

Radiative pion capture in $^{209}\text{Bi}^\dagger$

Helmut W. Baer, James A. Bistirlich, Nico de Botton,* Susan Cooper, and Kenneth M. Crowe
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

Peter Truöl

Physik-Institut der Universität Zürich, Zürich, Switzerland

John D. Vergados

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania

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The photon spectrum from stopped π^- absorption on ^{209}Bi was measured in the 50–150 MeV region with a high-resolution pair spectrometer. The total radiative branching ratio is $0.98 \pm 0.10\%$. Transitions to the single-particle states of ^{209}Pb are weak, in agreement with calculations presented. Considerable transition strength is observed to states of ^{209}Pb at 6–16 MeV excitation with evidence for a narrow peak at 7.9 ± 0.4 MeV. Present evidence favors the interpretation of this peak as excitation of the analog of a $T_>$ giant quadrupole resonance expected at ≈ 26 MeV in ^{209}Bi .

NUCLEAR REACTIONS $^{209}\text{Bi}(\pi^-, \gamma)$ with stopped π^- ; measured photon spectrum (50–150 MeV) and radiative branching ratio using pair spectrometer of 2 MeV resolution. Shell model calculations compared with measurements.

I. INTRODUCTION

Although it is now established¹ that the photon spectra from absorption of negative pions on light nuclei provide a quantitative probe of nuclear structure, only scant information exists on heavy nuclei. The single measurement for nuclei with $A > 70$ was performed by Petrukhin and Prokoshkin² who, using a natural Pb target and a Čerenkov total-absorption photon detector, measured a radiative branching ratio for stopped- π absorption of $2.1 \pm 0.5\%$. Neither the shape of the photon spectrum nor any specific nuclear structure features were determined. Measurements since then on light nuclei have shown¹ that (π^-, γ) reactions on $T=0$ target nuclei with pions in $1s$ or $2p$ atomic orbits induce strong and selective excitation of nuclear ($T=1$, $T_z=+1$) spin-isospin giant dipole vibrations as well as giant magnetic dipole states. It has also been predicted theoretically³ that for pion capture from $l=1$ orbits, giant quadrupole excitations are favored. In view of these results, a high-resolution measurement on a selected heavy nucleus seemed desirable to determine (1) an accurate radiative branching ratio and the general shape of the photon spectrum (for an isotopically pure target) and, (2) if additional evidence on excitation of giant electric multipoles and magnetic dipole states could be found for nuclei with large T_z ($43/2$ for ^{209}Bi) and pion capture orbitals of larger l ($l=3$ and 4 for ^{209}Bi). Additional theoretical motivation for the choice of ^{209}Bi is the existence of highly pure single-particle

states in ^{209}Bi and ^{209}Pb , the observation of which would determine the basic single-particle transition rates in heavy nuclei. With a resolution of 2 MeV full width at half-maximum (FWHM) these states cannot be clearly resolved; however, a comparison with summed theoretical transition strengths is still quite informative.

II. EXPERIMENT

The experiment was performed using the π^- beam of the Lawrence Berkeley Laboratory 184-inch cyclotron. A π^- beam of 190 MeV/ c was brought to rest in a metallic Bi plate ($17.1 \times 14.0 \times 0.32$ cm³) oriented 45° to the beam. Typical rates in the experiment were 10^4 (π^- 's/sec)/(g/cm²) stopping in the target. The photons were detected in a 180° pair-spectrometer⁴ employing a 3%(0.22 g/cm²) radiation length gold foil converter. The momenta of the electron-positron pairs are determined by magnetic analysis with coordinates given by three wire spark chambers with a total of 12 wire planes requiring 21 magnetostrictive wire delay line readouts. A PDP-15 computer was used on line to monitor the performance of the spark chambers and to record the data onto magnetic tape. The acceptance $[(\Delta\Omega/4\pi) \times \text{conversion probability} \times \text{detection efficiency}]$ of the spectrometer [Fig. 1(a)] is determined with a Monte Carlo calculation which includes the geometry, a field map, pair-production cross sections, energy loss due to radiation and ionization, and multiple scattering in the converter and chambers. Nu-

merous runs with a liquid hydrogen target were taken to check the acceptance and to determine the instrumental line shape [Fig. 1(a)] at 129.41 MeV from the monoenergetic photon of the $\pi^- + p \rightarrow n + \gamma$ reaction. The peak of the line shape is shifted by ~ 2 MeV from the photon energy due to energy loss in the converter and spark chambers. The mesonic capture $\pi^- + p \rightarrow \pi^0 + n$, $\pi^0 \rightarrow 2\gamma$ provides a check on the acceptance in the region $54.9 < E_\gamma < 83.0$ MeV.

III. RESULTS

As was found for light nuclei, the ^{209}Bi spectrum [Figs. 1(b) and 1(c)] exhibits the continuum as-

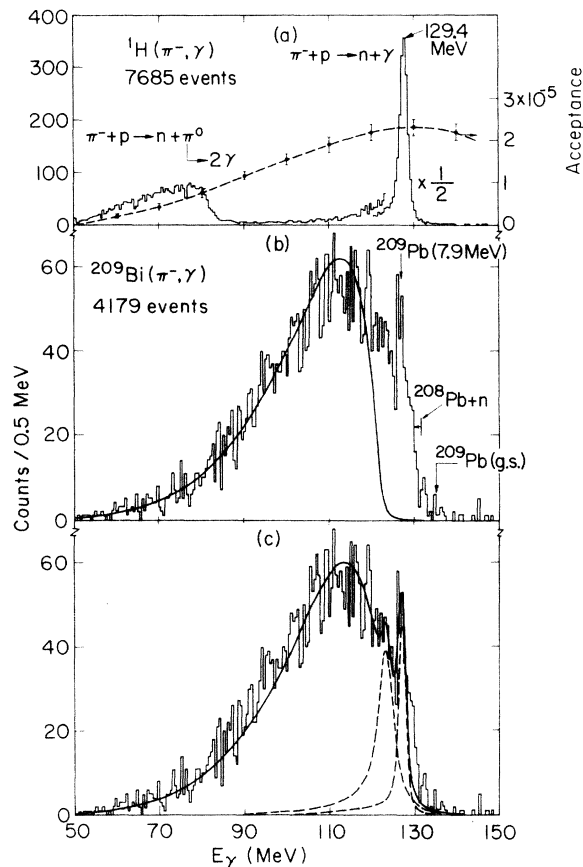


FIG. 1. Photon spectrum from π^- capture on hydrogen (a) and ^{209}Bi (b and c). Note from the $\pi^- + p \rightarrow n + \gamma$ reaction (a) that the instrumental line shape for 129.4 MeV photons has a full width at half maximum of 2 MeV and that its peak is shifted downward by ~ 2 MeV from the proton energy. [The broken curve in (a) shows the pair spectrometer acceptance as defined in the text.] The curve in (b) is a pole-model (Ref. 2) calculation ($\Delta = 16$ MeV) normalized to the data between 70–110 MeV; in (c) the curve is a fit to the full spectrum (pole + BW + line, Table I). The data give evidence for strong excitation of a state at 7.9 MeV in ^{209}Pb , and weak excitation of the single-particle states in ^{209}Pb (0–4 MeV).

sociated with quasifree capture $\pi^- + p \rightarrow n + \gamma$. The solid curve shows a calculation for the process $\pi^- + ^{209}\text{Bi} \rightarrow ^{208}\text{Pb}^* + n + \gamma$, using the pole model⁵ with an excitation energy $E_x^* = 11$ MeV ($\Delta = 16$ MeV, $\Delta \equiv M_{A-1} + m_n + E_{A-1}^* - M_A$, Ref. 1) for the recoil $^{208}\text{Pb}^*$. This energy was determined in a fit to the data between 70 and 110 MeV (corresponds to 27–67 MeV excitation in ^{209}Pb), a region which should be well above any nuclear resonance. It can be seen that a rather good description is obtained between 50–120 MeV. This, together with the excellent description¹ of the data on light nuclei ranging from ^3He to ^{40}Ca , demonstrates a rather general validity of this one-pole model.

Considerable transition strength is observed at 120–130 MeV (6–16 MeV excitation in ^{209}Pb) and there is a peak at $E_\gamma \approx 129$ MeV. To describe this transition strength to unbound states, two Breit-Wigner (BW) forms plus the pole model were fitted to the data. The curves shown in Fig. 1(c) and the R_γ given in Table I correspond to the fit with the pole-model $\Delta = 16$ MeV, and a fixed line width of zero for the peak at ~ 129 MeV. The BW form used to describe the data between the end point of the quasifree continuum and the line at $E_x = 7.9 \pm 0.4$ MeV serves primarily to define a background under the line, and cannot be regarded as evidence for a single “resonance” excitation in this region. By making reasonable variations of Δ , we find that the range of BW parameters $E_x = 11$ –15 MeV and $\Gamma_2 = 1$ –4 MeV are consistent with the data. Thus this transition strength falls into the region of the giant dipole resonance (GDR) [at 13.6 MeV in ^{208}Pb and 13.5 MeV in ^{209}Bi (Ref. 6)]; however, as discussed in Sec. IV, it seems unlikely that the GDR built on the ^{209}Pb ground state can be strongly excited in (π^-, γ) on ^{209}Bi . The peak at 7.9 MeV is at an energy 1–3 MeV lower than the reported giant quadrupole states which have been identified in this mass region.^{7–10} It is the same energy as the $M1$ state in ^{208}Pb observed¹¹ in 180° electron scattering and in the $^{208}\text{Pb}(\gamma, n)$ reaction.¹² Although studies¹ on ^6Li , ^{10}B , ^{12}C , and ^{14}N have shown the (π^-, γ) reaction to strongly select the analogs of $T_>$ (i.e., $\Delta T = 1$) $M1$ states built on the ground state of the target, in the $^{209}\text{Bi}(\pi^-, \gamma)$ reaction such transitions cannot occur since the isospin of the $M1$ states of ^{209}Bi have $T = T_c = 43/2$ and therefore they have no analog in ^{209}Pb .

The third region of interest is $E_x = 0$ –4 MeV where the pure single-particle states of ^{209}Pb occur. It is obvious from the spectrum that there is little transition strength to this region, and we measure $R_\gamma(0$ –4 MeV) = $(0.36 \pm 0.18) \times 10^{-4}$. An impulse approximation calculation for the single-particle transition was performed using the methods of Ref. 13 and A , B , C , and D coefficients of

TABLE I. Transition rates and branching ratios for radiative capture of stopped π^- on ^{209}Bi . The single-particle transitions $\pi^- + 1h_{9/2}$ (proton) $\rightarrow \gamma + j$ (neutron) with π^- in $4f$ or $5g$ orbitals are compared with the experimental value for the sum of all transitions up to 4 MeV excitation in ^{209}Pb . Experimental values for the total radiative branching ratio and energies and branching ratios for the line + BW fit [Fig. 1(c)] are given in the lower part of the table.

Single-particle state in ^{209}Pb	E_x (^{209}Pb) ^a (MeV)	E_γ ^b (MeV)	$\lambda_\gamma(4f)$ (10^{13} sec^{-1})	$\lambda_\gamma(5g)$ (10^{10} sec^{-1})	$R_\gamma(4f)$ ^c (10^{-4})
$2g \frac{9}{2}$	0	136.8	0.93	4.17	0.041 ± 0.012
$1i \frac{11}{2}$	0.78	136.0	1.17	3.11	0.052 ± 0.015
$1j \frac{15}{2}$	1.43	135.5	3.69	10.70	0.163 ± 0.048
$3d \frac{5}{2}$	1.57	135.2	0.31	1.51	0.014 ± 0.004
$4s \frac{1}{2}$	2.04	134.8	0.08	0.35	0.004 ± 0.001
$2g \frac{7}{2}$	2.50	134.3	0.32	0.14	0.014 ± 0.004
$3d \frac{3}{2}$	2.54	134.3	0.15	0.58	0.007 ± 0.002
			Theory	Sum =	0.30 ± 0.09
			Expt. $R_\gamma(0-4 \text{ MeV}) =$		0.36 ± 0.18
Experiment	7.9 ± 0.4 ^d	128.9 ± 0.4		($\Gamma_1 = 0-3 \text{ MeV}$)	4.7 ± 0.7 ^e
	13 ± 2	124 ± 2		($\Gamma_2 = 1-4 \text{ MeV}$)	9.6 ± 1.6 ^e
				$R_\gamma(\text{total}) =$	98 ± 10

^a Reference 29.

^b Assuming $4f$ capture; photons from $5g$ capture will appear at 0.59 MeV higher energies due to smaller π binding energy.

^c Assuming $4f$ capture only; $R_\gamma = C_f \lambda_\gamma(4f)/\lambda_a(4f)$ with distortion factor $C(4f) = 1.14 \pm 0.15$ (Ref. 22) and $\lambda_a(4f) = (1.7 \pm 0.5 \text{ keV})/\hbar = (2.6 \pm 0.8)10^{18} \text{ sec}^{-1}$ (Ref. 16).

^d Assuming $4f$ capture; $E_x = 8.5 \pm 0.4 \text{ MeV}$ if assume $5g$ capture.

^e Fitted with $\Delta \equiv 16$, $\Gamma_1 \equiv 0$, $\Gamma_2 = 2.7 \pm 0.8 \text{ MeV}$, $E_x = 7.9 \pm 0.4 \text{ MeV}$, and $E_x = 11.7 \pm 0.2 \text{ MeV}$.

Ref. 21. The radiative capture transition rates $\lambda_\gamma(4f)$ and $\lambda_\gamma(5g)$ for capture from $4f$ and $5g$ orbits connecting to the seven single-particle states of ^{209}Pb are given in Table I.

The radiative branching ratio is related to the transition rates by

$$R_\gamma = \sum_{nl} \frac{\lambda_\gamma(nl)}{\lambda_a(nl)} \omega(nl),$$

where $\lambda_a(nl)$ are the total strong absorption rates from orbits nl and $\omega(nl)$ the corresponding capture probabilities. Detailed capture schedules have not been published for heavy nuclei; however, from the available information^{14, 15}

$$\frac{\lambda^{\text{em}}(5g \rightarrow 4f)}{\lambda_a(5g)} = \frac{5.76 \times 10^{16} \text{ sec}^{-1}}{5.93 \times 10^{15} \text{ sec}^{-1}} = 9.7,$$

and¹⁶⁻¹⁸

$$\frac{\lambda^{\text{em}}(4f \rightarrow 3d)}{\lambda_a(4f)} = \frac{1.77 \times 10^{17}}{(2.6 \pm 0.8)10^{18}} = 0.07 \pm 0.02,$$

one can see that $4f$ capture dominates over $5g$ capture. However, from the yield measurement¹⁹

$Y(5g \rightarrow 4f) = 0.42 \pm 0.05$, one sees that the $n=4$ orbit accounts for less than half the captures. It is expected that some capture takes place from $l=0, 1$, and 2 orbits following the assumption²⁰ of a statistical $(2l+1)$ population as the initial condition in the pionic cascade; however, this fraction is small under the usual assumptions for an initial n [$n=14$ (Ref. 20)]. Thus the radiative branching ratios should be given quite accurately by summing over f orbits

$$\sum_n [\omega(nf)\lambda_\gamma(nf)/\lambda_a(nf)].$$

The dependence on n of λ_γ/λ_a for $l=0, 1$ capture (light nuclei) is thought²¹ to be small. We extend this assumption here to $l=3$, thus $R_\gamma = \lambda_\gamma(4f)/\lambda_a(4f)$. The values of λ_γ given in Table I were computed with hydrogenic wave functions. Comparing such wave functions with optical-model wave functions, Lucas and Werntz²² deduced the distortion correction factors $C(5g) = 1.38 \pm 0.20$ and $C(4f) = 1.14 \pm 0.15$. The latter was applied, i.e., $\lambda_\gamma(4f) = 1.14 \times \lambda_\gamma(4f; \text{hydrogenic})$, to compute the values of R_γ in the table. We obtain the

theoretical value $R_\gamma = (0.30 \pm 0.09)10^{-4}$ for the seven single-particle transitions. This is in good agreement with the measured value $R_\gamma(0-4 \text{ MeV}) = (0.36 \pm 0.18)10^{-4}$.

IV. DISCUSSION OF THE STATE AT 7.9 MeV

The shell-model configurations excited in ^{209}Bi (π^-, γ) belong to two classes which are illustrated in Fig. 2: (i) Those obtained by promoting the $1h_{9/2}$ proton to an unoccupied neutron orbital. These are the single-particle transitions going up to 4.25 MeV in ^{209}Pb for which transition rates are given in Table I. (ii) Transitions in which the $1h_{9/2}$ proton remains intact but one of the protons in the closed shells $j_2(N \leq 4)$ is promoted into a neutron shell $j_1(N \geq 6)$ resulting in configurations

$$[[j_1(n)j_2^{-1}(p)]J_1; 1h_{9/2}(p); J],$$

where J_1 is the angular momentum of the created p-h pair and J the total angular momentum of the state (doorway state). Such states are strongly mixed in the residual nucleus, possibly giving collective modes. It is not yet clear which type of collectivity is favored in (π^-, γ) on a heavy nucleus. In an effort to explain the peak at $E_x = 7.9 \pm 0.4$ MeV one might consider collective states built on the ^{209}Pb ground state such as the electric GDR, isoscalar quadrupole states (GQR), and magnetic dipole ($M1$) states, or $T_>$ ($=45/2$) collective modes built on the ^{209}Bi ground state, the analogs of which would be observed in ^{209}Pb .

The giant $M1$ state built on ^{209}Pb (g.s.) will be of the form

$$|2g_{9/2}(n), 1_2^+; J^\pi\rangle, \quad J^\pi = \frac{7}{2}^+, \frac{9}{2}^+, \frac{11}{2}^+,$$

where 1_2^+ is the $M1$ state in ^{209}Pb (illustrated in Fig. 2) given approximately²³ by

$$|1_2^+\rangle \simeq -0.62 |h_{9/2}(p)h_{11/2}^{-1}(p)\rangle \\ + 0.78 |i_{11/2}(n)i_{13/2}^{-1}(n)\rangle.$$

The proton component of this state contains configurations of type (ii). The transition rates to all three states were calculated, giving $\lambda_\gamma(4f) = 2.6 \times 10^{12} \text{ sec}^{-1}$ and $\lambda_\gamma(5g) = 1.1 \times 10^{10} \text{ sec}^{-1}$. These are less than the single-particle transitions in Table I, thus it does not seem possible to relate the state we observe to the state at 7.9 MeV seen¹¹ in electron scattering on ^{208}Pb . In addition it should be noted that the above state must be formed by an operator which changes the parity of the nuclear state, so it cannot be simply the Gamow-Teller operator. This makes the relation to electron scattering less direct than for light

nuclei.

The GDR and GQR excitations built on the ^{209}Pb (g.s.) (states $g_{9/2} \otimes L$, where L represents a collective p-h excitation of angular momentum L) have $T \geq \frac{45}{2}$ and $\pi = (-1)^L$. The number of components in such wave functions which have configurations of type (ii) is restricted by the condition that $j_1 = 2g_{9/2}$. Thus it is difficult to see how there could be much coherence in the transition amplitudes to such states.

To find ^{209}Pb wave functions which have large parentage to configurations (ii) and therefore could lead to considerable coherence in the transition amplitudes, one must look at the $T_>$ collective states built on the ^{209}Bi (g.s.). The $T_>$ components of the GDR are nearly nonexistent in nuclei with large neutron excess because the neutrons and protons occupy single-particle states of different parity. An interesting possibility is the $T_>$ component of the GQR in ^{209}Bi which can exist since it is formed by $2\hbar\omega$ excitations available to both neutrons and protons. The analog of a state at 7.9 MeV in ^{209}Pb is expected at 26.5 MeV in ^{209}Bi [Coulomb displacement energy = 18.8 MeV (Ref. 24)].

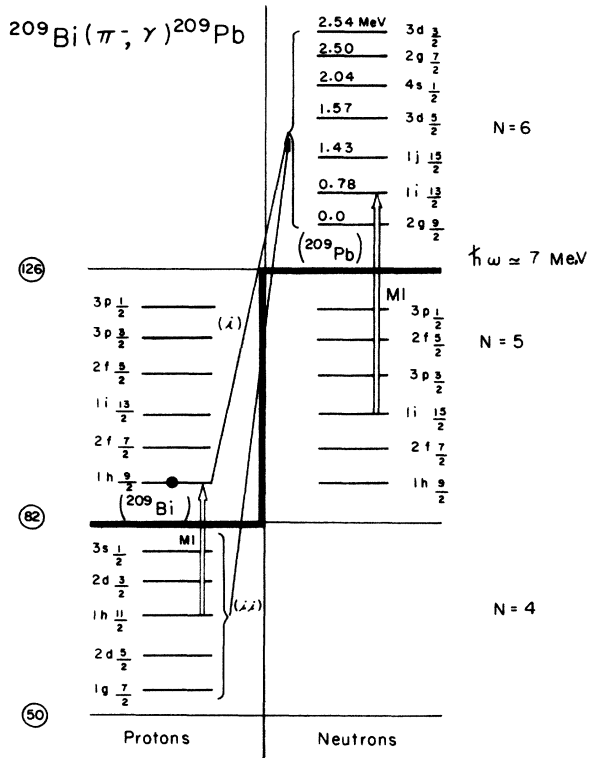


FIG. 2. Shell-model levels in the ^{209}Pb region. The single-particle transitions (i) of the $^{209}\text{Bi}(\pi^-, \gamma)^{209}\text{Pb}$ reaction and the (ii) neutron-particle, proton-hole excitations discussed in the text are illustrated. The $M1$ excitations of ^{209}Pb are also identified. Their relationship to the $^{209}\text{Bi}(\pi^-, \gamma)$ reaction is discussed in the text.

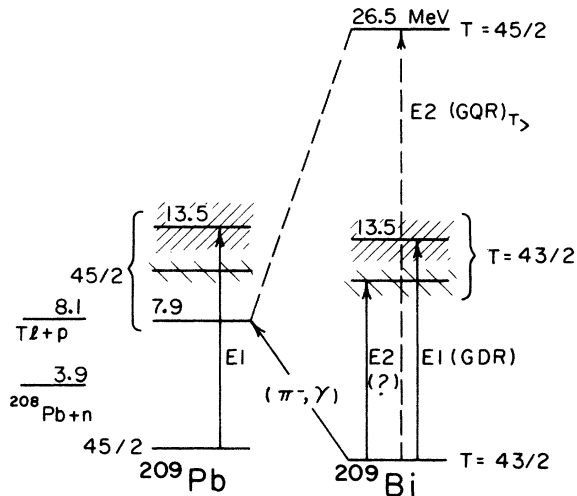


FIG. 3. Level diagram for giant resonance states of ^{209}Bi and ^{209}Pb . The peak in the γ spectrum of the ^{209}Bi - $(\pi^-, \gamma)^{209}\text{Pb}$ reaction at $E_x = 7.9 \pm 0.4$ MeV is identified as the excitation of the analog of a $T_>$ giant quadrupole resonance of ^{209}Bi at ≈ 26.5 MeV. The positions of the well-known electric dipole resonance and possible $T_<$ electric quadrupole resonance are also indicated.

A level diagram illustrating this transition is shown in Fig. 3. Evidence for a collective $E2$ state in ^{209}Bi at $E_x \approx 24$ MeV and with a width $\Gamma \approx 3.5$ MeV was reported recently by Snover *et al.*²⁵ in the reaction $^{208}\text{Pb}(p, \gamma)^{209}\text{Bi}$. Also, inelastic electron scattering cross sections show²⁶ some enhancement to states in this region in ^{208}Pb , and $E2$ or $E0$ character was suggested. The energies determined in both of these studies depend to some extent on the assumed background. Theoretical estimates^{27, 28} for the energies of isovector quadrupole states [$(\text{GQR})_{T_>}$] vary by several MeV, with $135 A^{-1/3} = 23$ MeV²⁸ being a typical value. Thus, on the basis of excitation energy it would appear that the identification of our 7.9 MeV state as the analog of a $T_>$ component of the GQR of ^{209}Bi is reasonable. The width of our state ($\Gamma_1 = 0-3$ MeV), though not too well determined by the data, is not inconsistent with the 3.5 MeV deduced from the (p, γ) study.²⁵ It should be noted that the decay width of the state in ^{209}Pb is much narrower than its analog in ^{209}Bi , because it is only 4 MeV above the neutron emission threshold (Fig. 3); also, due to its neutron-particle, proton-hole configuration, its decay to $^{208}\text{Pb} + n$ is greatly hindered.

The (π^-, γ) transition rate to the 7.9 MeV state can be calculated only after a detailed shell-model diagonalization is performed. For the present we can give only an approximate "sum rule limit." To calculate³⁰ this we summed the transition rates $\lambda_\lambda(4f)$ for all type (ii) doorway states (Fig. 2) with neutron-particle, proton-hole states coupled to $J_1^\pi = 1^+, 2^+, 3^+$, and $J^\pi = \frac{3}{2}^-, \dots, \frac{15}{2}^-$. The summed transition rate for all such states is $\lambda_\gamma(4f) = 84 \times 10^{13} \text{ sec}^{-1}$, giving $R_\gamma = (3.7 \pm 1.1) \times 10^{-4}$, where the indicated uncertainty is from the x-ray data and distortion factor. The experimental value is $(4.7 \pm 0.7) \times 10^{-4}$ assuming the background shown in Fig. 1(c). Notwithstanding the uncertainties in both the experimental and theoretical numbers, the calculation does demonstrate that the necessary strength can be obtained from the possible doorway states. The concentration of strength into a state (or states) at 7.9 MeV remains to be explained.

In conclusion we can say that the present study establishes that the strong state observed at 7.9 MeV in ^{209}Pb must be collective and its analog lies at ~ 26.5 MeV in ^{209}Bi . Although our study cannot establish unambiguously that this state corresponds to a $T_>$ component of the GQR built on the ground state of the target, the (π^-, γ) reaction clearly is a good means for the study of such states. First, because one observes the photon in the primary transition. This cannot be done in μ capture where the same states may also play a role.³¹ Second, because they occur at much lower energies in the final nucleus, and thus are in a region of lower level density and narrower decay widths.

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* Permanent address: DPHN/HE, C.E. N. Saclay, BP No. 2, 91-Gif sur Yvette, France.

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