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Analytic distorted waves for intermediate-energy alpha particles

R. T. Janus and I. E. McCarthy
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia 5042, Australia
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It is shown that physically consistent and realistic values of all the parameters can be found for a model that represents the distorted waves in the distorted-wave Born approximation for collective α inelastic scattering by a properly normalized decaying plane wave. The significance of this achievement in a thoroughly understood reaction is that we are now able to obtain analytic distorted waves that are sufficiently realistic and sufficiently simple to use in off-shell distorted-wave impulse approximation calculations of reactions of current interest requiring microscopic descriptions of the reaction mechanism.

[NUCLEAR REACTIONS ^{62}Ni(α,α'), $E=33$, 50, 100 MeV; ^{24}Mg, ^{48}Ti(α,α'), $E=41$ MeV; ^{32}S, ^{88}Sr(α,α'), $E=42$ MeV; analytic DWBA, calculated $\sigma(\theta)$.]

1. INTRODUCTION

We would like to take seriously an idea that has been little more than a curiosity in nuclear reaction theory for some years. This is the discovery by McCarthy and Pursey[1] that the shape of the angular distribution for α inelastic scattering at about 40 MeV could be well reproduced by an approximation to the distorted-wave Born approximation (DWBA) that represented the distorted waves by attenuated plane waves and restricted the reaction to the surface of the nucleus.

The attenuated plane wave model was suggested by the work of McCarthy[2] on the probability flux in the α-particle optical model and confirmed as realistic by Austern[3] for α particles and Amos[4] for nucleons. The local wave number of a computed distorted wave depends to a good approximation on radius but not on the angular coordinate in a spherical polar system centered at the nucleus. The real and imaginary parts of the wave number are given quite closely by the local WKB approximation for a given radius. The fact that the model gives an accurate representation of inelastic scattering is due to the restricted radial region involved. The effective wave number does not change much over the interaction region.

The reason for reviving the model distorted waves at this time is the growing need for absolute microscopic calculations of reactions. By this is meant the calculation of the requisite quasi-three-body terms in the matrix element using phenomenological representations of the interactions in each of the relevant quasi-two-body systems. The distorted-wave off-shell impulse approximation, which was developed for atomic physics by Hood, McCarthy, Teubner, and Weigold[5] and shown to give an excellent description of the $(e,2e)$ reaction, requires a description of the two-body interactions off the energy shell.

For investigation of the cluster structure of nuclei by $(\alpha,2\alpha)$ or $(p,p\alpha)$ or for microscopic (α,α') calculations, the $\alpha\alpha$ or $p\alpha$ t matrices must be computed from realistic potentials chosen to fit phase shifts. All such model t matrices are highly nonlocal. This means that the integrals in the distorted-wave approximation are 12-dimensional and very difficult to compute if the partial wave expansion of the distorted wave is used.

For many years we have been content to use unrealistic particle-particle interactions such as local finite range models or unrealistic reaction theories such as the on-shell impulse approximation, with very accurate distorted waves. For example the on-shell impulse approximation for $(\alpha,2\alpha)$ assumes that the $\alpha\alpha$ cross section is constant over the relevant energy-momentum interval. In fact it varies by more than an order of magnitude.

It is the purpose of this paper to show that in a completely understood reaction mechanism, the DWBA for collective (α,α') reactions, it is possible to relax somewhat the accuracy of the distorted waves without significantly upsetting the reaction description. The conclusion to be drawn from this is that we can use the same model distorted waves in calculations that are possible if we use them and impossible if we insist on partial wave expansions of optical model wave functions.

We will show that the parameters of the model distorted waves make sense physically and that consistencies over a wide range of nuclei and energies enable them to be determined in advance fairly accurately and refined by fitting (α,α') data.

2. COLLECTIVE INELASTIC SCATTERING

The distorted-wave theory of single-phonon excitations by α inelastic scattering is thoroughly understood. It thus provides an excellent testing ground for model distorted waves. The differential cross section is given for a phonon of angular momentum λ or for a rotational excitation by

$$\frac{d\sigma}{d\Omega} = \frac{k'}{k} \sum_{\mu} |T_{\lambda}^{\mu}(\vec{k}', \vec{k})|^2 ,$$

$$T_{\lambda}^{\mu} = \frac{m}{2\pi\hbar^2} \beta_{\lambda} R \int d^3r \, \chi^{(-)*}(\vec{k}', \vec{r}) \times \frac{dV(r)}{dr} Y_{\lambda}^{\mu}(\hat{r}) \chi^{(+)}(\vec{k}, \vec{r}) , \qquad (1)$$

where $V(r)$ is the optical model potential. For many reactions this cross section has been calculated. The optical model potential, characterized by Woods-Saxon parameters R and a and strength V_0, has proved to be essentially the same as for elastic scattering and the excitation parameter β_{λ} is roughly consistent with the corresponding number determined electromagnetically. The values of β_{λ} are improved[6] by coupled channel calculations, but for our purpose it is correct to use the values determined from the DWBA, since this is the theory we are approximating. All these parameters, which we will call reaction parameters, are well known for the reaction we will consider.

The model distorted wave to be used is

$$\chi^{(\pm)}(\vec{k}, \vec{r}) = e^{-\gamma k R_N} e^{i(1+\beta \pm i\gamma)\vec{k}\cdot\vec{r}} . \qquad (2)$$

The distortion parameters are:

β: wave number modification, allowing for the fact that the wave number in the interaction region is not the same as that for the incident beam. In the local WKB approximation

$$\beta \cong V(r)/2[E + V(r)] ; \qquad (3)$$

γ: absorption parameter, allowing for the fact that the distorted wave is attenuated by particle absorption as it enters the nucleus;

R_N: normalization parameter, chosen so that the magnitude of the wave function at the center of the nucleus is

$$\chi^{(\pm)}(\vec{k}, 0) = e^{-\gamma k R_N} . \qquad (4)$$

This form for the distorted wave has been confirmed[4] by calculating distorted waves in optical model potentials and plotting quantities such as $\ln|\chi^{(+)}|$, the phase ϕ, and the probability flux against radial and angular position. For example the plots of $\ln|\chi^{(+)}(\vec{r})|$ and $\phi(\vec{r})$ against $r\cos\theta$ are remarkably close to straight lines in the region where dV/dr is numerically significant.

3. PARAMETER DETERMINATION BY DATA FITTING

The optical model parameters for α elastic scattering have been thoroughly investigated. Although there are discrete and continuous ambiguities,[7] the values of R which make most sense in comparison with diffraction models are given in a compilation by Faivre, Krivine, and Papiau[8] as

$$R = (1.523A^{1/3} + 2.14) \text{ fm.}$$

In view of the ambiguities there does not seem to be any obvious way of choosing V_0. We have chosen for normalization purposes

$$V_0 = 60 \text{ MeV.}$$

Values of the surface parameter a vary with the nucleus, but they are usually quite close to

$$a \cong 0.35 \text{ fm.}$$

We will adopt the above values for R and V_0 and choose the value of β_{λ} determined by the DWBA for each reaction

By fitting the magnitude and shape of angular distributions we are able to determine β from the positions of maxima and minima, γ from the ratio of maxima to minima, a from the slope of the envelope of the maxima, and R_N from the magnitude. The fits are illustrated in Fig. 1.

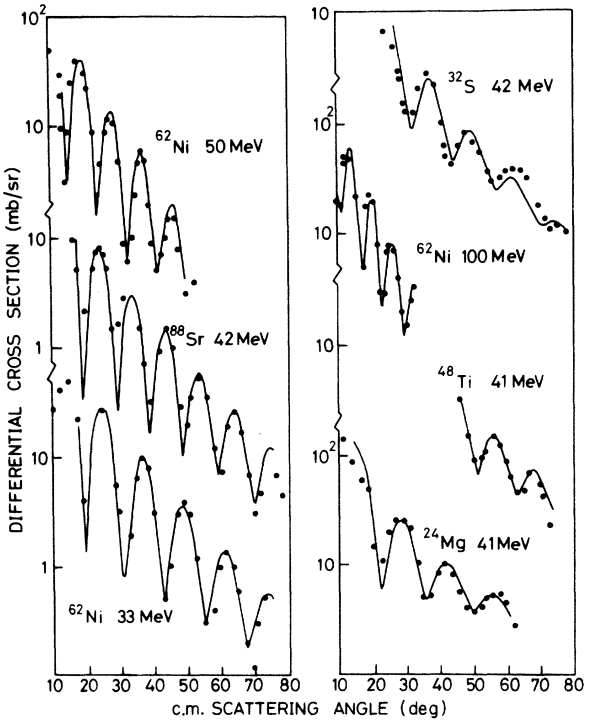

FIG. 1. Inelastic scattering of α particles to 2^+ states of nuclei (^{88}Sr is a 3- state). The experimental data are taken from Refs. 9–12.

TABLE I. Parameters determined by curve fitting. All excitations are 2+ except for ^{88}Sr, which is 3−.

Nucleus	E (MeV)	a (fm)	β	γ	$\gamma k R_N$	R_N (fm)
^{62}Ni	33	0.41	−0.41	0.023	1.12	20
^{62}Ni	50	0.41	−0.096	0.027	1.05	13
^{62}Ni	100	0.41	−0.106	0.031	1.03	8
^{24}Mg	41	0.33	−0.06	0.061	1.15	7
^{32}S	42	0.43	−0.06	0.055	1.19	8
^{48}Ti	41	0.35	−0.09	0.039	1.15	12.2
^{88}Sr	42	0.35	−0.16	0.019	1.02	19

The values of these parameters are given, for the cases we have examined, in Table I. The parameter β can be checked by Eq. (3). It is negative since the Coulomb potential dominates in the exterior region. In all cases it is close to the Coulomb value, e.g., for ^{32}S

$$-V_C/2(E-V_C) = -0.08 \text{ for } r = 8 .$$

The residual attractive nuclear interaction would increase this to the observed value −0.06. Since the wave function is attenuated the effective radius for calculating β is larger than the Woods-Saxon radius R.

The normalization parameter R_N normalizes the model distorted wave to a plane wave at a distance R_N before it enters the nucleus. Calculations of the probability flux[2] have shown that the Coulomb repulsion reduces the flux at the initial nuclear surface considerably so that R_N is quite large for large Z and small E. This is confirmed in Table I. For smaller Z or larger E, R_N is approximately the nuclear radius. The regularity observed in all cases is that the value of $\gamma k R_N$ is between 1.0 and 1.2. This enables a close estimate of the normalization of the wave function to be made, which can be improved by detailed fitting of inelastic scattering data.

4. CONCLUSIONS

By assuming realistic values, where possible, of the reaction parameters V_0, R, and β_λ we are able to obtain the reaction parameter a and the distortion parameters β, γ, and R_N by fitting inelastic scattering data. All parameters have physically meaningful values.

For the application of the model distorted waves to reactions that have not yet been calculated we use the converse of this proposition. By using physically meaningful parameters β, γ, and R_N, found by fitting inelastic scattering data, we obtain an accurate idea of the reaction parameters, in conjunction with values known from optical model studies of elastic scattering. We believe that the model distorted waves are good approximations to the distorted waves that actually describe the α-nucleus interaction, so that their use in more complicated reaction studies is justified.

In reaction studies using the off-shell distorted-wave impulse approximation, it will be necessary to use model distorted waves in a radial region larger than that for collective inelastic scattering. Since the local wave number modification parameter β is a function only of r to a good approximation, realistic treatment of its variation in the reaction volume involves a trivial extension of the method.

One question that is raised by the obvious success of this extremely simple model in fitting differential cross sections for inelastic scattering concerns the effect of the focus in the wave function intensity that is observed for nucleon[4,13] and α-particle[14] optical models. In the case of nucleons of energy about 100 MeV or less, the magnitude of the wave function in the focal region is up to three times that of the incident plane wave and the focus covers the surface region on the scattering axis. However, for α particles the stronger absorption reduces the intensity of the focus to no more than that of the incident plane wave and the size of the focus is reduced. This explains the fact that the present simple model suffices in the present case, whereas it would certainly not be suitable for nucleons below 100 MeV. The effect of the focus on angular distributions for nucleon inelastic scattering has been calculated by Kromminga and McCarthy.[15]

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