Separable potential for the α-α interaction

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A nonlocal separable interaction of Mongan type has been used to study S-, D-, and G-wave α-α scattering phase shifts up to a bombarding energy of 24 MeV. Calculations have been made in the coordinate representation in which the Coulomb effects have been taken into account exactly. A separable representation of the α-α interaction is found to be satisfactory.

[NUCLEAR STRUCTURE α-α interaction; used separable potential to study S-, D-, and G-wave α-α phase shifts.]

I. INTRODUCTION

One of the basic features of the α-α interaction which emerges from microscopic studies is its nonlocality.[1,2] A question that arises is: Is the nonlocality of a separable type? The interest in separability of the interaction has been aroused because of the solvability of the three-body problems (*e.g.* a 3-α model of ^{12}C) for such separable potentials. Usually, when three-body calculations are made using nonlocal separable interactions, separability is assumed as a matter of convenience. However, as far as the separability of the α-α interaction is concerned, it has already been shown by Leung and Park[3] that the real symmetric kernel $K_l(r, r')$ which arises when one builds up the α-α interaction from local N-N forces by the method of resonating group structure,[4] can be recast into a separable form using a formula due to Hill. This observation lends some support to the assumption of a separable form for the α-α interaction.[5] It is with these views that, in the present paper, we have used separable potentials in coordinate space for α-α scattering for $L=0$, 2, and 4. Some work on α-α scattering using separable potentials has recently been performed by Pigeon *et al.*[6] and by Meboniya and Surmava.[7] While the latter authors, working in momentum representation, take into account the effect of the Coulomb interaction in a rather qualitative way, the former authors, also working in the same representation, calculated the Coulomb effect exactly. The analysis of Pigeon *et al.*, however, is rather involved. Since the Coulomb effect can be taken into account exactly in a much simpler way,[8] in the coordinate representation, we decided to work in the latter representation. One added advantage of using the coordinate representation is that, for a given nonlocal separable potential, one may have some insight into the corresponding equivalent local potentials obtained by a suitable prescription.

II. METHOD

The method to be used in the present calculations has been presented in Ref. 8, and hence we shall outline it only briefly here.

The nonlocal Schrödinger equation with Coulomb force may be written as

$$\left(\frac{d^2}{dr^2}+k^2-\frac{l(l+1)}{r^2}-\frac{2\eta k}{r}\right)\psi_l(r) = \int_0^\infty K_l(r, r')\psi_l(r')dr' \tag{1}$$

in usual notations. For separable potentials of the type

$$K_l(r, r') = \sum_{i=1}^{N} \lambda_l^{(i)} g_l^{(i)}(r) g_l^{(i)}(r'), \tag{2}$$

the solution of Eq. (1) is

$$\psi_l(r) = F_l(r) + \sum_{i=1}^{N} a_l^{(i)} Z_l^{(i)}(r). \tag{3}$$

$F_l(r)$ and $Z_l^{(i)}(r)$ satisfy the equations

$$\left(\frac{d^2}{dr^2}+k^2-\frac{l(l+1)}{r^2}-\frac{2\eta k}{r}\right)F_l(r)=0, \tag{4}$$

$$\left(\frac{d^2}{dr^2}+k^2-\frac{l(l+1)}{r^2}-\frac{2\eta k}{r}\right)Z_l^{(i)}(r) = \lambda_l^{(i)} g_l^{(i)}(r), \tag{5}$$

with the boundary conditions

$$F_l(0)=0,$$
$$Z_l^{(i)}(0)=0,$$
$$Z_{l\,r\to\infty}^{(i)}(r) \to (\lambda_l^{(i)}/k) G_l(r) \times \int_0^\infty F_l(r) g_l^{(i)}(r) dr. \tag{6}$$

The functions $F_l(r)$ and $G_l(r)$ are the regular and irregular solutions of Eq. (4). $a_l^{(i)}$'s satisfy the algebraic equation

$$\sum_{j=1}^{N} (\delta_{ij} - \tau_l^{ij}) a_l^{(j)} = C_l^{(i)}, \tag{7}$$

where

$$\tau_l^{ij} = \int_0^\infty g_l^{(i)}(r) Z_l^{(j)}(r) dr, \tag{8}$$

and

$$C_l^{(i)} = \int_0^\infty g_l^{(i)}(r) F_l(r) dr. \tag{9}$$

From the asymptotic form of $\psi_l(r)$ we find

$$\tan\delta_l = -\frac{1}{k} \sum_i a_l^{(i)} c_l^{(i)} \lambda_l^{(i)}. \tag{10}$$

The formal solution of Eq. (5) is

$$Z_l^{(i)}(r) = \lambda_l^{(i)} \int_0^\infty G_l(r, r') g_l^{(i)}(r') dr', \tag{11}$$

with

$$G_l(r, r') = \frac{1}{k} F_l(r) G_l(r'), \quad r' > r$$
$$= \frac{1}{k} F_l(r') G_l(r), \quad r' < r. \tag{12}$$

III. RESULTS AND DISCUSSION

For computational purposes we find it more convenient to determine $Z_l^{(i)}(r)$ directly by solving Eq. (5) numerically. The phase shifts for α-α scattering for $L=0$, 2, and 4 were calculated using expression (10). In the present calculations we considered α-α bombarding energies up to about $E_{\text{c.m.}} \lesssim 12$ MeV and hence restricted ourselves to only real phase shifts. The potential chosen for S-wave α-α scattering was one of rank 2,

$$K_0(r, r') = \lambda_0^{(1)} g_0^{(1)}(r) g_0^{(1)}(r') + \lambda_0^{(2)} g_0^{(2)}(r) g_0^{(2)}(r'). \tag{13}$$

This is also the minimum possible rank[9] compatible with the trend of the experimental phase shifts[10-12] which become negative around $E_{\text{c.m.}} \approx 10$ MeV. For the sake of simplicity, the forms for $g_0^{(1)}$ and $g_0^{(2)}$ were taken to be the same, i.e.,

$$g_0^{(i)}(r) = e^{\beta_0^{(i)} r}. \tag{14}$$

Thus, for the S-wave phase shifts there were four adjustable parameters. For the above form factor, the best fit to the experimental phase shifts (as is shown in Fig. 1) was obtained with the fol-

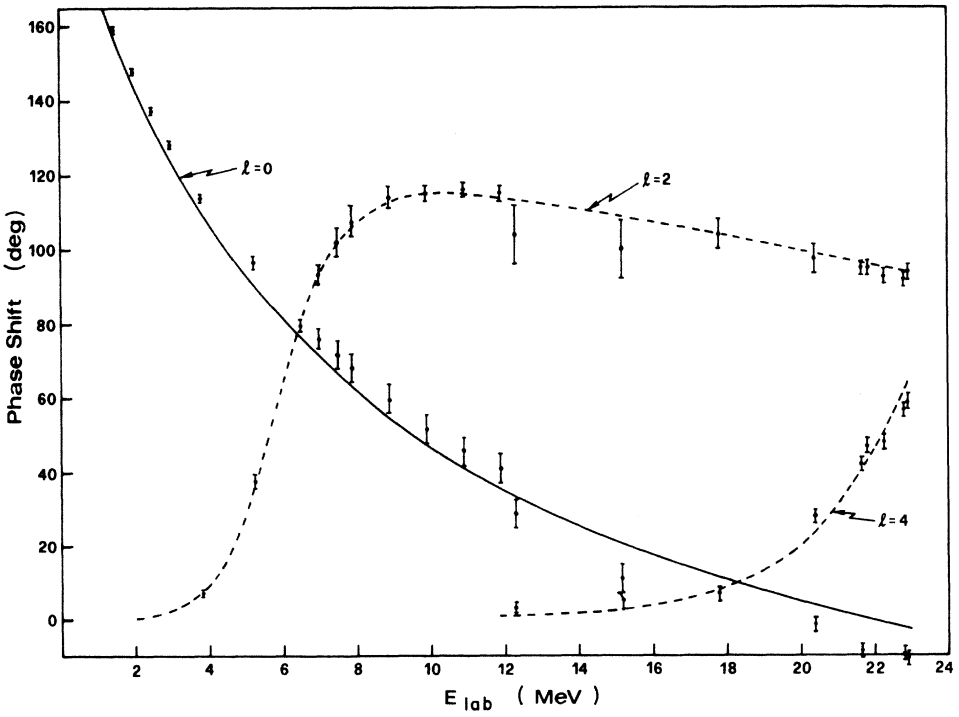

FIG. 1. S-, D-, and G-wave α-α phase shifts. Experimental points are from Refs. 10–12.

lowing values of the parameters:

$$\lambda_0^{(1)} = -2.4 \text{ fm}^{-3},$$
$$\beta_0^{(1)} = 0.4 \text{ fm}^{-1},$$
$$\lambda_0^{(2)} = 20.0 \text{ fm}^{-3},$$
$$\beta_0^{(2)} = 1.3 \text{ fm}^{-1}.$$

Thus a superposition of an attractive and a repulsive term was found to be necessary, as was also the case for local potentials.[13] For the D and G waves, however, a one-term separable potential of the form

$$K_l(r, r') = \lambda_l g_l(r) g_l(r'), \tag{15}$$

with

$$g_l(r) = r^l e^{-\beta_l r}, \tag{16}$$

seems to provide an adequate representation of the experimental data. The best fits to the phase shifts for D and G waves (shown in Fig. 1) were obtained: with $\lambda_2 = -31.5$ fm^{-7}, $\beta_2 = 1.5$ fm^{-1} for the D wave; and with $\lambda_4 = 1865$ fm^{-11}, $\beta_4 = 3$ fm^{-1} for the G wave. It is interesting to note that it is possible to reproduce the D and G wave phase shifts with an attractive part only. This is presumably due to the fact that for the D and G waves, the large centrifugal barrier masks any permissible repulsive part in the potential. Such a phenomenon was also observed in the case of local potentials,[13] where it was found possible to obtain the G-wave phase shifts with an attractive part alone and one needed repulsion progressively as one went to low l values. However, such a comparison between the situations for local and nonlocal separable potentials does not strictly make sense as the latter do not permit a graphical representation. A strict comparison can be made only when equivalent local potentials corresponding to separable ones have been obtained. In fact, such potentials when constructed by model prescriptions are found to be energy dependent.[14,15] Energy-independent local potentials may, however, be constructed from inverse scattering theory using the Marchenko prescription.[16] It would indeed be interesting to see how these energy-independent local potentials (corresponding to given separable ones) compare with the phenomenological ones and with the deductions made from the resonating group studies of the α-α interaction. Such a work is in progress.

In conclusion, we should like to remark that a one- or two-term separable representation of the α-α potential reproduces the experimental data quite satisfactorily. Since the Coulomb effects can be taken into account exactly and neatly, it is hoped that such a separable representation may also be profitably utilized in the mutual scattering of those light nuclei for which a cluster description is reasonably valid.

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