Statistical analysis of the energy dependence of $^{12}C + ^{12}C$ cross sections*

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A statistical analysis has been made of the narrow structure appearing in the excitation functions for $^{12}C + ^{12}C$ induced interactions for elastic scattering ($13.5 \le E_{c.m.} \le 37.5$ MeV), inelastic scattering ($20 \le E_{c.m.} \le 30$ MeV), and α particle production ($16 \le E_{c.m.} \le 21$ MeV). Average fluctuation widths, strengths and cross correlations predicted by the statistical models of nuclei and of nuclear reactions are compared with those obtained from the analysis of suitably reduced experimental data. Good agreement is found. The effects of gross structure, possible structure of intermediate width, and a small ratio of level width to spacing (Γ/D) on the analysis of the narrow structure were studied using synthetic excitation functions. Appropriate correction factors were obtained in this way for application to parameters extracted directly from the reduced data. The results of the studies with synthetic excitation functions support the validity of the present statistical analysis. Compound processes are found to contribute up to ~20% of the measured elastic scattering cross section at 90° c.m. New experimental results reported herein for $^{12}C(^{12}C, \alpha)^{20}Ne^*$ reactions also suggest a large direct component, in contrast to earlier measurements on this reaction at lower energies. Hauser-Feshbach predictions of absolute compound cross sections show over-all good agreement with the average fluctuating cross sections deduced from the experimental data. It is concluded that the structure with widths ~0.3 MeV observed in the experimental excitation functions studied here is of statistical origin, and that the statistical model can also explain the occasional structural features with individual widths up to ~0.8 MeV. Apart from the gross structure associated with potential scattering, no evidence is found in the elastic scattering data for structure requiring nonstatistical mechanisms for its explanation.

[NUCLEAR REACTIONS Statistical model analysis of $^{12}C(^{12}C, ^{12}C)^{12}C$, $13.5 \le E_{c.m.} \le 37.5$ MeV; $^{12}C(^{12}C, ^{12}C)^{12}C$ (4.43) $20 \le E_{c.m.} \le 30$ MeV; $^{12}C(^{12}C, \alpha)^{20}Ne^*$ $16 \le E_{c.m.} \le 21$ MeV. Deduced average widths, fluctuation intensity, cross correlations. Hauser-Feshbach calculations. Measured $d\sigma/d\Omega$ for $^{12}C(^{12}C, \alpha)^{20}Ne^*$ at $\theta_\alpha \simeq 0°$, $16 \le E_{c.m.} \le 21$ MeV.]

I. INTRODUCTION

One of the most interesting discoveries made in the early studies of heavy ion reactions was the observation of elastic scattering cross sections which varied rapidly with bombarding energy.[1] This structure in the excitation functions, which was particularly marked in systems such as $^{12}C + ^{12}C$, $^{12}C + ^{16}O$, and $^{16}O + ^{16}O$, has been the subject of continued study since its discovery some 14 years ago.[2]

Even in the earlier elastic scattering measurements several distinct classes of structure were identified. In the $^{12}C + ^{12}C$ system, for example, it was found that at energies near and below the Coulomb barrier certain pronounced resonances were observed; these also appeared in all the reaction channels.[3] They were first thought to reflect the formation of $^{12}C - ^{12}C$ quasimolecular configurations because of a large reduced width associated with the resonances observed in the elastic channel.[3,4] Subsequent measurements at lower energies on the reaction channels revealed the existence of additional resonances in the $^{12}C + ^{12}C$ system.[5] Michaud and Vogt[6] then suggested that additional degrees of freedom, associated with the formation of cluster-like $^{12}C + 3\alpha$ and $^{16}O + 2\alpha$ "doorway" configurations, were required in order to explain the presence of these additional resonances as well as other experimental features of $^{12}C + ^{12}C$ induced reactions.

Measurements at energies above the Coulomb barrier revealed fluctuating structure although no cross correlations such as those seen at lower energies were apparent. The structure in the $^{12}C(^{12}C, \alpha)^{20}Ne$ reaction exhibits a characteristic width of $\Gamma \approx 100$ keV and was shown to be consistent with Ericson fluctuations arising from formation of strongly overlapping compound states in ^{24}Mg.[7,8] Statistical analyses of the $^{12}C + ^{12}C$ and $^{16}O + ^{16}O$ elastic scattering[9] at bombarding energies just above the Coulomb barrier also supported the earlier conclusions that compound nucleus formation is an important, and probably the dominant, reaction mechanism producing the fluctuating cross sections. Studies on the α-particle and elastic exit channels in the $^{16}O + ^{16}O$ system resulted in similar conclusions.[10] Recently, however, nonstatistical behavior has been observed for the cross sections of the $^{12}C(^{12}C, \alpha)^{20}Ne$ reaction leading to certain states in ^{20}Ne.[11]

With the availability of the first MP tandem accelerator, these studies were extended to several times the Coulomb barrier energy.[12] It immediately became clear that yet a third type of structure of typical width in the range from 2 to 4 MeV was present. Detailed studies have shown that this gross structure corresponds to potential or shape elastic scattering and that it is reasonably well reproduced by appropriate optical model potentials.[13] Superposed on this broad structure, particularly in the case of ^{12}C + ^{12}C, to a lesser extent in ^{16}O + ^{16}O (but not at all for ^{14}N + ^{14}N), one finds additional structure having characteristic widths significantly less than 1 MeV.

The origin of this structure at higher energies has remained an important and open question. Greiner and collaborators[14,15] have proposed a double resonance mechanism wherein one or both of the interacting nuclei are temporarily excited to one of their quantum states during the interaction. Scheid, Greiner, and Lemmer[14] and Fink, Scheid, and Greiner[15] have shown that this mechanism can reproduce both the broad and the 200–300 keV structure observed in the ^{16}O + ^{16}O reaction, and some of the structure in the ^{12}C + ^{12}C reaction. It is also possible that the mechanisms advanced to explain the variation in the cross sections at lower energies, viz. α-cluster doorway states and statistical fluctuations, could also play a role in determining the structure observed at higher energies.

The present work concerns the origin of the nonpotential structure in the ^{12}C + ^{12}C elastic scattering excitation function over the higher energy range $E_{\text{c.m.}} = 13.5–37.5$ MeV. Our procedure is to perform detailed and careful statistical analyses of previously measured elastic[12] and inelastic[16,17] scattering cross sections and of new experimental data on the ^{12}C(^{12}C, α)^{20}Ne reaction. We obtain thereby experimental values for the characteristic widths Γ, the average value of the fluctuating component of the cross section, and the correlations of cross sections at different angles and for different channels. Independent estimates of these quantities based on the statistical models of nuclear structure and reactions are then compared to the experimental values. The extent to which these agree will then indicate the relative importance of compound nucleus formation in producing the observed fluctuating structure provided, of course, that the experimental data can be shown to satisfy the assumptions inherent in a statistical analysis. We also seek to determine whether the application of statistical methods can establish the presence in the ^{12}C + ^{12}C reaction of nonstatistical structures having an intermediate width such as predicted by Fink *et al.*[15]

In Sec. II we first mention briefly the assumptions made in performing our statistical analysis of the data. The method used to eliminate from the data the slow, modulating energy variation, characteristic of potential scattering, is then described. The results of the statistical analysis on the reduced cross sections, thus obtained, are presented together with a quantitative discussion of the consequences of analyzing the reduced rather than original data. Section III presents new experimental cross sections for the ^{12}C(^{12}C, α)^{20}Ne reaction and a similar statistical analysis of these data.

These results are compared, in Sec. IV, with the corresponding ones obtained from elastic scattering measurements in the same energy region. Also in this section, theoretical and semiempirical estimates for the values of Γ, compound cross sections, and cross correlations are compared to experimental values for these quantities extracted from the available data for the various channels. The question of nonstatistical intermediate structure (i.e., doorway states) is considered in Sec. V together with a statistical analysis of the theoretical cross sections for ^{12}C + ^{12}C elastic scattering.[15] A discussion is given of the sensitivity of the present statistical analysis to such types of structure. The last section presents a summary and conclusion.

II. STATISTICAL ANALYSIS OF ELASTIC SCATTERING DATA

The average characteristics of fluctuating cross sections are reflected in the autocorrelation function

$$R(\epsilon) = \frac{\overline{\sigma(E)\sigma(E+\epsilon)}}{\overline{\sigma(E)}\,\overline{\sigma(E+\epsilon)}} - 1\,, \qquad (1)$$

where $\overline{\sigma(E)}$ denotes the energy average of the quantity $\sigma(E)$.

The form of $R(\epsilon)$ has been specified by Ericson[18] within the framework of a compound nuclear reaction model which describes the fluctuations arising in the measured cross sections. In particular he has shown that

$$R(\epsilon) = R(0)\frac{\Gamma^2}{\Gamma^2 + \epsilon^2}\,, \qquad (2)$$

$$R(0) = \frac{1 - {y_d}^2}{N_{\text{eff}}}\,, \qquad (3)$$

and

$$P(x) = \frac{1}{1 - y_d}\exp\left(-\frac{x + y_d}{1 - y_d}\right) I_0\left(\frac{2(xy_d)^{1/2}}{1 - y_d}\right), \qquad (4)$$

where $R(0)$ is the average normalized variance,

Γ is the average width of levels populated in the compound nucleus, and y_d is the ratio of the average direct component of the reaction cross section to the average of the measured cross section. N_{eff} is the effective number of independent channels contributing to the observed cross section and is equal to or less than the number of different positive spin projections. Equation (4) gives the distribution of the fluctuating cross sections, $x=\sigma/\overline{\sigma}$, for the case of $N_{\text{eff}}=1$. I_0 here denotes the modified Bessel function of zero order.

Ericson's model and its assumptions may be summarized as follows: The scattering matrix for the transition from an initial state α with total angular momentum J to a final state α' and J is given by

$$S^J_{\alpha\alpha'}(E)=\overline{S}^J_{\alpha\alpha'}+i\sum_{\lambda}\frac{g^J_{\alpha\lambda}g^J_{\lambda\alpha'}}{E-E_\lambda+\frac{1}{2}i\Gamma^J_\lambda}\,, \tag{5}$$

where it is assumed that
(i) $\overline{S}^J_{\alpha\alpha'}$ is approximately energy independent, such that the change in $\overline{S}^J_{\alpha\alpha'}$ over an energy range of the order of many Γ^J_λ's can be neglected.
(ii) The resonances in the compound system overlap strongly, i.e., $\Gamma^J/D^J\gg 1$ where $\Gamma^J=\langle\Gamma^J_\lambda\rangle_\lambda$ is the average level width in the compound system and $D^J=\langle D^J_{\lambda\lambda'}\rangle_{\lambda\lambda'}$ is the average spacing between levels of the same spin and parity.
(iii) The quantities $g^J_{\alpha\lambda}$ are in general complex constants whose phases vary randomly with respect to the channel α and the level index λ.
(iv) The distribution of the level widths Γ^J_λ has a small dispersion, i.e., $\Gamma^J_\lambda\approx\Gamma^J$.

According to Ericson the relation $\Gamma^J_\lambda=\sum_\alpha\Gamma^J_{\lambda\alpha}$ remains valid in the region of overlapping levels and the partial widths $\Gamma^J_{\lambda\alpha}$ have the same distribution as that observed for isolated neutron resonances. Assumption (iv) then holds for the case of many open channels. Furthermore, for the case of strong fluctuations, i.e., if $\langle\sigma^{\text{fl}}\rangle$ is comparable to the nonfluctuating component of σ, the complex quantities $g^J_{\alpha\lambda}$ may be identified with the partial width amplitudes $\gamma^J_{\alpha\lambda}$ and $\overline{S}^J_{\alpha\alpha'}$ becomes $S^{(P)J}_{\alpha\alpha'}$, the matrix element for direct or potential scattering.

Moldauer[19] studied the fluctuations in nuclear reaction cross sections for arbitrary Γ^J and D^J. He confirmed Ericson's results in the limit of large Γ/D and evaluated corrections to Eqs. (2) and (3) which were shown to arise from partial width fluctuations and resonance-resonance interference phenomena. Similar corrections appear also in the formulas for the average cross sections and these will be discussed in Sec. IV.

It is clear that the experimental data which we consider here do not satisfy all of the above criteria. In particular, the presence of gross structure arising from shape elastic or potential scattering[12,13] implies that $\overline{S}^J_{\alpha\alpha'}$ has a strong energy dependence when $\alpha=\alpha'$. This specific energy dependence must be removed from the data before a proper statistical analysis can be attempted. Also it is not obvious whether assumption (ii) is satisfied since the $^{12}C+{}^{12}C$ entrance channel can populate compound nuclear levels in the vicinity of the yrast line, where the density of states is low. Finally, the presence of intermediate structure could be reflected in large ranges of widths in violation of assumption (iv). These effects will be investigated quantitatively using synthetic excitation functions. The present section considers the effects of removing the gross structure. The effects of small values of Γ/D, and possible effects of intermediate structure will be discussed in Appendix A.

Throughout this paper the term "intermediate structure" is used to denote structure of a nonstatistical origin, e.g., as would be caused by the presence of a doorway state or some configuration representing a simple structure in the continuum. The term "intermediate width" here refers to a width in the range of approximately ~400–1000 keV without specifying the physical origin of the structure. "Narrow" structure denotes a characteristic width of ≲400 keV.

Figure 1(a) displays the $^{12}C+{}^{12}C$ differential cross section for elastic scattering at 90° in the center of mass over an energy range $E_{\text{c.m.}}=13.5$ to 37.5 MeV.[12] The data points are at intervals of 0.125 MeV, a spacing which is also comparable

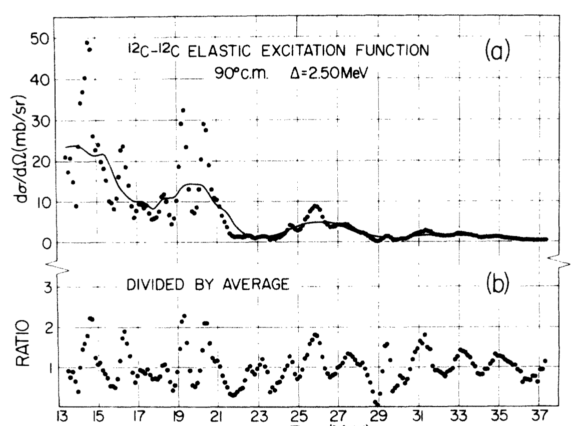

FIG. 1. (a) The experimental $^{12}C+{}^{12}C$ elastic excitation function (Ref. 12). The full curve is a running average of the data taken over an interval $\Delta=2.5$ MeV. (b) The experimental excitation function divided by the running average. In the text these ratios are referred to as "reduced" data.

to the experimental energy resolution. Also shown is a running average of the cross section determined with an averaging interval $\Delta = 2.5$ MeV which shows clearly the modulating gross structure which must be removed. The method used here, first suggested by Papallardo,[20] is to divide the experimental cross section by this empirical average. Figure 1(b) shows the reduced data obtained in this way for $\Delta = 2.5$ MeV. In principle the size of the averaging interval Δ is chosen such that the resulting average contains all or most of the broad structure which we wish to remove and very little of the fluctuating component. Thus we would choose $\Gamma \ll \Delta \ll \Gamma_{SE}$ where Γ and Γ_{SE} represent the characteristic widths of the narrow and the gross structure, respectively. For a case in which $\Gamma_{SE} > 20\Gamma$, we would then expect that the value of $R(0)$, obtained from data which have been reduced with an averaging interval Δ, would increase as a function of Δ until $\Delta \sim 10\Gamma$. At this point $R(0)$ should vary little with increasing Δ and this "plateau region" should extend until $\Delta \sim \Gamma_{SE}$. Figure 2 shows $R(0)^{\text{obs}}$ [the superscript "obs" denotes that these are observed values of $R(0)$ obtained directly from the reduced data with no corrections applied for averaging by Δ]. Only a suggestion of a plateau is apparent for the data at 70, 80, and 90°, while the 50 and 60° data give no indication of a plateau. This shows that for $^{12}C + ^{12}C$ elastic scattering, at these energies, the difference in widths between the narrow and the broad structure is not sufficiently large to permit a complete separation of the two in the autocorrelation function.

The dependence of $R(0)^{\text{obs}}$ on Δ for the 70, 80, and 90° data indicates that Δ should be chosen somewhere in the range of $1.5 \leq \Delta \leq 3$ MeV but that even within this limit there is a large variation in the values of $R(0)^{\text{obs}}$. In order to obtain true values of $R(0)$ and Γ, which are by definition independent of Δ, it is necessary to know, for a given value of Δ, the extent to which $R(0)^{\text{obs}}$ and Γ^{obs} are affected by the fact that the reduced data do not contain all the fluctuating strength and, moreover, still retain some of the gross structure. This can be studied quantitatively by generating synthetic excitation functions whose true statistical characteristics [$R(0), \Gamma, \Gamma/D$, etc.] are known, *a priori,* performing the autocorrelation analysis, and then comparing the resulting values of $R(0)^{\text{obs}}$ and Γ^{obs} with the "true" values. Studies performed with known fluctuations superposed on known gross structure (see Appendix A) also show

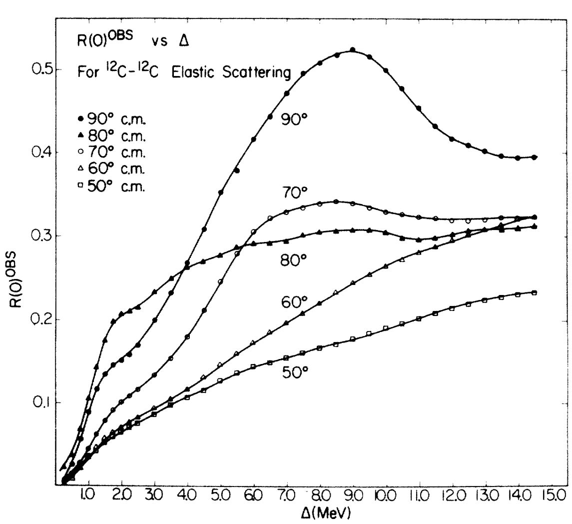

FIG. 2. Dependence of the average normalized variance, $R(0)^{\text{obs}}$, on the averaging interval Δ.

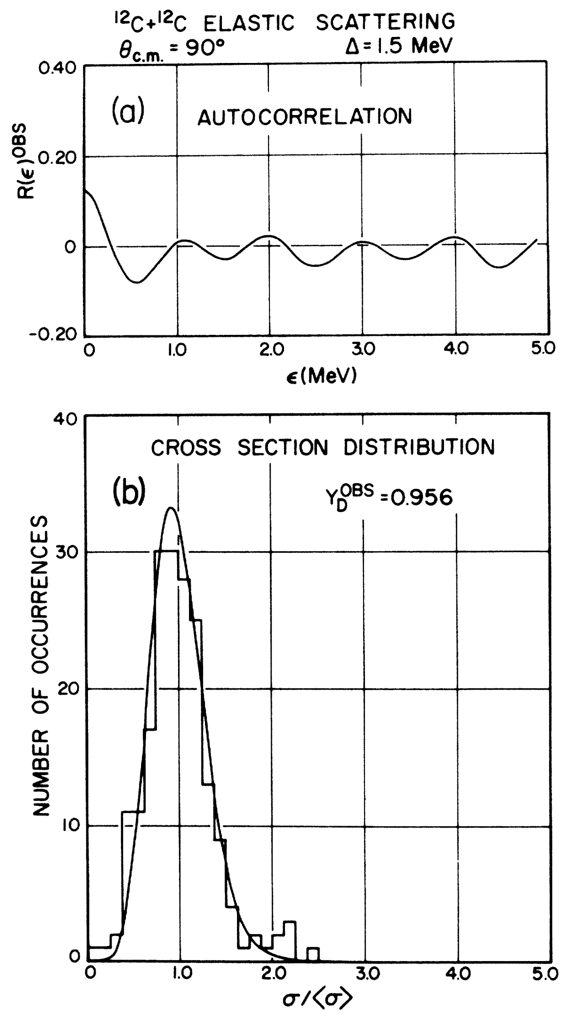

FIG. 3. (a) The autocorrelation function $R(\epsilon)^{\text{obs}}$ for the reduced elastic scattering data at 90° c.m. (b) The distribution of cross sections. The full curve is obtained from Eq. (4) with y_d^{obs} adjusted to 0.956. The value of χ^2 per degree of freedom is 0.9.

TABLE I. Results of autocorrelation analyses. $\Delta = 1.5$ MeV.

c.m. angle (deg)	$R(0)^{obs}$	y_d^{obs}			y_d	Γ^{obs} (keV)	$\bar{\Gamma}^{FRD}$ (keV)	Γ	Γ^{PC}	Sample size
50	0.051	0.984 [a]	0.974 [b]	0.965 ± 0.007 [c]	$0.945^{+0.012}_{-0.023}$ [d]	160 [e]	171 ± 22 [f]	330 ± 48 [d]	306 ± 37	23
60	0.057	0.970	0.971	0.970 ± 0.007	$0.938^{+0.014}_{-0.026}$	157	160 ± 22	327 ± 47	341 ± 40	23
70	0.078	0.967	0.960	0.957 ± 0.008	$0.914^{+0.020}_{-0.037}$	178	191 ± 24	367 ± 53	341 ± 40	21
80	0.176	0.923	0.908	0.901 ± 0.018	$0.794^{+0.049}_{-0.101}$	178	191 ± 24	367 ± 53	341 ± 40	21
90	0.134	0.956	0.932	0.926 ± 0.018	$0.848^{+0.036}_{-0.070}$	191	205 ± 26	393 ± 57	371 ± 45	21

[a] Using the autocorrelation method [Eq. (3)].
[b] Using the distribution of cross sections [Eq. (4)].
[c] The value of y_d^{obs} from Eq. (3), corrected only for the finite range of data (FRD).
[d] Includes correction for FRD and the effect of the averaging interval $\Delta = 1.5$ MeV.
[e] Using the autocorrelation method [Eq. (2)].
[f] The value of Γ^{obs} from Eq. (2), corrected only for the FRD.

that, because of the gross structure, the optimum value for Δ is 1.5 MeV. Put, Roeders, and van der Woude[21] have noted that the corrections to be applied to $R(0)^{obs}$ and Γ^{obs} can be quite large even if Δ is as large as 10Γ. This is also the case in the present work.

Examples of $R(\epsilon)^{obs}$ and $P(x)^{obs}$ obtained from analysis of the reduced data are given in Figs. 3(a) and (b), respectively, for the excitation function measured at 90°. In Fig. 3(a), the oscillations in $R(\epsilon)$ for $\epsilon > 2\Gamma$ are associated with the finite range of data available for analysis. The value of Γ obtained from Eq. (2) is $\Gamma^{obs} = 191$ keV. The value of y_d obtained from Eq. (3) with $N_{eff} = 1$ is $y_d^{obs} = 0.932$. (N_{eff} is identically unity for elastic scattering of spinless particles.) Figure 3(b) compares the distributions of cross sections for the reduced data with Eq. (4) for a best-fit value of $y_d^{obs} = 0.956$, in close agreement with the value obtained from the autocorrelation analysis, Eq. (3).

The results of the autocorrelation analyses for all scattering angles are given in Table I. Columns 2 and 3 give the values of $R(0)^{obs}$ and y_d^{obs} as obtained from Eqs. (1) and (3), respectively, and column 4 lists for comparison the value of y_d^{obs} obtained from Eq. (4). The values in columns 3 and 4 are in reasonable agreement. The fifth column gives the value of y_d^{obs} obtained from Eq. (3) and corrected for the bias introduced by the finite range of data. This correction is discussed in Appendix B. The error on y_d given in column 5 is based on the sample size (column 11) which is defined by $n = \Delta E/\pi\Gamma$ where ΔE is the energy range of the excitation function. The value of y_d presented in column 6 includes the correction discussed in Appendix A for the effect of the averaging interval Δ. For $\Delta = 1.5$ MeV, it is seen that this correction is sizeable: the fluctuation contribution to the reaction is changed by a factor of 2. The larger error given in column 6 reflects the uncertainty in the size of this correction combined with the effect of the finite range of data. The compound or fluctuating fraction of the cross sections is a maximum of ~20% at 80 and 90°. As is expected, the ratio of the average fluctuating part of the cross section to the total cross section, $(1 - y_d)$, decreases at the more forward

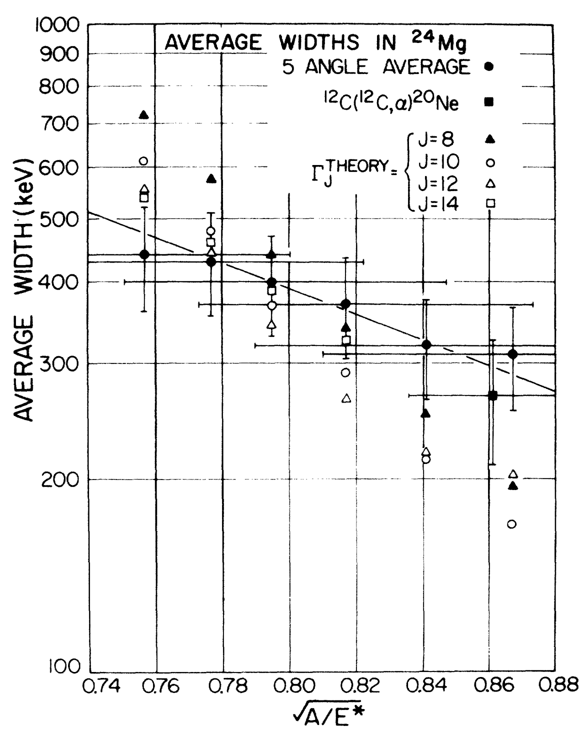

FIG. 4. The experimental dependence of Γ on $\sqrt{A/E^*}$ (filled circles). A and E^* are the mass (amu) and excitation energy (MeV) in ^{24}Mg, respectively. The theoretical values of Γ are obtained from Eq. (6).

angles where the potential scattering is much larger.

The values of Γ^{obs} obtained from Eq. (2) are given in column 7. When corrections for the bias accompanying the finite range of data are applied, the values in column 8 are obtained. As is the case with $R(0)$ the correction associated with the averaging interval $\Delta = 1.5$ MeV is much larger, again about a factor of 2. The final value is given in column 9. It is interesting to compare these values of Γ with those obtained from the peak counting method introduced by Brink and Stephen[22] (column 10). In this case the relation $\Gamma = 0.50/N$ was used where N is the average number of peaks per MeV. The agreement between the values of Γ given in columns 9 and 10 serves to validate the use of synthetic excitation functions to determine the effects of using reduced data.[23]

It is apparent from an inspection of Fig. 1(b) that the typical width of the fluctuations is larger in the portion of the excitation function at higher energies than at lower energies. A quantitative study was therefore undertaken to determine the dependence of the average width on the excitation energy in the compound nucleus. Figure 4 shows values of Γ obtained from analyzing 10 MeV wide subintervals and averaging over the five scattering angles. The abscissa is the value of $(A/E^*)^{1/2}$ where E^* is the excitation energy in ^{24}Mg in MeV and $A = 24$. The horizontal bars denote the extent of the 10 MeV wide subintervals. Within the larger errors on Γ (associated with the smaller sample size of the subinterval) the logarithm of Γ is seen to increase linearly as $\sqrt{E^*}$. In this analysis the values of Γ obtained at each angle were assumed to be statistically independent since the coherence angle given by $\theta_c = 1/kR \lesssim 6°$ is less than the 10° spacing between adjacent angles.

III. ^{12}C(^{12}C,α)^{20}Ne REACTION EXPERIMENTAL RESULTS AND STATISTICAL ANALYSIS

The ^{12}C(^{12}C, α)^{20}Ne reaction was investigated for two reasons. First, the study of exit channels other than elastic scattering provides important additional information on the reaction mechanism.

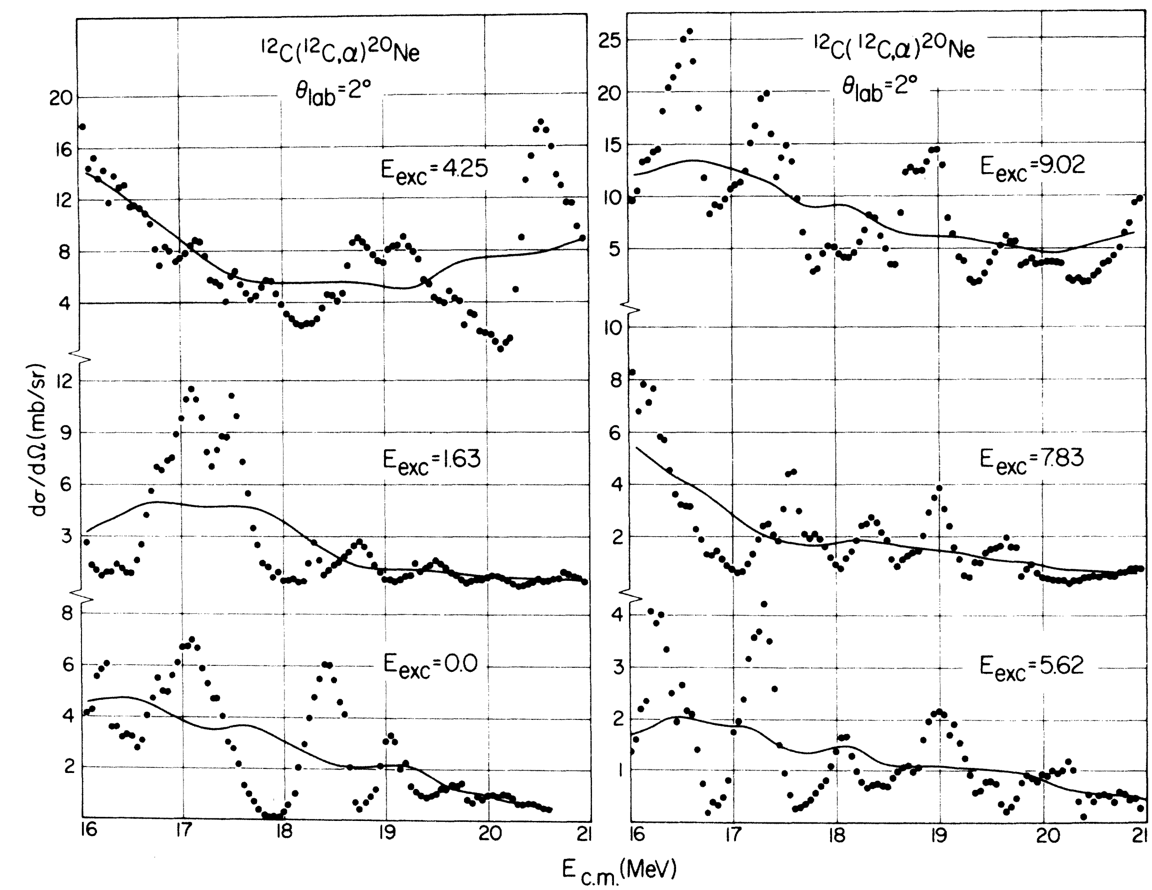

FIG. 5. Excitation functions for ^{12}C(^{12}C, α)^{20}Ne*. Also shown is a running average of the data taken with $\Delta = 2.0$ MeV.

The statistical model predicts the average behavior of all energetically allowed channels, and each additional measurement provides another independent determination of the statistical properties of the compound nucleus. The $\alpha + {}^{20}$Ne channels also have the advantage of being strongly populated in this reaction. Further, any nonstatistical effects arising from possible α-particle intermediate structure[24] would be expected to be prominent in the α-particle channel.

Absolute cross sections were measured for the ^{12}C(^{12}C, α)^{20}Ne reaction at 50 keV intervals over the energy range $E_{c.m.} = 16-21$ MeV using the ^{12}C(5$^+$) beam from the Yale MP tandem accelerator. The target consisted of a 20 μg/cm^2 natural carbon foil and resulted in an energy resolution in the incident channel of ~50 keV c.m. The α particles were detected at an average angle of 3° c.m. which facilitates the fluctuation analysis by eliminating any uncertainty in the value of N_{eff}. However, energy straggling in an absorber, which is required to prevent the forward scattered ^{12}C ions from entering the forward detector, reduces the attainable energy resolution for the emergent α particles. A specially designed absorber system using hydrogen gas[25] minimized this effect and made possible an energy resolution of $\leq$130 keV.

Excitation functions for six strongly populated, well resolved final levels in ^{20}Ne were analyzed. The cross sections for the levels with excitation energies and spins (MeV, J^π) of (0.0, 0$^+$), (1.63, 2$^+$), (4.25, 4$^+$), (5.63, 3$^-$), (7.83, 2$^+$), and (9.02, 4$^+$) are shown in Fig. 5. The absolute normalization is accurate to ±20%.

The statistical analysis of the above experimental results proceeded in the same manner as described in Sec. II for the case of elastic scattering. The observation of the α particles at angles close to 0° ensures that only $m = 0$ magnetic substates make appreciable contributions to the cross section and, hence, $N_{eff} = 1$ is a good approximation. The data were first divided by a running average obtained with $\Delta = 1.5$ MeV in order to reduce the effects of possible gross structure. The averaged cross sections, shown in Fig. 5, suggest that some modulating structure is present.

The results of the analysis performed on the reduced data are listed in Table II. A comparison of y_d with y_d^{obs} and Γ with Γ^{obs} reveals the large corrections associated with analyzing reduced data and, in this case, over a very limited range of energy. The value of Γ obtained when the six individual values are averaged is 263 ± 42 keV. This compares favorably with the average value 225 ± 49 keV obtained from counting maxima. An analysis of the elastic scattering data over the same range of bombarding energy yields a value of $\Gamma = 277 \pm 68$ keV. These widths are comparable even though the α-particle data were obtained with an experimental resolution which was less than one half that used for the elastic scattering. This demonstrates that in this energy region there is no structure finer than the ~250 keV structure observed here.

A remarkable feature of the present results is the large direct or nonfluctuating component in the cross sections: y_d is greater than 0.5 in all cases. This result has also been obtained by Greenwood, Segel, and Erskine[26] and is in contrast to the results obtained for this reaction at lower bombarding energies[7,8] where the fluctuations in the data were consistent with $y_d = 0$.

IV. COMPARISON OF THE STATISTICAL MODEL PREDICTIONS WITH THE EXPERIMENTAL RESULTS

Independent estimates of the average width, the distribution of the cross sections, and the magnitude of their fluctuations, obtained from the statistical model of nuclei and nuclear reactions, will be compared with the experimental results.

A. Comparison of widths

The average width of the levels of a given angular momentum J and energy E_x in the compound nucleus must be independent of the exit channel for which this width is deduced. This is the case for the channels studied here. The six reaction channels listed in Table II yield an average coherence width of $\Gamma = 263 \pm 42$ keV. The elastic data at five different angles yield, when analyzed over the same excitation energy range in ^{24}Mg ($30 \leq E_{exc} \leq 35$ MeV), an average coherence width of 277 ± 68 keV. Hauser-Feshbach calculations (Sec. IV C) of the dependence of the cross sections for elastic and for ^{12}C(^{12}C, α)^{20}Ne scattering on the total angular momentum suggest that the various angular momenta make similar contributions to the observed width and that $J = 12$ is the dominant

TABLE II. Analyses performed on the reduced data. $\Delta = 1.5$ MeV. Sample size ~7.

E_{exc}	$R(0)^{obs}$	y_d^{obs}	y_d	Γ^{obs} (keV)	Γ	Γ^{PC}
0.0	0.306	0.83	$0.60^{+0.21}_{-0.60}$	148	281 ± 70	227 ± 49
1.63	0.338	0.81	$0.55^{+0.25}_{-0.55}$	136	258 ± 68	227 ± 49
4.25	0.169	0.91	$0.80^{+0.10}_{-0.14}$	175	333 ± 85	208 ± 45
5.62	0.323	0.82	$0.57^{+0.23}_{-0.57}$	124	236 ± 62	233 ± 51
7.83	0.232	0.88	$0.72^{+0.14}_{-0.24}$	109	207 ± 55	227 ± 49
9.02	0.210	0.89	$0.75^{+0.13}_{-0.20}$	140	266 ± 70	227 ± 49

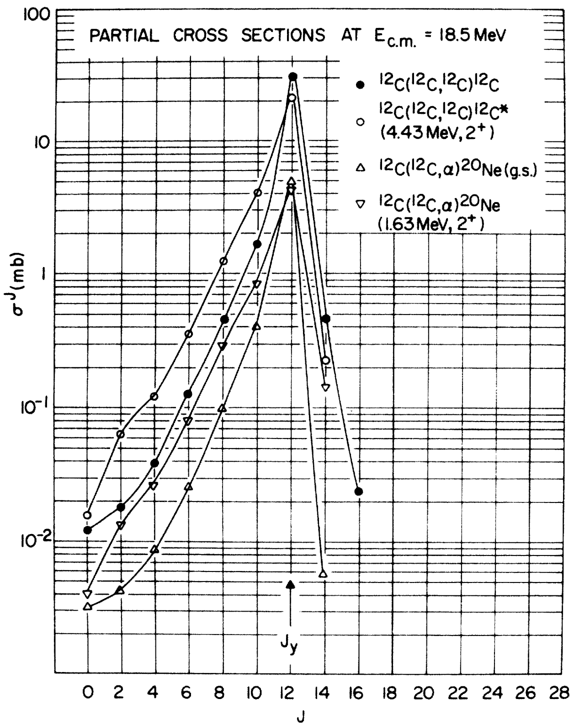

FIG. 6. Partial cross sections at $E_{c.m.}=18.5$ MeV calculated with a Hauser-Feshbach statistical model. At this energy, the maximum angular momentum allowed by our parametrization of the yrast cutoff is $J=12$.

spin. Figure 6 shows a plot of σ_J for the elastic, inelastic, and α-particle channels at an average bombarding energy of 18.5 MeV c.m. The rapid increase in the cross section with increasing angular momentum illustrates the sensitive dependence of the cross section on the yrast angular momentum cutoff.

The dispersion in the widths in Table II extracted for individual channels from the average width most probably reflects the finite range of data which has been analyzed to obtain these results. The results shown in Table I were obtained from an analysis of a larger sample of data and indeed the individual widths for each angle shown in Table I do show smaller deviations from their common average of 377 ± 60 keV.

The different values for the average widths given in Tables I and II (377 keV and 263 MeV, respectively) reflect the fact that the average width varies with excitation energy and that different ranges of excitation energy have been used in obtaining these averages. In Tables I and II, the ranges of excitation energy in ^{24}Mg were 30 to 35 MeV and 27.5 to 51.5 MeV, respectively.

A theoretical expression for the energy dependence of the average width can be obtained from simple considerations on the assumption that the nucleus is in thermal equilibrium.[27] The average width depends on the temperature, T, and thresh-

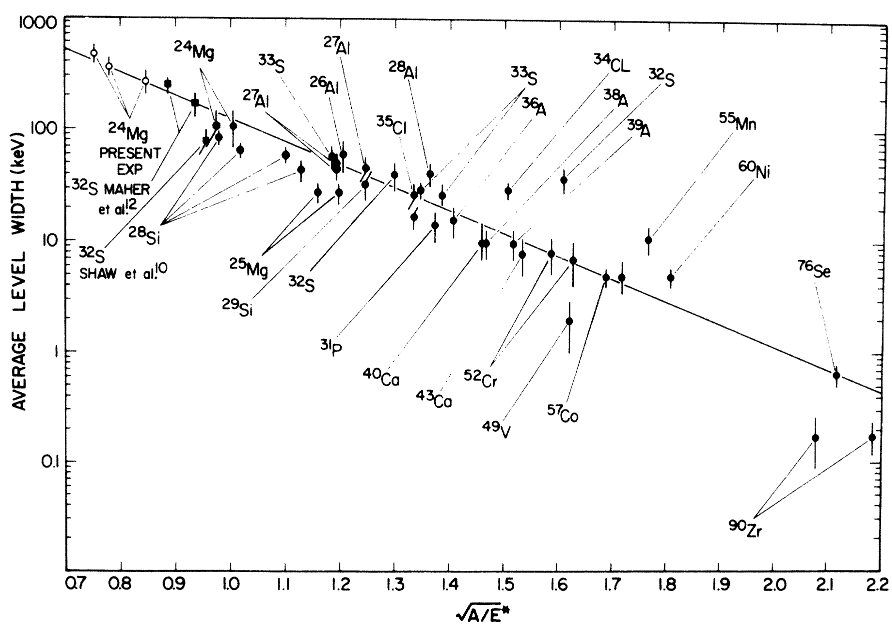

FIG. 7. The dependence of experimental widths on $\sqrt{A/E^*}$ (see caption to Fig. 4). The straight line fit to the data points has the same slope as the line shown in Fig. 4. Only the data shown as filled circles have been included in the least squares fit.

old energy for particle emission, W, and is given by $\Gamma \sim \exp(-W/kT)$. Since the temperature varies as $\sqrt{E^*}$, we expect $\ln\Gamma \propto (E^*)^{-1/2}$ where E^* is the excitation energy. Figure 4 shows that the present results for the energy dependence of Γ are in agreement with this simple model. If one takes into account the variation of temperature with mass number and neglects the variations of pairing energies and particle emission thresholds from nucleus to nucleus, it follows that $\Gamma = C\exp[-\alpha \times (A/E^*)^{1/2}]$ where A is the mass of the nucleus and C and α are constants. Figure 7 presents the logarithms of the average widths taken from a recent compilation[28] plotted versus $(A/E^*)^{1/2}$ for various nuclei. The straight line is least-squares fitted to these data (filled circles only) and the open circles represent the widths for ^{24}Mg obtained in the present work. The excellent agreement with the values predicted by this semi-empirical evaluation of coherence widths obtained for other nuclei strongly suggests that the origin of narrow structure in the ^{12}C(^{12}C, ^{12}C)^{12}C excitation functions is compound elastic scattering.

The above comparison neglects the dependence of the average width on angular momentum. Thus, we would not expect, *a priori*, a coherence width deduced from an analysis of ^{23}Na(p,α)^{20}Ne to agree with the value obtained from ^{12}C(^{12}C, α)^{20}Ne because of the different angular momenta in the entrance channel, and hence in the compound system. Estimates of this angular momentum dependence for widths in ^{24}Mg have been made using the statistical model relation

$$\frac{\langle\Gamma^J\rangle}{\langle D^J\rangle} = \frac{1}{2\pi}\sum_{c''l} T_l^{Jc''} \tag{6}$$

and a Fermi gas model to compute the density of levels $(D^J)^{-1}$. The procedure for the evaluation of the number of open channels $(\sum_{c''l} T_l^{Jc''})$ and the level density will be described in Sec. IV C. The results for Γ^J are shown in Fig. 4 for spins $J = 8$, 10, 12, and 14. The theoretical values compare favorably with the experimental values over the energy range studied. The angular momentum dependence is not pronounced, a result which has been noted by several workers,[27] and this probably accounts for the general agreement shown in Fig. 7.

While the agreement between the theoretical and the experimental widths shown in Fig. 4 is good, the uncertainty in the level density parameter "a" ($a/A = 0.14$ MeV^{-1} was used in the Fermi-gas formula to obtain D^J) places a $\pm 50\%$ confidence limit on the theoretical values of Γ^J. Thus the agreement between the widths predicted on a semi-empirical basis and the measured ones shown in Fig. 7 provides the stronger argument in favor of the compound origin of the fluctuating structure.

B. Comparison of cross section distributions

The statistical model predicts that the fluctuations in the cross sections should be distributed in the form specified by Eq. (3). Figure 3(b) shows that the agreement obtained for $\theta_{c.m.} = 90°$ is quite good when y_d^{obs} is adjusted to the value 0.956. Similar agreement was found at the other four angles, and the values of y_d^{obs} obtained from this analysis agreed well with those from the autocorrelation analysis (see Table I).

The fluctuations in the cross sections for excitation functions at angles differing by more than the coherence angle $\theta_c \sim 1/kR$ should be uncorrelated[29] provided several partial waves contribute to the cross section. Since the data are at 10° intervals and $\theta_c \lesssim 6°$ for the present case, no strong correlations would be expected on this basis. However, Fig. 6 shows that the cross section can be dominated by a single partial wave at some ener-

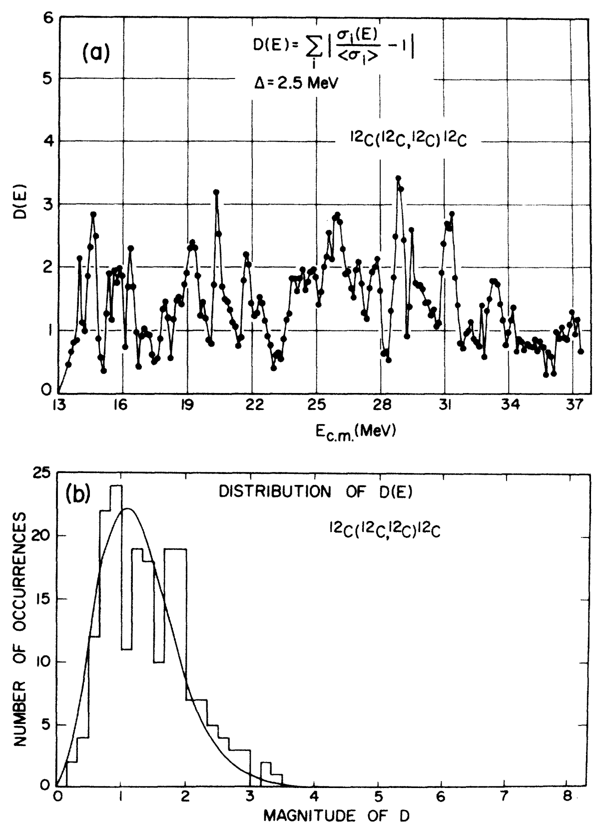

FIG. 8. (a) The cross correlation function $D(E)$ for elastic scattering at five angles. (b) The distribution of $D(E)$. The calculated distribution (continuous curve) is obtained by assuming statistical independence of the fluctuations in the five individual excitation functions.

gies, which might place this assumed independence in question. The degree of independence may be tested by considering the fractional deviations of the cross sections from their average values at each angle. Figure 8(a) shows the quantity

$$D(E)=\sum_{i=1}^{5}\left|\frac{\sigma_i(E)}{\overline{\sigma_i(E)}}-1\right|,$$

where $\sigma_i(E)$ is the cross section at the ith angle at energy E.[30] Although no strong cross correlations are apparent in this figure, the statistical independence of the fluctuations in the five elastic excitation functions can be tested more quantitatively by comparing predictions for the distribution of $D(E)$, based on such an independence hypothesis, to the observed distribution. The distribution of $D(E)$ can be predicted in a combinatorial calculation since the distribution of the fluctuations at each angle is known [Eq. (4)] and the data at each angle are assumed to be statistically independent. In Fig. 8(b) we present the experimental and the calculated distributions of $D(E)$ as a histogram and a full curve, respectively; the good agreement shown here does not indicate any significant angular cross correlations. The results obtained from a cross correlation analysis of the six α-particle channels listed in Table II are shown in Fig. 9. Again, the agreement is fairly good. The lack of cross correlations in the data and the predicted domination of the cross section by a single partial wave at some energies (see Fig. 6) is most likely explained in the following way. In the theoretical expression for a fluctuating differential cross section, amplitudes for the individual partial waves were added coherently. In terms of *amplitudes*, the contributions of the adjacent partial waves become relatively larger than would be indicated by Fig. 6, which gives angle-integrated cross sections. These contributions in amplitude may well be sufficient to remove the correlation which would be obtained if only one partial wave amplitude were present.

C. Comparison of average magnitudes of fluctuations

In this subsection we compare the predictions of the Hauser-Feshbach statistical model for nuclear reactions with the experimentally deduced fraction of the cross section which proceeds via nondirect reaction mechanisms. The average fluctuating cross section is related to the average normalized variance by Eq. (3) and the relation $\langle\sigma_{fl}\rangle=\langle\sigma\rangle(1-y_d)$ where $\langle\sigma\rangle$ and $\langle\sigma_{fl}\rangle$ are the averages of the measured cross section and its fluctuating component, respectively. The magnitude of y_d is assumed constant over the entire range of bombarding energy, while the average value of the cross section, $\langle\sigma\rangle$, is determined at each energy by a 2.5 MeV wide running average. (The value of y_d is in fact energy dependent as is the case with Γ. However, analysis of subintervals shows this dependence to be small and imprecisely determined so that we are justified in using a constant value in this analysis.) The experimental cross sections determined in this way for the fluctuating component of the $^{12}C(^{12}C,{}^{12}C)^{12}C$ and $^{12}C(^{12}C,\alpha)^{20}Ne$ reactions are presented in Figs. 10 and 11.

The Hauser-Feshbach theory has had considerable success recently in predicting absolute cross sections for compound nuclear reactions in this mass region and well above the Coulomb bar-

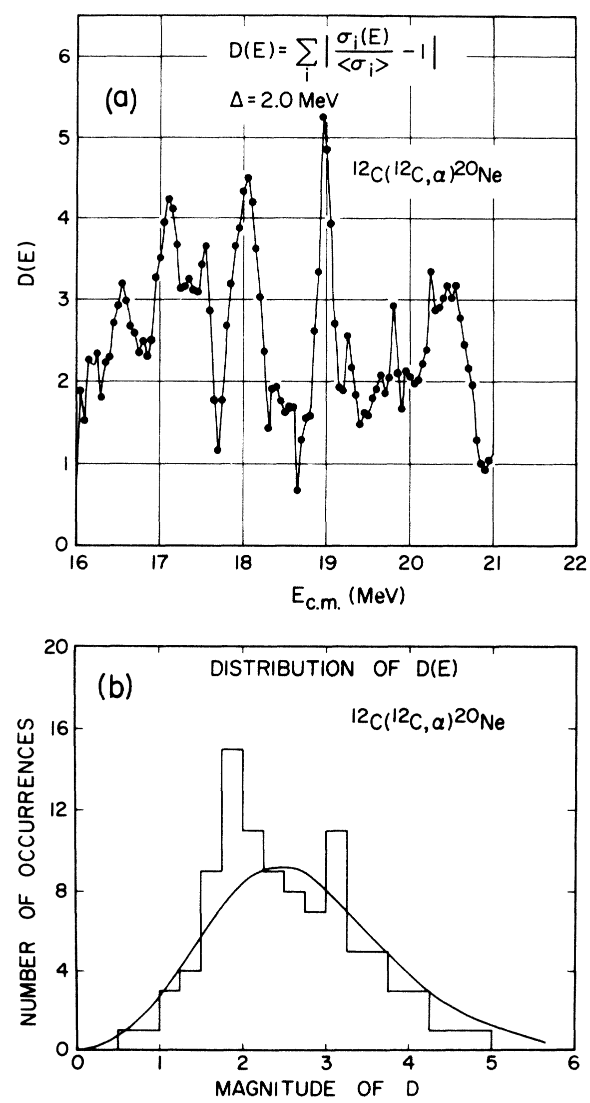

FIG. 9. Cross correlation analysis of the $^{12}C(^{12}C,\alpha)^{20}Ne$ reaction channels; see caption to Fig. 8.

rier.[31-33] Earlier studies[7,8,34] also emphasized the promise of this method for estimating compound nuclear cross sections for heavy ion reactions. We refer here to the full expression for the theoretical cross section as given by Hauser and Feshbach[35] and not to an approximate form of the theory as developed by Eberhard *et al.*[36] which is also used frequently[17] and which is usually normalized to the experimental data at some point. A discussion of the difficulties presented by this particular approximation in evaluating the number of open channels in conjunction with heavy ion reactions has been given by Greenwood *et al.*[32]

The calculations were performed with the computer code STATIS[37] which evaluated the following expression for the differential cross section for scattering from channel α to channel α'.

$$\frac{d\sigma}{d\Omega_{\alpha\alpha'}}(\theta)=W_{\Pi}W_{s}W_{D}\sum_{L}\frac{1}{4k_{\alpha}^{2}}\sum_{J,\pi}\frac{1}{(2i+1)(2I+1)}\left[\sum_{s,l}T_{l}(\alpha)\right]\times\sum_{s'l'}\left[\frac{T_{l'}(\alpha')}{\sum_{c''l''}T_{l''}(c'')}\right]\bar{Z}(lJlJ;sL)\bar{Z}(l'Jl'J;s'L)(-)^{s-s'}P_{L}(\cos\theta). \tag{7}$$

Except for the first three factors in Eq. (7), the notation used in conjunction with the optical model transmission coefficient T_l, and angular momentum recoupling coefficients $\bar{Z}$ is standard[8,35] and self-explanatory. The denominator $\sum_{c''l''}T_{l''}(c'')$ includes a summation over all possible outgoing channels. Low lying states of known spin in the residual nuclei were summed explicitly and a level density expression derived by Lang[38] was used for higher excitation energies in these nuclei. This level density expression and a general description of this type of calculation are given in Ref. 31. The factors W_s, W_D, and W_Π are associated with the symmetrization of the scattering amplitude required by the presence of identical particles, the inability of the detector to distinguish the two reaction products in the exit channel, and the fluctuation width correction, respectively. The presence of identical bosons in the entrance channel requires $W_s=2$ in all calculations described here and limits the summation in Eq. (7) to even l values, J values, and positive parity. The identity of ^{12}C ions in the exit channel for elastic scattering obviously requires that $W_D=2$ for this case. Although the detection systems used by Wieland *et al.*[16] and Emling *et al.*[17] separate elastic and inelastic scattering, they do not distinguish for the case of inelastic scattering which ^{12}C ion is in the excited state and which is in the ground state. Thus $W_D=2$ for inelastic scattering and $W_D=1$ for the $^{12}C(^{12}C,\alpha)^{20}Ne$ reaction. The origin and value of W_Π is discussed below.

In his treatment of fluctuating cross sections Moldauer[39] obtained the following expression for the average compound cross section, similar in appearance to the Hauser-Feshbach formula, but valid for arbitrary values of Γ and D.

$$\sigma_{\alpha\alpha'}=\frac{\pi}{k_{\alpha}^{2}}\left(\left\langle\frac{\theta_{\lambda\alpha}\theta_{\lambda\alpha'}}{\theta_{\lambda}}\right\rangle_{\lambda}-M_{\alpha\alpha'}\right). \tag{8}$$

The fluctuation width correction $W_{\Pi}^{\alpha\alpha'}$ arises from replacing the first term in Eq. (8) by $\langle\theta_{\lambda\alpha}\rangle\langle\theta_{\lambda\alpha'}\rangle/\langle\theta_{\lambda}\rangle$ where $\langle\theta_{\lambda\alpha}\rangle=(2\pi/D)N_{\lambda}|g_{\lambda\alpha}|^{2}$, and $\theta_{\lambda}=\sum_{\alpha}\theta_{\lambda\alpha}$ and $M_{\alpha\alpha'}$ is a term arising from resonance-resonance interference effects. (N_λ is a volume integral of the wave function for the resonant state.) Assuming the amplitudes $g_{\lambda\alpha}$ for different channels to be uncorrelated, Moldauer showed that, in the limit of large Γ/D *or* many competing direct transitions, $M_{\alpha\alpha'}$ becomes small and $\langle\theta_{\lambda\alpha}\rangle_{\lambda}$ may be replaced by the transmission coefficient for compound nucleus formation. Furthermore the fluctuation width correction W_Π in the limit of large Γ/D becomes $W_{\Pi}^{\alpha\alpha'}=1+a\delta_{\alpha\alpha'}$ with $1\leq a\leq 2$ and, in the presence of many competing direct transitions, $a\to 1$. Since in the present study Γ/D as given by Eq. (6) is usually greater than 5 *and* there is significant competition from direct reactions, the resonance-resonance interference term has been neglected and W_Π has been set equal to $1+\delta_{\alpha\alpha'}$.

The unitarity of the S matrix requires the existence of correlations among the amplitudes $g_{\lambda\alpha}$, a fact which is neglected in the above treatment. Gibbs[39] has noted that the consequences of including unitarity in the statistical model are small except possibly for the case of elastic scattering. Although he has treated elastic scattering quantitatively within a simple model, the uncertainties in the resultant formulas, associated with the removal of large amounts of flux from the entrance channel by direct reactions, prevent their application to $^{12}C+^{12}C$ scattering at these energies.

The transmission coefficients were calculated with the optical model code ABACUS-2.[40] Standard optical potential parameters were used for protons and neutrons.[41] For the $^{12}C+^{12}C$ and $\alpha+^{20}Ne$ channels optical potentials taken from coupled channel fits to the elastic and inelastic scattering in these channels[42,43] were used. In addition the higher partial waves near to and including the grazing partial wave were excluded at higher energies by the introduction of a yrast limit on the total angular momentum in the compound nucleus. This effectively set $T_l = 0$ for the partial waves that are expected to contribute mainly to direct reactions. The requirement that the transmission coefficient used in Eq. (7) should correspond only to compound nuclear absorption is thus approximately met by the above choices of optical potentials for $\alpha + {}^{20}\text{Ne}$ and ${}^{12}\text{C} + {}^{12}\text{C}$ and by including a yrast limit cutoff in the compound nucleus. The parameters determining this cutoff (see Table I) resulted in yrast limits for $J = 14$ and $J = 16$ at energies below $E_{\text{c.m.}} \simeq 21.5$ MeV and $E_{\text{c.m.}} \simeq 28$ MeV, respectively.

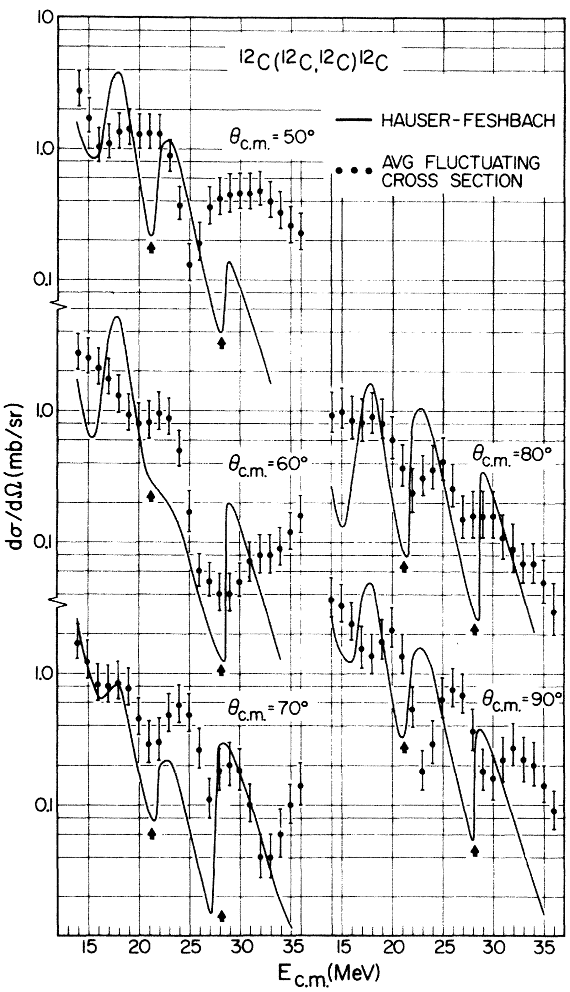

FIG. 10. The average fluctuating component of the ${}^{12}\text{C} + {}^{12}\text{C}$ elastic cross section (energy averaged over 2.5 MeV) and corresponding Hauser-Feshbach predictions. The indicated errors reflect the uncertainty in the values of y_d obtained in the fluctuation analysis. The discontinuities at the arrows are caused by the sharp yrast cutoff which allows the values $J = 14$ and $J = 16$ to contribute beginning at ~21 and 28 MeV, respectively.

The parameters are similar to those used in an analysis of ${}^{12}\text{C} + {}^{14}\text{N}$ reactions[31] and the resulting cutoff values agree closely with the limiting angular momenta in the entrance for fusion of ${}^{12}\text{C} + {}^{12}\text{C}$ predicted by a semiclassical model recently suggested by Bass.[44]

Table III lists all the parameters used in the optical model calculations, in the level density formula, and in the yrast cutoff calculations. A discussion of the sensitivity of the calculated cross sections to the various input parameters is given in Ref. 31 and applies here as well. The very shallow imaginary potential depths for the ${}^{12}\text{C} + {}^{12}\text{C}$ and ${}^{20}\text{Ne} + \alpha$ channels arise from the inclusion of coupled channel effects in their derivation from a comparison to experimental data.[42,43] The weak absorption of the ${}^{12}\text{C} + {}^{12}\text{C}$ potential results in some degree of transparency even for $l = 0$ partial waves. This unphysical result for the lower partial waves is not of real consequence for the present comparison because these low partial waves make a negligible contribution to the predicted cross sections (see Fig. 6). The important feature is that compound nuclear absorption be reduced for the near-grazing partial waves, and reducing the magnitude of W is one way of accomplishing this.[42]

Uncertainties in the above parameters and in the value of W_{fl} (especially at energies where Γ/D is not very large) lead us to expect the absolute normalization of the theoretical compound nuclear cross sections to be accurate to about a factor of 2. Figures 10 and 11 show the calculated compound nuclear and experimental fluctuation cross sections for the elastic channel and six reaction channels. The agreement for elastic scattering and for several ${}^{12}\text{C}({}^{12}\text{C}, \alpha){}^{20}\text{Ne}$ channels is good. In some cases, however, the predicted ${}^{12}\text{C}({}^{12}\text{C}, \alpha)$-${}^{20}\text{Ne}$ cross sections are higher than the experimental cross section (by a factor of 2 at most). Discontinuities in the calculated cross sections in Fig. 10 occur at a bombarding energy where an additional partial wave is suddenly allowed by the sharp yrast cutoff. A modulating structure in addition to this, and which is also apparent in Fig. 10, arises from the use of very weakly absorbing potentials which permit "shape resonances" in the variation of T_l with energy.

It should be emphasized again that these Hauser-Feshbach predictions are absolute and not normalized to these data. Thus, for elastic scattering, the amount of compound scattering which is theoretically predicted is consistent with the average size of the fluctuations present in the data.

Fluctuating structure in the excitation functions for ${}^{12}\text{C} + {}^{12}\text{C}$ inelastic scattering has also been observed. The measurements of Emling *et al.*[17] were carried out in sufficiently fine steps to per-

TABLE III. Parameters used in optical model calculations. The spin cutoff factor is evaluated using a rigid body moment of inertia $\mathscr{I} \sim \frac{2}{5} mR^2$ where $R = r_0 A^{1/3}$ and $r_0 = 1.5$ fm. For the yrast cutoff in ^{24}Mg, values of $r_0 = 1.25$ at lower energies and $r_0 = 1.30$ for higher excitation (see Ref. 31) energies were used. Δ_p is the pairing energy.

	Optical model parameters						
Channel	$V_{(\text{real})}$ (MeV)	V_{imag} (MeV)	R_{real}	a_{real}	R_{imag}	a_{imag}	Ref.
^{20}Ne + α	50.0	2.0 [a]	4.94	0.59	4.94	0.46	42
^{16}O + ^{8}Be	14.0	$0.4 + 0.15E^*$ [a]	6.10	0.49	6.10	0.49	
^{12}C + ^{12}C	14.0	0.82 [a]	6.18	0.35	6.41	0.56	43
^{23}Na + p	$56.0 - 0.55E^*$	13.5 [b]	3.56	0.65	3.56	0.47	41
^{23}Mg + n	$48.2 - 0.3E^*$	11.5 [b]	3.56	0.65	3.55	0.47	41
	Level density parameters [c]						
	^{24}Mg (CN)	^{20}Ne	^{16}O		^{12}C	^{23}Na (^{23}Mg)	
a/A	0.140	0.149	0.149		0.149	0.167	
Δ_p	5.13	5.13	5.13		5.13	2.67	

[a] Volume absorption.
[b] Surface absorption.
[c] See Refs. 31 and 45.

mit an autocorrelation analysis, and they obtained a value of $R(0)^{\text{obs}} = 0.257 \pm 0.061$ for the ^{12}C(^{12}C,^{12}C)-^{12}C* (4.43) reaction at $\theta_{\text{c.m.}} = 90°$ over the energy range $E_{\text{c.m.}} = 10$–18 MeV using a running average of $\Delta = 2.1$ MeV. If we correct their value of $R(0)^{\text{obs}}$ for the finite range of data and for the effect of averaging with $\Delta = 2.1$ MeV, a value of $R(0) = 0.290^{+0.10}_{-0.07}$ results. Emling *et al.*[17] use $N_{\text{eff}} = 2$ which then yields a value of $y_d = 0.65^{+0.10}_{-0.18}$. The average fluctuating cross section in the region $E_{\text{c.m.}} = 13$–18 MeV is thus $\langle \sigma_{\text{fl}} \rangle \sim 0.7^{+0.4}_{-0.2}$ mb/sr, and the corresponding Hauser-Feshbach prediction for this energy region is 0.6 mb/sr. On the assumption that y_d does not change appreciably with energy we used the same y_d value for ^{12}C + ^{12}C inelastic cross sections measured by Wieland *et al.*[16] over the energy range $E_{\text{c.m.}} = 20$–30 MeV. Figure 12 shows the cross sections and the corresponding Hauser-Feshbach prediction for this region. The agreement is excellent and the statistical model therefore accounts for the magnitude of the fluctuations in the inelastic as well as the elastic scattering.

In summary, the width, strength, and cross correlations predicted by the statistical models of nuclei and of nuclear reactions agree very well with the characteristics of the narrow structure appearing in the measured cross sections for the elastic, inelastic, and reaction channels. It seems reasonable to conclude on this basis that the origin of this fluctuating structure is statistical compound nucleus formation.

V. INTERMEDIATE STRUCTURE

In the preceding section we have shown that the fluctuating structure with a characteristic width of

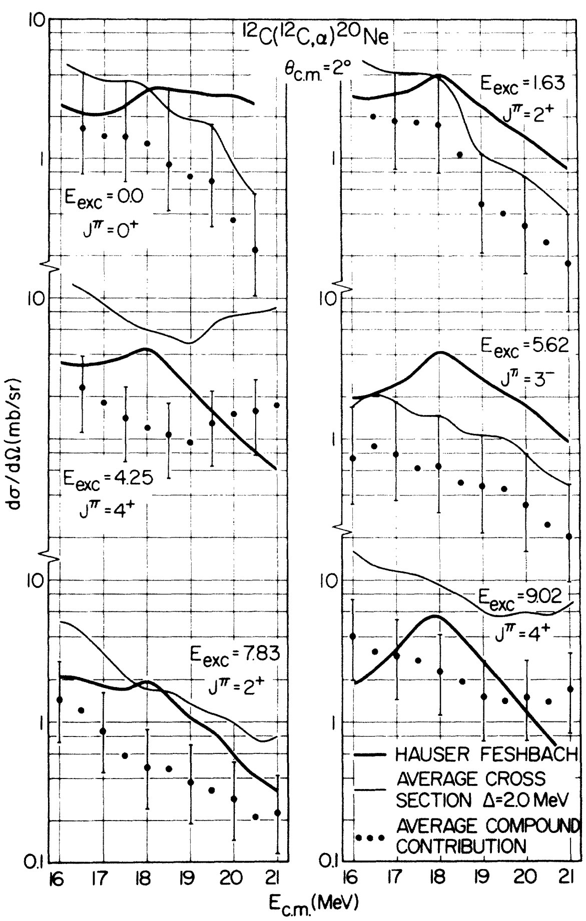

FIG. 11. The average fluctuating components of cross sections for ^{12}C(^{12}C, α)^{20}Ne and corresponding Hauser-Feshbach predictions.

~300 keV originates with statistical compound nucleus formation. It is also well known[46,47] that cross sections exhibiting statistical fluctuations may, with reasonable probability, contain occasional individual peaks in an excitation function with widths or spacings two or even three times larger than the characteristic width. The only way to positively identify structures of nonstatistical origin in the presence of strong fluctuations is to observe structure which falls outside the range of the statistical model, i.e., a correlation or fluctuation which, statistically, is very unlikely. Although the present data indicate no such correlations in the elastic scattering or in the $\alpha + {}^{20}Ne$ reaction channels, Jansen and Scheid[48] have suggested that this test may be insufficient to establish the presence of intermediate structure in this case. In particular, they have performed a cross correlation analysis on theoretical excitation functions of Fink *et al.* in which the structure necessarily arises solely from known direct and intermediate mechanisms. They report no apparent correlations and that the "compound" component of the cross section, deduced by applying an autocorrelation analysis to the results of their original calculation, was not unlike the value of $(1-y_d^{obs})$ deduced from the experimental data.[17] They conclude, therefore, that the results of an autocorrelation analysis for ${}^{12}C + {}^{12}C$ are not a reliable indicator of the reaction mechanism.[48]

It is clear that an autocorrelation analysis, i.e., the extraction of $R(\epsilon)$ from the experimental data, determines only the average width and strength of the component of the cross section which varies with energy; by itself it provides no indication of the physical origin of this energy variation. It is only through a comparison of $R(\epsilon)$ with independent theoretical predictions (such as is made in Sec. IV) that information on the origin of the fluctuating structure is gained.

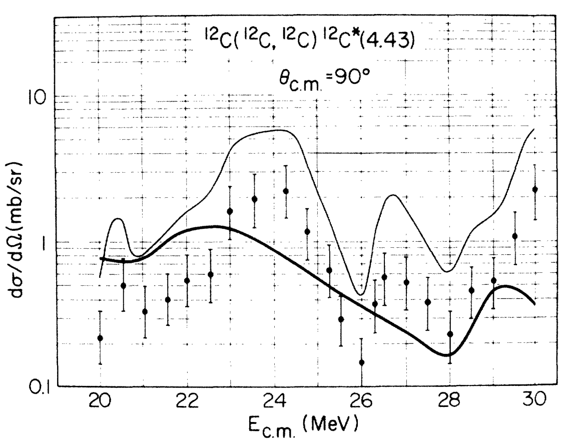

FIG. 12. ${}^{12}C({}^{12}C,{}^{12}C){}^{12}C^*$ $(2^+, 4.43)$ inelastic cross sections and Hauser-Feshbach predictions (heavy line). The measured cross section (Ref. 16) (light line) and the average fluctuating component of the cross section (points) are related by a value of y_d which is based on the value given by Emling *et al.* (Ref. 17) for lower energies.

We have performed a statistical analysis of these same theoretical cross sections[49]; several comments in addition to Ref. 48 are in order. The analysis was performed in a manner analogous to that discussed above in our analysis of the elastic data; the slow energy variation was removed by dividing out a running average of the cross section obtained with $\Delta = 2.5$ MeV. The range of energy included in the analysis was from 13.5 to 20 MeV. Predicted cross sections at higher energies were not included because the coupling of the elastic channel and 2^+ (4.43 MeV) inelastic channels does not produce structure above 20 MeV. (Higher excited states would have to be included in the calculation before a comparison could be made.) The results of such an analysis are given in Table IV. We obtain values of $R(0)^{obs}$ comparable to those reported by Jansen and Scheid.[48] From peak counting we obtained $\Gamma \approx 800$ keV for the characteristic width of the structure in their theoretical calculations. Since these theoretical cross sections should be analyzed in exactly the same manner as were the experimental data, this large value of Γ results in very large corrections that must be applied to account for the effect of using an averaging interval $\Delta = 2.5$ MeV. Such corrections result in values of $R(0)$ which yield a rather small direct component (consistent with 0 for the 70, 80, and 90° theoretical cross sections). Such large "compound" cross sections and large widths are not indicated by the statistical analysis of the experimental data and also could not be accounted for

TABLE IV. Fluctuation analysis of theoretical ${}^{12}C - {}^{12}C$ data. $\Delta = 2.5$ MeV.

Angle (deg)	$R(0)^{obs\,a}$	$R(0)^{b}$	y_d [b]	$\Gamma^{obs\,a}$ (keV)	$\Gamma^{obs\,b}$ (keV)	$\Gamma^{PC\,c}$ (keV)
50	0.10	$0.34^{+0.20}_{-0.18}$	$0.81^{+0.10}_{-0.13}$	241	687^{+101}_{-94}	830
60	0.13	$0.42^{+0.25}_{-0.22}$	$0.76^{+0.13}_{-0.49}$	357	1017^{+145}_{-139}	750
70	0.22	$0.76^{+(0.24)}_{-0.40}$	$0.49^{+0.31}_{-0.49}$	316	900^{+133}_{-123}	940
80	0.24	$0.82^{+(0.18)}_{-0.43}$	$0.42^{+0.36}_{-0.42}$	335	955^{+141}_{-131}	750
90	0.28	$0.93^{+(0.07)}_{-0.49}$	$0.26^{+0.49}_{-0.26}$	316	900^{+133}_{-123}	750

[a] Using the autocorrelation method [Eq. (3)].
[b] Includes corrections for FRD and the effect of an averaging interval of $\Delta = 2.5$ MeV.
[c] Using the peak counting method.

within the framework of any reasonable statistical models of nuclear structure and reactions. Thus a self-consistent statistical analysis of these theoretical cross sections (treated as statistical fluctuations) yields results which are at variance with both the experimental data and statistical models. Two related questions present themselves at this point:

(i) What would be the effect on the present statistical analysis of the experimental data if there were a significant component of intermediate structure of the type predicted by Fink *et al.*[15] present in the experimental data?

(ii) Could the present statistical analysis give any positive indication of such structure if it were present in the experimental data, given the presence of statistical fluctuations with $\Gamma \sim 300$ keV.

The answers to these questions are obtained from a study of synthetic excitation functions presented in Appendix A. Synthetic excitation functions were generated which contained both a 300 and an 800 keV fluctuating component. These components had equal intensities, i.e., equal average cross sections. Values of $R(0)^{obs}$ and Γ^{obs} obtained from an analysis of these excitation functions were then compared with those obtained from excitation functions containing only the 300 keV component. The results of the comparison depend on the value of Δ used in the respective analyses since the 800 keV structure is attenuated more than the 300 keV structure, particularly for smaller values of Δ. We find that Γ^{obs} increases by 15% when the 800 keV structure is added and when $\Delta = 1.5$ MeV. If $\Delta = 2.5$ MeV, this increase is 22%. Given the uncertainties in the values of Γ^{obs} and $R(0)^{obs}$ extracted from the experimental data, such changes in Γ^{obs} or in $R(0)^{obs}$ are not sufficient to seriously affect the analysis of the ~300 keV structure. Similarly, the present analysis is not sufficiently sensitive to either detect the presence or demonstrate the absence of 800 keV structure at a level of intensity equal to that of the ~300 keV fluctuating component. It is thus possible that some of the structure in the excitation function for the elastic scattering of $^{12}C + ^{12}C$ originates with the virtual excitation of quasibound states.[14,15] However, the positive evidence for the effects of this reaction mechanism, which has been sought in this study and in studies of the inelastic scattering,[17] has not yet been found. The main definitive result coming from the statistical analysis is that the compound mechanism accounts, with reasonable probability, for all of the fluctuating structure observed in the data including the occasional peaks which are spaced 600–800 keV apart.

Low and Tamura[46] have noted that the amount of structure predicted by Fink *et al.*[15] depends on the strength of the imaginary potential in the coupled or quasibound channel. This was taken to be zero, and other coupled channel calculations, in which a nonzero imaginary potential was used, did not produce any intermediate structure.[43] Furthermore, an assumption which is apparently not valid for this particular reaction is the "never-come-back" approximation[15] which implies that flux leaving the entrance channel and forming a compound nucleus does not return to the entrance channel. The amount of compound elastic scattering predicted by the Hauser-Feshbach model indicates that an amount of this flux returns which is sufficient to account for the structure observed in the elastic scattering. It would be interesting with regard to the question of intermediate structure to have a nuclear reaction in which the coupling to excited states of the projectile and target were strong, and the compound elastic scattering were weak. In this case, compound nuclear fluctuations would not hinder the verification of the mechanism suggested by Fink *et al.*[15]

Although no evidence for nonstatistical behavior has been observed in the experimental data studied here, Van-Bibber *et al.*[50] have observed an anomaly in the cross section for $^{12}C(^{12}C,p)^{23}Na$ at $E_{c.m.} = 19.3$ MeV. The origin of this structure is not known and the absence of any corresponding anomaly in the elastic channel and in the $\alpha + ^{20}Ne$ channels investigated here is noteworthy (see Figs. 8 and 9).

The α-cluster "doorway" state model[6] may also predict resonant structure in the excitation function. The difference between such a picture and the statistical compound nucleus may not be that great if the α-cluster model contains sufficient degrees of freedom and the intermediate levels overlap strongly. Clustering effects may also explain the large direct components observed in some of the $^{12}C(^{12}C,\alpha)^{20}Ne$ reaction channels studied.

Finally, there has been considerable discussion in the literature on the possible existence in $^{16}O + ^{16}O$ scattering of two types of structure in addition to the gross structure.[9,10,12-14] The argument for two types of structure was based on a fluctuation analysis by Shaw *et al.*[10] which yielded $\Gamma \approx 80$ keV at an average c.m. energy of 18.5 MeV, and on the observation of broader structure, 200–300 keV wide, at higher bombarding energies (see Fig. 7 of Maher *et al.*[12]). It should be noted, however, that much of this variation in width could be accounted for in the framework of the statistical model of nuclei. Included in Fig. 7 as filled squares are the values of Γ obtained by Shaw *et al.*[10] and values of Γ obtained by counting the maxima in Fig. 7 of Ref. 12 (Maher *et al.*). The latter widths are in the range ~200–300 keV, even though the data were taken with 25 keV c.m. resolution and the agreement with the semiempirical predictions of the compound nuclear fluctuation widths is good. The verification of an intermediate mechanism in this system would require a simultaneous observation of both narrow and intermediate width structure over the same range of excitation energy and a demonstration that the wider peaks associated with the intermediate width structure could not result, with reasonable probability, from the narrower statistical fluctuations.

VI. SUMMARY AND CONCLUSIONS

A complete statistical analysis of the ^{12}C-$(^{12}C, ^{12}C)^{12}C$ scattering at five center of mass angles and of the $^{12}C(^{12}C, \alpha)^{20}Ne$ reaction at $\theta \sim 3^\circ$ for six exit channels has been performed. An examination of the assumptions underlying the statistical analysis has shown that the (unmodified) experimental data do not meet all the standard requirements for such an analysis. The consequences of small Γ/D were investigated (Appendix A) and were found to validate the analysis. The effects of gross structure on the statistical analysis have been treated by analyzing modified or reduced data obtained by dividing the experimental excitation function data by a running average of the cross section taken over an interval Δ. The effect of this averaging procedure has been studied quantitatively by an extensive investigation of synthetic excitation functions. The average compound contribution to the elastic cross section was found to be ~20% at 80 and 90° and smaller at the more forward angles. A large direct reaction component was observed in some of the $^{12}C(^{12}C, \alpha)^{20}Ne^*$ reaction channels. Predictions of the statistical models of nuclear structure and of nuclear reactions for the characteristic widths, compound component of the cross section, distribution of cross section fluctuations, and their cross correlations were compared with experimental values extracted from the data. Good over-all agreement for elastic, inelastic, and reaction channels was obtained, thereby demonstrating that the dominant mechanism producing the narrow fluctuations in the cross sections is statistical compound nucleus formation. This mechanism accounts for *all* of the fluctuating structure observed in the present experimental data. No evidence was found for nonstatistical phenomena. The sensitivity of statistical methods to the presence of theoretically predicted intermediate structure in this reaction was also studied. Given the presence of strong fluctuations ($\Gamma \sim 300$ keV) and strong gross structure ($\Gamma_{SE} \sim 2$–3 MeV) and the present finite range of data, it was not possible to conclusively demonstrate the presence or absence of the type of structure predicted by Fink *et al.*[15] The Hauser-Feshbach statistical model,[35] on the other hand, does predict, with remarkable precision, the amount of fluctuating structure in the elastic and inelastic scattering deduced from an analysis of the experimental data.

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APPENDIX A: SYNTHETIC EXCITATION FUNCTION

Synthetic excitation functions were generated and used to evaluate the correction factors and uncertainties associated with the fluctuation analysis of the data. The synthetic excitation function consists of a superposition of many S-matrix pole terms of the form[47,51,52]:

$$a^J_{\alpha\alpha'}(E) = \sum_\lambda \frac{g^J_{\alpha\lambda} g^J_{\lambda\alpha'}}{E - E^J_\lambda + (i/2)\Gamma^J_\lambda} \qquad \text{(A1)}$$

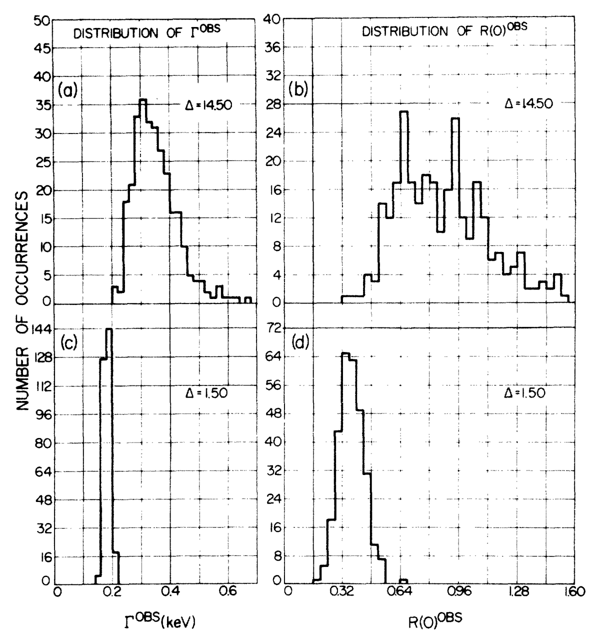

FIG. 13. Typical distributions obtained for $R(0)^{obs}$ and Γ^{obs} from the study of a set of ~300 synthetic excitation functions, all generated with the same input parameters [$\Gamma = 0.4$ MeV, $R(0) = 1.0$] but with different sets of random numbers. Note the shift in the distributions when Δ is varied.

with the partial cross sections given by

$$\sigma_{\alpha\alpha'}(E)=\frac{\pi}{k_\alpha^2}\frac{2J+1}{(2i+1)(2I+1)}\,|a^J_{\alpha\alpha'}(E)|^2. \quad \text{(A2)}$$

The quantities $g^J_{\alpha\lambda}$ and E^J_λ are real random numbers with the following distributions: The $g^J_{\alpha\lambda}$ are normally distributed around a mean value of zero. The level spacing $D_\lambda = E_{\lambda+1} - E_\lambda$ follows the Wigner distribution and Γ_λ is taken constant. The above statistics are consistent with Ericson's fluctuation model wherein the quantities $g^J_{\alpha\lambda}$ represent the partial width amplitudes, $\Gamma/D \gg 1$ and there are many open channels ($\Gamma_\lambda \approx \Gamma$). Moldauer[53] has pointed out, however, that the distributions of the resonance parameters that describe the region of isolated resonances do not necessarily prevail in the region of overlapping resonances, and has demonstrated, for the case of strong compound absorption that a wide distribution of level widths occurs even though many channels are open. In the case studied here a large part of the total absorption is due to direct processes and not to compound formation; thus we expect the distributions of widths and pole strengths to behave according to the Ericson limit for many open channels.

Our investigation consisted of analyzing sets of independent excitation functions, each generated with the same input parameters $R(0)$, D, and Γ. These were analyzed as outlined in Sec. II and distributions were obtained for Γ^{obs} and $R(0)^{obs}$. The centroids of these distributions then determine the average values of Γ^{obs} and $R(0)^{obs}$ and the variances determine the uncertainties associated with the extraction of these parameters from an excitation function with the same sample size. Figure 13 shows an example of a study of ~300 cases. Figures 13(a) and (b) show the distributions of $R(0)$ and Γ obtained in a regular fluctuation analysis in which, effectively, no running average is used, while parts (c) and (d) show the distribution occurring when $\Delta = 1.5$ MeV is used in the analysis. Figure 14 shows, as

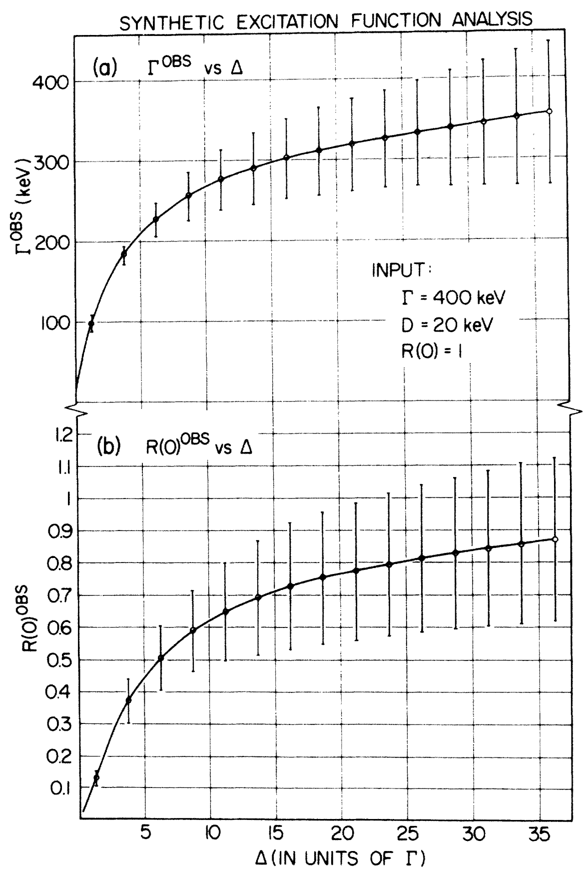

FIG. 14. Dependence of Γ^{obs} and $R(0)^{obs}$ on the value of Δ used in the analysis of the same excitation functions. The variation with Δ becomes small for values beyond $\Delta \gtrsim 20\,\Gamma$.

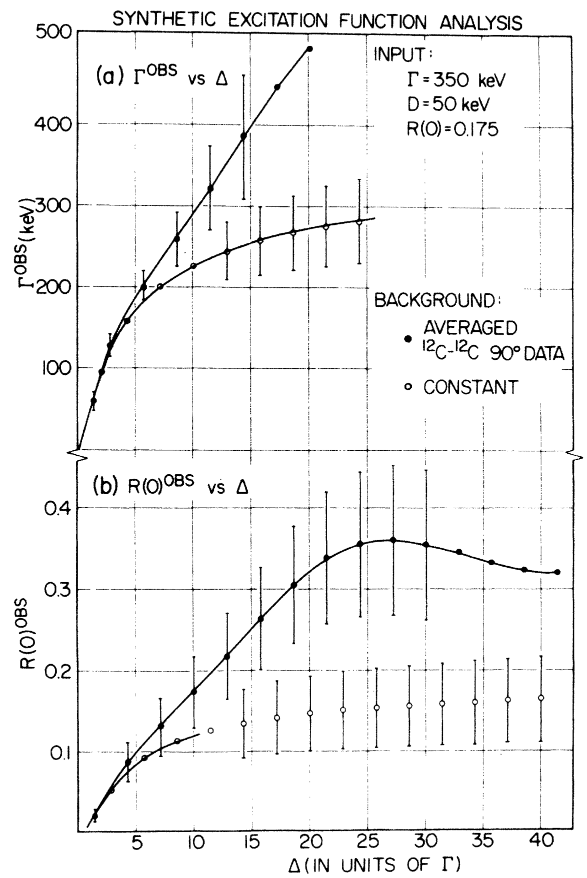

FIG. 15. $R(0)^{obs}$ and Γ^{obs} versus Δ for excitation functions with and without gross structure in the background. At small values of Δ the gross structure has very little effect.

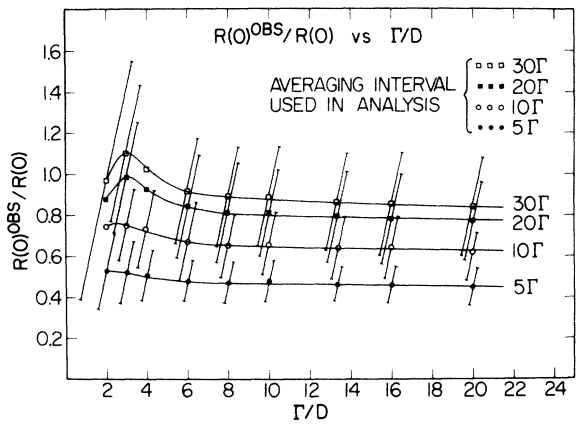

FIG. 16. Dependence of the correction factor $R(0)^{obs}/R(0)$ on the value of Γ/D used to generate the excitation functions. This dependence is shown for different values of the averaging intervals, Δ, used in the analysis.

a function of Δ, the variation of values of Γ^{obs} and $R(0)^{obs}$ extracted from a set of excitation functions all with input values of $\Gamma = 400$ keV, $D = 20$ keV, and $R(0) = 1.0$.

From studies of synthetic excitation functions with different amounts of constant ("direct") background it is found that the corrections to be applied to $R(0)^{obs}$ and Γ^{obs} are independent of the magnitude of the energy independent background.

In order to study the effect of gross structure on our analysis of data, fluctuating amplitudes were superposed on an averaged $^{12}C - ^{12}C$ excitation function (an interval of $\Delta = 2.5$ MeV was used). (A similar procedure has been used by Low and Tamura.[46]) The results of analyzing such composite excitation functions are compared in Fig. 15 with the analysis of synthetic excitation functions having the same direct to fluctuating cross section ratio but no gross structure. The advantage of working with small averaging intervals, i.e., small Δ, is demonstrated here. Thus, to analyze fluctuating cross sections with modulating structure, an initial estimate of the average width of these fluctuations is necessary and can be obtained using an independent estimate such as peak counting. Once Γ is known Δ/Γ can be used in conjunction with Figs. 14 or 15 to determine the proper correction factor.

Evaluation of Eq. (6) indicates that values of Γ/D are as low as 5 (corresponding to 60 open channels) at the lower $^{12}C + ^{12}C$ center of mass energies which would apparently place the present analysis in question. The effects of small values of Γ/D, however, can be studied with synthetic excitation functions. By repeating such studies with groups of excitation functions having different values of Γ and D one obtains the dependence of the correction factors for the extracted values of $R(0)^{obs}$ and Γ^{obs} on the value of Γ/D. Figure 16 shows the results of $R(0)$. $R(0)^{obs}/R(0)$ changes by ~25% for small Γ/D values but when small averaging intervals ($\Delta \sim 5\Gamma$) are used in the analysis this change is not as pronounced (~15%). For Γ^{obs}/Γ no such changes were observed.

A change expected in the distribution of the widths, Γ, due to small values of Γ/D could have some effect[47]; this was studied by analyzing several excitation functions using a distribution of widths centered around Γ instead of constant Γ; this spreading in Γ_λ was found to have only a small effect. Using distributions of pole-strength parameters $g^J_{\lambda\alpha}$ with different dispersions also had no effect on the correction factors derived.

The effect on the statistical analysis of the presence of additional (i.e., intermediate width) structure has been studied by superposing pole terms with two different distributions for the values of $g_{\alpha\lambda}$ and E_λ and different widths Γ_1 and Γ_2. Analyses of the resulting synthetic excitation functions revealed changes in $R(0)^{obs}$ and Γ^{obs} which were not sufficiently large to permit identification of such structure in the presence of strong gross structure and ~300 keV fluctuations.

APPENDIX B: EFFECTS OF THE FINITE RANGE OF DATA

The corrections applied to $R(0)^{obs}$ and Γ^{obs} for the $^{12}C - ^{12}C$ elastic data fluctuation analysis (Table I) were taken directly from the synthetic excitation function studies since the generated samples, by design, had the same sample size as did the experimental data. For the analysis of the ^{12}C-$^{12}C, \alpha)^{20}Ne$ data, however, we had to take into account the effect of a much smaller sample size.

The effects of the finite range of data on the values of $R(0)^{obs}$ and Γ^{obs} extracted in the autocorrelation analysis were calculated using the general methods and assumptions of Ref. 52. For the present work, where y_d is large, the results of Ref. 52, which were derived for $y_d = 0$, had to be modified. The following relation was derived for $R(0)$

$$R(0) = \frac{\alpha_d}{N} - \frac{\alpha_d}{N} a \frac{N + \alpha_d}{N + a\alpha_d} \pm (\overline{B}^2)^{1/2}, \qquad \text{(B1)}$$

where N is the number of effective channels, $\alpha_d = 1 - y_d^2$,

$$a = \frac{2}{m} \arctan m - \frac{1}{m^2} \ln(1 + m^2),$$

and $m = \Delta E/\Gamma$ is the sample size.

$$\overline{B}^2 = \alpha_d^{\,2}\,\frac{N+\alpha_d}{(N+a\alpha_d)^2}\,\frac{(2/m)(N+a\alpha_d)^2\arctan m - 2a^2N(N+\alpha_d) + 24\pi^2\alpha_d(N+\alpha_d)}{N^3+6a\alpha_d N^2+3(a\alpha_d)^2N+24\pi^2\alpha_d^{\,3}b}\,, \tag{B2}$$

where

$$b = \frac{1}{m^3}\arctan\left(\frac{m}{2}\right) - \frac{1}{m^4}\ln\left(1+\frac{m^2}{4}\right).$$

For the case of $y_d = 0$ and arbitrary m these expressions reduce to those given in Ref. 52.

For $y_d = 0$ and $m \gg 1$ (large sample size) the same bias correction and uncertainties quoted in Refs. 8 and 34 are obtained.

The effect of finite experimental energy resolution, δE, has been treated by several authors[52,54] for the case of $\delta E < \Gamma$. The first correction term is of the order of $(\delta E/2\Gamma)^2$ which is $\lesssim 3\%$ in our case for the $^{12}C - {}^{12}C$ elastic data and $\lesssim 1\%$ for our $^{12}C(^{12}C, \alpha)^{20}Ne$ data. An exact treatment of this effect for the case of a rectangular resolution function[55] showed it to be $\lesssim 3\%$ for all the data considered here. The uncertainties in $R(0)^{obs}$ arising from counting statistics could also be neglected, since in all cases they were less than 5%.

Finally, in order to obtain the combined effects of finite range and averaging interval Δ, the overall correction factor was assumed equal to the product of the individual correction factors. The latter were evaluated using Eq. (B1), the approximate formulas given in Refs. 8 and 34, and the results shown in Fig. 15.

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