Galilean-invariance ambiguity in the nonrelativistic pion-nucleon absorption operator*

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In the model of a nucleon interacting with a massive object via a potential in the Dirac equation, we have demonstrated that the behavior of the nonrelativistic pion absorption operator under Galilean transformations depends upon the properties of the potential under Lorentz transformations. In particular, the operator has a Galilean invariant form if the potential is the time-like component of a four-vector, but not if the potential is a scalar.

Barnhill[1] pointed out that there exists an ambiguity in the nonrelativistic reduction of the pseudoscalar interaction which is assumed to represent the pion-nucleon interaction. If the nucleon is taken to be static, the reduction leads to the usual form[2]

$$H \to -\frac{if}{\mu}\vec{\sigma}\cdot\vec{q}, \tag{1}$$

where $\vec{q}$ is the pion momentum operator. If one views this interaction in a frame in which the nucleon is moving nonrelativistically and if $\vec{q}$ is small, one might expect from Galilean invariance that

$$H \to -\frac{if}{\mu}\sigma\cdot\left[\vec{q}-\frac{\mu}{2m}(\vec{p}_i+\vec{p}_f)\right], \tag{2}$$

where $\vec{p}_i(\vec{p}_f)$ is the initial (final) nucleon momentum operator. Barnhill obtained the result

$$H \to -\frac{if}{\mu}\vec{\sigma}\cdot\left[\vec{q}-\lambda\frac{\mu}{2m}(\vec{p}_i+\vec{p}_f)\right], \tag{3}$$

where λ is arbitrary. Thus Barnhill concluded that the size of the "Galilean invariant" term is undetermined.

One may note that pion absorption can only take place on an "off-shell" nucleon i.e., there must be a third body to provide energy-momentum conservation. Thus it seems possible that this ambiguity in the operator reduction might arise because the "third body" has been ignored. In addition, we are particularly interested in the description of processes such as the absorption of negative pions from atomic orbitals. There the reduced operator will be used as a nonrelativistic operator corresponding to very small momentum transfer and the matrix elements will be taken with respect to two Schrödinger wave functions describing the nucleon moving initially and finally in the field of the nucleus. For these reasons we have studied the following problem:

Consider a nucleon interacting with a massive object. Assume that this interaction is described by a potential in the Dirac equation. Treat the pseudoscalar $\pi - N$ interaction as a perturbation and compute the matrix element

$$\mathfrak{M} \equiv ig\bar{\chi}_f\gamma_5\chi_i ,$$

where $g \equiv 2Mf/\mu$.

Take the nonrelativistic limit of this matrix element and identify the operator that corresponds to the one to be used with Schrödinger wave functions.

We will consider two types of interaction potentials: either the fourth component of a four-vector or a scalar. One might believe off-hand that the transformation character of the potential is immaterial. That this is not so is the essential point of this paper.

Writing

$$\chi = \begin{pmatrix} u \\ v \end{pmatrix}$$

we have the relations

$$v = \frac{1}{E+m\pm V}\vec{\sigma}\cdot\vec{p}u, \tag{4}$$

$$u = \frac{1}{E-m-V}\vec{\sigma}\cdot\vec{p}v, \tag{5}$$

where + applies when V is a scalar and − applies when V is the fourth component of a four-vector. Substituting relation (4) into (5) and dropping terms of order p^2V/m^3 and $p^2(E-m)/m^3$ we obtain the Schrödinger equation

$$(p^2/2m + V + m - E)u = 0 . \tag{6}$$

Using Eq. (4) to eliminate v the matrix element becomes

$$\mathfrak{M}/ig = u_f^\dagger(E_i+m\pm V)^{-1}\vec{\sigma}\cdot\vec{p}u_i - (\vec{p}u_f^\dagger)\cdot\vec{\sigma}(E_f+m\pm V)^{-1}u_i . \tag{7}$$

Using (6) to make the replacement

$$Vu_{i,f} = (E_{i,f}-m-p^2/2m)u_{i,f} \tag{8}$$

and the notational convenience that p_i acts only on u_i and p_f acts only on $u_f^\dagger$ we have

$$\mathfrak{M}/ig \to u_f^\dagger(E_i+m\mp m\pm E_f\mp p_f^2/2m)^{-1}\vec{\sigma}\cdot\vec{p}_i u_i - u_f^\dagger(E_f+m\mp m\pm E_i\mp p_i^2/2m)^{-1}\vec{\sigma}\cdot\vec{p}_f u_i . \tag{9}$$

We can now identify the desired operator, and if we write $\epsilon_{i,f}=E_{i,f}-m$ it is

$$H/ig\to\left(2m+\epsilon_i\pm\epsilon_f\mp\frac{p_f^2}{2m}\right)^{-1}\vec{\sigma}\cdot\vec{p}_i$$
$$-\left(2m+\epsilon_f\pm\epsilon_i\mp\frac{p_i^2}{2m}\right)^{-1}\vec{\sigma}\cdot\vec{p}_f$$
$$\to(2m)^{-1}\left[\left(1-\frac{\epsilon_i}{2m}\mp\frac{\epsilon_f}{2m}\pm\frac{p_f^2}{(2m)^2}\right)\vec{\sigma}\cdot\vec{p}_i\right.$$
$$\left.-\left(1-\frac{\epsilon_f}{2m}\mp\frac{\epsilon_i}{2m}\pm\frac{p_i^2}{(2m)^2}\right)\vec{\sigma}\cdot\vec{p}_f\right]. \quad (10)$$

Defining $\vec{q}\equiv\vec{p}_f-\vec{p}_i$; $\vec{s}\equiv(\vec{p}_i+\vec{p}_f)/2$; $\omega\equiv\epsilon_f-\epsilon_i$ =pion total energy (=pion mass, μ, to the order of the calculation); and $\eta\equiv\epsilon_i+\epsilon_f$, and keeping through quadratic terms we obtain the result

$$H\to-\frac{if}{\mu}\vec{\sigma}\cdot\left[\vec{q}\left(1-\frac{\eta\pm\eta}{4m}\pm\frac{(s^2+q^2/4)}{(2m)^2}\right)\right.$$
$$\left.-\frac{\vec{s}(\omega\mp\omega\mp\vec{s}\cdot\vec{q}/m)}{2m}\right]. \quad (11)$$

For $\vec{q}$ small we may drop additional terms and obtain:

$$H\to-\frac{if}{\mu}\vec{\sigma}\cdot\vec{q};\quad V=\text{scalar} \quad (12)$$

$$H\to-\frac{if}{\mu}\vec{\sigma}\cdot\left[\vec{q}-\frac{\mu}{2m}(\vec{p}_i+\vec{p}_f)\right];\quad V=\text{four-vector}. \quad (13)$$

Equation (13) gives the result that Galilean invariance is preserved to the order expected. Equation (12) shows no Galilean invariance in the scalar case.

We do not know exactly what conclusions to draw from this result but some of the possibilities are:
(1) Galilean invariance is *not* to be expected *a priori* and therefore the Lorentz character of the nucleon-nucleon interaction can be determined by doing pion-absorption experiments.
(2) Galilean invariance *is* to be expected *a priori* and therefore *all* nucleon-nucleon interactions must behave as a Lorentz four-vector.
(3) It is not proper to use a scalar potential in the Dirac equation.
(4) The use of the Dirac equation to describe a nucleon is wrong in some essential aspect.
The reader will no doubt think of other possibilities.

We have demonstrated that, for a given Lorentz transformation property of the potential V appearing in the Dirac equation, there is no ambiguity in the nonrelativistic reduction of the pion absorption operator (in pseudoscalar coupling to first order) to be used in the calculation of a matrix element with the Schrödinger wave functions generated by that potential V. There remains an uncertainty in the nonrelativistic operator, however, due to our lack of knowledge of V.

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[1]M. V. Barnhill, III, Nucl. Phys. A131, 106 (1969).
[2]Isospin and pion field operators have been omitted since they are not essential to the argument.