leading to the same excitation energy in the compound nucleus  $^{24}\text{Mg}$  as in the  $^{12}\text{C} + ^{12}\text{C}$  reaction. For comparison, an excitation function of the  $^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}$  reaction is also shown. The arrows mark the positions of the  $^{12}\text{C} + ^{12}\text{C}$  resonances.

tem.<sup>1</sup> These measurements resulted in totally smooth excitation functions. The question then was why are there correlated resonances in the carbon system and not in the oxygen system, or why do there exist  $^{12}\text{C} + ^{12}\text{C}$  molecules but not  $^{16}\text{O} + ^{16}\text{O}$  molecules?

Vogt and McManus<sup>5</sup> were the first who tried to answer this question. According to these authors a potential with a secondary minimum at large nuclear separation is due to the molecular states observed in the  $^{12}\text{C} + ^{12}\text{C}$  system [see Fig. 4(a)]. This secondary minimum is due to a decrease in the surface energy of the  $^{12}\text{C} + ^{12}\text{C}$  system which originates when the two carbon nuclei are soldered together at one surface point during a grazing collision. Thus both carbon nuclei, fixed on one surface point, can perform vibrations along their molecular axis within this secondary minimum. During this vibration both nuclei have to change their shapes permanently which is only possible if both are easy to deform (as the  $^{12}\text{C} + ^{12}\text{C}$  system) and almost impossible if both nuclei are very rigid because of a closed shell structure ( $^{16}\text{O} + ^{16}\text{O}$  system).

Another commonly used explanation has been given by Imanishi.<sup>6</sup> This explanation, however, deals only with the  $^{12}\text{C} + ^{12}\text{C}$  case and gives no idea

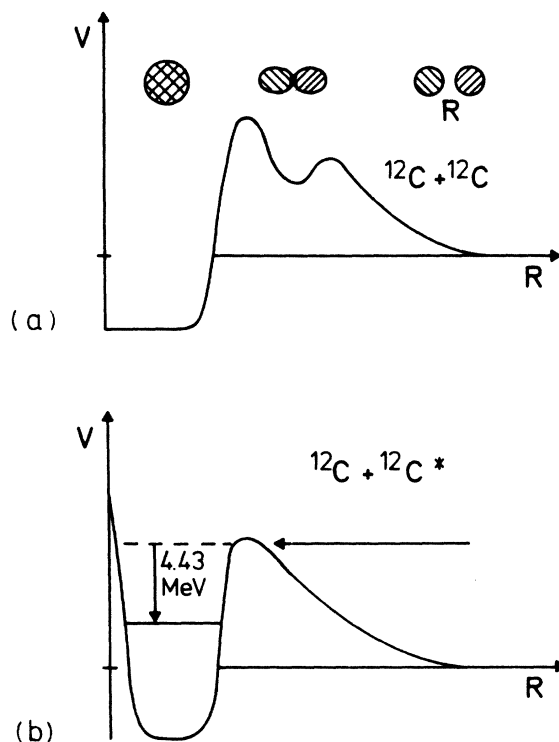


FIG. 4. Nuclear potentials of the reaction  $^{12}\text{C} + ^{12}\text{C}$  according to (a) Vogt and McManus and (b) Imanishi.

why  $^{16}\text{O}+^{16}\text{O}$  molecules do not exist. According to Imanishi the following mechanism is responsible for the  $^{12}\text{C}+^{12}\text{C}$  resonances: During the collision one of the two carbon nuclei will make a transition to the first excited state at 4.43 MeV losing a large amount of kinetic energy. This results in a quasibound state in the effective potential of both nuclei [see Fig. 4(b)]. The positions and the widths of the three resonances just below the Coulomb barrier could be reproduced with detailed coupled channel calculations. However, there is no *a priori* reason that the same mechanism should not also be applicable to the  $^{16}\text{O}+^{16}\text{O}$  system.

### III. FURTHER EXPERIMENTAL INVESTIGATIONS

Since the early experiments of ABK, much more experimental work has been done on this subject, and more data are now available. In this section most of these experiments are summarized in order to find an explanation for the existence and the lack of correlated resonances.

The experiments following the  $^{12}\text{C}+^{12}\text{C}$  work were the total  $\gamma$ -yield measurements of ABK<sup>1,7</sup> for similar heavy-ion systems at projectile energies in the vicinity of the Coulomb barrier:  $^{12}\text{C}+^{14}\text{N}$ ,  $^{12}\text{C}+^{16}\text{O}$ ,  $^{12}\text{C}+^{19}\text{F}$ ,  $^{12}\text{C}+^{20}\text{Ne}$ ,  $^{16}\text{O}+^{20}\text{Ne}$ , and  $^{14}\text{N}+^{14}\text{N}$ . All excitation functions were smooth with the exception of the  $^{14}\text{N}+^{14}\text{N}$  case.<sup>7</sup> Here a single peak has been found. This peak has been a serious obstacle to the interpretation of sub-Coulomb resonances for many years. However, this difficulty has been recently overcome by measurements of Stokstad *et al.* who have found smooth excitation functions for the  $\alpha$ -particle and proton exit channels of the  $^{14}\text{N}+^{14}\text{N}$  reaction.<sup>8</sup> In a very recent measurement of the total  $\gamma$  yield of the  $^{14}\text{N}+^{14}\text{N}$  system a smooth excitation function has been found by the same authors.<sup>9</sup>

Smooth excitation functions have been found for the  $\alpha$ -particle and  $^{12}\text{C}$  exit channels of the  $^{14}\text{N}+^{10}\text{B}$  reaction resulting in the same compound nucleus  $^{24}\text{Mg}$  as the  $^{12}\text{C}+^{12}\text{C}$  reaction.<sup>10</sup> Smooth excitation functions exist for the  $\alpha$ -particle and proton exit channels of the  $^{14}\text{N}+^{12}\text{C}$  reaction,<sup>10</sup> for the  $\alpha$ -particle exit channel of the  $^{12}\text{C}+^9\text{Be}$  system,<sup>8</sup> and for the total  $\gamma$  yield of the  $^{13}\text{C}+^{13}\text{C}$  reaction.<sup>11</sup> A re-measurement of the  $^{16}\text{O}+^{16}\text{O}$  system gave as a result smooth functions for the proton, neutron, deuteron, and  $\alpha$ -particle exit channels as well as for the total  $\gamma$  yield.<sup>12</sup>

Excitation functions for the  $^{13}\text{C}+^{12}\text{C}$  system<sup>10,11</sup> ( $\alpha$ -particle and proton exit channels, total  $\gamma$  yield) are fairly smooth and exhibit no correlated structure in spite of the fact that this system is closely akin to the  $^{12}\text{C}+^{12}\text{C}$  system and that one should expect correlated structure in the framework of the

two explanations mentioned above. Figure 5 shows some of the excitation functions of the  $^{13}\text{C}+^{12}\text{C}$  system measured at Erlangen. In the  $^{12}\text{C}+^{14}\text{C}$  system excitation functions for the  $\alpha$ -particle, deuteron, and triton exit channels have been measured at one angle.<sup>13</sup> Whereas the  $d$  and  $t$  excitation functions are almost smooth, the  $\alpha$ -particle excitation functions exhibit some structure which might not be of statistical origin. However, as long as there is no proof that this structure is correlated among different exit channels and several angles, one should be very careful to interpret this structure as an entrance channel effect as in  $^{12}\text{C}+^{12}\text{C}$  case [there exists for instance some structure in the  $^{12}\text{C}+^{13}\text{C}$  system which is by no means correlated among different exit channels and angles (see Ref. 10)].

The only systems, besides  $^{12}\text{C}+^{12}\text{C}$ , for which correlated resonances have been found for all exit channels and angles measured are the  $^{16}\text{O}+^{12}\text{C}$

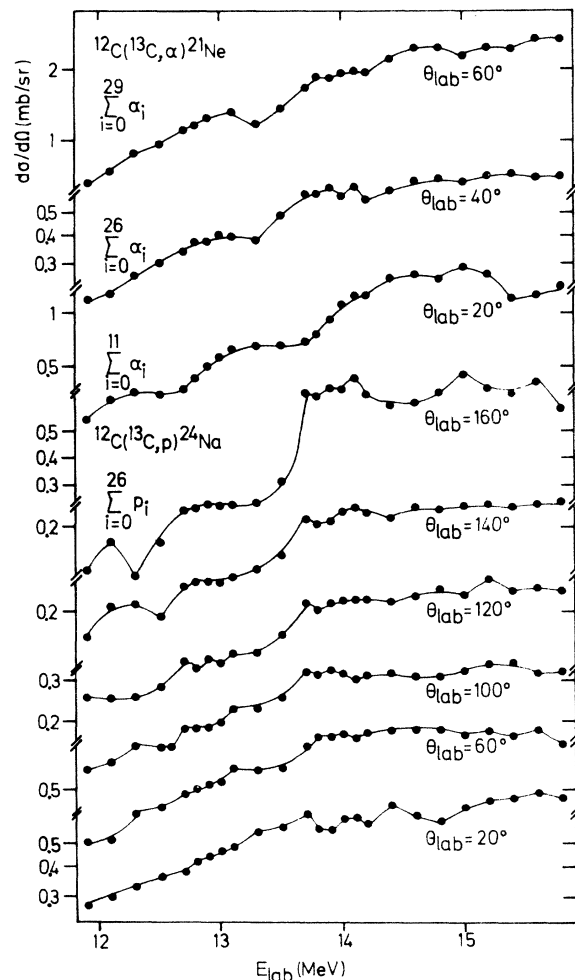


FIG. 5. Excitation functions of the  $\alpha$ -particle and proton exit channels for the reaction  $^{13}\text{C}+^{12}\text{C}$  (Ref. 10).

system<sup>14,15</sup> and the  $\alpha + {}^{20}\text{Ne}$  system.<sup>16</sup> In the former case excitation functions have been measured for the  $\alpha$ -particle and proton exit channels at several angles as well as the total  $\gamma$  yield. Several correlated resonances have been found in the region of the Coulomb barrier having widths of about 220–350 keV. Figure 6 shows some of these excitation functions.<sup>15</sup> For the  $E_{\text{lab}} = 21.6$  MeV resonance the deviations of the cross section from the average cross section are indicated. In Fig. 7 the statistical probabilities are given for these deviations for different values of the damping coefficient  $N$ .

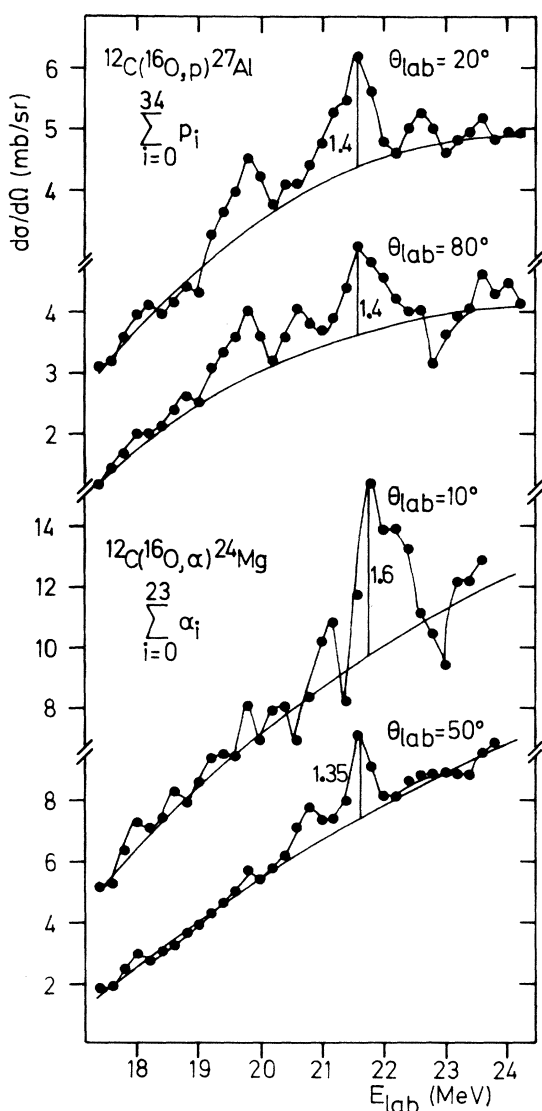


FIG. 6. Excitation functions for the  ${}^{16}\text{O} + {}^{12}\text{C}$  system (Ref. 15). The smooth solid lines show an average cross section; the values on the vertical lines indicate the measured deviation from the average cross section.

In the  $\alpha + {}^{20}\text{Ne}$  system excitation functions have been measured for the proton exit channel as well as for the  ${}^{12}\text{C} + {}^{12}\text{C}$  exit channel at several angles. The projectile energy has been chosen so that the compound nucleus  ${}^{24}\text{Mg}$  is formed at an excitation energy  $E_x \approx 20$  MeV, which corresponds to the excitation energy of the  ${}^{12}\text{C} + {}^{12}\text{C}$  reaction. Figure 8 shows some of these excitation functions. Correlated peaks can be clearly seen in all excitation functions measured. The  $\theta_{\text{lab}} = 40^\circ$  measurement has been performed with high-energy resolution. As a result, the excitation function exhibits considerable fine structure. Using an energy-averaging procedure (averaging interval is 100 keV) only the intermediate structure survives. The widths of these resonances are almost the same as in the  ${}^{12}\text{C} + {}^{12}\text{C}$  case.

Table I contains systems which have been investigated until now in order to see if there exist

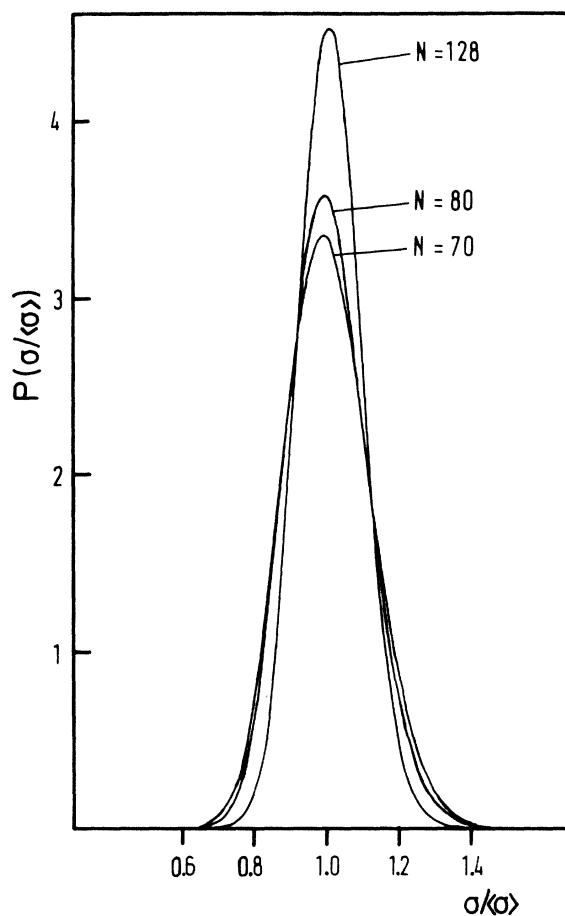


FIG. 7. Statistical probability distributions  $P(\sigma/\langle\sigma\rangle)$  for different damping coefficients  $N$ .  $N = 128$  belongs to the proton excitation functions of Fig. 6,  $N = 70$  to the  $\theta_{\text{lab}} = 10^\circ$ , and  $N = 80$  to the  $\theta_{\text{lab}} = 50^\circ$   $\alpha$ -particle excitation functions of Fig. 6.

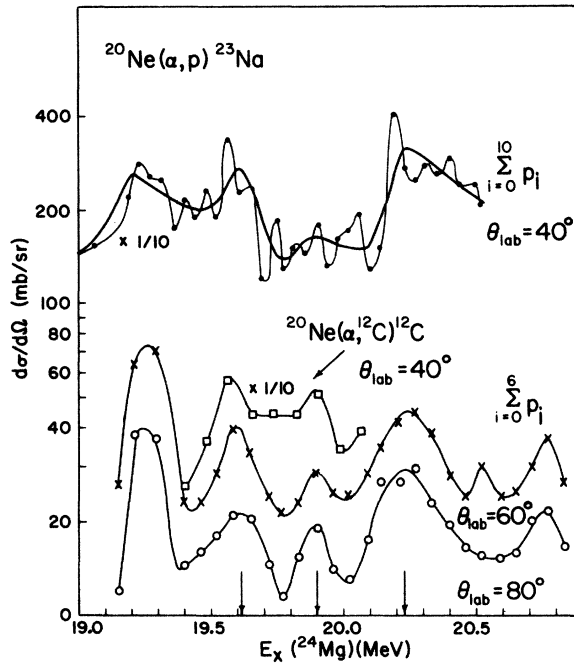


FIG. 8. Excitation functions for the reactions  $^{20}\text{Ne}(\alpha, p)^{23}\text{Na}$  and  $^{20}\text{Ne}(\alpha, ^{12}\text{C})^{12}\text{C}$  for projectile energies leading to  $E_x \approx 20$  MeV in  $^{24}\text{Mg}$ . The arrows indicate the positions of the  $^{12}\text{C} + ^{12}\text{C}$  resonances. In the case of the  $\theta_{\text{lab}} = 40^\circ$  excitation function a good resolution function and an energy averaged function (averaging interval is 100 keV) is shown by the heavy solid line.

correlated structures in the excitation functions similar to those of the  $^{12}\text{C} + ^{12}\text{C}$  system.

Besides the several new systems that have been investigated since the early  $^{12}\text{C} + ^{12}\text{C}$  experiment,

two additional experiments provide very useful information for the study of the  $^{12}\text{C} + ^{12}\text{C}$  resonances. The aim of both experiments was to extend the energy range of the  $^{12}\text{C} + ^{12}\text{C}$  excitation functions down to very low energies. This provides a more accurate extrapolation of the  $^{12}\text{C} + ^{12}\text{C}$  total cross section down to the low energies which are important for carbon burning in stellar matter. As a result of these measurements a continuation of the correlated resonances down to about 4 MeV below the Coulomb barrier with a spacing of less than 0.5 MeV could be found.<sup>17,18</sup>

#### IV. $\alpha$ -PARTICLE MOLECULES

In this section we formulate conditions for both the existence and absence of correlated structure for the systems shown in Table I. In Sec. V it is shown that these conditions are reasonable in the light of the mechanism which is responsible for the correlated resonances.

First of all, looking at Table I, it seems incomprehensible that the  $^{16}\text{O} + ^{12}\text{C}$  system exhibits resonances whereas the  $^{13}\text{C} + ^{12}\text{C}$  system, which is very akin to the  $^{12}\text{C} + ^{12}\text{C}$  system, does not. In particular, if one accepts the interpretation of Vogt and McManus for the  $^{12}\text{C} + ^{12}\text{C}$  resonances in which the deformability of the entrance channel nuclei plays the dominant role, this behavior is far from being obvious.

If one attempts an explanation within the framework of the doorway state model<sup>19</sup> this could be due to different values for the decay width  $\Gamma_d^\dagger$  and the distance  $D_d$  of the intermediate states for different systems.

TABLE I. Reactions which have been investigated in order to see whether correlated resonances exist. Column 6 contains the level densities of the compound nuclei compared with  $^{24}\text{Mg}$  formed in the  $^{12}\text{C} + ^{12}\text{C}$  reaction.

System	Correlated resonances	Compound nucleus	$E_x$ (MeV)	Exit channels	$\rho/\rho_{^{12}\text{C}-^{12}\text{C}}$	Reference
$^{12}\text{C} + ^{12}\text{C}$	+	$^{24}\text{Mg}$	20.6	$\alpha, p, n, \gamma$	1	1
$\alpha + ^{20}\text{Ne}$	+	$^{24}\text{Mg}$	20.6	$^{12}\text{C}, p$	2	16
$p + ^{23}\text{Na}$	-	$^{24}\text{Mg}$	20.6	$\alpha$	2	16
$^{14}\text{N} + ^{10}\text{B}$	-	$^{24}\text{Mg}$	35.3	$\alpha, ^{12}\text{C}$	210	10
$^{13}\text{C} + ^{12}\text{C}$	-	$^{25}\text{Mg}$	22.9	$\alpha, p, \gamma$	11	10, 11
$^{13}\text{C} + ^{13}\text{C}$	-	$^{26}\text{Mg}$	29.3	$\gamma$	70	11
$^{12}\text{C} + ^{14}\text{C}$	-	$^{26}\text{Mg}$	26.0	$\alpha, d, t$	24	13
$^{14}\text{N} + ^{12}\text{C}$	-	$^{26}\text{Al}$	22.6	$\alpha, p$	26	10
$^{12}\text{C} + ^9\text{Be}$	-	$^{21}\text{Ne}$	21.9	$\alpha$	2	8
$^{16}\text{O} + ^{12}\text{C}$	+	$^{28}\text{Si}$	25.4	$\alpha, p, \gamma$	36	14, 15
$^{14}\text{N} + ^{14}\text{N}$	-	$^{28}\text{Si}$	36.2	$\alpha, p, \gamma$	$1, 3 \times 10^3$	8, 9
$^{16}\text{O} + ^{16}\text{O}$	-	$^{32}\text{S}$	27.4	$\alpha, p, n, d, \gamma$	130	1, 12
$^{12}\text{C} + ^{20}\text{Ne}$	-	$^{32}\text{S}$	29.6	$\gamma$	580	1
$^{12}\text{C} + ^{19}\text{F}$	-	$^{31}\text{P}$	32.6	$\gamma$	$2, 2 \times 10^3$	1
$^{16}\text{O} + ^{20}\text{Ne}$	-	$^{36}\text{Ar}$	32.0	$\gamma$	$4, 8 \times 10^3$	1

If the decay width  $\Gamma_d^\dagger$  for the decay of an intermediate state into the compound nucleus is large, the intermediate state will decay so fast that it cannot be observed as a resonance structure experimentally. On the other hand, if there exist many overlapping intermediate states with  $\Gamma_d > D_d$ , where  $\Gamma_d$  is the total width of the intermediate state, one also would expect smooth excitation functions.

The total width  $\Gamma_d$  of an intermediate state is composed of  $\Gamma_d = \Gamma_d^\dagger + \Gamma_d^\ddagger$ , where  $\Gamma_d^\dagger$  is the width for the direct decay in all open channels (escape width). For the decay width  $\Gamma_d^\dagger$  and the escape width  $\Gamma_d^\ddagger$  the following relations are valid<sup>19</sup>:

$$\Gamma_d^\dagger(J^\pi) \propto \rho(E_x, J^\pi),$$

$$\Gamma_d^\ddagger(J^\pi) \propto G(E_x, J^\pi),$$

where  $\rho(E_x, J^\pi)$  is the level density of the compound nuclear states with spin  $J$  and parity  $\pi$  and  $G(E_x, J^\pi)$  is the number of open channels to which the intermediate state with spin  $J$  and parity  $\pi$  is able to decay.

The number of open channels  $G(E_x, J^\pi)$  for some of the systems in Table I have been calculated (see Appendix) and are shown in Fig. 9 as a function of the angular momentum  $J$ . The calculated level densities  $\rho(E_x)$  for the compound nuclei are included in column 6 of Table I. These values are normalized to the level density of  $^{24}\text{Mg}$  which one

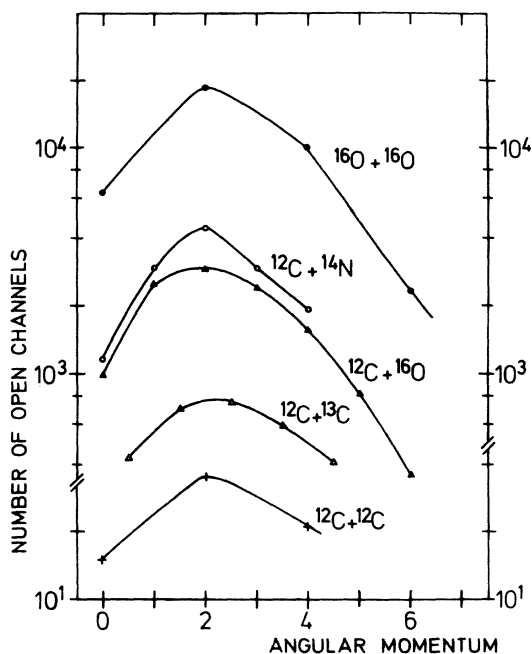


FIG. 9. Number of open channels a state with spin  $J$  is able to decay into for some of the heavy-ion systems of Table I.

obtains in the case of the  $^{12}\text{C} + ^{12}\text{C}$  reaction.

The correlated resonances measured in the  $^{16}\text{O} + ^{12}\text{C}$  system are much less pronounced than in the  $^{12}\text{C} + ^{12}\text{C}$  system. If one therefore takes the number of open channels and the level density calculated for the  $^{16}\text{O} + ^{12}\text{C}$  system as an upper limit for a possible detection of molecular intermediate states, then it is obvious from Fig. 9 and Table I that systems like  $^{16}\text{O} + ^{16}\text{O}$ ,  $^{14}\text{N} + ^{14}\text{N}$ , and  $^{14}\text{N} + ^{10}\text{B}$  do not exhibit any structure in their excitation functions. It is, however, far from obvious, for instance, why the  $^{13}\text{C} + ^{12}\text{C}$  system has smooth excitation functions.

One possible explanation could be that the intermediate states (which act as doorway states between entrance channel and compound nucleus formation) in the  $^{12}\text{C} + ^{12}\text{C}$ ,  $^{16}\text{O} + ^{12}\text{C}$ , and  $\alpha + ^{20}\text{Ne}$  systems do not exhibit such a simple structure like a two-atomic molecule. Perhaps the doorway states consist of a more complicated structure which is closely correlated with the structure of the entrance channel nuclei. In fact all three systems mentioned above contain pure  $\alpha$ -particle nuclei so that one may suppose that these intermediate states are related to some kind of  $\alpha$ -particle structure, for instance a  $4n$  nuclear core and one or more  $\alpha$  particles around it. This type of intermediate structure was proposed for the first time by Michaud and Vogt<sup>20</sup> in order to explain the  $^{12}\text{C} + ^{12}\text{C}$  resonances.

In Fig. 10 one possible  $\alpha$ -particle intermediate state (hereafter we refer to it as an  $\alpha$ -particle molecule) is schematically shown for the  $^{12}\text{C} + ^{12}\text{C}$  system. In the entrance channel two carbon nuclei interact via an optical potential. The  $\alpha$ - $^{12}\text{C}$  residual interaction then induces a transition to the  $\alpha$ -particle molecule which acts as a doorway state between entrance and exit channels. The mean lifetime of this  $\alpha$ -particle molecule is determined by the  $\alpha$ - $\alpha$  interaction (and also by the  $\alpha$ - $^{12}\text{C}$  interaction) and by the residual nucleon-nucleon interaction (between nucleons which belong to different  $\alpha$  particles). In Ref. 21 an  $\alpha$ - $\alpha$  interaction

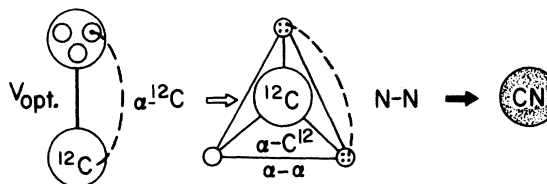


FIG. 10. Schematic picture of the possible reaction mechanism involved in the  $^{12}\text{C} + ^{12}\text{C}$  reaction. From left to right: entrance channel, intermediate state, compound nucleus. The intermediate state shown is one of the possible  $\alpha$ -particle molecules. The dashed lines represent the residual interaction.

is derived from a fundamental nucleon-nucleon interaction showing a repulsive soft core for small  $\alpha$ - $\alpha$  distances. The same holds for the  $\alpha$ - $^{12}\text{C}$  interaction calculated from this  $\alpha$ - $\alpha$  interaction in Ref. 20. This repulsive core prevents the  $\alpha$ -particle molecule from collapsing immediately and gives rise to a finite lifetime. However, the nucleon-nucleon interaction mentioned above generates more and more complicated states until the compound nucleus is created with its full complexity.

In this picture it seems reasonable that one or more additional nucleons (especially an odd number) added to pure  $\alpha$ -particle nuclei (as in the  $^{13}\text{C} + ^{12}\text{C}$  system) cause a rapid breakup of a possible  $\alpha$ -particle molecule.

It is obvious that one should expect correlated resonances in the framework of this  $\alpha$ -particle molecule picture if the following conditions are fulfilled:

- (1) Both nuclei in the entrance channel must be pure  $\alpha$ -particle nuclei.
- (2) The number of open channels and the level density of the related compound nucleus must be small (less than or comparable to the values for the  $^{16}\text{O} + ^{12}\text{C}$  system).

With these two conditions, all the experimental results for the reactions shown in Table I can be explained.

So far, the interpretation of the observed correlated structure in terms of  $\alpha$ -particle molecules instead of nuclear molecules is somewhat tentative. However, there exists experimental evidence on the one hand which supports the existence of  $\alpha$ -particle molecules (see Sec V) and on the other hand, measurements which show that this structure can hardly be due to the formation of nuclear molecules.

The latter experiments are those of Patterson, Winkler, and Zaidins<sup>17</sup> and Mazarakis and Stephens<sup>18</sup> which have extended the  $^{12}\text{C} + ^{12}\text{C}$  excitation functions to very low incident energies. These measurements show a continuation of the correlated structure down to 3 MeV with a spacing of less than 0.5 MeV.

While the three resonances measured by ABK could have been due to single-particle states in the  $^{12}\text{C} + ^{12}\text{C}$  system which are accidentally close together (as supposed in Ref. 23) the continuation of this structure shows clearly that it must be due to intermediate states. The spacing of single-particle resonances is mainly determined by the radius of the interacting potential, by the reduced mass, and only weakly by the depth of the potential. For reasonable values of the  $^{12}\text{C} + ^{12}\text{C}$  radius one expects single-particle levels to be separated at least by 1 MeV, which is in contrast to the experi-

mental results mentioned above. The same arguments hold for the  $^{16}\text{O} + ^{12}\text{C}$  system.<sup>24</sup>

In the paper of Michaud and Vogt<sup>20</sup> arguments are given which also contradict the interpretation given by Imanishi<sup>6</sup> in terms of single-particle states. These authors show that the energy-averaged, total-absorption cross section of the  $^{12}\text{C} + ^{12}\text{C}$  reaction is fairly well reproduced down to very low energies with an optical potential having an imaginary part of about 1 MeV. Imanishi, however, has to take an imaginary part of 0.1 MeV in order to reproduce the correct width of the assumed single-particle states. This very small imaginary part, however, results in a total-absorption cross section which is several orders of magnitude higher than the experimental value at low energies. Besides this, Imanishi's calculations are not able to reproduce all these resonances which have been measured in Refs. 17 and 18 (for instance the resonance  $E_{c.m.} \approx 5$  MeV does not appear in Imanishi's calculations).

#### V. EXPERIMENTAL EVIDENCE FOR $\alpha$ -PARTICLE MOLECULES

The measurements of Patterson *et al.*<sup>17</sup> and Mazarakis and Stephens<sup>18</sup> have established the intermediate character of the  $^{12}\text{C} + ^{12}\text{C}$  resonances. The object of this section is to show that this intermediate structure is due to  $\alpha$ -particle molecules as introduced in the previous chapter.

The basic idea for identifying the intermediate states with  $\alpha$ -particle molecules is the following: If  $\alpha$ -particle molecules are indeed the reason for the observed intermediate structure, then excited states in the corresponding residual nuclei having a rather pure  $\alpha$ -particle parentage should be preferentially populated at the resonance energies because of the large overlap between intermediate state and residual nucleus. This test is therefore very closely related to the question if there exist excited states with pure  $\alpha$ -particle configurations. This question has been investigated by many authors, both theoretically<sup>25</sup> and experimentally<sup>26,27</sup> with the result that such states do exist in some  $4n$  nuclei.

The nuclei which are considered in this paper to have excited states with more or less pronounced  $\alpha$ -particle parentage are  $^{24}\text{Mg}$ ,  $^{20}\text{Ne}$ , and  $^{23}\text{Na}$ . In  $^{24}\text{Mg}$  the first excited  $J^\pi = 0^+$  state at 6.44 MeV is assumed according to Refs. 28 and 29 to have a  $4p$ - $4h$  configuration (one  $\alpha$  particle surrounding a  $^{20}\text{Ne}$  core). In  $^{20}\text{Ne}$  excited states with  $^{12}\text{C} + 2\alpha$  and  $^{16}\text{O} + \alpha$  parentage are considered.

The former configuration is assumed for the 7.20 MeV,  $J^\pi = 0^+$  and the 7.84 MeV,  $J^\pi = 2^+$  states and has been shown to be valid experimentally.<sup>27</sup> The latter configuration ( $4p$ - $4h$  configuration) is assumed for



the 6.72 MeV,  $J^\pi = 0^+$  and the 7.43 MeV,  $J^\pi = 2^+$  states. The interpretation of these two states in terms of a 4p-4h configuration as in the work of Satpathy, Schmid, and Faessler<sup>29</sup> is, however, not so obvious. Usually these states are considered to be shell model states with  $(sd)^4$  configuration according to their large cross sections in single-nucleon<sup>30</sup> and two-nucleon<sup>31</sup> transfer reactions. This interpretation is, however, contradicted by the large reduced  $\alpha$ -particle width of these states<sup>32</sup> which is difficult to explain within the framework of the shell model.<sup>33</sup> To account for this large width a  $(fp)^4$  admixture has been suggested by Fortune, Middleton, and Betts<sup>34</sup> which supports the assumption of a  $^{16}\text{O} + \alpha$  parentage for the two  $^{20}\text{Ne}$  states mentioned above.

Assuming some kind of  $\alpha$ -particle configuration for excited states in  $^{23}\text{Na}$  appears at first sight to be rather artificial. However, recent studies<sup>35,36</sup> of the structure of the lowest lying negative parity states in  $^{23}\text{Na}$  at 2.64, 3.68, and 3.85 MeV with spin assignment  $J = \frac{1}{2}^-, \frac{3}{2}^-,$  and  $\frac{5}{2}^-$ , respectively,

gave strong evidence for this assumption.<sup>37</sup> The  $J^\pi = \frac{3}{2}^-$  and  $\frac{5}{2}^-$  levels fit the energy formula for a  $K = \frac{1}{2}$  rotational band based on the  $J^\pi = \frac{1}{2}^-$  band head.<sup>35</sup> The  $J^\pi = \frac{1}{2}^-$  band head seems to have an 8p-1h configuration which can be explained in the framework of the Nilsson model with the excitation of one proton from the  $K = \frac{1}{2}^-$  Nilsson orbit 4 to orbit 7.<sup>38</sup> Middleton *et al.*<sup>37</sup> assume for this  $J^\pi = \frac{1}{2}^-$  state a configuration which consists of the coupling of a  $1p_{1/2}$  hole to the 7.20 MeV,  $J^\pi = 0^+$  state in  $^{20}\text{Ne}$  having a 8p-4h configuration. For the  $J^\pi = \frac{3}{2}^-$  and  $\frac{5}{2}^-$  levels they assume the coupling of a  $1p_{1/2}$  hole to the 7.84 MeV,  $J^\pi = 2^+$  state in  $^{20}\text{Ne}$  which also has a 8p-4h configuration. The equivalent assumption is made by the same authors concerning the configuration of the lowest negative parity states in  $^{21}\text{Ne}$  (coupling of a  $1p_{1/2}$  nucleon to the  $^{20}\text{Ne}$  states with 8p-4h configuration). This assumption has been proved experimentally to be valid for the  $J^\pi = \frac{3}{2}^-$  and  $\frac{5}{2}^-$  members of the lowest lying  $K = \frac{1}{2}^-$  rotational band in  $^{21}\text{Ne}$ .<sup>37</sup> On account of the close similarity of the level diagrams for  $^{21}\text{Ne}$  and  $^{23}\text{Na}$  this experimental result supports the assumption concerning the configuration of the lowest negative parity states in  $^{23}\text{Na}$  (one  $p_{1/2}$  hole coupled to the 8p-4h states in  $^{20}\text{Ne}$ ).

We have measured excitation functions for transitions to all these states mentioned above. In the  $^{12}\text{C} + ^{12}\text{C}$  case the energy range of the early mea-

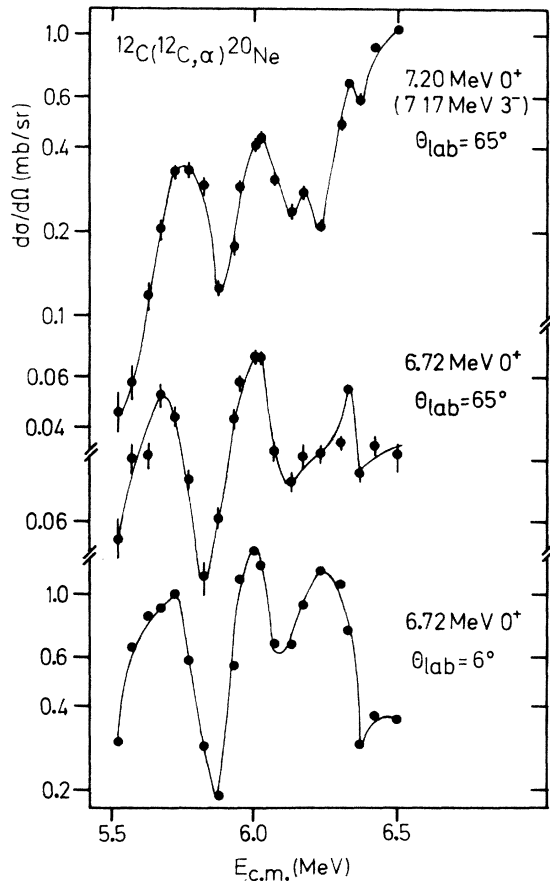


FIG. 11. Excitation functions for the reaction  $^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}$  for transitions to excited states in  $^{20}\text{Ne}$  with an assumed  $\alpha$ -particle parentage.

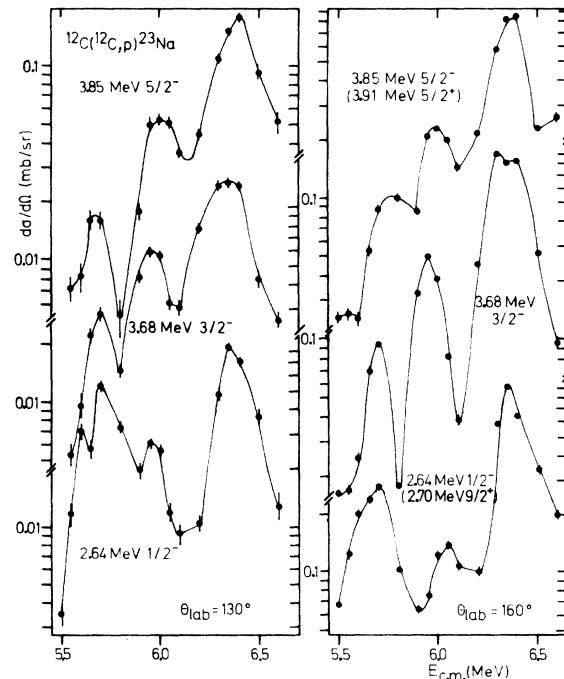


FIG. 12. Excitation functions for the reaction  $^{12}\text{C}(^{12}\text{C}, p)^{23}\text{Na}$  for transitions to the lowest lying negative parity states in  $^{23}\text{Na}$  with an assumed 8p-1h configuration.

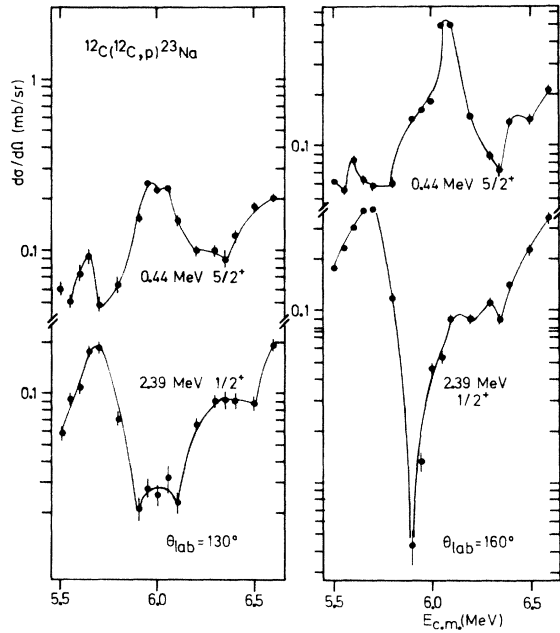


FIG. 13. Excitation functions for the reaction  $^{12}\text{C}(^{12}\text{C},p)^{23}\text{Na}$  for transitions to excited states in  $^{23}\text{Na}$  having no  $8p\text{-}1h$  configuration.

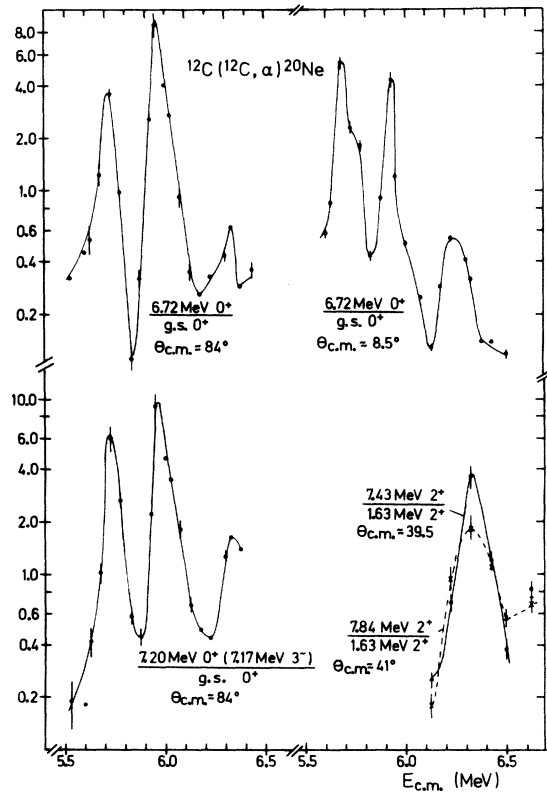


FIG. 14. Cross section ratios for transitions to excited states in  $^{20}\text{Ne}$  with an assumed  $\alpha$ -particle parentage compared with transitions to states having the same  $J^\pi$  assignment but no  $\alpha$ -particle parentage. The reaction is  $^{12}\text{C}(^{12}\text{C},\alpha)^{20}\text{Ne}$ .

measurements of ABK was chosen. In the  $^{16}\text{O}+^{12}\text{C}$  case the energy range reported in Ref. 15 was taken. The experimental results for the  $^{12}\text{C}+^{12}\text{C}$  reaction are shown in Figs. 11 and 12. The excitation functions exhibit pronounced structure at the resonance energies of ABK. In contrast to this result excitation functions for transitions to excited states in the residual nuclei having no  $\alpha$ -particle configuration do not show this regular resonance pattern. This is illustrated in Fig. 13 for the  $^{12}\text{C}(^{12}\text{C},p)^{23}\text{Na}$  reaction.

The possibility that the resonance structure in the excitation functions of Figs. 11 and 12 is due to statistical fluctuations can be ruled out at once. The resonances appear correlated in excitation functions measured for different angles which differ by more than the coherence angle. Furthermore, deviations from an average cross section are too large to give reasonable statistical probability for their occurrence. An additional argument against a statistical interpretation stems from the fact that these resonances are still present if one compares the c.m. yields for transitions to excited states having  $\alpha$ -particle configurations with transitions to normal states having the same  $J^\pi$  assignment. The statistical model predicts that the cross section for any two excited states of the same spin and parity (comparable excitation energies) should fluctuate in a random way, with the

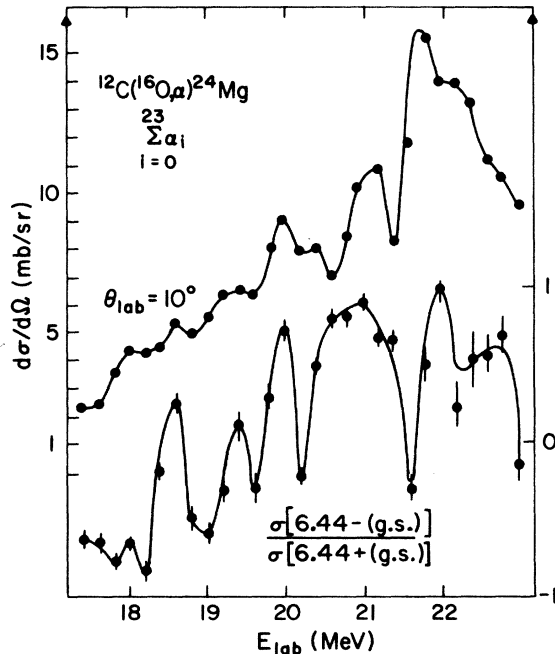


FIG. 15. Excitation function for the reaction  $^{12}\text{C}(^{16}\text{O},\alpha)^{24}\text{Mg}$  (upper curve) and comparison of the c.m. yield for the transition to the 6.44 MeV,  $0^+$  state in  $^{24}\text{Mg}$  with the transition to the  $0^+$  ground state.

TABLE II. Excited states in  $^{24}\text{Mg}$  and  $^{28}\text{Si}$  having  $\alpha$ -particle parentage.

Compound nucleus	$E_{\text{exp}}$ (MeV)	$E_{\text{theor}}$ (MeV)	Parentage	Reference
$^{24}\text{Mg}$	20	19.22	$^{12}\text{C} + 3\alpha$	29
$^{24}\text{Mg}$	20	20	$^{16}\text{O} + 2\alpha$ ( $fp$ shell)	28
$^{28}\text{Si}$	25	22	$^{20}\text{Ne} + 2\alpha$	28

result that, on the average, their cross sections should be comparable. In Figs. 14 and 15 cross section ratios for the reactions  $^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}$  and  $^{12}\text{C}(^{16}\text{O}, \alpha)^{24}\text{Mg}$  are shown which still exhibit resonance structure at the resonance energies.

This preferentially strong population of  $\alpha$ -particle states in the residual nuclei gives evidence for the existence of intermediate states having the configuration of an  $\alpha$ -particle molecule. In the case of the  $^{12}\text{C}(^{16}\text{O}, \alpha)^{24}\text{Mg}$  reaction intermediate states with a  $^{20}\text{Ne} + 2\alpha$  configuration exist at the resonance energies. These states have a large overlap with the 6.44 MeV,  $J^\pi = 0^+$  excited state in  $^{24}\text{Mg}$  with  $^{20}\text{Ne} + \alpha$  parentage. In the case of the  $^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}$  reaction intermediate states with a  $^{16}\text{O} + 2\alpha$  and  $^{12}\text{C} + 3\alpha$  configuration should exist. These states preferentially decay into excited states in  $^{20}\text{Ne}$  having  $^{16}\text{O} + \alpha$  and  $^{12}\text{C} + 2\alpha$  parentage, respec-

tively.

The population of  $\alpha$ -particle states in the residual nucleus takes place either via a direct process or via a compound nucleus formation. It is interesting to note that excited states with  $^{16}\text{O} + 2\alpha$ ,  $^{12}\text{C} + 3\alpha$ , and  $^{20}\text{Ne} + 2\alpha$  parentages are assumed to exist in the corresponding compound nuclei  $^{24}\text{Mg}$  and  $^{28}\text{Si}$ , respectively. The excitation energy of these states is almost identical to the excitation energies attained in the  $^{12}\text{C} + ^{12}\text{C}$  and  $^{16}\text{O} + ^{12}\text{C}$  reactions (see Table II).

A detailed analysis of the  $^{16}\text{O} + ^{12}\text{C}$  reaction has shown that the main contribution to the cross section proceeds via compound nucleus formation.<sup>15</sup> In the case of the  $^{12}\text{C} + ^{12}\text{C}$  reaction decay via the compound nucleus also takes place. This can be seen from Fig. 16 where excitation functions are shown for transitions to normal states in  $^{20}\text{Ne}$  and  $^{23}\text{Na}$ . These excitation functions exhibit resonances at the expected energies.

The situation in the case of the  $^{12}\text{C}(^{12}\text{C}, p)^{23}\text{Na}$  reaction seems to be somewhat more complicated. One has to assume doorway states with a  $^{12}\text{C} + 3\alpha$  parentage being responsible for the resonances of Fig. 12. The residual nucleon-nucleon interaction can cause the breakup of an  $\alpha$  particle in such a way that one proton will be emitted while three other nucleons are captured into the empty  $1p_{1/2}$  level of the remaining  $^{12}\text{C} + 2\alpha$  configuration in  $^{20}\text{Ne}$ . This process is equivalent to the coupling of a  $p_{1/2}$  hole to the  $8p-4h$  configuration in  $^{20}\text{Ne}$  which we assume to be valid for the lowest negative parity states in  $^{23}\text{Na}$ .

## VI. CONCLUSION

Among the many heavy-ion systems for which excitation functions have been measured in the vicinity of the Coulomb barrier, only three systems exhibit correlated resonances in the excitation functions of all exit channels and angles measured: the  $^{12}\text{C} + ^{12}\text{C}$ ,  $^{16}\text{O} + ^{12}\text{C}$ , and  $\alpha + ^{20}\text{Ne}$  systems. These three systems have, as a common feature, exclusively pure  $\alpha$ -particle nuclei in the entrance channels. The supposition that both nuclei in the entrance channel have to be  $\alpha$ -particle nuclei is, however, not sufficient in order to obtain correlated resonances. This is clearly seen from the  $^{16}\text{O} + ^{16}\text{O}$ ,  $^{12}\text{C} + ^{20}\text{Ne}$ , and  $^{16}\text{O} + ^{20}\text{Ne}$  systems for which smooth excitation functions have been measured.

The additional condition which must be fulfilled concerns the number of open channels and the density of levels in the corresponding compound nucleus. Both values must be less than or comparable to the values for the  $^{16}\text{O} + ^{12}\text{C}$  system. These two conditions are sufficient to explain both the absence and the existence of resonances for

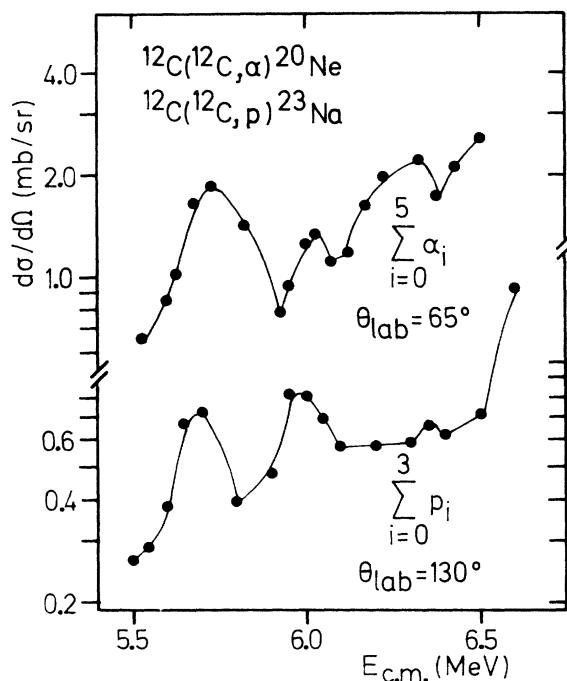


FIG. 16. Excitation functions of the  $^{12}\text{C} + ^{12}\text{C}$  reaction for transitions to excited states in the residual nuclei  $^{20}\text{Ne}$  and  $^{23}\text{Na}$  having no  $\alpha$ -particle parentage.

all systems measured so far.

The continuation of resonance structure in the excitation functions of the  $^{12}\text{C} + ^{12}\text{C}$  system down to very low energies has raised serious objections to the early interpretation of these resonances in terms of single-particle states, called nuclear molecules. This continuation of structure shows clearly that it must be due to more complicated intermediate states. From the fact that only systems having  $\alpha$ -particle nuclei in the entrance channel exhibit correlated resonances, one might deduce that these intermediate states are  $\alpha$ -particle doorway states. The selective population of excited states with a presumed  $\alpha$ -particle parentage in several residual nuclei seems to confirm this interpretation.

*Note added:* After submission of this paper the authors' attention has been drawn to Ref. 39. In Ref. 39 excitation functions of the  $\alpha$ -particle exit channel have been measured at two angles ( $\theta_{\text{lab}} = 10$  and  $150^\circ$ ) for the  $^{12}\text{C} + ^{13}\text{C}$  system. Two resonances are reported at about 5 MeV above the Coulomb barrier for  $^{12}\text{C} + ^{13}\text{C}$ . For these energies the level density in the compound nucleus  $^{25}\text{Mg}$  and the number of open channels have already increased considerably compared to the sub-Coulomb case. Both values are now larger compared to the  $^{12}\text{C} + ^{16}\text{O}$  case [ $\rho(^{25}\text{Mg})/\rho(^{12}\text{C} - ^{12}\text{C}) \approx 40$ ] and also larger than in the cases of  $^{12}\text{C} + ^9\text{Be}$  and  $^{14}\text{N} + ^{12}\text{C}$  where smooth excitation functions have been measured. It might be the case that for these high energies the considerations given briefly in the Introduction are already valid. On the other hand, however, it is not clear from Ref. 39 if the attempt was made to eliminate the interpretation in terms of statistical fluctuations for these two resonances (for instance by summing over, as much as possible, transitions to the residual nucleus). It is well known that excitation functions for single transitions exhibit strong fluctuations (especially at forward angles where the damping coefficients are rather small) which, however, disappear if one takes the sum over several transitions.

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#### APPENDIX

##### A. Number of open channels

The number of open channels into which a certain nuclear state with spin  $J$  is able to decay was cal-

culated according to

$$G(J) = \sum_{\alpha} \sum_{l=0}^{l_{\text{max}}} \sum_{s=|J-l|}^{J+l} \sum_{i=|S-l|}^{S+i} \left[ N_l(I) + \int_{E_0}^{E(l)} \rho(E, I) dE \right].$$

In this expression,  $\alpha$  characterizes the decay mode (only two-particle decay, no  $\gamma$  emission),  $l$  is the relative angular momentum in the exit channel,  $I$  and  $i$  are the intrinsic spins of the emitted particle, and  $S$  is the channel spin in the exit channel.  $N_l(I)$  is the number of excited states with spin  $I$  in the residual nuclei which are known from the literature<sup>32,35,40</sup> (the lowest lying level of these nuclei, up to an excitation energy  $E_0$ ). The number of states in the residual nuclei above  $E_0$  was calculated according to the level density formula of Gilbert and Cameron<sup>4</sup>

$$\rho(E, I) = \rho(E) [(2I + 1)/2\sigma^2] \exp[-(I + \frac{1}{2})^2/2\sigma^2],$$

$$\rho(E) = \exp 2(aE)^{1/2} [12(2\sigma^2)^{1/2} (aE)^{1/4} E]^{-1},$$

$$E = E_x - P(N) - P(Z).$$

$E_x$  is the excitation energy of the residual nucleus.

The parameters  $P(N)$ ,  $P(Z)$ ,  $a$ , and  $\sigma$  are those of Ref. 4. For all those cases where parameters are not known from Ref. 4 the following values have been taken:  $a = A/7$  and  $\sigma^2 = 0.0888(aE)^{1/2} A^{2/3}$ . The maximum value of  $I$  which has to be considered for  $\rho(E, I)$  depends for a given  $J$  value on the angular momentum  $l$ . Each  $l$  value also determines the upper limit  $E(l)$  for the integral.  $E(l) = E_{\text{max}} - \epsilon$ , where  $E_{\text{max}}$  is the highest possible excitation energy of the residual nucleus for a given projectile energy. The energy  $\epsilon$  was defined as the energy for which the transmission coefficient  $T_l$  in the exit channel has the value 0.3. For the  $T_l$  calculations optical potentials have been used which are given in the literature. The parity  $\pi$  of the nuclear state with spin  $J$  was taken into account only in the case of the  $^{12}\text{C} + ^{12}\text{C}$  and the  $^{16}\text{O} + ^{16}\text{O}$  systems assuming an equal distribution of levels with  $\pi = +1$  and  $\pi = -1$ .

##### B. Level density of the compound nucleus

For the calculation of the level densities the formula of Gilbert and Cameron<sup>4</sup> has been used. However, in the case of the high excitation energies which apply for all compound nuclei considered, other parameters have to be chosen than those given by Gilbert and Cameron. In Refs. 41 and 42, it is shown that the following parameters will give reasonable results:

$$\sigma^2 = \theta \hbar^{-2} (E/a)^{1/2}, \quad a = A/7, \quad E = E_x - b\delta$$

with

$$\theta = \theta_{\text{rigid}}, \quad b = \begin{cases} 2 \\ 1 \\ 0 \end{cases} \left. \vphantom{b} \right\} \text{for } \begin{cases} g-g \\ u-g \\ u-u \end{cases} \text{ nuclei.}$$

$\delta = 1.8$  MeV is the average of the neutron and proton pairing parameters of Ref. 43.

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