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$E2/M1$ multipole mixing ratios of $2'\to 2$ gamma transitions in even-even spherical nuclei*

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A systematic survey is presented of $E2/M1$ mixing ratios of γ transitions between the $2^{+\prime}$ and 2^+ levels of even-even nuclei in the mass range $58 \le A \le 152$. Particular attention is given to the variations in the phase of the mixing ratios, which are deduced from the literature in a systematic manner. It is shown that the systematics of both magnitudes and phases of the mixing ratios are explained quite well for a number of nuclei by a model proposed by Greiner, in which the magnitude of the mixing ratio is parametrized in terms of the deviation of the g factor of the first 2^+ state from the value Z/A. It is further shown that a semimicroscopic description, in terms of small admixtures of two-particle components to the phonon basis states, yields reasonable agreement with the observed phase variations and absolute magnitudes, even when only very few two-particle states are considered.

[NUCLEAR STRUCTURE ^{58,60,62}Ni, ^{64,66,68}Zn, ^{70,72,74,76}Ge, ^{74,76,78}Se, ^{80,82,84}Kr, ^{84,86,88}Sr, ^{92}Zr, ^{94,96,98}Mo, ^{100,102,104}Ru, ^{104,106,108,110}Pd, ^{106,108,110,112,114,116}Cd, ^{116}Sn, ^{122,124,126}Te, ^{126,128,132}Xe, ^{132,134}Ba, ^{140,142}Ce, ^{144}Nd, ^{150}Sm, ^{152}Gd; calculated $E2/M1$ mixing ratio δ.]

I. INTRODUCTION

The interpretation of the magnitudes and relative phases of the electromagnetic transition matrix elements between the low-lying excited states of even-even nuclei in the mass range $40 < A < 150$ may be attempted in the basis states of three nonequivalent models: The excited states may be described as vibrations about a spherical equilibrium shape, as rotations of a "soft" deformed core, or as excitations of two particles (quasiparticles) from the ground state. These various models lead to quite different predictions for the static and dynamic electromagnetic multipole moments, and it is to be expected that detailed study of the systematic behavior of these moments can result in an indication of the extent to which the low-lying levels can be understood in terms of collective or single-particle effects.

In a previous communication by the author,[1] a study was presented of $E2/M1$ mixing ratios in deformed even-Z, even-N nuclei ($150 < A < 190$). The results of that study indicated that a phenomenological interpretation of the mixing ratios between states of the β- or γ-vibrational bands and the ground-state band was possible if the appropriate mixing of intrinsic states were included. In the present work, a similar study of previously measured $2'\to 2$ $E2/M1$ mixing ratios of even-even nuclei in the mass range $60 \le A \le 150$ is presented; the compiled mixing ratios are compared with other experimentally determined static and dynamic electromagnetic multipole moments and with the structure of the spectrum of excited states in order to determine the applicability of the appropriate model. Particular attention is given to the phases of the mixing ratios, which are deduced in a systematic manner from the literature; these phases yield additional insight into the structure of the excited states.

The properties of the levels of nuclei of the f-p shell ($40 < A < 60$) cannot, in general, be dealt with in phenomenological terms, and must be treated more microscopically; the same is true of most neutron- or proton-closed-shell nuclei in other mass regions. These will be discussed in a subsequent publication. In the present communication, we deal with those nuclei for which the lower excited states can be interpreted primarily in collective terms.

A number of similar compilations have been undertaken in the past.[2] However, in view of the success of high-resolution detectors and electronics in eliminating ambiguities and conflicts in the angular distribution and angular correlation literature in recent years, it seems worthwhile to offer a more current compilation.

II. COMPILATION OF VALUES

The $E2/M1$ mixing ratios have been obtained from a survey of the angular distribution and correlation literature. In extracting the mixing ratios from the quoted angular correlation coefficients, the phase convention of Krane and Steffen (KS)[3] has been employed, in which emission matrix elements are always used for the multipole operators. The amplitude mixing ratio is given in this convention by the ratio of the reduced emission matrix elements of the multipole operators as

$$\delta = \frac{\langle J_f || \vec{j}_N \vec{A}(E2) || J_i \rangle}{\langle J_f || \vec{j}_N \vec{A}(M1) || J_i \rangle}, \tag{1}$$

where $\vec{A}$ represents the appropriate electromagnetic vector field and $\vec{j}_N$ is the nuclear current. The angular correlation coefficients for the case in which the initial transition in a cascade $J_1 \to J_2 \to J_3$ is of mixed $E2/M1$ character are written as

$$A_{kk} = \frac{F_k(11\,J_1J_2) - 2\delta F_k(12\,J_1J_2) + \delta^2 F_k(22\,J_1J_2)}{1+\delta^2} \times F_k(L_2L_2J_3J_2). \tag{2}$$

If the mixed transition is the final transition in a cascade (or if the transition is observed following the decay of a nuclear state oriented by, for example, nuclear reactions or cryogenic methods), the interference term is written with a + sign. The phase of the mixing ratio so defined may be compared with the frequently employed Biedenharn-Rose (BR)[4] and Rose-Brink (RB)[5] conventions for a γ_1-γ_2 cascade as follows:

$$\begin{aligned} \delta(\gamma_1)_{BR} &= -\delta(\gamma_1)_{KS}, \quad \delta(\gamma_1)_{RB} = -\delta(\gamma_1)_{KS}, \\ \delta(\gamma_2)_{BR} &= \delta(\gamma_2)_{KS}, \quad \delta(\gamma_2)_{RB} = -\delta(\gamma_2)_{KS}. \end{aligned} \tag{3}$$

The mixing ratios may be compared with theoretical values through the expression

$$\frac{\delta}{E_\gamma} = 0.835 \frac{\langle J_f || \mathfrak{M}(E2) || J_i \rangle}{\langle J_f || \mathfrak{M}(M1) || J_i \rangle}, \tag{4}$$

where the reduced matrix elements of the multipole operators are those used, for example, by Bohr and Mottelson,[6] and are given in units of electron-barns (eb) for $E2$ and nuclear magnetons (μ_N) for $M1$. The γ-ray energy E_γ is measured in MeV.

For purposes of theoretical comparisons, it is useful to define the mixing ratio Δ:

$$\Delta = \frac{\langle J_f || \mathfrak{M}(E2) || J_i \rangle}{\langle J_f || \mathfrak{M}(M1) || J_i \rangle}, \tag{5}$$

where Δ is given in units of $e\text{b}/\mu_N$.

A selection of values of the $2' \to 2$ mixing ratios deduced from the angular correlation literature is given in Table I. In general, the most recent value has been selected; however, in a number of instances earlier values exhibit smaller experimental uncertainties and these values have been used when available. Also shown in Table I are the reduced $E2$ transition probabilities $B(E2)$ for the decay of the first excited 2^+ states, as well as the ratios of the reduced $E2$ transition probabilities describing the decay modes of the $2'$ states.

A cursory inspection of Table I illustrates a number of systematic features of the $2' \to 2$ mixing ratios. In general, the magnitudes are large, in agreement with the expected forbiddance of $M1$ transitions between collective states. The phases seem to show little systematic variation and, indeed, one seems to find nearly equal frequencies for the occurrence of positive or negative phases. This is illustrated by the histograms shown in Fig. 1, from which it can be seen that for nuclei at least four valence particles (or holes) away from a closed shell, the mixing ratios have their largest values and also have roughly equal numbers of cases with positive as with negative phases. As one approaches a closed shell, one phase clearly begins to dominate. As shown below, in the lowest order approximations, the two-particle contribution to the $M1$ matrix element depends on the single particle g factor; the largest contributions are expected to arise from proton states, which always have positive g factors. Hence near closed shells one expects a unique phase for Δ, arising from the dominant contribution from the two-proton states; the data are consistent with this expectation.

The dependence of the magnitudes of the mixing ratios on shell effects is illustrated in Fig. 2, which shows the data of Table I as a function of A. A decrease amounting to a factor of 10–100 in the magnitude of Δ as one approaches a shell closure is apparent. One can also infer again the dominance of proton over neutron contributions; the minima of the data are smaller (i.e., less collective and more two-particle) for closed-neutron configurations (in which the proton configurations dominate) than for closed-proton configurations.[7]

In the following section these tabulated mixing ratios and transition probabilities are compared with the predictions based on the interpretation of the low-lying even-parity levels in terms of various nuclear models.

III. COMPARISON WITH NUCLEAR MODELS

A. Phonon (vibrational) model

In the harmonic vibrational model, the low-lying even-parity excited states are treated as arising from quadrupole vibrations of the nuclear surface.[8] The energy spectrum of the excited states expected in this model is shown in Fig. 3; in practice, the degeneracy of the N-phonon levels is split by various residual interactions. Also shown in Fig. 3, for comparison, is the energy spectrum of ^{120}Te,[9] with the energy spacing nor-

TABLE I. $E2/M1$ mixing ratios of $2' \rightarrow 2$ γ transitions.

Nucleus	$E(2)$ (MeV)	$B(E2, 2\rightarrow 0)$ [a] $[10^{-2}\ (eb)^2]$	$E(2')/E(2)$	$\frac{B(E2,2'\rightarrow 0)}{B(E2,2'\rightarrow 2)}$ [a]	E_γ (MeV)	δ	Δ (eb/μ_N)
$^{58}_{28}Ni_{30}$	1.454	1.4	1.91	0.0018 [b]	1.321	−1.1(2) [b]	−1.0
$^{60}_{28}Ni_{32}$	1.333	1.9	1.62	0.0042 [c]	0.826	+0.7(2) [c]	+1.0
$^{62}_{28}Ni_{34}$	1.172	1.5	1.96	0.039 [c]	1.129	+3.2(1) [c]	+3.4
$^{64}_{30}Zn_{34}$	0.992	3.4	1.82	0.0062	0.812	−4(1) [d]	−6
$^{66}_{30}Zn_{36}$	1.039	2.9	1.80	0.000 [e]	0.828	−1.9(3) [e]	−2.7
$^{68}_{30}Zn_{38}$	1.077	2.4	1.75	0.031 [f]	0.806	−1.5(1) [f]	−2.3
$^{70}_{32}Ge_{38}$	1.040	3.6	1.64	0.0086	0.670	$-(5^{+4}_{-1})$ [g]	−9
$^{72}_{32}Ge_{40}$	0.835	4.6	1.75	0.0019	0.630	−10.3(1.3) [h]	−20
$^{74}_{32}Ge_{42}$	0.596	6.3	2.04	0.018	0.609	+2.9(8) [i]	+5.7
$^{76}_{32}Ge_{44}$	0.563	5.6	1.97	0.023 [j]	0.546	+3.5(15) [j]	+8
$^{74}_{34}Se_{40}$	0.635	9.2	2.00	0.072 [k]	0.635	$-(6^{+2}_{-1})$ [k]	−11
$^{76}_{34}Se_{42}$	0.559	8.7	2.18	0.034	0.657	+5.5(5) [l]	+10
$^{78}_{34}Se_{44}$	0.614	7.2	2.13	0.021	0.695	$+(6^{+7}_{-2})$ [m]	+11
$^{80}_{36}Kr_{44}$	0.618	6.8 [o]	2.04	0.016	0.640	$+(17^{+80}_{-9})$ [n]	+31
$^{82}_{36}Kr_{46}$	0.777	3.5 [o]	1.90	0.018	0.698	+2.6(2) [p]	+4.5
$^{84}_{36}Kr_{48}$	0.883	3.0 [o]	2.14	0.073	1.018	$-(40^{+\infty}_{-30})$ [q]	−48
$^{84}_{38}Sr_{46}$	0.795	6.9	1.83	0.0075 [r]	0.661	+0.8(2) [r]	+1.4
$^{86}_{38}Sr_{48}$	1.078	3.9	1.72	0.17	0.778	+0.27(4) [s]	+0.42
$^{88}_{38}Sr_{50}$	1.836	3.4	1.75	2.7 [t]	1.383	−0.04(2) [t]	−0.04
$^{92}_{40}Zr_{52}$	0.934	1.6	1.98	5.6	0.913	+0.05(2) [u]	+0.07
$^{94}_{42}Mo_{52}$	0.871	5.7	2.14	0.005 [v]	0.993	−2.0(4) [v]	−2.4
$^{96}_{42}Mo_{54}$	0.778	5.9	1.92	0.068 [w]	0.717	+0.44(4) [w]	+0.73
			2.09	0.0063 [w]	0.847	−1.1(1) [w]	−1.6
$^{98}_{42}Mo_{56}$	0.787	5.2	1.82	0.018 [x]	0.645	+0.58(3) [x]	+1.1
			2.24	0.006 [x]	0.971	−2.2(2) [x]	−2.6
$^{100}_{44}Ru_{56}$	0.540	11	2.52	0.059 [y]	0.826	+6(2) [y]	+9
$^{102}_{44}Ru_{58}$	0.475	15	2.32	0.038 [z]	0.628	−60(20) [z]	−110
$^{104}_{44}Ru_{60}$	0.358	19	2.50	0.049 [aa]	0.535	−9(2) [aa]	−19
$^{104}_{46}Pd_{58}$	0.556	11	2.42	0.060 [bb]	0.786	$+(30^{+\infty}_{-22})$ [bb]	+46
$^{106}_{46}Pd_{60}$	0.512	13	2.20	0.024	0.616	−7(2) [cc]	−14
$^{108}_{46}Pd_{62}$	0.434	15	2.14	0.013 [dd]	0.497	−3.1(4) [ee]	−7.5
$^{110}_{46}Pd_{64}$	0.374	19	2.18	0.014 [dd]	0.440	$-(5^{+2}_{-1})$ [dd]	−12
$^{106}_{48}Cd_{58}$	0.633	9.3	2.71	0.31 [ff]	1.084	−0.9(2) [gg]	−1.0
$^{108}_{48}Cd_{60}$	0.633	11	2.53	0.097 [ff]	0.973	$-(1.5^{+1.5}_{-0.6})$ [ff]	−1.8
$^{110}_{48}Cd_{62}$	0.658	10	2.23	0.041	0.818	−1.2(2) [hh]	−1.8
$^{112}_{48}Cd_{64}$	0.617	11	2.12	0.036 [ff]	0.695	−0.77(6) [gg]	−1.3

TABLE I *(Continued)*

Nucleus	$E(2)$ (MeV)	$B(E2,2\rightarrow 0)$ [a] $[10^{-2}\ (e\text{b})^2]$	$E(2')/E(2)$	$\frac{B(E2,2'\rightarrow 0)}{B(E2,2'\rightarrow 2)}$ [a]	E_γ (MeV)	δ	Δ $(e\text{b}/\mu_N)$
$^{114}_{48}Cd_{66}$	0.558	12	2.16	0.022 [ff]	0.650	$-(1.4^{+0.7}_{-0.3})$ [gg]	−2.8
$^{116}_{48}Cd_{68}$	0.513	12	2.38	0.052 [ff]	0.710	$-(1.5^{+0.9}_{-0.4})$ [ff]	−2.5
$^{116}_{50}Sn_{66}$	1.293	3.9	1.63	0.015 [ii]	0.820	−1.8(2) [ii]	−2.6
$^{122}_{52}Te_{70}$	0.564	12	2.23	0.011	0.691	−3.5(1) [jj]	−6.0
$^{124}_{52}Te_{72}$	0.603	18	2.20	0.0074	0.722	−3.4(1) [kk]	−5.6
$^{126}_{52}Te_{74}$	0.667	11	2.13	0.0036	0.754	−5.5(4) [kk]	−8.7
$^{126}_{54}Xe_{72}$	0.386	16 [l]	2.28	0.020	0.491	$+(9^{+4}_{-2})$ [mm]	+22
$^{128}_{54}Xe_{74}$	0.441	13 [l]	2.20	0.013	0.528	+6.4(10) [nn]	+14
$^{132}_{54}Xe_{78}$	0.668	8 [l]	1.94	0.0014	0.630	$+(5^{+2}_{-1})$ [oo]	+10
$^{132}_{56}Ba_{76}$	0.464	16 [o]	2.24	0.026	0.573	$+(9^{+7}_{-3})$ [pp]	+19
$^{134}_{56}Ba_{78}$	0.605	14	1.93	0.0063	0.563	−7.4(9) [qq]	−16
$^{140}_{58}Ce_{82}$	1.596	6.9	1.47	0.0070	0.752	+0.33(3) [rr]	+0.53
			1.58	11	0.923	+0.014(14) [ss]	+0.02
$^{142}_{58}Ce_{84}$	0.642	8.4 [o]	2.39	2.9	0.894	−0.08(3) [tt]	−0.11
$^{144}_{60}Nd_{84}$	0.695	8.3 [o]	2.24	0.004 [uu]	0.865	−1.6(5) [uu]	−2.3
$^{150}_{62}Sm_{88}$	0.334	27 [o]	3.13	0.017 [vv]	0.712	$-(4^{+3}_{-1})$ [ww]	−7
			3.49	0.25 [vv]	0.860	$+(7^{+4}_{-2})$ [ww]	+10
$^{152}_{64}Gd_{88}$	0.344	24	2.70	0.021 [xx]	0.586	−3.1(1) [yy]	−6.4
			3.23	0.15 [xx]	0.765	+4.3(7) [yy]	+6.7

[a] Unless otherwise indicated, $B(E2)$ values are derived from lifetimes and branching ratios given by C. M. Lederer, J. M. Hollander, and I. Perlman, *Table of Isotopes* (Wiley, New York, 1967).

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malized such that the excitation energy of the first 2^+ state is equal to the phonon energy. This is a rather unique example of a vibrator; in practice one seldom finds such close spacing of the three-phonon quintuplet.

In the zeroth-order harmonic model, both the state vectors and the multipole operators are treated collectively. The state vector of the N-phonon level of spin J is given by

$$|J_N M\rangle = \frac{1}{\sqrt{N}} \{b^\dagger_{2m_1} b^\dagger_{2m_2} \cdots b^\dagger_{2m_N}\}_{JM} |0\rangle , \qquad (6)$$

where the brackets indicate that the phonon creation operators $b^\dagger_{2m}$ are coupled together to give a resultant J and M, including appropriate angular momentum and parentage coefficients. The collective form of the $M1$ operator is given by

$$\mathfrak{M}(M1,\mu) = (3/4\pi)^{1/2} g_R \mu_N J_\mu , \tag{7}$$

$$= (3/4\pi)^{1/2} g_R \mu_N (\hbar\sqrt{10}) \{(-1)^{m_1} b_{2-m_1} b^\dagger_{2m_2}\}_{1\mu} . \tag{8}$$

Here μ_N represents the nuclear magneton ($=e\hbar/2Mc$). If the $2'$ and 2 levels are interpreted as two- and one-phonon states respectively, it is apparent that the $2'\to2$ $M1$ transition must not exist in this model, since the $M1$ operator cannot change the number of phonons.

The lowest order perturbation which can be applied to this model is allowing for configuration mixing of $\Delta N=1$ phonon levels. For example

$$\begin{aligned} |2\rangle &= a|2_1\rangle + b|2_2\rangle , \\ |2'\rangle &= a|2_2\rangle - b|2_1\rangle , \end{aligned} \tag{9}$$

where $a^2+b^2=1$, and where the state vectors on the right-hand side of Eqs. (9) are the pure-phonon states J_N given by Eq. (6). In this approximation, the static and dynamic properties of the levels may be computed to be

$$\frac{E(2')}{E(2)} = \frac{2-b/a}{1+2b/a} , \tag{10a}$$

$$\frac{B(E2,2'\to2)}{B(E2,2\to0)} = \frac{2(1-2b^2)^2}{1-b^2} , \tag{10b}$$

$$\frac{B(E2,2'\to0)}{B(E2,2'\to2)} = \frac{1}{2}\left(\frac{b}{1-2b^2}\right)^2 , \tag{10c}$$

$$|eQ(2)| = |ab|[\tfrac{256}{35}\pi B(E2,2\to0)]^{1/2} . \tag{10d}$$

The deviations of the energy ratios, $B(E2)$ ratios, and quadrupole moments from the predictions of the pure phonon model ($b=0$) may be reasonably well accounted for by phonon mixing. Singh, Mehta, and Waghmare[10] have recently done a similar type of calculation based on configuration mixing and have obtained reasonable agreement with experimental values.

However, this type of mixing does not give rise to $M1$ transitions. It is apparent from the very nature of the $M1$ operator that $M1$ transitions of the type $J\to J\pm1$ must vanish, since the J_μ operator cannot change the value of J ($J_\mu|JM\rangle \to |JM+\mu\rangle$). Additionally, the vanishing of the $M1$ component of the $2'\to2$ transition follows from Eqs. (7) and (9):

$$\langle 2||\mathfrak{M}(M1)||2'\rangle \propto ab[g_R(2_2)-g_R(2_1)] . \tag{11}$$

As long as the g factors of the phonon states are identical, the $M1$ amplitude vanishes.

Nonvanishing $M1$ transitions may be obtained by introducing noncollective contributions into the state vectors, Eq. (6), or into the $M1$ operator, Eq. (7). These nonphonon contributions will be discussed in succeeding sections. It should be noted, however, that the reasonable success obtained from a calculation of the $B(E2)$ ratios including phonon mixing suggests that such states may provide a useful basis from which to proceed.

B. Rotational models

In this section we consider the excited states as members of quasirotational bands. Figure 3 illustrates how the multiple-phonon levels may be decomposed into various intrinsic excitations

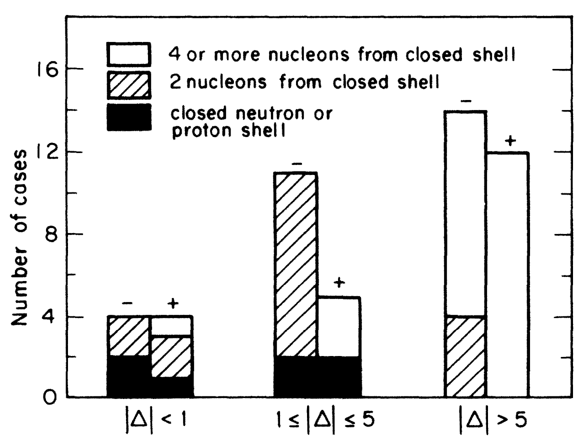

FIG. 1. Histogram of $E2/M1$ mixing ratios Δ of $2'\to2$ transitions in even-even nuclei $60 \lesssim A \lesssim 150$. The labels + and − refer to the phase of Δ, as defined in the present work.

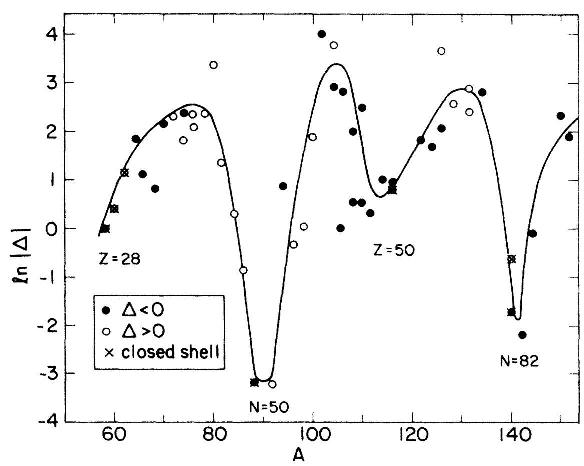

FIG. 2. $E2/M1$ mixing ratios of $2'\to2$ transitions in even-even nuclei $60 \lesssim A \lesssim 150$. The solid curve indicates the trend of the measured values and shows pronounced minima in the vicinity of closed shells.

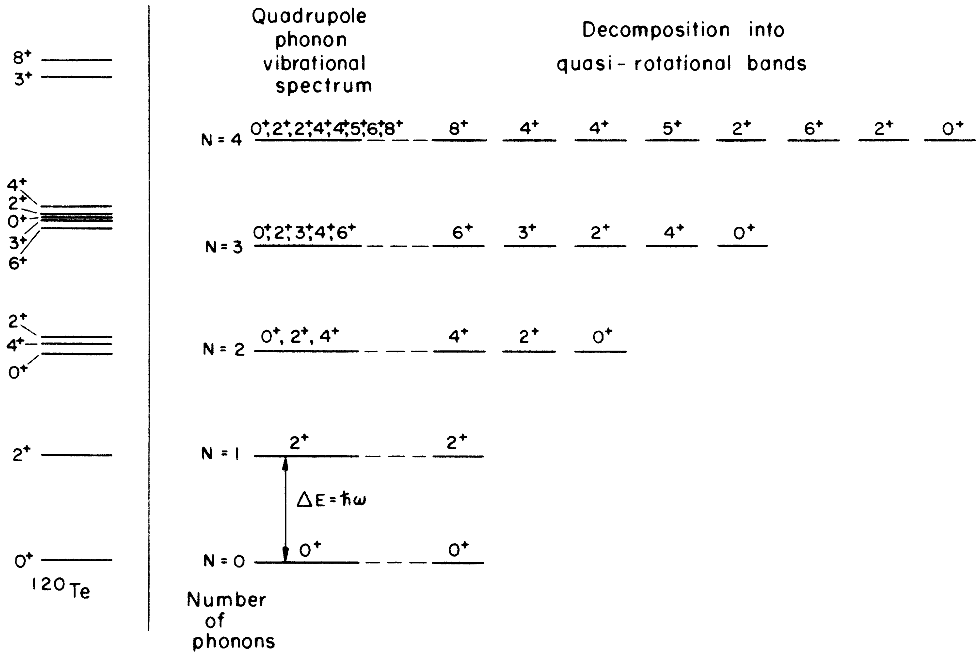

FIG. 3. Spectrum of excited states expected on the basis of the phonon vibrational model. The actual spectrum of ^{120}Te is shown to the left. To the right is indicated the decomposition of the phonon states into quasirotational bands, which might be identified (in order) as the ground state, γ, β, $\gamma\gamma$, $\beta\beta$, $\gamma\gamma\gamma$, $\beta\gamma$, and $\beta\beta\beta$ quasibands.

and rotational bands. These rotational bands deviate considerably from the $J(J+1)$ spacing expected for a rigid rotor, indicating the "softness" of the nuclear deformation. (This is the basis for such considerations as the variable moment of inertia model[11] or the higher-order cranking model.[12]) The 2′ excitations may then be considered as states of the quasi-γ or quasi-β bands. The $E2/M1$ mixing ratios then obtain a collective (i.e., $E2$) character, in agreement with observations. However, contrary to observation, the crossover $2' \to 0$ $E2$ transition would not be strongly forbidden by this type of model, and thus it is to be expected that limited success in interpreting $E2/M1$ mixing would be obtained.

An alternative possibility is to consider an asymmetric rotor model, for example, that of Davydov and Filippov.[13] However, as was shown by Lipas,[14] collective $M1$ transitions must be identically vanishing in such a model.

C. Phonon-plus-particles model

As discussed in Sec. III A, the phonon model disallows all $M1$ transitions; in the present approach, we introduce a small admixture of a two-particle state into the state vector. The approximate success of the phonon model in accounting for the lower $B(E2)$ ratios indicates that this admixture may be treated as a perturbation of the phonon state vectors.

In the present calculation we employ the method developed by Tamura and Yoshida.[15] A pair of nucleons in the quasiparticle states $|j_1\rangle$ and $|j_2\rangle$ is excited from the ground state. The coupling of the particle and collective motions is described by the interaction

$$H_{\text{int}} = -\tfrac{1}{2}\chi \sum_{\mu} \hat{Q}_\mu Q_\mu^* , \tag{12}$$

where Q_μ is the collective quadrupole operator (i.e., the $\alpha_{2\mu}$ in the model of nuclear surface vibrations), and $\hat{Q}_\mu$ is a two-quasiparticle quadrupole operator, given by

$$\hat{Q}_\mu = \sum_{\substack{jm\\ j'm'}} \langle jm|r^2 Y_{2\mu}|j'm'\rangle (u_j v_{j'} + u_{j'} v_j) \times (\alpha_{jm}^\dagger \beta_{j'm'}^\dagger + \beta_{jm}\alpha_{j'm'}) , \tag{13}$$

where u and v are respectively the usual quasiparticle non-occupation and occupation amplitudes, and $\alpha^\dagger$ and $\beta^\dagger$ (α and β) are the quasiparticle creation (destruction) operators.

Under such an interaction, assumed to be treatable by standard methods of first-order perturbation theory, the state vectors of the 2 and 2′ levels can be written as perturbations of the collective states 2_n (with n regarded as a seniority index of the collective states, rather than as a phonon number):

$$|2\rangle = |2_1\rangle + \sum_{j_1 j_2} b(2_1;(j_1 j_2)2)\,|(j_1 j_2)2\rangle\,,$$
$$|2'\rangle = |2_2\rangle + \sum_{j_1 j_2} b(2_2;(j_1 j_2)2)\,|(j_1 j_2)2\rangle\,. \tag{14}$$

We employ a notation slightly different from that of Tamura and Yoshida, but preserve the spirit of their work. The mixing amplitudes b are given by

$$b(2_n;(j_1 j_2)2) = \frac{\chi\langle 0\|Q\|2_n\rangle\langle j_1\|\hat{Q}\|j_2\rangle(u_{j_1}v_{j_2}+u_{j_2}v_{j_1})}{10[E_{j_1}+E_{j_2}-E(2_n)]}\,, \tag{15}$$

where the E_j are the quasiparticle energies. The $M1$ matrix element may then be computed to be

$$\langle 2\|\mathfrak{M}(M1)\|2'\rangle = \tfrac{1}{10}\chi^2\langle 0\|Q\|2_2\rangle \times\langle 0\|Q\|2_1\rangle \sum_{\substack{j_1 j_2\\ j_1' j_2'}} B_{j_1' j_2';\,j_1 j_2}\,. \tag{16}$$

The quantity $B_{j_1' j_2';\,j_1 j_2}$ depends on the couplings of the single-particle states and on the $M1$-matrix elements between the single-particle states. The dominant contributions to the total $M1$ transition probability will arise from cases in which $j_1'=j_1$ and $j_2'=j_2$ (including as a special case, $j_1'=j_2'=j_1=j_2$); that is, the identical configuration $(j_1 j_2)$ is admixed into both the 2 and 2′ levels. The total $M1$ transition probability is then proportional to $\langle j_1\|\mathfrak{M}(M1)\|j_1\rangle$ or $\langle j_2\|\mathfrak{M}(M1)\|j_2\rangle$, which are at least an order of magnitude larger than the matrix element $\langle j_1\|\mathfrak{M}(M1)\|j_2\rangle$; that is, empirical values of the former matrix elements do not differ greatly from the single-particle estimate (Schmidt limit) for the static $M1$ moments, while the empirical $M1$ transition probabilities are generally retarded by 2–3 orders of magnitude relative to single-particle (Weisskopf) estimates. Thus, the major contribution to the $2'\to 2$ $M1$ transition matrix element is proportional to

TABLE II. Computed two-particle contributions to $2'\to 2$ $E2/M1$ mixing ratios.

Nucleus	Δ (eb/μ_N)	$\sum B_{j_1 j_2;\,j_1 j_2}$ (fm^4/MeV^2) Protons	Neutrons	$\Delta_{\rm I}$	$\lvert\Delta_{\rm II}\rvert$
^{58}Ni	−1.0		−0.5	−250	180
^{60}Ni	+1.0		3.9	−150	20
^{62}Ni	+3.4		1.1	−350	20
^{64}Zn	−6	77	1		4
^{66}Zn	−2.7	84	−0.1		10
^{68}Zn	−2.3	88	−0.3		2
^{70}Ge	−9	148	−0.4	−5	2.5
^{72}Ge	−20	105	−0.5		
^{74}Ge	+5.7	76	−1.4	+5	2.5
^{76}Ge	+8	75	−3	+6	3
^{74}Se	−11	105	−0.4		0.6
^{76}Se	+10	111	−1.5		1.2
^{78}Se	+11	150	−3.7		1.2
^{80}Kr	+31	215	−4		2
^{82}Kr	+4.5	283	−9		2
^{84}Kr	−48	353	−11		
^{84}Sr	+1.4	150	−9		4
^{86}Sr	+0.42	326	27		0.6
^{88}Sr	−0.04	−1027			
^{92}Zr	+0.07	160	45		0.2
^{94}Mo	−2.4	312	44		4
^{96}Mo	+0.73	236	−43		2
^{98}Mo	+1.1	283	−13		3
^{100}Ru	+9	293	−7		1.2
^{102}Ru	−110	261	−3		1.2
^{104}Ru	−19	202	−1	+1.4	1.9
^{104}Pd	+46	425	−4	+1.7	1.2
^{106}Pd	−14	371	−1	+1.3	1.9
^{108}Pd	−7.5	283	2	+1.6	2.5
^{110}Pd	−12	237	3	+2.2	4.0
^{106}Cd	−1.0	2500	−2		
^{108}Cd	−1.8	2600	1	+0.1	0.2
^{110}Cd	−1.8	4000	3	+0.1	0.2
^{112}Cd	−1.3	2100	4	+0.5	0.6
^{114}Cd	−2.8	1300	3	+0.6	0.7
^{116}Cd	−2.5	930	1	+0.3	0.7
^{116}Sn	−2.6		−2	+300	160
^{122}Te	−6.0	1500	−3	+0.6	1.2
^{124}Te	−5.6	3200	−6	+2.0	0.7
^{126}Te	−8.7	−5600	−5	−0.6	0.7
^{126}Xe	+22	250	−4		5
^{128}Xe	+14	310	−3		5
^{132}Xe	+10	1800	4		4
^{132}Ba	+19	510	−1		3
^{134}Ba	−16	930	2	+1.2	4
^{140}Ce	+0.53	−1600			3
^{142}Ce	−0.11	1200	108	+1.2	0.2
^{144}Nd	−2.3	2400	650	+1.3	2.5
^{150}Sm	−7	430	0.8	+2.0	5
^{152}Gd	−6.4	230	0.1		12

$$B_{j_1 j_2;\,j_1 j_2} = \frac{(u_{j_1}v_{j_2}+u_{j_2}v_{j_1})^2\,|\langle j_1\|\hat{Q}\|j_2\rangle|^2}{[E_{j_1}+E_{j_2}-E(2_2)][E_{j_1}+E_{j_2}-E(2_1)]}\left\{\frac{j_1(j_1+1)+6-j_2(j_2+1)}{2[30j_1(j_1+1)(2j_1+1)]^{1/2}}\langle j_1\|\mathfrak{M}(M1)\|j_1\rangle + \frac{j_2(j_2+1)+6-j_1(j_1+1)}{2[30j_2(j_2+1)(2j_2+1)]^{1/2}}\langle j_2\|\mathfrak{M}(M1)\|j_2\rangle\right\}\,. \tag{17}$$

We expect that the detailed structure of Δ, in particular its phase and most of its variation in magnitude between neighboring even-even nuclei, will be contained in the $B_{j_1 j_2; j_1 j_2}$ term of Eq. (16). The remaining $E2$ matrix elements of Eq. (16) and of $\langle 2 \| \mathfrak{M}(E2) \| 2' \rangle$ are assumed to be highly collective and thus to vary relatively slowly.

Values of $B_{j_1 j_2; j_1 j_2}$ have been computed from Eq. (17). The single-particle $M1$ matrix elements have been computed in a manner similar to the Schmidt limits for the magnetic moments, except we have taken $g_s = 0.6(g_s)_{\text{free}}$ as giving a more realistic estimate of the empirical moments. For the matrix elements of $\hat{Q}$ we have used the Weisskopf estimate of the $E2$ transition intensity, modified by taking the neutron and proton effective charges to be 0.7 and 1.7 e, respectively. The pairing factors and quasiparticle energies have been computed using the single-particle energies, Fermi energies, and gap parameters given by Kisslinger and Sorensen.[16] For each even-even nucleus we have computed the five largest contributions to $B_{j_1 j_2; j_1 j_2}$ from two-neutron and also from two-proton states. The sum of these five values is shown in Table II.

The tabulated values of $\sum B_{j_1 j_2; j_1 j_2}$ illustrate the dominance of the contributions from the two-proton configurations over the two-neutron configurations. This dominance follows from four causes: (1) The single-particle matrix elements of $\hat{Q}$ are proportional to the assumed effective charges; the proton contributions would thus be expected to dominate over the neutron contributions by a factor of the square of the ratio of the effective charges, which amounts to a factor of 8. (2) The single-particle $M1$ moments (i.e., g factors) are generally larger for protons than for neutrons by a factor of 3. (3) The two-proton excitation energies are generally lower than the two-neutron energies; this produces another factor of at least 2. (4) The single-neutron $M1$ moment (g factor) is negative when $j = l + \frac{1}{2}$, leading to cancellations in the summation; this does not occur for protons.

The mixing ratio Δ may be expressed as

$$\Delta = \frac{\langle 2 \| \mathfrak{M}(E2) \| 2' \rangle}{\langle 2 \| \mathfrak{M}(M1) \| 2' \rangle} = \frac{[(3/4\pi) Z e R_0^2]^2}{\frac{1}{10}\chi^2} \frac{\langle 2 \| \mathfrak{M}(E2) \| 2' \rangle}{\langle 0 \| \mathfrak{M}(E2) \| 2' \rangle \langle 0 \| \mathfrak{M}(E2) \| 2 \rangle} \times \frac{1}{\sum_{j_1 j_2} B_{j_1 j_2; j_1 j_2}} . \tag{18}$$

We take $R_0 = 1.2 A^{1/3}$ fm and $\chi = 40$ MeV$/R_0^2$. The $E2$ matrix elements may be computed from either of two methods:

Method I. It follows from Eqs. (9) and (10b) that

$$\frac{\langle 2 \| \mathfrak{M}(E2) \| 2' \rangle}{\langle 0 \| \mathfrak{M}(E2) \| 2' \rangle} = -\sqrt{2} \left(\frac{1 - 2b^2}{b} \right), \tag{19}$$

and that

$$eQ(2) = ab \left(\frac{256\pi}{175} \right)^{1/2} \langle 0 \| \mathfrak{M}(E2) \| 2 \rangle, \tag{20}$$

and thus

$$\frac{\langle 2 \| \mathfrak{M}(E2) \| 2' \rangle}{\langle 0 \| \mathfrak{M}(E2) \| 2' \rangle \langle 0 \| \mathfrak{M}(E2) \| 2 \rangle} = \frac{-a(1 - 2b^2)}{eQ(2)} \left(\frac{512\pi}{175} \right)^{1/2} . \tag{21}$$

We take $a \approx +1$ and compute b^2 from Eq. (10c) and the crossover-to-cascade $B(E2)$ ratios. If the value of $Q(2)$ is known,[17] the value of Δ may be computed. The sign of Δ is determined by the sign of $\sum B$ as well as that of $Q(2)$.

Method II. In the event $Q(2)$ is unknown, we may obtain a value for the magnitude of the $2 \to 0$ matrix element as

$$|\langle 0 \| \mathfrak{M}(E2) \| 2 \rangle| = [5B(E2, 2 \to 0)]^{1/2} . \tag{22}$$

We again use Eq. (19) for the ratio of the $E2$ matrix elements and compute b from Eq. (10c). In this case the signs of the $2 \to 0$ matrix element and of b are undetermined, and hence the sign of Δ is undetermined.

Values of Δ computed according to Methods I and II are shown in Table II. The agreement of the calculated values with the absolute magnitudes of the experimental values is not unreasonable, considering the approximations made in selecting a broad set of parameters for each mass range. Detailed comparisons between theory and experiment for a given nucleus must be based on calculations which employ a set of single-particle energies and moments more appropriate to that nucleus (for example, deduced from the neighboring odd-mass nuclei). One could also improve the agreement in the present case by employing a reduced value of the coupling constant χ, which was selected somewhat arbitrarily[15]; a reduction by a factor of 3, for example, would increase the calculated values of Δ by an order of magnitude and give rather good agreement with experiment for the Ru, Pd, Cd, and Te isotopes. However, these cases are even more sensitive (in magnitude as well as phase) to a resonance-type behavior arising from the closeness of the energies of the two-particle and 2_2 levels, and thus such a discussion of the value of χ must await a detailed calculation based on a more realistic and suitable choice of single-particle energies.

The phase of Δ is not predicted uniquely, but rather is subject to a number of estimates of the $E2$ matrix elements of Eq. (18). The magnitudes of these $E2$ matrix elements vary relatively little over the range of even-even isotopes of a given atomic number; it may be assumed that the relative phases of these matrix elements do likewise. We therefore assume that the information on variation in the phase of Δ is contained in the $B_{j_1 j_2; j_1 j_2}$ terms, and from Eq. (17) we see that this in turn depends on the $M1$ matrix elements and on the energy differences in the denominator. Since the dominant contributions to $B_{j_1 j_2; j_1 j_2}$ come from proton states, the $M1$ matrix elements (i.e., the g factors) are all positive, and thus the phase variations will be characterized by the energy differences between the two-quasiparticle states and the unperturbed collective states 2_n. We expect that $(E_{j_1}+E_{j_2})$ will always exceed $E(2_1)$, and so we examine the relationship between $(E_{j_1}+E_{j_2})$ and $E(2_2)$. The unperturbed energies of the 2_2 states have not been computed, but their relative systematic behavior can be inferred from that of the perturbed $2'$ states. The energy relationships between the $2'$ states and the lowest two-quasiparticle states are illustrated in Fig. 4, for a number of sequences of even-even isotopes. In all cases a suitable selection of 2_2 states, highly correlated with the $2'$ states, could be made to cross with the two-quasiparticle energies at a point corresponding to an observed change in phase of the mixing ratio. It is interesting to note in support of this contention that, in four illustrated sequences of isotopes in which this phase change occurs, it always occurs only once in each sequence; that is, phase sequences such as $(+-+)$ or $(-+-)$ do not occur. Furthermore, it is possible that the phase difference of the $2' \rightarrow 2$ mixing ratio between the Te and Xe isotopes may simply result from the relationship between the $2'$ level and the two-proton states, as indicated in Fig. 4.

The theory does predict phase changes at $^{134-136}$Ba, $^{100-102}$Pd and $^{94-96}$Ru, while the observed changes occur at $^{132-134}$Ba, $^{104-106}$Pd, and $^{100-102}$Ru. It is indeed possible that a more refined calcula-

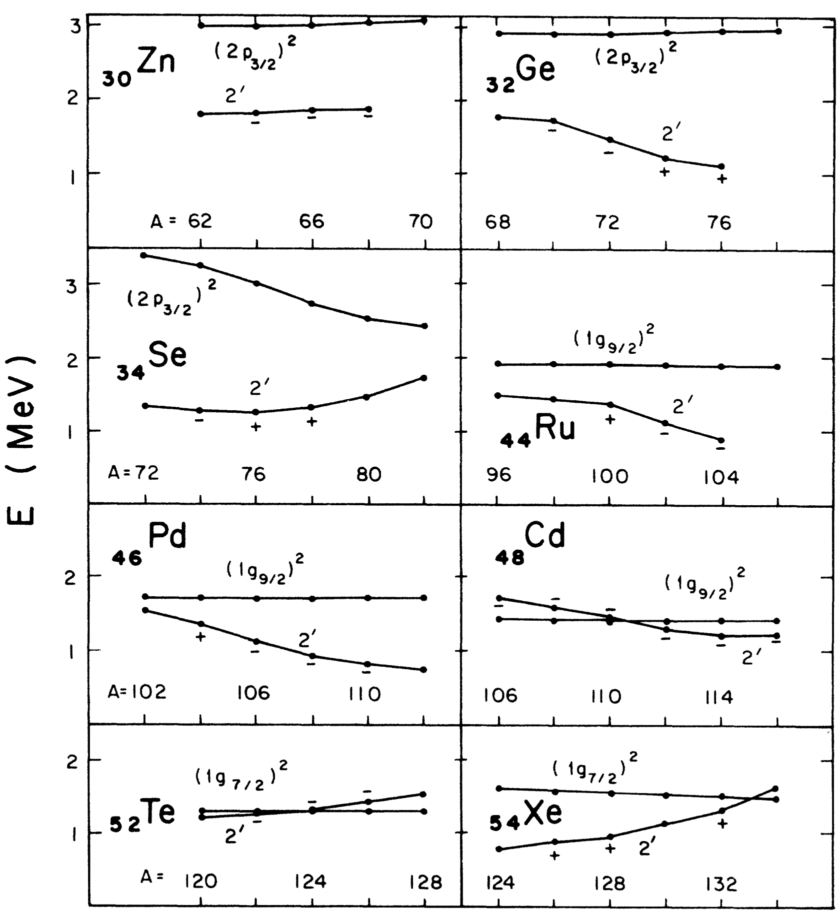

FIG. 4. Relationship between energies of $2'$ states and lowest two-proton configuration. The $2'$ states are labeled with + or − to indicate the phase of the $2' \rightarrow 2$ $E2/M1$ mixing ratio.

tion using single-particle energies more suited to each particular nucleus (rather than an average set for a larger mass region) could provide more successful predictions of the level crossings and, thus, of the change in phase of Δ. Further conclusions in this respect must await additional measurements of the 2′→2 mixing ratios, particularly those of the more neutron-deficient isotopes.

An additional point of interest in the comparison of relative phases is the degree to which the phases (and possibly also the magnitudes) of Δ correlate with $Q(2)$. Unfortunately, the quadrupole moment data available[17] is not sufficient to draw detailed conclusions. It would, for example, be of interest to attempt to account for the difference in the phase of Δ between the Te and Xe isotopes with a change in sign of $Q(2)$, indicating the Xe isotopes may be somewhat oblate. However, the lack of values of $Q(2)$ for the Xe isotopes makes such comparisons impossible at present.

An alternate approach to the phonon-plus-particles model has been given by Korolev.[18] In this approach the nucleus is treated as a core plus one or more zero-spin pairs which excite collective modes of the core. One can then compute, in terms of an interaction constant and a suitable set of unperturbed energy levels, the $M1$ and $E2$ matrix elements to be expected for transitions to the first 2^+ state (assumed to be a one-phonon state) from various possible structures of the 2′ state (two-phonon, one-phonon plus pair, one "excited" pair, etc.). The success of this model then depends strongly on the interpretation of the physical 2′ state, in particular its admixtures of paired states, although Korolev[18] has obtained reasonable agreement with ratios of reduced $E2$ transition probabilities in the Cd isotopes.

D. Higher-order $M1$ operators

In our discussion of the phonon model in Sec. IIIA, it was pointed out that if both the state vectors and the $M1$ operators are treated in their lowest-order phonon modes, the $M1$ matrix elements must vanish. In the previous section, the effects of relaxing this restriction for the state vectors was considered; in the present section, we consider the effect of higher-order terms in the $M1$ operators. Here "higher order" refers to more sophisticated couplings than that suggested by Eq. (8). [We note that Eq. (7) contains the implicit assumption that the nuclear mass and charge distributions are identical, and thus we are presently concerned with cases in which the mass and charge distributions differ.]

A generalized $M1$ operator may be obtained by including higher-order phonon contributions of the form

$$\mathfrak{M}(M1,\mu)=(3/4\pi)^{1/2}\mu_N[g^{(0)}J_\mu^{(0)}+g^{(1)}J_\mu^{(1)}+\cdots], \tag{23}$$

where we can make the identifications [cf. Eq. (7)]

$$g^{(0)}=g_R\,,\qquad J_\mu^{(0)}=J_\mu\,, \tag{24}$$

and where we can define a first-order coupling of the form

$$J_\mu^{(1)}=\{\alpha_{2m_1}J_{m_2}^{(0)}\}_{1\mu}=\sum_{m_1m_2}\langle 2m_1 1m_2|1\mu\rangle\alpha_{2m_1}J_{m_2}^{(0)}, \tag{25}$$

where α_{2m_1} is the collective quadrupole operator which can be represented as a linear combination of the phonon creation and annihilation operators $b_{2m_1}^\dagger$ and b_{2m_1}. This operator $J_\mu^{(1)}$ can now connect states differing by one phonon number, and thus we expect nonvanishing $M1$ matrix elements between the 2′ and 2 levels. The coefficient $g^{(1)}$ may either be computed on the basis of the interaction which is assumed to give rise to the coupling of Eq. (25), or else may be regarded as a single parameter of the theory to be determined from comparisons with experiment.

Greiner[19] has used such a model to compute g factors and $E2/M1$ mixing ratios in vibrational nuclei by assuming that the existence of a stronger pairing force for protons than for neutrons results in a smaller proton deformation, which in turn causes the $M1$ operator to obtain the tensorial structure described by Eq. (23). The factor $g^{(1)}$ is given by

$$g^{(1)}=-\frac{Z}{A}(1-2f)\,\frac{2}{3}\sqrt{10}\,\frac{f}{\beta_0}\,, \tag{26}$$

where β_0 is the root-mean-square amplitude of the vibration, defined by

$$B(E2,2\to0)=\tfrac{1}{5}(\tfrac{3}{4}ZeR_0^{\,2})^2\beta_0^{\,2}\,, \tag{27}$$

and where f gives the difference in proton and neutron deformations,

$$f\approx\frac{N}{A}\left(\frac{\beta_0(n)}{\beta_0(p)}-1\right)\,. \tag{28}$$

We then obtain

$$\Delta=\frac{ZA^{2/3}}{g^{(1)}}(2.3\times10^{-3})\,, \tag{29}$$

or, from Eq. (26),

$$\Delta=(1.1\times10^{-3})\,\frac{A^{5/3}\beta_0}{f(1-2f)}\,. \tag{30}$$

An alternative formulation of this type has been given in a series of papers by Grechukhin.[20] The result obtained is similar in form to Eq. (29); however, $g^{(1)}$ is ultimately regarded as a free parameter to be determined from comparisons with the empirical Δ values. Grechukhin does attempt a semiclassical calculation of $g^{(1)}$, which again depends on the difference between the nuclear mass and charge distributions. Whereas in Greiner's model this difference is simply represented by the parameter f, in Grechukhin's calculations the proton distribution is computed semiclassically by considering the competition between the Coulomb repulsion and the proton-neutron attraction, from which one obtains

$$g^{(1)} = 8\times10^{-3}\frac{Ze^2}{R_0\kappa}, \tag{31}$$

where κ is the symmetry energy constant in the Weiszacker semiempirical mass formula ($\kappa \approx 20$ MeV). This leads to values of $g^{(1)}$ of order 5×10^{-3}, whereas from a comparison of Eq. (29) with the tabulated values of Δ, it can be seen that values of $g^{(1)}$ of order 0.5 are required. In Greiner's model, the parameter f may be determined from the difference between the empirical value of $g(2)$, the g factor of the first 2^+ level, and the hydrodynamical value Z/A. The deduced values of f generally lie to the range 0.1–0.2, and hence in Greiner's model, $g^{(1)}$ is of order Z/A, leading to reasonable agreement between the predicted values of Δ and the measured values. Grechukhin takes $g^{(1)}$ as a parameter of the model, and sets Z/A as a limiting value. In the present calculation we will follow Greiner's method.

In Table III are shown the predicted values of Δ based on Greiner's[19] parameter f which we have derived from the empirical $g(2)$ values, according to the relationship $g(2)=(Z/A)(1-2f)(1+\frac{2}{3}f)$. The $g(2)$ values were obtained from the compilation of Shirley.[17] The data are shown for the medium weight nuclei in the mass range $100 \leq A \leq 126$, for which the most reliable $g(2)$ values are available. The agreement both in sign and in magnitude is striking, particularly in ^{100}Ru and ^{102}Ru, in which cases the theory correctly predicts the change in sign at ^{100}Ru and the resonance-like effect observed for ^{102}Ru. [It should be noted that $f<0$ requires that the neutron pairing force be stronger than the proton pairing force. This can be circumvented by regarding f as a free parameter of the theory, on which $g(2)$ and Δ both depend, but arising from some undetermined facet of the nuclear structure.]

A more systematic indication of the relationship between $g(2)$ and Δ is illustrated in Fig. 5. Again the $g(2)$ values are as given by Shirley,[17] with additional results for the Xe isotopes from Gordon *et al.*[21] The uncertainties of many of the $g(2)$ values are sufficiently large as to prevent drawing definite conclusions regarding the applicability of this model. However, it should be noted that there is no case in clear disagreement with the expected relationship; there is, for example, no instance of a positive Δ for which the error bars do not allow positive values of $[g(2)-Z/A]$. More quantitative conclusions of this nature require considerable reduction in the experimental uncertainties of the $g(2)$ values.

TABLE III. Comparison of experimental $2'\to2$ $E2/M1$ mixing ratios with computed values.

Nucleus	$g(2)$	Z/A	f	Δ Theory	Δ Experiment
^{100}Ru	0.55(7)	0.44	−0.16	+2.5	+9
^{102}Ru	0.41(3)	0.43	0.03	−20	−110
^{104}Ru	0.29(4)	0.42	0.20	−5.8	−19
^{106}Pd	0.38(3)	0.43	0.09	−7.8	−14
^{108}Pd	0.30(4)	0.43	0.20	−5.5	−7.5
^{110}Pd	0.25(3)	0.42	0.25	−5.9	−12
^{110}Cd	0.35(8)	0.44	0.14	−4.8	−1.8
^{112}Cd	0.30(6)	0.43	0.20	−4.5	−1.3
^{122}Te	0.34(5)	0.43	0.14	−5.6	−6.0
^{124}Te	0.29(4)	0.42	0.21	−4.5	−5.6
^{126}Te	0.25(7)	0.41	0.25	−4.5	−8.7

IV. DISCUSSION

We have shown that agreement of at least a qualitative nature may be obtained between experi-

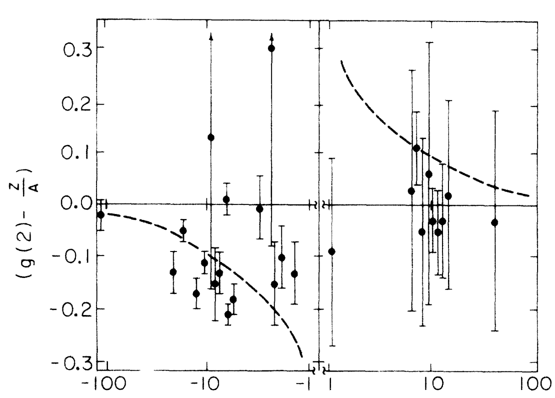

FIG. 5. The relationship between the reduced $2'\to2$ $E2/M1$ mixing ratios Δ (in units of eb/μ_N) and the g factors of the first 2^+ states. Experimental uncertainties on the g factors are as shown; the uncertainities of the Δ values have been omitted for clarity. The dashed lines illustrate the general trend of the relationship which might be expected based on Eq. (29).

mental values of $2' \to 2$ $E2/M1$ mixing ratios and theoretical values computed either (1) on the basis of a semimicroscopic model in which the $M1$ amplitudes are obtained from two-quasiparticle admixtures to the collective vibrational states, or (2) on the basis of a phenomenological description in which the mixing ratio Δ is strongly related to the deviation of the g factor of the first excited 2^+ state from the hydrodynamical value Z/A. It is perhaps somewhat surprising that these two approaches should both appear promising for such a calculation, since they represent fundamentally different ways of interpreting nuclear structure. In calculation (1), the $M1$ amplitudes are due purely to the dynamics of the two "valence" particles. Although there are effects present due to the "core", (i.e., the nucleus minus the two "valence" particles), such effects represent at most a renormalization of the coupling constant χ or perhaps of the $E2$ amplitudes. (Of course, the energy levels available for the two quasiparticles are determined by the properties of the core; however, the core itself does not take part in the transition.) The phenomenological approach represented by calculation (2) considers the dynamics of the core (in this case the entire nucleus) through the collective variables. In effect the latter approach includes implicitly a number of effects not considered in calculation (1), including variations in the structure of the core induced by the transition and higher-seniority configurations (four quasiparticles, etc.) whose effects might be significant.

In an attempt at a more quantitative comparison of the two calculations, we have computed the g factor of the first excited 2^+ state using the state vectors $|2\rangle$ given by Eq. (14), obtaining after some manipulation

$$g = g_R + \Delta g , \tag{32}$$

where

$$\Delta g = \left(\frac{2}{15}\right)^{1/2} \left(\frac{2\pi}{3ZeR_0^{\,2}}\right)^2 \chi^2 B(E2,\ 2\to 0) \sum_{j_1 j_2} B_{j_1 j_2 ; j_1 j_2} \,. \tag{33}$$

The expression for $B_{j_1 j_2 ; j_1 j_2}$ is identical to that given by Eq. (17), except that the energy factor in the denominator becomes $[E_{j_1} + E_{j_2} - E(2_1)]^2$. Evaluating Δg for a medium-weight nucleus ($A \approx 100$) we obtain $\Delta g \approx 0.5$, using parameters identical to those used in Sec. IIIA for the calculation of Δ. Equation (33) requires that Δg have the same sign as the $B_{j_1 j_2 ; j_1 j_2}$ term; as discussed above, the dominance of two-proton contributions leads to positive values for $B_{j_1 j_2 ; j_1 j_2}$ and thus to $\Delta g > 0$. [In this case the previous discussion regarding the prediction of phase changes in Δ according to crossings between $E(2_2)$ and $E_{j_1} + E_{j_2}$ does not apply, owing to the different energy factor which appears in the $B_{j_1 j_2 ; j_1 j_2}$ expression for Δg.] Furthermore, the computed magnitude of Δg is too large, although the reduction in χ^2 by an order of magnitude as suggested above will yield a reasonable size for Δg. (An alternative explanation, in which the angular momenta of the "core" nucleons are somehow quenched such that $g_R \approx 0$, with all of the magnetic properties of the nucleus arising from the "valence" pair, is unreasonable.) One, therefore, concludes that although the dynamics of the "valence" pair may possibly give a reasonable explanation for the $M1$ transition probabilities, it is unable to account for the static nuclear magnetic dipole moment. This failure may arise from either (a) the dynamics of the core, which does not contribute directly to the $M1$ transition probability in the present formulation, or (b) higher seniority configurations which would influence both the static and dynamic $M1$ moments. It appears that perhaps the phenomenological model may be successful in accounting for the microscopic effects using few independent parameters.

V. CONCLUSIONS

A complete understanding of the $E2/M1$ mixing ratios of the low-lying even-parity states of even-even nuclei in the mass range $60 \lesssim A \lesssim 150$ obviously requires a detailed calculation which considers all multiple-seniority configurations of a complete set of single-particle basis states. The success of such a calculation for a given nucleus, however, requires knowledge of the appropriate single-particle states and transition probabilities. We have shown in the present work that the systematic behavior of the $E2/M1$ mixing ratios can be interpreted with reasonable success across the entire mass range by considering only a few low-lying two-particle states as perturbations of the phonon basis states (presumably the correlations among the multiple-seniority configurations are implicitly included in the phonon state vectors such that their effects are included in the single-phonon $E2$ transition matrix elements). The variations in phase, and, to a lesser extent, the variations in magnitude can be successfully accounted for using a relatively small number of parameters.

Additionally, as will be subsequently demonstrated,[22] nearly all of the "single-phonon" transitions in the Ru, Pd, Cd, and Te isotopes may be interpreted in terms of a single-parameter model based on the higher-phonon terms in the magnetic moment operator, where that single parameter is deduced according to the method of Greiner from the difference between the g factor of the 2^+ level and its expected hydrodynamical values of Z/A. Such a model not only is successful in predicting the relative magnitudes over a large range of mass numbers, but also seems to hold for transitions from levels of up to four phonons.[22]

The phase of the $E2/M1$ mixing ratio is a nuclear observable which has in the past not been widely used as a probe of the nuclear structure. This situation has resulted in part from the confusion resulting from the several different phase conventions which can be used to extract the mixing ratio from the angular correlation data (along with a corresponding failure on the part of numerous investigators to specify which convention they have adopted). In the present work these phases have been determined in a consistent manner and can be related to the intrinsic electromagnetic matrix elements. We have shown how this phase can be related in a model-dependent way to details of the nuclear structure. It is hoped that considerations such as these can lead to a better understanding of the phase relationships between nuclear electromagnetic matrix elements; for example, it should be possible to employ the observed mixing ratios and suitably computed (model-dependent) $M1$ matrix elements to determine the phase of the corresponding $E2$ matrix element, which might be important in understanding a similar phase-dependent problem such as the interference term which arises from measurements of quadrupole moments by the reorientation effect following Coulomb excitation. In any event, it is apparent that both the phase and the magnitude of the $E2/M1$ mixing ratio can be successfully employed as a probe of the nuclear structure.

Note added in proof: Hubler, Kugel, and Murnick[23] have recently recomputed a number of $g(2)$ values derived from perturbed-angular-correlation measurements employing implantation into ferromagnetic hosts following Coulomb excitation (IMPAC). Although their results suggest substantial changes in a number of previously published g factors (some of which are represented in Table III and Fig. 5), the qualitative conclusions of the accompanying discussion regarding Greiner's model are unchanged. In the quantitative comparison of Table III, the revised g factors yield slightly better agreement between theory and experiment for the mixing ratios. More recent IMPAC results on the even-even Xe isotopes by Norlin *et al.*[24] suggest $g(2) < Z/A$, in apparent contradiction to the expectation based on the mixing ratios. A recent measurement[25] of $\delta(2' \rightarrow 2)$ in ^{120}Sn of -1.43 ± 0.25 complements the data of Table I.

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