

## Coulomb excitation of vibrational-like states in the even- $A$ actinide nuclei

F. K. McGowan, C. E. Bemis, Jr., W. T. Milner, J. L. C. Ford, Jr., R. L. Robinson, and P. H. Stelson  
*Oak Ridge National Laboratory,\* Oak Ridge, Tennessee 37830*

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Coulomb excitation of vibrational-like states in the even- $A$  actinide nuclei ( $230 \leq A \leq 248$ ) was measured using  $^4\text{He}$  ions in order to test nuclear models describing these states. In particular, the one-phonon octupole vibrational interpretation of the low-lying negative parity states provides an interesting theoretical framework with which to compare the experimental information. The excitation probabilities were determined relative to the elastic scattering by the observation of elastically and inelastically scattered  $^4\text{He}$  ions using a split-pole magnetic spectrometer equipped with a position-sensitive proportional detector. The values of  $B(E\lambda, 0_g \rightarrow J = \lambda)$  range from 0.5 to 4 single-particle units for  $\lambda = 2$  and from 10 to 30 single-particle units for  $\lambda = 3$ . For those cases, where the  $K, J^\pi$  assignments are known, the agreement between the experimental results and the microscopic calculations by Neegard and Vogel of the  $B(E3, 0 \rightarrow 3)$  for the  $3^-$  members of the one-phonon octupole quadruplet is good when the Coriolis coupling between the states with  $K$  and  $K \pm 1$  is taken into account. The magnitudes of the reduced  $E0$  nuclear matrix elements,  $\rho(E0, 2' \rightarrow 2_g)$ , extracted from the  $E0$  transition probabilities,  $T(E0, 2' \rightarrow 2_g)$ , for decay of the  $\beta$ -vibrational-like  $2^+$  state are  $0.37 \pm 0.06$  and  $0.43 \pm 0.06$  for  $^{232}\text{Th}$  and  $^{238}\text{U}$ , respectively. Several  $2^+$  states observed in this Coulomb excitation reaction survey are presumed to be  $2^+$  members of rotational bands based on  $0^+$  excited states which are strongly populated in the  $(p, t)$  reaction.

[ NUCLEAR REACTIONS  $^{230, 232}\text{Th}(\alpha, \alpha')$ ,  $^{236, 238}\text{U}(\alpha, \alpha')$ ,  $E=16$  and  $17$  MeV,  
 $^{234}\text{U}(\alpha, \alpha')$ ,  $E=16$  to  $18$  MeV,  $^{238, 240, 242, 244}\text{Pu}(\alpha, \alpha')$ ,  $^{244, 246, 248}\text{Cm}(\alpha, \alpha')$ ,  $E=17$   
 MeV; measured  $\sigma(E_{\alpha'}; \theta=150)$ ; deduced  $B(E2)$ ,  $B(E3)$ . Enriched targets. ]

### I. INTRODUCTION

In an earlier communication,<sup>1</sup> we presented information on  $E2$  and  $E4$  transition moments deduced from Coulomb excitation of the ground-state rotational bands in the even- $A$  actinide nuclei ( $230 \leq A \leq 248$ ) with  $^4\text{He}$  ions. Model-dependent deformation parameters,  $\beta_{20}$  and  $\beta_{40}$ , were extracted from the measured  $E2$  and  $E4$  transition moments for distributions of nuclear charge represented by deformed Fermi distributions and by a deformed homogeneous distribution. States at higher excitation energies than the  $2^+$  and  $4^+$  states in the ground-state rotational band were also observed in the spectra of the  $^4\text{He}$  ions elastically and inelastically scattered from these actinide nuclei. The Coulomb excitation reaction induced by light ions selectively excites  $2^+$  and  $3^-$  states by direct  $E2$  and  $E3$  Coulomb excitation. In many cases where information from nuclear spectroscopy existed, these higher-lying states could be identified with states of known spin and parity. This communication summarizes the  $B(E\lambda, 0_g \rightarrow J = \lambda)$  values deduced from the Coulomb excitation probabilities.

Of particular interest in the even- $A$  actinide nuclei, is the occurrence of negative parity states at quite low excitation energies in contrast to nuclei in the rare-earth deformed region. For the Ra and lighter Th nuclei for example, the  $1^-$  states occur in the energy range 200–400 keV above the ground

state. The negative parity states in the actinide region could possibly be interpreted as rotational states for the reflection asymmetric octupole deformation or the related case of octupole shape isomeric minima or, alternatively, as octupole vibrational states built upon a stable quadrupole deformation. The one-phonon octupole vibrational spectrum contains four states with  $K=0, 1, 2,$  and  $3$  and their associated rotational spectra. The theoretical calculations by Vogel<sup>2</sup> which minimize the total potential energy with respect to deformations indicate a stable equilibrium shape with  $\beta_{30}=0$ . This vibrational interpretation provides an interesting theoretical framework with which to compare the experimental information. The microscopic calculations for the  $B(E3, 0 \rightarrow 3)$  values for the  $3^-$  members of the one-phonon octupole quadruplet by Neegard and Vogel,<sup>3</sup> which also included the influence of the Coriolis coupling between states with  $K$  and  $K \pm 1$ , are in good agreement with the experimental data for nuclei in the rare-earth region. The inclusion of the Coriolis interaction in the microscopic calculations is required to explain satisfactorily the distribution of the  $B(E3)$  strength<sup>4</sup> among the one-phonon octupole vibrational states. On the other hand, for nuclei in the actinide region, there are very few experimental  $B(E3, 0_g \rightarrow 3)$  values to compare with the microscopic calculations.<sup>5</sup> For the cases where  $K, J^\pi$  are known from other nuclear spectroscopic infor-

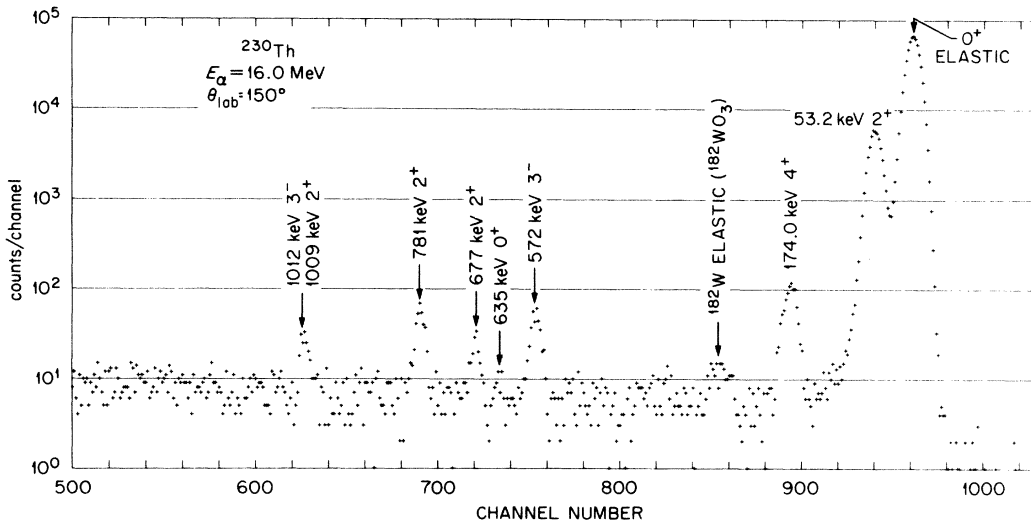


FIG. 1. Elastically and inelastically scattered 16-MeV  ${}^4\text{He}$  ions from  ${}^{230}\text{Th}$  at a lab angle of  $150^\circ$  using a carbon foil as the target backing. The  ${}^{182}\text{W}$  contamination arises from the  ${}^{182}\text{W}$   $0_3^+$  molecular ion which was collected at the  ${}^{230}\text{Th}$  position in the isotope separator.

mation, our experimental  $B(E3, 0_g \rightarrow 3)$  values are compared with theoretical predictions.

Our  $B(E2, 0_g \rightarrow 2')$  values for excitation of the  $\beta$ -vibrational-like states in  ${}^{232}\text{Th}$  and  ${}^{238}\text{U}$ , taken together with results from an analysis of  $\gamma$ -ray spectroscopy of the Coulomb excitation reaction,<sup>6,7</sup> provide information on the  $E0$  transitions  $2' \rightarrow 2_g$ . The absolute values of the reduced  $E0$  nuclear matrix elements,  $\rho(E0; 2' \rightarrow 2_g)$ , extracted from the  $E0$  transition probabilities,  $T(E0; 2' \rightarrow 2_g)$ , are compared with theoretical predictions.

## II. EXPERIMENTAL METHOD AND RESULTS

The experiments were performed using  ${}^4\text{He}$  ions accelerated in the EN tandem Van de Graaff at Oak Ridge National Laboratory. The elastically and inelastically scattered  ${}^4\text{He}$  ions were detected at the focal plane of an Enge split-pole spectrometer by a 20-cm-long position-sensitive gas proportional detector. Details of the targets and calibration of the Enge spectrometer and detector system have been discussed in an earlier communication.<sup>1</sup>

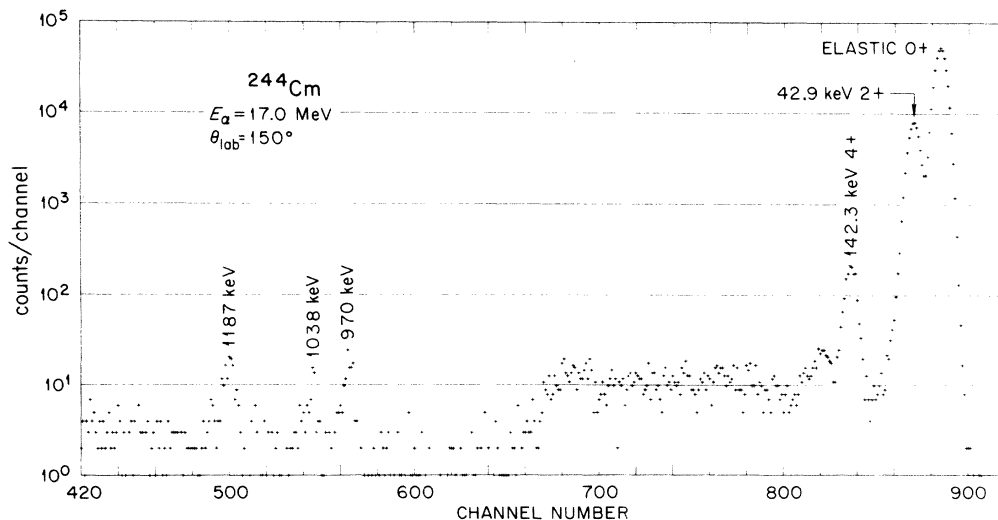


FIG. 2. Elastically and inelastically scattered 17-MeV  ${}^4\text{He}$  ions from  ${}^{244}\text{Cm}$  at a lab angle of  $150^\circ$ . The background below the  ${}^{244}\text{Cm}$   $4^+$  peak arises from  $\sim 40$  ppm of Pb and other heavy-element impurities uniformly distributed in the Ni target backing.

Figures 1 and 2 show typical spectra of  $^4\text{He}$  ions scattered from  $^{230}\text{Th}$  and  $^{244}\text{Cm}$  at a lab angle of  $150^\circ$ . In addition to Coulomb excitation of the  $2^+$  and  $4^+$  states in the ground-state rotational band, states at higher excitation energies are observed in  $^{230}\text{Th}$ . The  $K, J^\pi$  assignments for these states are known from the decay of  $^{230}\text{Pa}$  to levels in  $^{230}\text{Th}$ ,  $^{8-10}$  viz. (572 keV  $0, 3^-$ ), (635 keV  $0, 0^+$ ), (677 keV  $0, 2^+$ ), (781 keV  $2, 2^+$ ), (1009 keV  $?, 2^+$ ), and (1012 keV  $1, 3^-$ ). The  $3^-$  and  $2^+$  states are primarily excited by direct  $E3$  and  $E2$  Coulomb excitation and the  $0^+$  state at 635 keV is excited by

double  $E2$  excitation. In the case of  $^{244}\text{Cm}$ , information from nuclear spectroscopic studies<sup>11</sup> is insufficient to make any spin and parity assignments to the states observed at 970, 1038, and 1187 keV.

The experimental excitation probabilities for the states at higher excitation energies, determined relative to the elastic scattering peak, are tabulated in Table I. Calculations of the Coulomb excitation cross sections, used to extract the transition matrix elements from the experimental data, were performed using the semiclassical  $E2$  coupled-channels code of Winther and de Boer<sup>12</sup> which has

TABLE I. Experimental excitation probabilities for the states of higher excitation energies with  $^4\text{He}$  ions at a lab scattering angle of  $150^\circ$ .

Target	$E_\alpha$ (MeV)	Level (keV)	$10^4[\sigma(J)/\sigma(0^+)]$	Target	$E_\alpha$ (MeV)	Level (keV)	$10^4[\sigma(J)/\sigma(0^+)]$			
$^{230}\text{Th}$	16.00	572	$5.57 \pm 0.39$	$^{236}\text{U}$	16.00	688	$0.28 \pm 0.15$			
		635	$0.35 \pm 0.17$			745	$3.00 \pm 0.36$			
		677	$2.25 \pm 0.29$			848	$0.67 \pm 0.16$			
		781	$5.59 \pm 0.39$			959	$5.64 \pm 0.42$			
		1009 } 1012 }	$2.42 \pm 0.29$			1040	$1.27 \pm 0.32$			
	17.06	572	$6.58 \pm 0.56$		1150	$0.54 \pm 0.19$				
		677	$3.94 \pm 0.52$		745	$4.69 \pm 0.61$				
		781	$8.34 \pm 0.76$		959	$7.20 \pm 0.52$				
		$^{232}\text{Th}$	16.00		714 } 730 }	$0.52 \pm 0.13$	$^{238}\text{U}$	16.00	680	$0.57 \pm 0.22$
					774 } 785 }	$12.58 \pm 0.57$			732	$3.55 \pm 0.32$
873 } 884 }	$0.78 \pm 0.26$			827	$0.37 \pm 0.23$					
1106	$1.11 \pm 0.28$			966	$0.58 \pm 0.19$					
998	$1.16 \pm 0.22$									
17.00	714 } 730 }		$0.98 \pm 0.19$	1037 } 1060 }	$4.24 \pm 0.37$					
	774 } 785 }		$19.3 \pm 0.7$	1169	$1.07 \pm 0.25$					
	873 } 884 }		$0.78 \pm 0.17$	1224	$0.36 \pm 0.22$					
	1106		$1.73 \pm 0.29$	680	$0.68 \pm 0.24$					
	732		$5.32 \pm 0.53$							
$^{234}\text{U}$	16.00	849.6 } 851.6 }	$3.34 \pm 0.40$	827	$0.76 \pm 0.24$					
		927	$3.82 \pm 0.46$	998	$1.28 \pm 0.29$					
		1023	$0.76 \pm 0.29$	1037 } 1061 }	$6.76 \pm 0.61$					
		17.00	849.6 } 851.6 }	$4.10 \pm 0.74$	1169	$1.16 \pm 0.30$				
			927	$5.20 \pm 0.76$	1169	$6.33 \pm 0.82$				
	1023		$1.60 \pm 0.59$	983	$5.93 \pm 0.71$					
	1312		$1.00 \pm 0.46$	649	$3.68 \pm 0.51$					
	17.50		849.6 } 851.6 }	$6.25 \pm 0.63$	938	$3.23 \pm 0.58$				
		927	$6.77 \pm 0.68$	833 $\pm 5$	$2.99 \pm 0.39$					
		1023	$1.75 \pm 0.42$	1020 $\pm 6$	$2.53 \pm 0.33$					
		1312	$1.09 \pm 0.33$	1102 $\pm 4$	$4.36 \pm 0.44$					
		18.00	849.6 } 851.6 }	$5.35 \pm 0.59$	708 $\pm 4$	$2.52 \pm 0.83$				
	927		$7.96 \pm 0.64$	960 $\pm 4$	$2.28 \pm 0.39$					
	1023		$1.26 \pm 0.36$	1020 $\pm 4$	$6.52 \pm 0.55$					
	1312		$1.26 \pm 0.34$	1111 $\pm 4$	$2.98 \pm 0.42$					
$^{244}\text{Cm}$	17.00		970 $\pm 4$	$2.72 \pm 0.33$	970 $\pm 4$	$2.72 \pm 0.33$				
		1038 $\pm 6$	$1.53 \pm 0.29$	1038 $\pm 6$	$1.53 \pm 0.29$					
		1187 $\pm 4$	$3.56 \pm 0.37$	1187 $\pm 4$	$3.56 \pm 0.37$					
		1124 } 1128 }	$5.40 \pm 1.08$	1124 } 1128 }	$5.40 \pm 1.08$					
		1050 $\pm 7$	$5.01 \pm 0.50$	1050 $\pm 7$	$5.01 \pm 0.50$					
	$^{246}\text{Cm}$	17.00	1100 $\pm 7$	$1.83 \pm 0.42$	1100 $\pm 7$	$1.83 \pm 0.42$				
			1100 $\pm 7$	$1.83 \pm 0.42$	1100 $\pm 7$	$1.83 \pm 0.42$				
			1100 $\pm 7$	$1.83 \pm 0.42$	1100 $\pm 7$	$1.83 \pm 0.42$				
			1100 $\pm 7$	$1.83 \pm 0.42$	1100 $\pm 7$	$1.83 \pm 0.42$				
			1100 $\pm 7$	$1.83 \pm 0.42$	1100 $\pm 7$	$1.83 \pm 0.42$				

TABLE II. Experimental results for  $B(E\lambda, 0 \rightarrow J = \lambda)$ . For those cases where the  $K, J^\pi$  assignments of the states are not known from other nuclear spectroscopy studies, the  $B(E\lambda, 0 \rightarrow J = \lambda)$  are given for both assignments  $J = 2^+$  and  $J = 3^-$ .

Nucleus	Level (keV)	$K, J^\pi$	$E\lambda$	$B(E\lambda, 0 \rightarrow J)$ ( $e^2 b^\lambda$ )	$B(E\lambda)/B(E\lambda)_{s.p.}^a$	
$^{230}\text{Th}$	572	$0, 3^-$	$E3$	$0.64 \pm 0.06$	$29 \pm 3$	
	677	$0, 2^+$	$E2$	$0.046 \pm 0.006$	$1.10 \pm 0.14$	
	781	$2, 2^+$	$E2$	$0.123 \pm 0.013$	$2.9 \pm 0.3$	
	1009 or 1012	$? , 2^+$ $1, 3^-$	$E2$ $E3$	$0.084 \pm 0.013$ $0.50 \pm 0.07$	$2.0 \pm 0.3$ $23 \pm 3$	
$^{232}\text{Th}$	774	$0, 2^+$	$E2$	$0.10 \pm 0.04$	$2.4 \pm 0.9$	
	774	$0, 3^-$	$E3$	$0.45 \pm 0.05$	$20 \pm 2$	
	785	$2, 2^+$	$E2$	$0.122 \pm 0.008$	$2.9 \pm 0.2$	
	1106	$1, 3^-$	$E3$	$0.26 \pm 0.05$	$11.5 \pm 2.3$	
$^{234}\text{U}$	849.6 or 851.6	$0, 3^-$ $0, 2^+$	$E3$ $E2$	$0.59 \pm 0.07$ $0.098 \pm 0.013$	$26 \pm 3$ $2.3 \pm 0.3$	
	926.9	$2, 2^+$	$E2$	$0.123 \pm 0.013$	$2.9 \pm 0.3$	
	1023	$2, 3^-$	$E3$	$0.22 \pm 0.05$	$9.5 \pm 2.3$	
	1312	$? , 3^-$	$E3$	$0.22 \pm 0.07$	$9.7 \pm 2.9$	
	$^{236}\text{U}$	745	$0, 3^-$	$E3$	$0.53 \pm 0.07$	$23 \pm 3$
959		$2, 2^+$	$E2$	$0.18 \pm 0.02$	$4.2 \pm 0.4$	
1040		$? , 3^-$	$E3$	$0.31 \pm 0.08$	$13.5 \pm 3.4$	
1150		$? , 3^-$	$E3$	$0.16 \pm 0.06$	$7 \pm 2.5$	
$^{238}\text{U}$	732	$0, 3^-$	$E3$	$0.64 \pm 0.06$	$27 \pm 2.5$	
	966	$0?, 2^+$	$E2$	$0.017 \pm 0.007$	$0.4 \pm 0.1$	
	998	$? , 3^-$	$E3$	$0.24 \pm 0.05$	$10 \pm 2$	
	1037	$0, 2^+$	$E2$	$0.063 \pm 0.009$	$1.4 \pm 0.2$	
	1060	$2, 2^+$	$E2$	$0.127 \pm 0.009$	$2.9 \pm 0.2$	
	1169	$? , 3^-$	$E3$	$0.28 \pm 0.07$	$12 \pm 3$	
	1224	$? , 2^+$	$E2$	$0.022 \pm 0.013$	$0.5 \pm 0.3$	
$^{238}\text{Pu}$	661	$0, 3^-$	$E3$	$0.71 \pm 0.12$	$30 \pm 5$	
	983	$0, 2^+$	$E2$	$0.166 \pm 0.022$	$3.8 \pm 0.5$	
$^{240}\text{Pu}$	649	$0, 3^-$	$E3$	$0.41 \pm 0.06$	$17 \pm 2.5$	
	938	$2, 2^+$	$E2$	$0.079 \pm 0.018$	$1.8 \pm 0.4$	
$^{242}\text{Pu}$	833	$0, 3^-$	$E3$	$0.42 \pm 0.07$	$17 \pm 3$	
	1020	$? , 3^-$	$E3$	$0.45 \pm 0.07$	$19 \pm 3$	
	1102	$(2, 2^+)$	$E2$	$0.157 \pm 0.018$	$3.5 \pm 0.4$	
$^{244}\text{Pu}$	708	$? , 3^-$ or $? , 2^+$	$E3$ $E2$	$0.30 \pm 0.10$ $0.045 \pm 0.013$	$12 \pm 4$ $1.0 \pm 0.3$	
	960	$(2, 3^-)$ or $? , 2^+$	$E3$ $E2$	$0.37 \pm 0.07$ $0.059 \pm 0.013$	$15 \pm 3$ $1.3 \pm 0.3$	
	1020	$(0 \text{ or } 1, 3^-)$ or $? , 2^+$	$E3$ $E2$	$1.16 \pm 0.12$ $0.195 \pm 0.018$	$47 \pm 5$ $4.3 \pm 0.4$	
	1111	$(0 \text{ or } 1, 3^-)$ or $? , 2^+$	$E3$ $E2$	$0.59 \pm 0.10$ $0.104 \pm 0.018$	$24 \pm 4$ $2.3 \pm 0.4$	
	$^{244}\text{Cm}$	970	$? , 3^-$ or $? , 2^+$	$E3$ $E2$	$0.52 \pm 0.07$ $0.082 \pm 0.014$	$21 \pm 3$ $1.8 \pm 0.2$
		1038	$? , 3^-$ or $? , 2^+$	$E3$ $E2$	$0.32 \pm 0.07$ $0.054 \pm 0.014$	$13 \pm 3$ $1.2 \pm 0.3$
		1187	$? , 3^-$ or $? , 2^+$	$E3$ $E2$	$0.96 \pm 0.12$ $0.168 \pm 0.023$	$39 \pm 5$ $3.7 \pm 0.5$
$^{246}\text{Cm}$		1124 or 1128	$2, 2^+$ $1, 3^-$	$E2$ $E3$	$0.224 \pm 0.046$ $1.31 \pm 0.03$	$4.9 \pm 1.0$ $52 \pm 11$
	$^{248}\text{Cm}$	1050	$? , 3^-$ or $? , 2^+$	$E3$ $E2$	$1.07 \pm 0.13$ $0.180 \pm 0.023$	$42 \pm 5$ $3.9 \pm 0.5$
1100		$? , 3^-$ or $? , 2^+$	$E3$ $E2$	$0.41 \pm 0.10$ $0.069 \pm 0.019$	$16 \pm 4$ $1.5 \pm 0.4$	

$${}^a B(E\lambda)_{s.p.} = \frac{2\lambda+1}{4\pi} \left(\frac{3}{\lambda+3}\right)^2 (0.12A^{1/3})^{2\lambda} e^2 b^\lambda \text{ for } J_i=0, J_f=\lambda.$$

been expanded to include  $E1$ ,  $E3$ , and  $E4$  excitations. The  $0^+$ ,  $2^+$ ,  $4^+$ , and  $6^+$  members of the ground-state rotational band were included in the calculations. The intraband matrix elements included all possible reduced  $E2$  and  $E4$  matrix elements in the rigid-rotor limit which connect these states. The interband  $E\lambda$  matrix elements, which connect the higher excited states with states in the ground-state rotational band, were chosen according to the Bohr-Mottelson collective model with the intrinsic transition matrix elements taken from the experimental  $B(E\lambda, 0_g \rightarrow J=\lambda)$  values. However, for  $^{232}\text{Th}$  and  $^{238}\text{U}$  the magnitude of most of the interband  $E\lambda$  matrix elements were available from the analyses of  $\gamma$ -ray spectroscopy<sup>6,7</sup> of the Coulomb excitation reaction and these were used in the calculations. The experimental results for the reduced transition probabilities,  $B(E\lambda, 0_g \rightarrow J=\lambda)$ , are summarized in Table II. For those cases where the  $K, J^\pi$  assignments of the states are not known from other nuclear spectroscopic studies, the  $B(E\lambda, 0_g \rightarrow J=\lambda)$  values are given for both assignments,  $J=2^+$  and  $J=3^-$ . The excitation of states with these spin assignments results primarily from direct  $E2$  and  $E3$  Coulomb excitation. The dependence of the calculated excitation probabilities on the choice of  $K$  assignment for these states is within the accuracy (10 to 15%) of the measured excitation probabilities.

### III. DISCUSSION

#### A. $^{230}\text{Th}$

The decay of  $^{230}\text{Pa}$  to levels in  $^{230}\text{Th}$  has been investigated by  $\gamma$ -ray and internal-conversion electron spectroscopy.<sup>9,10,13</sup> A striking feature of the level scheme for  $^{230}\text{Th}$  is occurrence of vibration-like spectra with associated rotational spectra, viz. rotational bands based on the  $\beta$ - and  $\gamma$ -vibrational states at 635 and 781 keV and the  $K=0$  and 1 members of the one-phonon octupole quadruplet based on the  $J=1^-$  states at 508 and 952 keV. The  $K, J^\pi$  assignments were deduced primarily from measured internal-conversion coefficients and ratios of reduced transition probabilities. The deviations of the  $B(E1)$  branching ratios from the Alaga rules<sup>14</sup> for transitions from members of the octupole quadruplet to the ground-state band do, however, imply a Coriolis interaction between states with  $K$  and  $K\pm 1$ . The  $B(E\lambda, 0_g \rightarrow J=\lambda)$  deduced from our measurements for  $^{230}\text{Th}$  are summarized in Table II. The weak excitation of the  $0'^+$  state at 635 keV of the  $\beta$ -vibrational band is consistent with the  $B(E2, 0_g \rightarrow 2')$  value for excitation of the 677 keV state in Table II, if we assume that the intrinsic  $E2$  moments of the ground-state and  $\beta$ -vibrational bands are equal and that the interband reduced

transition probability  $B(E2, 2_g \rightarrow 0') = B(E2, 2' \rightarrow 0_g)$ . Here  $0'$  and  $2'$  designate states in the  $\beta$ -vibrational band. The resulting  $B(E2, 2_g \rightarrow 0')$  value deduced from the measured excitation probability of the  $0'$  state is  $1.9 \pm 1.0$  single-particle units,  $B(E2)_{\text{s.p.}}$ . The  $B(E2, 0_g \rightarrow 2')$  from excitation of the  $2'^+$  state at 677 keV is  $(1.10 \pm 0.14)B(E2)_{\text{s.p.}}$ . This  $0'$  state at 635 keV was observed to be strongly populated in the  $^{232}\text{Th}(p, t)^{230}\text{Th}$  reaction by Maher *et al.*<sup>15</sup> Since the states at 1009 and 1012 keV are both likely to be Coulomb excited to some extent and are unresolved in our measurements, the  $B(E\lambda, 0_g \rightarrow J=\lambda)$  values given in Table II for these states should be considered as upper limits.

#### B. $^{232}\text{Th}$

The spectrum of  $^4\text{He}$  ions scattered from  $^{232}\text{Th}$  contained several composite peaks (see Fig. 2 in Ref. 1), namely a composite peak from excitation of the  $1^-$  state at 714 keV and the  $0'^+$  state at 730 keV, a composite peak from excitation of the  $3^-$  state at 774 keV, the  $2'^+$  state at 774 keV, and the  $2''^+$  state at 785 keV, and a composite peak from excitation of the  $4'^+$  state at 873 keV and the  $5^-$  state at 884 keV. By making use of the results<sup>6</sup> from the analysis of  $\gamma$ -ray spectroscopy of the Coulomb excitation reaction, these composite peaks have been analyzed to obtain the reduced transition probabilities from the excitation of the  $0'$ ,  $2'$ , and  $4'$  states of the  $K=0$   $\beta$ -vibrational band. The excitation yields of the  $1^-$  and  $5^-$  states of the  $K=0$  octupole band from  $\gamma$ -ray spectroscopy are consistent with the excitation processes  $E3E2$  via the  $3^-$  state and  $E2E3$  via the  $2^+$  state. Assuming equality of the intrinsic quadrupole moments of the ground-state band and the  $\beta$ -vibrational band, we obtain from the excitation yield of the  $0'$  state for  $E_\alpha=16$  and 17 MeV  $B(E2, 2_g \rightarrow 0')$  values of  $(1.6 \pm 0.7)B(E2)_{\text{s.p.}}$  and  $(1.9 \pm 0.7)B(E2)_{\text{s.p.}}$ , respectively. For this deduction we have made the additional assumption that  $B(E2, 2_g \rightarrow 0') = B(E2, 2' \rightarrow 0_g)$ . This equality seems to be reasonable. For instance, the  $2'$  state is primarily excited by direct  $E2$  Coulomb excitation. From the excitation yield of the  $2'$  state for  $E_\alpha=16$  and 17 MeV we obtain  $B(E2, 0_g \rightarrow 2')$  values of  $(2.7 \pm 0.5)B(E2)_{\text{s.p.}}$  and  $(3.5 \pm 0.4)B(E2)_{\text{s.p.}}$ , respectively. Finally, the  $B(E2, 0_g \rightarrow 2')$  value from the excitation yield of the  $4'$  state for  $E_\alpha=17$  MeV is  $(2.4 \pm 0.9)B(E2)_{\text{s.p.}}$ . Combining these various results our final value for  $B(E2, 0_g \rightarrow 2')$  is  $(2.4 \pm 0.4)B(E2)_{\text{s.p.}}$ . The  $B(E2, 0_g \rightarrow 2')$  for excitation of the  $2'^+$  state from  $\gamma$ -ray spectroscopy data is only  $(0.83 \pm 0.07)B(E2)_{\text{s.p.}}$ , i.e., excitations which decay by  $\gamma$  rays and internal conversion electrons (excluding  $E0$  decay). This implies that an appreci-

able fraction of the excitations decay by  $E0$  radiation via the  $2' \rightarrow 2_g$  transition. The ratio of the  $E0$  transition probability to the  $E2$   $\gamma$  transition probability,  $T(E0, 2' \rightarrow 2_g)/T(E2, 2' \rightarrow 2_g)$ , is  $16.7 \pm 5.5$ . This ratio is frequently referred to as the "total  $E0$  internal-conversion coefficient." With a knowledge of  $T(E2, 2' \rightarrow 2_g)$  which is available from the  $\gamma$ -ray spectroscopy data,<sup>6</sup> we have derived  $T(E0, 2' \rightarrow 2_g) = (2.34 \pm 0.77) \times 10^{11} \text{ sec}^{-1}$  and  $|\rho(E0, 2' \rightarrow 2_g)| = 0.37 \pm 0.06$ . The tables of electronic factors of the  $E0$  conversion probability by Bell *et al.*<sup>16</sup> have been used in the extraction of the reduced  $E0$  nuclear matrix element from the experimental value of  $T(E0, 2' \rightarrow 2_g)$ . This experimental result for  $|\rho(E0, 2' \rightarrow 2_g)|$  compares favorably with calculations<sup>17</sup> within the framework of the phenomenological models of Bohr-Mottelson and Davydov, viz.  $\rho(E0, 2' \rightarrow 2_g) = 0.46$  for  $^{232}\text{Th}$ .

#### C. $^{234}\text{U}$

The  $K, J^\pi$  assignments in Table II for the states in  $^{234}\text{U}$  are based on nuclear spectroscopy results<sup>18</sup> from the decay of  $^{234}\text{Pa}$  and  $^{234}\text{Np}$  and on recent data<sup>19</sup> from the  $^{234}\text{U}(d, d')$  reaction. The  $3^-$  state at 849 keV and the  $2^+$  state at 851.6 keV are both likely to be Coulomb excited and as a result the  $B(E\lambda, 0_g \rightarrow J=\lambda)$  values given in Table II must be considered as upper limits as these states were unresolved in our experiment. The  $2^+$  states at 851.6 and 926.9 keV have also been observed in the  $^{236}\text{U}(p, t)^{234}\text{U}$  reaction by Maher *et al.*<sup>15</sup>

#### D. $^{236}\text{U}$

The spin and parity assignments in Table II for the states in  $^{236}\text{U}$  are based primarily on the results from the  $^{236}\text{U}(d, d')$  reaction by Boyno *et al.*<sup>19</sup> They also suggest that  $K, J^\pi = 0, 1^-$  and  $0, 5^-$  for the 688- and 848-keV states. Our Coulomb excitation yields for the 688- and 848-keV states are consistent with these assignments, i.e., consistent with the excitation processes  $E3E2$  via the  $3^-$  state and  $E2E3$  via the  $2^+$  state. The interband  $E3$  matrix elements were chosen according to the Bohr-Mottelson collective model with the intrinsic transition matrix elements taken from the  $B(E3, 0_g \rightarrow 3)$  value for the  $3^-$  state at 745 keV in Table II. The use of reduced  $E3$  matrix elements based on the collective model seems justified because the  $B(E3, 0_g \rightarrow 3)$  values in Table II are large, 10 to  $30 B(E3)_{s.p.}$  Lederer, Jaklevic, and Prussin<sup>20</sup> have, however, assigned  $K, J^\pi = 2, 2^-$  to the 688-keV state from internal-conversion electron and  $\gamma$ -ray spectroscopy of the  $^{236}\text{Np} \rightarrow ^{236}\text{U}$  decay. Analyses of the Coulomb excitation probabilities, assuming  $K=2$  and  $J=2^-, 3^-,$  and  $4^-$  for the 688-, 745-, and 848-keV states in accord with Lederer *et al.*, require intrinsic

interband  $E3$  matrix elements corresponding to  $B(E3, 0_g \rightarrow 3) = 6.6 \times 10^2 B(E3)_{s.p.}$  and  $2.8 \times 10^4 B(E3)_{s.p.}$  for an assignment of  $J=2^-$  and  $4^-$  to the 688- and 848-keV states, respectively. These results are clearly inconsistent with a  $B(E3, 0_g \rightarrow 3) = 20 B(E3)_{s.p.}$  deduced from the excitation yield for the 745-keV state assuming  $K=2$ . This is a reflection of the selection rule that unnatural parity states are very weakly excited in even- $A$  nuclei at backward scattering angles. Our results, therefore, support  $J^\pi$  assignments of  $1^-, 3^-$ , and  $5^-$  with  $K=0$  for the 688-, 745-, and 848-keV states, respectively. The possible existence of a state with  $K, J^\pi = 2, 2^-$  at 688 keV is not ruled out by our studies but seems unlikely as its associated rotational states should have been observed in our experiments. The  $2^+$  state at 959 keV and the  $3^-$  state at 1040 keV have also been observed in the  $^{238}\text{U}(p, t)^{236}\text{U}$  and  $^{235}\text{U}(d, p)^{236}\text{U}$  reactions,<sup>15, 21</sup> respectively.

#### E. $^{238}\text{U}$

The  $K, J^\pi$  assignments in Table II for the states in  $^{238}\text{U}$  are based on nuclear spectroscopy results from the decay<sup>22</sup> of  $^{238}\text{Pa} \rightarrow ^{238}\text{U}$ , from the  $^{238}\text{U}(n, n'\gamma)$  reaction,<sup>23</sup> from  $\gamma$ -ray spectroscopy of the Coulomb excitation reaction,<sup>7</sup> and from the  $^{238}\text{U}(d, d')$  reaction.<sup>24</sup> In addition to the  $B(E\lambda, 0_g \rightarrow J=\lambda)$  values for excitation of states with  $J=\lambda$  in Table II, the  $K, J^\pi = 0, 1^-$  and  $0, 5^-$  states at 680 and 827 keV were excited by  $E3E2$  via the  $3^-$  state and  $E2E3$  via the  $2^+$  state. The interband  $E3$  matrix elements, corresponding to  $B(E3, 0_g \rightarrow 3) = (33 \pm 12) B(E3)_{s.p.}$  as deduced from the excitation probabilities for the  $J=1^-$  and  $5^-$  states, are consistent with the value  $B(E3, 0_g \rightarrow 3) = (27 \pm 2.5) B(E3)_{s.p.}$  extracted from the excitation probability for the  $K, J^\pi = 0, 3^-$  state at 732 keV. The composite peak from excitation of the  $K, J^\pi = 0, 2'^+$  and  $2, 2''^+$  states at 1037 and 1060 keV, respectively (see Table I), was analyzed using the result<sup>7</sup> from  $\gamma$ -ray spectroscopy that the value of  $B(E2, 0_g \rightarrow 2'')$  is  $(2.85 \pm 0.15) B(E2)_{s.p.}$ . The value of the  $B(E2, 0_g \rightarrow 2')$  for excitation of the  $K, J^\pi = 0, 2'^+$  is  $(1.45 \pm 0.20) B(E2)_{s.p.}$ . From the  $\gamma$ -ray spectroscopy data,<sup>7</sup> the  $B(E2, 0_g \rightarrow 2')$  value for excitations which decay by  $\gamma$  rays and internal-conversion electrons is  $(0.69 \pm 0.04) B(E2)_{s.p.}$ . Again, as was the case for  $^{232}\text{Th}$ , this result implies that an appreciable fraction (52%) of the excitations decay by  $E0$  radiation via the  $2' \rightarrow 2_g$  transition. The transition probability ratio,  $T(E0, 2' \rightarrow 2)/T(E2, 2' \rightarrow 2_g)$  is  $4.39 \pm 1.14$ . Combining this result with the  $T(E2, 2' \rightarrow 2_g)$  value from  $\gamma$ -ray spectroscopy data,<sup>7</sup> we have  $T(E0, 2' \rightarrow 2_g) = (5.62 \pm 1.46) \times 10^{11} \text{ sec}^{-1}$  and  $|\rho(E0, 2' \rightarrow 2_g)| = 0.43 \pm 0.06$ . This experimental result for  $|\rho(E0, 2' \rightarrow 2_g)|$  compares favorably with

the value  $\rho(E0, 2' \rightarrow 2_g) = 0.49$  for  $^{238}\text{U}$  from phenomenological model calculations.<sup>17</sup>

F.  $^{238, 240, 242, 244}\text{Pu}$

The assignments of  $K, J^\pi$  in Table II for the states in  $^{238}\text{Pu}$ ,  $^{240}\text{Pu}$ , and  $^{242}\text{Pu}$  are based on nuclear spectroscopy results<sup>25-28</sup> from the decay of  $^{238}\text{Np}$ ,  $^{238}\text{Am}$ , and  $^{240}\text{Np}$  and from the  $^{242}\text{Pu}(d, d')$  reaction.<sup>24</sup> Although  $E0$  radiation via the  $2' \rightarrow 2_g$  transition from the  $K, J^\pi = 0, 2'^+$  state at 983 keV has been observed by Ahmad *et al.*<sup>27</sup> in the  $^{238}\text{Am}$

$\rightarrow ^{238}\text{Pu}$  decay, it is not possible to deduce  $T(E0, 2' \rightarrow 2_g)$  from a knowledge of  $B(E2, 0_g \rightarrow 2')$  because the intensity of the  $2' \rightarrow 0_g$  transition relative to  $2' \rightarrow 2_g$  and  $2' \rightarrow 4_g$  transitions is not known. The difficulty in detecting the transition at 983.0 keV results from the presence of the strong  $2'' \rightarrow 2_g$  transition at 984.4 keV from the decay of the  $K, J^\pi = 2, 2''^+$  state at 1028.5 keV in  $^{238}\text{Pu}$ , e.g., see the relative  $\gamma$ -ray intensities from the  $^{238}\text{Np} \rightarrow ^{238}\text{Pu}$  by Winter *et al.*<sup>26</sup> We did not observe the excitation of the  $K, J^\pi = 2, 2''^+$  state at 1028.5 keV in  $^{238}\text{Pu}$ . Our upper limit for excitation of this

TABLE III. Comparison of measurements and microscopic calculations of  $B(E3, 0 \rightarrow 3)$  for the  $3^-$  members of the one-phonon octupole quadruplet. The label  $\alpha$  is the  $K$  quantum number corresponding to the largest component of the Coriolis coupled octupole state wave function.

Nucleus	$K, J^\pi$	Experiment		$\alpha, J^\pi$	Theory	
		$E(3^-)$ (keV)	$B(E3, 0 \rightarrow 3)$ ( $10^{-2}e^2b^3$ )		$E(3^-)$ (keV)	$B(E3, 0 \rightarrow 3)$ ( $10^{-2}e^2b^3$ )
$^{230}\text{Th}$	$0, 3^-$	572	$64 \pm 6$	$0, 3^-$	430	73
	$1, 3^-$	1012	$\leq 50$	$1, 3^-$	940	28
				$2, 3^-$	1120	4
				$3, 3^-$	1324	10
$^{232}\text{Th}$	$0, 3^-$	774	$45 \pm 5$	$0, 3^-$	546	59
	$1, 3^-$	1106	$26 \pm 5$	$1, 3^-$	945	26
				$2, 3^-$	1100	1.3
				$3, 3^-$	1323	9
$^{234}\text{U}$	$0, 3^-$	850	$\leq 59$	$0, 3^-$	801	36
	$2, 3^-$	1023	$22 \pm 5$	$1, 3^-$	1298	1.4
	$? , 3^-$	1312	$22 \pm 7$	$2, 3^-$	1041	19
				$3, 3^-$	1507	5
$^{236}\text{U}$	$0, 3^-$	745	$53 \pm 7$	$0, 3^-$	650	51
	$? , 3^-$	1040	$31 \pm 8$	$1, 3^-$	1078	0.06
	$? , 3^-$	1150	$16 \pm 6$	$2, 3^-$	905	29
				$3, 3^-$	1165	10
$^{238}\text{U}$	$0, 3^-$	732	$64 \pm 6$	$0, 3^-$	778	44
	$? , 3^-$	998	$24 \pm 5$	$1, 3^-$	1161	0.7
	$? , 3^-$	1169	$28 \pm 7$	$2, 3^-$	922	18
				$3, 3^-$	1236	11
$^{238}\text{Pu}$	$0, 3^-$	661	$71 \pm 12$	$0, 3^-$	912	48
				$1, 3^-$	1186	0.0
				$2, 3^-$	1031	9
				$3, 3^-$	1273	9
$^{240}\text{Pu}$	$0, 3^-$	649	$41 \pm 6$	$0, 3^-$	878	53
				$1, 3^-$	1183	0.0
				$2, 3^-$	939	4
				$3, 3^-$	1221	12
$^{242}\text{Pu}$	$(0, 3^-)$	833	$42 \pm 7$	$0, 3^-$	881	12
	$? , 3^-$	1020	$45 \pm 7$	$1, 3^-$	1137	4
				$2, 3^-$	791	41
				$3, 3^-$	1123	9
$^{244}\text{Pu}$	$(2, 3^-)$	960	$37 \pm 7$	$0, 3^-$	1204	11
	$(0 \text{ or } 1, 3^-)$	1020	$116 \pm 12$	$1, 3^-$	820	30
	$(0 \text{ or } 1, 3^-)$	1111	$59 \pm 10$	$2, 3^-$	925	4
				$3, 3^-$	1285	10
$^{246}\text{Cm}$	$1, 3^-$	1128	$\leq 131$	$0, 3^-$	1293	9
				$1, 3^-$	890	1.5
				$2, 3^-$	774	32
				$3, 3^-$	1271	12

state corresponds to the value  $B(E2, 0_g^- \rightarrow 2^-) \leq 0.8B(E2)_{s.p.}$  which is a factor of 3.6 times smaller than the corresponding  $B(E2, 0_g^- \rightarrow 2^-)$  values for the  $K, J^\pi = 2, 2^+$  states in  $^{230,232}\text{Th}$  and  $^{234,236,238}\text{U}$ . The  $K, J^\pi = 0, 2^+$  state at 983 keV has also been observed in the  $^{240}\text{Pu}(p, t)^{238}\text{Pu}$  reaction.<sup>29</sup>

There is insufficient spectroscopic information to make definite  $K$  assignments to the  $3^-$  and  $2^+$  states at 1020 and 1102 keV in  $^{242}\text{Pu}$  but the  $2^+$  state at 1102 keV probably corresponds to the 1107-keV level observed in the  $^{244}\text{Pu}(p, t)^{242}\text{Pu}$  reaction by Maher *et al.*<sup>15</sup> However, we did not see any appreciable Coulomb excitation of a  $2^+$  state at 995 keV which was observed in the  $(p, t)$  reaction.<sup>15</sup>

Apart from preliminary data<sup>30</sup> from the  $^{244}\text{Pu}(d, d')$  reaction, there is not any other spectroscopic information for the four states observed in  $^{244}\text{Pu}$ . The  $K$  assignments for the  $3^-$  states at 960, 1020, and 1111 keV in Table II are based on intensity patterns and cross-section ratios from the  $(d, d')$  reaction.<sup>30</sup> For completeness we give also the  $B(E2, 0_g^- \rightarrow 2^-)$  values for the alternative assignment,  $J = 2^+$ , to these states.

#### G. $^{244, 246, 248}\text{Cm}$

Aside from a little  $\gamma$ -ray spectroscopy data<sup>31</sup> from the decay of  $^{244}\text{Bk} \rightarrow ^{244}\text{Cm}$  and some preliminary data<sup>30</sup> from the  $^{248}\text{Cm}(d, d')$  reaction, there is insufficient information to make  $K, J^\pi$  assignments for the states observed in  $^{244}\text{Cm}$  and  $^{248}\text{Cm}$ . In Table II, the  $B(E\lambda, 0_g^- \rightarrow J = \lambda)$  values are given for both assignments  $J = 3^-$  and  $J = 2^+$ .

The  $K, J^\pi$  assignments for the states in  $^{246}\text{Cm}$  are based on the extensive nuclear spectroscopy results<sup>32</sup> from the decay of  $^{246}\text{Am} \rightarrow ^{246}\text{Cm}$ . The  $2^+$  state at 1124 keV and the  $3^-$  state at 1128 keV, unresolved in our spectra, are both likely to be Coulomb excited and the  $B(E\lambda, 0_g^- \rightarrow J = \lambda)$  values given in Table II must be considered as upper limits.

### IV. COMPARISON WITH THEORETICAL CALCULATIONS

#### A. Octupole states

A comparison between the microscopic calculations<sup>5,33</sup> of the  $B(E3, 0^- \rightarrow 3^-)$  values for the  $3^-$  members of the one-phonon octupole quadruplet and the experimental results is presented in Table III. These microscopic calculations of the  $3^-$  state energies and the corresponding  $B(E3, 0_g^- \rightarrow 3^-)$  values include the Coriolis coupling between the states of the octupole quadruplet. The label  $\alpha$  in column 5 of Table III is the  $K$  quantum number corresponding to the largest component of the Coriolis coupled octupole state wave function. For those cases where the  $K, J^\pi$  assignments are known, the gen-

eral features of the experimental data are reproduced by the Coriolis coupled wave functions. The expected accuracy of the calculations for the  $3^-$  state energies is not better than  $\sim 200$  keV but the agreement between the measurements and calculations for the  $B(E3, 0_g^- \rightarrow 3^-)$  values is rather good. For instance, without the inclusion of the Coriolis interaction in the microscopic calculations, the  $B(E3, 0_g^- \rightarrow 3^-)$  values from calculations<sup>5,33</sup> with unperturbed random phase approximation states in  $^{238}\text{U}$  are 24, 19, 18, 14 (in units of  $10^{-2}e^2b^3$ ) for  $K = 0, 1, 2, 3$ , respectively. The inclusion of the Coriolis interaction in the calculations increases the  $B(E3, 0_g^- \rightarrow 3^-)$  values by approximately 60% over that for the unperturbed random phase approximation  $K = 0$  state in the Th and U nuclei, i.e., the  $B(E3)$  strength is redistributed by the Coriolis interaction. For the lighter nuclei in the actinide region, the  $K = 0$  states of the octupole quadruplet come lowest in excitation energy and the inclusion of the Coriolis coupling improves the agreement between theory and experiment for the  $B(E3, 0^- \rightarrow 3^-)$  value for the  $3^-$  member in the approximate  $K = 0$  band. This is similar to that observed for the lighter rare-earth nuclei.<sup>4</sup> For the heavier actinide nuclei, the calculations by Neergard and Vogel predict that the bands with approximate  $K = 1$  and 2 are lowest in excitation energy and the most of the  $B(E3)$  strength is in the excitation of the  $3^-$  member of the approximate  $K = 2$  band. Unfortunately, the experimental information is not extensive enough to test these predictions in the heavier actinide nuclei.

The properties of collective vibrational states in the actinide nuclei have also been investigated theoretically by Komov, Malov, and Soloviev.<sup>34</sup> These authors describe one-phonon vibrational states within the framework of the superfluid nuclear model taking into account the multipole-multipole residual interactions. The state energies for the  $3^-$  members of the one-phonon octupole quadruplet,  $K = 0, 1, 2, 3$ , and the reduced transition probabilities,  $B(E3, 0_g^- \rightarrow 3^-)$ , are given by Komov *et al.* Since the inclusion of the Coriolis interaction between states of the intrinsic octupole quadruplet is important to improve agreement with experiment and is not explicitly taken into account by Komov *et al.*, we have not attempted detailed comparisons. For instance, the  $B(E3, 0_g^- \rightarrow 3^-)$  values from their calculations for the  $K = 0$  state range from only 3 to 12  $B(E3)_{s.p.}$  for the actinide nuclei.

#### B. Quadrupole vibrational-like states

A recent survey<sup>15</sup> of the  $(p, t)$  reaction on  $^{230,232}\text{Th}$ ,  $^{234,236,238}\text{U}$ ,  $^{242,244}\text{Pu}$ , and  $^{248}\text{Cm}$  revealed uniformly strong population of excited  $0^+$  state



with cross sections  $\sim 15\%$  of the  $0^+$  ground-state cross section. Maher *et al.*<sup>15</sup> suggested that the  $2^+$  states of the excited  $0^+$  state bands did not have the properties of  $\beta$  vibrations because the  $B(E2, 0_g \rightarrow 2)$  values for transitions connecting the ground-state band to the excited state band were of the order of 1 to 3  $B(E2)_{s.p.}$  or about an order of magnitude smaller than would normally be expected. A striking feature of this present survey of the Coulomb excitation reaction on the actinide nuclei is that the  $B(E2, 0_g \rightarrow 2)$  values for excitation of the  $2^+$  states outside the ground-state rotational band are between 1 and 4  $B(E2)_{s.p.}$ . Any  $\beta$ -vibrational  $2^+$  states with  $B(E2, 0_g \rightarrow 2)$  values an order of magnitude larger and located below 1200 keV in excitation energy most certainly would have been observed in our studies of the Coulomb excitation reaction. The  $2^+$  states at 774 and 1037 keV in  $^{232}\text{Th}$  and  $^{238}\text{U}$ , respectively, have properties characteristic of  $\beta$ -vibrational states, namely the ratio of reduced  $E2$  transition probabilities for decay to members of the ground-state rotational band and large  $E0$  transition probabilities for the transitions  $0' \rightarrow 0_g$  and  $2' \rightarrow 2_g$ . The magnitudes of the reduced  $E0$  nuclear matrix elements,  $\rho(E0, 2' \rightarrow 2_g)$ , deduced from our measurements are consistent with that expected for  $\beta$ -vibrational  $2^+$  states. Although  $^{232}\text{Th}$  and  $^{238}\text{U}$  were not residual nuclei in the  $(p, t)$  reaction survey by Maher *et al.*,<sup>15</sup> the  $B(E2, 0_g \rightarrow 2)$  values for excitation of the  $2^+$  states are 2.4 and 1.4  $B(E2)_{s.p.}$  and are similar to those for  $2^+$  states in  $^{230}\text{Th}$ ,  $^{234,238}\text{U}$ , and  $^{238}\text{Pu}$  which were observed in the  $(p, t)$  reaction survey.

The calculations of Komov *et al.*<sup>34</sup> also support  $B(E2, 0_g \rightarrow 2')$  values in the range 1 to 4  $B(E2)_{s.p.}$  for the actinide nuclei in agreement with our results. Komov *et al.* also present from their calculations results for the transition probabilities to the  $\gamma$ -vibrational  $2^+$  states,  $B(E2, 0_g \rightarrow 2'')$  and these values range from 0.1 to 7  $B(E2)$  in general agreement with our results.

A noncollective description of the low-lying  $0^+$  states strongly excited in the  $(p, t)$  reaction but weakly excited in the  $(t, p)$  reaction<sup>35</sup> in the actinide nuclei has been presented by van Rij and Kahana.<sup>36</sup> Their pairing model is based on an oblate-prolate pairing force which is weak compared to the oblate-oblate and the prolate-prolate pairing forces. For a choice of the pairing strengths,

$G_{op} = 0.1G_{pp}$  and  $G_{pp} = G_{oo}$ , van Rij and Kahana obtain  $B(E2)$  values up to one  $B(E2)_{s.p.}$  and  $\rho$  values up to 0.1 which are appreciably smaller than our experimental results. Apparently, no attempt was made in their calculations to determine the best value of  $G_{op}/G_{pp}$  but increasing  $G_{op}/G_{pp}$  to 0.2 decreases the cross section for population of the excited  $0^+$  states in the  $(p, t)$  reaction and would also increase the  $B(E2)$  and  $\rho$  values improving the agreement with our experimental results.

Bés, Broglia, and Nilsson<sup>37</sup> have proposed a multipole pairing model (monopole and quadrupole pairing interactions) which also explains the strong population of excited  $0^+$  states in the  $(p, t)$  reaction and the weak population in the  $(t, p)$  reaction for the actinide nuclei. However, no results from their calculations were presented concerning the  $E2$  and  $E0$  transition probabilities.

## V. SUMMARY

Vibrational-like states, predominantly  $2^+$  and  $3^-$  states, in even- $A$  actinide nuclei have been investigated via the Coulomb excitation reaction with  $^4\text{He}$  ions. In general, the trends in the experimental reduced transition probabilities,  $B(E\lambda, 0_g \rightarrow J=\lambda)$  are reproduced by available theoretical calculations of collective vibrational states in deformed nuclei. Particularly good agreement is achieved with the microscopic calculations of Neegard and Vogel<sup>5,33</sup> for the  $B(E3, 0_g \rightarrow 3)$  values for members of the one-phonon octupole quadruplet when the Coriolis coupling between the states of the octupole quadruplet is included in the calculations. The  $B(E2, 0_g \rightarrow 2)$  values for the  $K=0$  and  $K=2$  vibrational states are in the range of 1 to 4  $B(E2)_{s.p.}$ , characteristic of those expected for collective quadrupole vibrational states. For  $^{232}\text{Th}$  and  $^{238}\text{U}$ , the magnitudes of the reduced  $E0$  matrix elements,  $\rho(E0, 2' \rightarrow 2_g)$ , deduced from a combination of these results with  $\gamma$ -ray spectroscopy results from the Coulomb excitation reaction, are consistent with those expected for  $\beta$ -vibrational  $2^+$  states. There is definitely a need for more extensive experimental information from nuclear spectroscopy to test theoretical predictions in the heavier actinide nuclei and for further theoretical investigations of the vibrational-like states for deformed nuclei in the actinide region.

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