the correction gives rise to oscillations as a function of $A^{1/3}$ in the depth of V_d in the nuclear surface which are similar in amplitude and wavelength to those found by Perey and Satchler[44] in their accurate analysis of the elastic data. Further investigations of this point are planned.

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^{28}Si Nucleus in the Projected Hartree-Fock Model*

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The structure of the ^{28}Si nucleus is studied using a variational procedure. The method of angular-momentum projection from a deformed intrinsic state is applied, and each J^π state is projected from a determinant which is variationally "best" for that state. This more general variational procedure includes important vibrational correlations, which in ^{28}Si have hexadecapole character. The level spacings in the energy spectrum improve considerably compared to those in the Hartree-Fock method followed by angular momentum projection, and this leads to a much better agreement with the experimental spectrum. The $E2$ transition probabilities, except for the $6^+ \rightarrow 4^+$ transition, are also in good agreement with the experiments.

1. INTRODUCTION

A NUMBER of calculations[1–3] have been done in recent years to study the low-lying states in ^{28}Si. All the calculations which consider ^{28}Si to be deformed and obtain the energy spectrum in either the $SU(3)$ model or the Hartree-Fock (HF) model have the shortcoming that the energy spectrum is too dense by about a factor of 2. Das Gupta and Harvey[2] discuss the possible reasons for this compression and suggest that for the oblate HF solution there are residual correlations (β vibration) which are of importance. They then show that the states in ^{28}Si associated with the oblate minimum can in fact be explained in terms of the rotation-vibration collective model, and suggest that this additional degree of freedom should be included in a many-body calculation.

Recently, Rowe[4] has discussed several methods for describing "vibrational" correlations in finite nuclei. A comparison of these methods by Parikh and Rowe[5] in the model of Lipkin *et al.*[6] has shown that the projected Hartree-Fock (PHF) approximation in which the variation is carried out after projection, gives good results for a whole range of situations. This goes from the case where the nucleus is vibrational to the other extreme where it has static deformation and includes the transitional region in between. Similar results in

* Work supported in part by the National Research Council of Canada (B.C.) and by the U.S. Atomic Energy Commission (J.C.P.).

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[2] S. Das Gupta and M. Harvey, Nucl. Phys. **A94,** 602 (1967).
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a more realistic situation have been reported by Ullah and Rowe.[7] They have investigated the $J^\pi=3^-$, $T=0$ state and the associated octupole correlations in the ground state of ^{16}O.

In view of these findings, it seems natural to apply the PHF method to the ^{28}Si nucleus. We have carried out such a calculation and report the results here. More precisely, it is the purpose of this paper (i) to study the influence of vibrational correlations on the energy spectrum in ^{28}Si and (ii) to further test the PHF technique in a realistic nuclear case.

We find that by taking account of the additional correlations the energy spectrum turns out to have about the right density of levels. The electromagnetic transition probabilities between the lowest few states in the band work out, but the BE2($6^+\to4^+$) seems to be in conflict with recent measurements. An interesting feature we find is that the residual correlations are of the hexadecapole variety instead of the quadrupole variety (β vibrations) suggested in Ref. 2.

2. PROJECTED HARTREE-FOCK METHOD

We present in this section a summary of the PHF method. More details can be found elsewhere.[8–10]

The method of angular momentum projection from a deformed determinant has been applied extensively to calculate properties of nuclei in the $2s$-$1d$ shell.[11–12] The nuclei are considered to have an inert ^{16}O core plus active particles in the $2s$-$1d$ shell. The Hamiltonian of the system is written as

$$H=\sum_\alpha \epsilon(\alpha)\,a_\alpha^+a_\alpha+\tfrac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta\mid V_A\mid\gamma\delta\rangle a_\beta^+a_\alpha^+a_\gamma a_\delta, \quad (1)$$

where $\langle\alpha\beta\mid V_A\mid\gamma\delta\rangle$ is the antisymmetrized two-body interaction matrix element. The HF theory provides a prescription for finding determinantal states χ_κ which minimize the expectation value of H. In general, the intrinsic states are deformed and are therefore not eigenstates of the total angular momentum (a.m.) operator. We take them to be axially symmetric, so that K, the projection of total a.m. on the symmetry axis, is a good quantum number. A state ψ_{MK}^J with good a.m. J, projection M along the z axis in the lab frame, and projection K along the body-fixed z axis can be obtained from the state χ_K.

The projection method of Peierls and Yoccoz,[13] which used the Hill-Wheeler integral,[14] is applied. We have

$$|\psi_{MK}^J\rangle=[(2J+1)/8\pi^2(N_{JK})^{1/2}]\times\int d\Omega\, D_{MK}^{J*}(\Omega)\,R(\Omega)\,|\chi_\kappa\rangle, \quad (2)$$

where $R(\Omega)=\exp(-i\alpha J_z)\ \exp(-i\beta J_y)\ \exp(-i\gamma J_z)$ and $D_{MK}^J(\Omega)$ is the matrix element of the rotation operator $R(\Omega)$,

$$D_{MK}^J(\Omega)=\langle JM\mid R(\Omega)\mid JK\rangle,$$

and N_{JK} is the normalization constant. From the wave functions $|\psi_{MK}^J\rangle$, one can then obtain energies, moments, and transition probabilities for the low-lying states in the nucleus.

This is the HF method followed by projection (HFP method) and has the restrictive feature that the intrinsic state is the same for all the states in a band. The PHF method is an improved variational procedure where the variation is carried out after the a.m. projection. It allows for the possibility of having different deformed determinants for various states in a band and can give important corrections to the HFP procedure.

More precisely, we have used a modified Hamiltonian H' with

$$H'=H-\lambda_2Q_{20}-\lambda_4Q_{40}, \quad (3)$$

where H is defined in Eq. (1),

$Q_{20}=(16\pi/5)^{1/2}r^2Y_{20}$, (quadrupole operator)

$Q_{40}=(4\pi/9)^{1/2}r^4Y_{40}$, (hexadecapole operator)

and λ_2 and λ_4 are Lagrange multipliers. We carry out a HFP calculation described above for different values of the Lagrange multipliers λ_2 and λ_4 and obtain separate minima (with respect to the parameters λ_2 and λ_4) for each angular momentum state in the band. While this is only a two-parameter variation which preserves axial symmetry, it nonetheless allows for important quadrupole and hexadecapole correlations.

The results of our calculation are shown in Sec. 3. The single-particle energies in the Hamiltonian H [Eq. (1)] are taken to be $\epsilon(1d_{5/2})=-7.0$ MeV, $\epsilon(2s_{1/2})=-4.2$ MeV, and $\epsilon(1d_{3/2})=0.0$ MeV, corresponding to a one-body spin-orbit coupling strength of -2.8 MeV. The two-body interaction V is a Rosenfeld exchange mixture with a Yukawa radial dependence and is given by

$$V_{12}=V_0\tfrac{1}{3}(\tau_1\cdot\tau_2)\,(0.3+0.7\sigma_1\cdot\sigma_2)\,e^{-r_{12}/a}/(r_{12}/a), \quad (4)$$

with $V_0=40$ MeV and $\lambda=(m\omega/2h)^{1/2}a=0.589$.

3. ANALYSIS OF RESULTS AND DISCUSSION

From the energy spectra shown in Fig. 1, it is immediately seen that the quality of the spectrum is improved for the "rotational" band by the inclusion of residual correlations. This comes about (see Table I)

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[8] M. Bouten and P. Van Leuven, Physica **34**, 461 (1967); M. Bouten, P. Van Leuven, H. Depuydt, and L. Schotsmans Nucl. Phys. **A100**, 90 (1967).
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[14] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1106 (1953).

TABLE I. Calculated properties of ^{28}Si for different values of the Lagrange multipliers λ_2 and λ_4. Each column represents a solution. The type of the HF solution and the values of the Lagrange multipliers are indicated. E_{HF}, $\langle Q_{20}\rangle$, $\langle Q_{40}\rangle$, and $\langle J\cdot J\rangle$ denote, respectively, the energy, the quadrupole moment, the hexadecapole moment, and the expectation value of $\mathbf{J}\cdot\mathbf{J}$ for the intrinsic determinant. $E(J^\pi)$ gives the energy of projected J^π state. $N_p(d_{5/2})$, $N_p(s_{1/2})$, and $N_p(d_{3/2})$ are the total number of protons (in the spherical orbits in the bracket) for the $J^\pi=0^+$ ground state. The minimum energy for each J^π is in italics (see Fig. 1).

Type	Oblate	Oblate	Oblate	Prolate	Spherical
λ_2	0.0	0.0	0.0	0.0	0.0
λ_4	0.0	0.2	0.4	0.0	0.0
E_{HF}	−137.280	−136.990	−136.147	−135.116	−136.0400
$E(0^+)$	−139.981	−140.550	*−140.798*	*−137.294*	*−136.0400*
$E(2^+)$	−138.922	*−138.955*	−138.236	−136.546	···
$E(4^+)$	*−136.856*	−136.737	−136.093	−134.539	···
$E(6^+)$	−134.303	*−134.339*	−133.925	−132.633	···
$E(8^+)$	−133.973	*−134.002*	−133.496	−132.603	···
$\langle Q_{20}\rangle$	−19.66	−18.68	−16.25	21.18	0.0
$\langle Q_{40}\rangle$	10.90	14.08	16.96	−13.86	0.0
$\langle J\cdot J\rangle$	20.87	23.90	26.81	24.90	0.0
$N_p(d_{5/2})$	4.53	4.69	4.93	3.72	6.00
$N_p(s_{1/2})$	0.89	0.61	0.32	1.18	0.0
$N_p(d_{3/2})$	0.58	0.70	0.74	1.09	0.0

because $J=0_1^+$, 2_1^+, 6_1^+, and 8_1^+ states move away from the $J=4_1^+$ state. The latter is not affected by the additional degrees of freedom included in the PHF variation. One can understand this by a somewhat crude argument. Since the expectation value of $\langle J^2\rangle$ when both λ_2 and λ_4 are equal to zero is 20.87 units (which gives a value of $J\approx 4$), the $J^\pi=4^+$ state is not influenced by the correlations. The first excited 0_2^+ state shown in Fig. 1 is obtained by projecting states of good a.m. from the prolate HF minimum. It comes at an energy which is about 1 MeV below the energy of the lowest observed 0^+ excited state, and its prolate intrinsic structure would imply that there is no $E2$ transition probability from $0_2^+\rightarrow 2_1^+$ because the 2_1^+ state has oblate intrinsic structure. This is in disagreement with the experiment[15] which gives an enhancement of about a factor of 8 over the single-particle estimate. The second excited 0_3^+ state shown in Fig. 1 has a spherical nature [closed $(d_{5/2})^{12}$ configuration] and lies more than 1 MeV above the 0_2^+ state. It should be emphasized that while the 0_1^+ and 0_2^+ states are orthogonal to each other, the 0_3^+ state has nonzero overlaps with them. In order to understand the position and the γ-decay rate of the first two excited 0^+ states, an "inverted coexistence" model similar to that of Bar-Touv and Goswami[16] involving the three calculated 0^+ states can be invoked. Das Gupta and Harvey[2] have suggested that the 0_2^+ state is a one-particle–one-hole (Tamm-Dancoff) state based on the oblate minimum.

The $E2$ transition probabilities have been previously calculated by Gunye and Warke[17] in the HFP approximation. The $B(E2)$ values in the PHF calculation shown in Table II differ somewhat from those calculated in Ref. 17, but have essentially the same kind of agreement with the experiments except for the $6^+\rightarrow 4^+$ transition, which neither of the calculations predicts correctly. A comparison of the experimental $E2$ strengths for the $K=0$ ground-state band in ^{20}Ne, ^{24}Mg, and ^{28}Si done by Lam *et al.*[18] shows that the strengths in ^{28}Si are much smaller than those in ^{20}Ne and ^{24}Mg, and particularly the $6^+\rightarrow 4^+$ strength is down by nearly a factor of 10, having a value of only 2.9 W.u. (Weisskopf units). This indicates a substantial difference in structure between the 6^+ and the 4^+ states. Now, the prolate 6_1^+ state has about the same excitation energy above the prolate HF minimum as the oblate 6_1^+ state has above the oblate HF minimum. One could therefore in a semiclassical picture expect considerable mixing of the two 6_1^+ states. A similar admixture of the 4^+(oblate) and 4^+(prolate) states would be much smaller because of the larger potential barrier. Such a model has been discussed by Castel and Svenne[19] for ^{28}Si, and would have the consequence of reducing the $E2$ decay rate from $6^+\rightarrow 4^+$ state. For example, an equal mixture of the two 6^+ states and no mixture of the two 4^+ states can reduce the $B(E2)$ by a factor of 4 compared to the rate shown in Table II.

The last three rows in Table I show for the $J=0_1^+$ ground state the number of protons (neutrons) in the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ shells and the manner in which they change with λ_2 and λ_4. It is quite surprising to find that for $\lambda_2=0.0$ and $\lambda_4=0.4$, which is variationally the "best" set of parameters for the ground state, there is very little occupation of the $s_{1/2}$ shell. The Q_{40} opera-

[15] T. K. Alexander *et al.*, in *Proceedings of International Nuclear Physics Conference, Gatlinburg, Tennessee*, edited by R. L. Becker *et al.* (Academic Press Inc., New York, 1967).
[16] J. Bar Touv and A. Goswami, Phys. Letters **28B**, 391 (1969).
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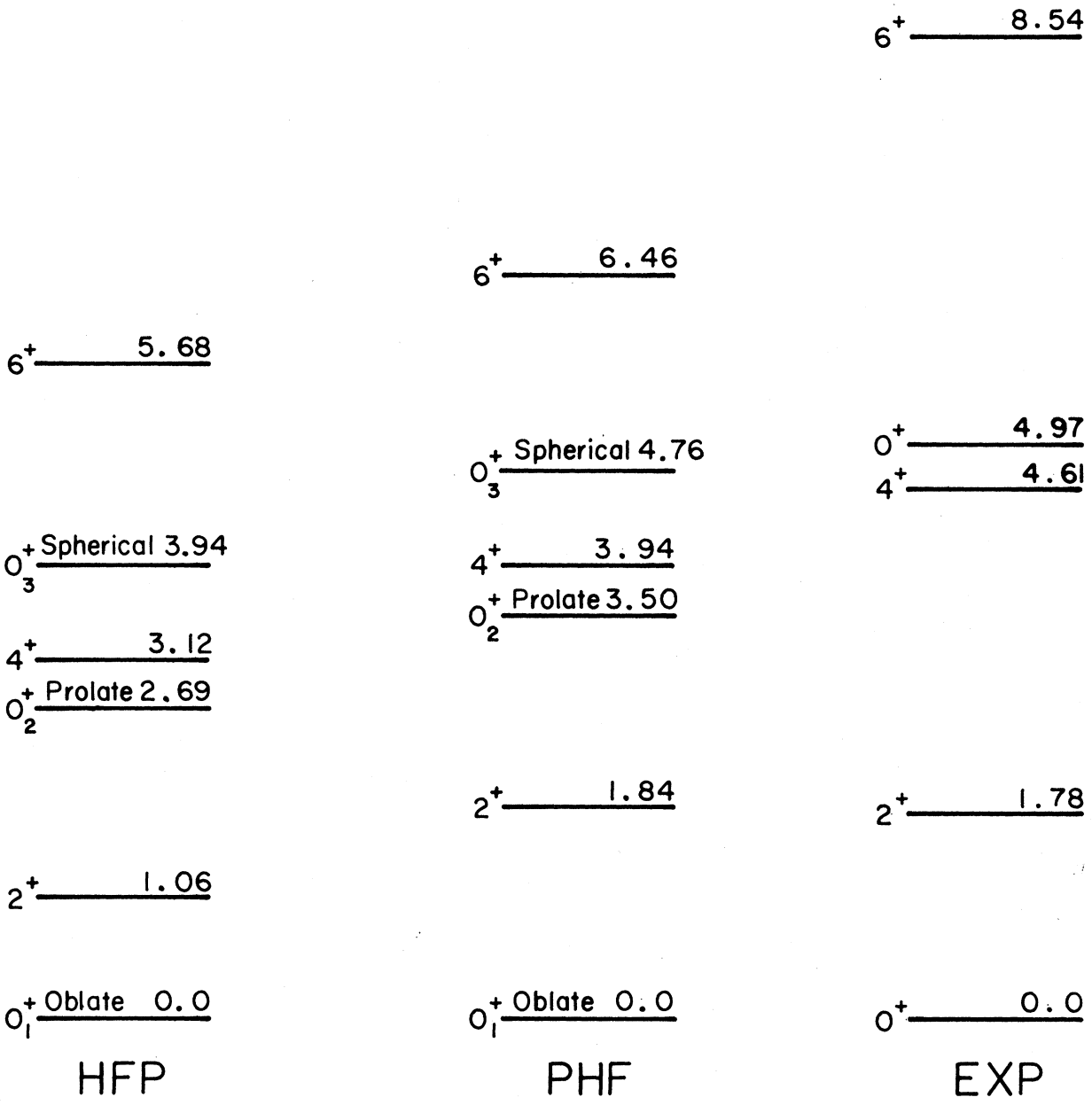

FIG. 1. Energy spectrum of ^{28}Si. HFP shows the spectrum obtained by projecting from the oblate HF solution (column 2 of Table I). The PHF spectrum is obtained by using the improved variational method. The minimum energy for each J^π is italicized in Table I. For comparison the experimental spectrum is shown on the right side.

TABLE II. The calculated and experimental $E2$ transition probabilities between some of the low-lying states in ^{28}Si. Column 3 gives the values in units of e^2 (fm)^4. Columns 4 and 5 are in Weisskopf units.

J_i	J_f	$B(E2;J_i\to J_f)_{\text{calc}}$	$B(E2)_{\text{calc}}/B(E2)_{\text{s.p.}}$	$B(E2)_{\text{expt}}/B(E2)_{\text{s.p.}}$
2_1^+	0_1^+	78.75	11.4	13±1
4_1^+	2_1^+	67.11	9.75	9±2
6_1^+	4_1^+	47.71	6.93	2.9±0.6
0_2^+	2_1^+	~0	...	8
0_3^+	2_1^+	≤1	...	

tor does not connect $s_{1/2}$ with either $d_{5/2}$ or $d_{3/2}$, but it does connect $d_{5/2}$ with itself and $d_{5/2}$ with $d_{3/2}$, with the result that an increase in hexadecapole correlations would tend to populate $d_{5/2}$ and $d_{3/2}$ shells at the expense of the $s_{1/2}$ shell. It would be of interest to check experimentally whether in the ground state of ^{28}Si there is little occupation of the $s_{1/2}$ shell compared to the $d_{3/2}$ shell.

The existence of hexadecapole correlations could have been expected, but the absence of quadrupole correlations is not very well understood. The oblate and the prolate intrinsic states have large quadrupole deformations, though they are nowhere near having perfect overlaps with the (0, 12) and (12, 0) $SU(3)$ representations, respectively. In view of this, one would expect that the inclusion of collective quadrupole correlations in PHF would improve the spectrum. A PHF calculation done by Dreizler *et al.*[20] for ^{20}Ne showed very little influence of residual correlations, but the ^{20}Ne intrinsic state has deformation which is almost exactly equal to that of the (8, 0) representation of $SU(3)$.

[20] R. Dreizler, P. Federman, B. Giraud, and E. Osnes, Nucl. Phys. **A113,** 145 (1968).

It should be mentioned that we have also done similar (two-parameter) calculations for ^{24}Mg and ^{32}S, but with no substantial improvement in the spectrum. We think that this might be due to inclusion of only those correlations that preserve axial symmetry. It has been argued[21] that correlations which do not preserve axial symmetry could be of importance. We are at present unable to include such correlations in our calculations.

In conclusion, we feel that the PHF method is a very useful technique for spectroscopic calculations and provides an extremely good approximation to the lowest few states in a nucleus. In the future, we intend to include more general correlations.

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Nuclear Structure Effects in the γ Decay of Isobaric Analog States in Mg^{24}, Al^{27}, and Si^{28}†

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The $Na^{23}(d, n)$, the $Mg^{26}(d, n)$, and the $Al^{27}(d, n)$ reactions were used to populate isobaric analog states. Their γ decays were measured with a Ge(Li) detector (i) in coincidence with the neutron groups, (ii) in singles arrangements using a bare detector, and (iii) in a three-crystal configuration. The observed branching ratios were compared to the approximate selection rules operating in the rotational and the Hartree-Fock model. It is found that the 4.23-MeV state in Mg^{24} has a wave function with admixtures of angular momentum projection $K=0$ and $K=2$. This fact supports the model proposed by Kelson *et al.* which gives an ellipsoidally symmetric deformation to Mg^{24}. The observed transitions in Si^{28} indicate that the analog of the ground-state doublet in Al^{28} has an oblate deformation.

1. INTRODUCTION

DURING the last few years, Hartree-Fock (HF) calculations[1] have been successfully applied to nuclear structure calculations in the 1d-2s shell, in particular, to Ne^{20}, Mg^{24}, and Si^{28}. Prior to these calculations, the Nilsson single-particle model[2] ($A\approx24$) and the weak coupling collective single-particle model[3] ($A=27$) were found to give reasonable fits to low-lying levels in this mass region. The popularity of the latter

† Work supported by the U.S. Atomic Energy Commission.

[1] See, e.g., G. Ripka, in *Advances in Nuclear Physics,* edited by M. Baranger and E. Vogt (Plenum Press, Inc., New York, 1968), Vol. I; I. Kelson, Phys. Rev. **132,** 2189 (1968).

[2] H. E. Gove, in *Proceedings of the International Conference on Nuclear Physics Kingston, 1960,* edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, Canada, 1960).

[3] R. M. Lombard and G. R. Bishop, Nucl. Phys. **A101,** 625 (1967).