# Contribution of Deuteron Breakup Channels to Deuteron Stripping and Elastic Scattering

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We present a model of deuteron stripping and elastic scattering which treats explicitly the contributions from channels in which the deuteron is broken up into a relative $S$ state and the target is in its ground state. An adiabatic treatment of these channels leads to a description of deuteron stripping which resembles the distorted-wave Born approximation, although a deuteron optical potential plays no role. The adiabatic approximation is shown to give a good account of 21.6-MeV elastic deuteron scattering from Ni, at least for surface partial waves, and is expected to apply to other nuclei in this mass and energy region, as well as at higher energies. The calculations assume that the effective two-nucleon–nucleus interaction is the sum of the nucleon optical potentials evaluated at one-half the incident deuteron kinetic energy. Some possible corrections to this assumption are discussed.

## I. INTRODUCTION

THE distorted-wave Born approximation (DWBA) has had considerable success in describing $(d,p)$ and $(p,d)$ cross sections[1] and polarization data.[2,3] Reference 1 contains an excellent critique of the DWBA method in its most sophisticated form, including a discussion of the difficulties the method runs into as the deuteron energy involved is increased from 12 MeV to the 20-MeV region.

It has often been remarked that some of the difficulties of the DWBA may be associated with the inadequate treatment of three-body effects, i.e., effects arising from the breakup of the deuteron in the field of the nucleus. It is clear that a complete assessment of the DWBA treatment can only come from an exact three-body calculation and considerable progress has been made along these lines.[4] However, in view of the widespread use of stripping reactions as a tool for nuclear-structure analysis, it is felt that the development of an extension of the DWBA method is desirable which, while not requiring an exact three-body treatment of every stripping reaction, does contain the dominant contributions from the three-body channels.

In this paper, we examine the role of the coupling between the deuteron elastic and breakup channels in an adiabatic approximation.[5] We are led to propose a modified prescription for the analysis of stripping and pickup reactions that involves only a minor modification of the standard DWBA matrix element.

It is a key feature of the DWBA that the predictions for stripping reactions are closely linked with the analysis of elastic deuteron scattering. In the method proposed here, this link is no longer so direct and a deuteron optical potential plays no role in the stripping calculation. However, the physical picture involved does have implications for elastic deuteron scattering. We have therefore developed a theory of elastic deuteron scattering using our adiabatic approximations. We have shown that, at least for the partial waves of primary interest in stripping, the elastic scattering of deuterons in the 20-MeV region for nuclei with $A\sim50$ is accounted for very well. We believe that this result provides considerable justification for our prescription for stripping involving deuterons in this energy region.

## II. STRIPPING REACTIONS

### A. Three-Body Model

It is well known[6] that when the explicit coupling between nuclear states in both the incident and outgoing channels is neglected, the transition matrix $T(d, p)$ for a stripping reaction reduces to an expression of the form[7]

$$T(d, p) = \int d\mathbf{R}\, d\mathbf{r}[\chi^{(-)}(p)]^*\phi^*(n)\, V_{np}\psi^{(+)}(p, n), \quad (1)$$

where $\chi^{(-)}(p)$ is the usual[6] proton distorted wave, $\phi(n)$

* Royal Society (London) research fellow.

[1] R. J. Philpott, W. T. Pinkston, and G. R. Satchler, Nucl. Phys. **A119,** 241 (1968).

[2] T. J. Yule and W. Haeberli, Nucl. Phys. **A117,** 1 (1968).

[3] P. J. Bjorkholm, W. Haeberli, and B. Mayer, Phys. Rev. Letters **22,** 955 (1969).

[4] R. D. Amado, Phys. Rev. **132,** 485 (1963); A. N. Mitra, *ibid.* **139,** B1472 (1965); **150,** 839 (1966); A. S. Reiner and A. I. Jaffe, *ibid.* **161,** 935 (1967); W. Bierter and K. Dietrich, Z. Physik **202A,** 75 (1967); R. Aaron and P. E. Shanley, Phys. Rev. **142,** 608 (1966); J. V. Noble, *ibid.* **157,** 939 (1967); A. I. Noble, *ibid.* **157,** 939 (1967); A. I. Baz, V. F. Demin, and I. I. Kuz'min, Yadern. Fiz. **4,** 1131 (1966) [English transl.: Soviet J. Nucl. Phys. **4,** 815 (1967)], and references therein; T. G. Efimenko, B. N. Zakhariev, and V. P. Zhigunov, Ann. Phys. (N.Y.) **47,** 275 (1968).

[5] The sense in which we use the term adiabatic will be clarified in Sec. II B.

[6] See, e.g., G. R. Satchler, in *Lectures in Theoretical Physics,* edited by P. D. Kunz, D. A. Lind, and W. E. Brittin (University of Colorado Press, Boulder, 1965), Vol. VIIIC.

[7] We also neglect the explicit contribution from proton exchange terms.

is the form factor associated with the initial and final nuclear states,[8] and $V_{np}$ is the neutron-proton interaction. The vectors $\mathbf{R}$ and $\mathbf{r}$ are, respectively, the position vector of the center of mass of the neutron and proton relative to the target and the position vector of the neutron relative to the proton. The arguments of the wave functions in Eq. (1) denote the spin and space coordinates of which they are functions.

Our primary concern here is with the function $\psi^{(+)}(p, n)$ in Eq. (1). This function is the projection, on to the ground state of the target nucleus, of the many-body wave function appropriate to a deuteron in the incident channel. It therefore has outgoing waves corresponding to elastic deuteron scattering, breakup of the deuteron without target excitation, as well as the contribution to all possible stripping reactions not involving target excitation. The equation satisfied by $\psi^{(+)}(p, n)$ may be written

$$[E-T_R-H_{np}-V_e(n, p)]\psi^{(+)}(p, n)=0, \quad (2)$$

$$H_{np}=T_r+V_{np}, \quad (3)$$

where $T_R$ and $T_r$ are kinetic energy operators, and $E$ is the energy of the system ($E=E_d-\epsilon_0$, where $E_d$ is the incident deuteron kinetic energy and $\epsilon_0$ is the binding energy of the deuteron). The effective interaction $V_e(n,p)$ has been studied in Refs. 9–11. In this paper we shall assume, mainly on the grounds of simplicity, that

$$V_e(n, p)=V_n(\mathbf{R}+\tfrac{1}{2}\mathbf{r})+V_p(\mathbf{R}-\tfrac{1}{2}\mathbf{r})+V_C(R), \quad (4)$$

where $V_n$ and $V_p$ are nucleon optical potentials corresponding to one half of the deuteron kinetic energy, and $V_C(R)$ is the Coulomb field of the target nucleus evaluated at the center of mass of the deuteron.

A discussion of the corrections to the assumption (4) will be deferred until Sec. IV B and we confine ourselves here to two remarks only. In the first place, we note that the interaction (4) gives rise to breakup in the *nuclear* field of the target only. The contributions to deuteron elastic scattering[12] and stripping[13] from deuteron breakup in the Coulomb field are probably very small for the reactions above the Coulomb barrier which we shall be interested in. Second, we shall neglect the nonlocality of the nucleon potentials. It will be clear that, provided the ranges of nonlocality of the $V_n$ and $V_p$ are not large compared with the range of the $n$-$p$ force, our development is not altered in any essential way by the lifting of this assumption. Unless otherwise stated, the calculations reported here have been performed with the proton potential given by Perey[14] and the local equivalent of the neutron potential given by Perey and Buck.[15]

### B. Adiabatic Approximation

In the usual[6] DWBA treatment of the transition matrix element (1) two approximations are made.

(i) The function $\psi^{(+)}(p, n)$ is replaced by its elastic component. Thus, defining the projectors

$$P_0=|\phi_0\rangle\langle\phi_0|, \qquad Q_0=1-P_0, \quad (5)$$

where $|\phi_0\rangle$ is the deuteron ground state, we have

$$\psi^{(+)}(p, n)=P_0\psi^{(+)}(p, n)+Q_0\psi^{(+)}(p, n), \quad (6)$$

$$P_0\psi^{(+)}(p, n)=\phi_0(\mathbf{r})\chi_0(\mathbf{R}), \quad (7)$$

where $\chi_0(\mathbf{R})$ describes elastic deuteron scattering.[16] In the DWBA, the explicit contribution from $Q_0\psi^{(+)}$ is neglected.

(ii) In the evaluation of the matrix element (1) the elastic wave function $\chi_0(\mathbf{R})$ is required for $R$ lying inside the nucleus. The extrapolation from the asymptotic region, where $\chi_0(\mathbf{R})$ is determined by the elastic phase shifts, is performed by assuming that $\chi_0(\mathbf{R})$ can be generated by an optical potential of a largely conventional form with parameters adjusted to fit elastic scattering data.[17] It has often been questioned how far this is a realistic procedure for deuterons.

We first examine the neglect of the breakup component $Q_0\psi^{(+)}$. In this connection it is important to note that the evaluation of the matrix element (1) requires accurate knowledge of $\psi^{(+)}(p, n)$ only in a very restricted region of configuration space, i.e., (a) for $r<$ range of $V_{np}$, (b) for $R$ within the range of the bound state neutron form factor $\phi(n)$.

We first consider the case when $V_{np}$ is a zero-range force. In this limit, an immediate consequence of (a) is that only the components of $\psi^{(+)}(p, n)$ with the neutron and proton in a relative $S$ state give a contribution to the stripping matrix element.[18] Furthermore, if we assume that the spin-dependent terms in the effective interaction $V_e(n, p)$ [Eq. (2)] are symmetric under interchange of neutron and proton space and spin coordinates,[19] then it is easy to see that only $^3S$ states of $n$-$p$ relative motion can contribute to

[8] See Ref. 1 and references therein.
[9] W. F. Junkin and F. Villars, Ann. Phys. (N.Y.) **45,** 93 (1967).
[10] S. Mukherjee, Nucl. Phys. **A118,** 423 (1968).
[11] N. Austern and K. C. Richards, Ann. Phys. (N.Y.) **49,** 309 (1968).
[12] C. F. Clement, Phys. Rev. **128,** 2724 (1962); J. K. Dickens and F. G. Perey, *ibid.* **138,** B1083 (1965); G. Bencze and E. Pietarinen, Phys. Letters **19,** 586 (1965).
[13] C. F. Clement, Nucl. Phys. **66,** 241 (1965); F. P. Gibson and A. K. Kerman, Phys. Rev. **145,** 758 (1966).
[14] F. G. Perey, Phys. Rev. **131,** 745 (1963).
[15] F. G. Perey and B. Buck, Nucl. Phys. **32,** 353 (1962).
[16] In the interest of clarity we have omitted explicit reference to spin degrees of freedom.
[17] A recent review is P. E. Hodgson, Advan. Phys. **15,** 329 (1966).
[18] A. I. Baz, Nucl. Phys. **51,** 145 (1964).
[19] In the approximation (4), this means we are neglecting the *difference* between the spin-orbit terms in the neutron and proton optical potentials.

$\psi^{(+)}(n, p)$. We can therefore write

$$\psi^{(+)}(p, n)=\phi_0(r)\chi_0(\mathbf{R}) + \int d\mathbf{k}\phi^{(+)}(\epsilon_k, r)\chi(\epsilon_k, \mathbf{R})+\psi'(p, n), \quad (8)$$

where the $\phi^{(+)}(\epsilon_k, r)$ are scattering states of the $n$-$p$ system which, together with $\phi_0$ (the deuteron) form a complete set of $^3S$ states,

$$H_{np}\phi^{(+)}(\epsilon_k)=\epsilon_k\phi^{(+)}(\epsilon_k), \qquad \epsilon_k\geq 0 \quad (9a)$$

$$H_{np}\phi_0=-\epsilon_0\phi_0, \quad (9b)$$

$$\langle\phi_0 \mid \phi^{(+)}(\epsilon_k)\rangle=0, \qquad \langle\phi^{(+)}(\epsilon_k) \mid \phi^{(+)}(\epsilon_{k'})\rangle=\delta(\mathbf{k}-\mathbf{k}'), \quad (9c)$$

$$\mid\phi_0\rangle\langle\phi_0\mid+\int d\mathbf{k}\mid\phi^{(+)}(\epsilon_k)\rangle\langle\phi^{(+)}(\epsilon_k)\mid=1. \quad (9d)$$

With the assumptions above concerning $V_e(n, p)$, the function $\psi'(p, n)$ in (8) denotes the contribution to $Q_0\psi^{(+)}(p, n)$ from singlet-odd and non-$S$-wave triplet eigenstates of $H_{np}$, and gives no contribution to the stripping matrix element in the zero-range limit.

From Eqs. (2), (4), (8), and (9), we therefore have at $r=0$

$$(E+\epsilon_0-T_R-\bar{V}(R))\phi_0(0)\chi_0(\mathbf{R})+\int d\mathbf{k} \times[E-\epsilon_k-T_R-\bar{V}(R)]\phi^{(+)}(\epsilon_k, 0)\chi(\epsilon_k, \mathbf{R})=0, \quad (10)$$

where

$$\bar{V}(R)=V_n(R)+V_p(R)+V_C(R). \quad (11)$$

It is at this point that we introduce our adiabatic approximation. We shall assume that the error involved in replacing the quantity in square brackets in (10) by

$$[E+\epsilon_0-T_R-\bar{V}(R)] \quad (12)$$

is small. With this approximation, Eq. (10) becomes a differential equation for $\psi^{(+)}(r=0, \mathbf{R})$. Thus, we have

$$\psi^{(+)}(\mathbf{r}=0, \mathbf{R})=\phi_0(0)\tilde{\chi}(\mathbf{R}), \quad (13a)$$

$$[E+\epsilon_0-T_R-\bar{V}(R)]\tilde{\chi}(\mathbf{R})=0, \quad (13b)$$

$$\tilde{\chi}(\mathbf{R})=\chi_0(\mathbf{R})+\int d\mathbf{k}\ \phi^{(+)}(\epsilon_k, 0)\chi(\epsilon_k, \mathbf{R})/\phi_0(0), \quad (13c)$$

and the stripping matrix element becomes

$$[T(d, p)]_{\text{Z.R.}}=D_0^0\int d\mathbf{R}[\chi^{(-)}(\mathbf{R})]^*\phi^*(\mathbf{R})\tilde{\chi}(\mathbf{R}), \quad (14)$$

where

$$D_0^0=\int d\mathbf{r}\ V_{np}\phi_0(r) \quad (15)$$

is the usual constant that appears in the zero-range DWBA limit.[6]

It is difficult to put precise limits on the accuracy of our adiabatic approximation. The replacement (12) in Eq. (10) clearly requires that the range of values of $\epsilon_k$ associated with relative $S$ states in $\psi^{(+)}(p, n)$ should be small compared with the energy associated with motion of the center of mass of the neutron and proton. We shall produce evidence in Sec. IV that the dominant $\epsilon_k$ satisfy $\epsilon_k\lesssim 10$ MeV for the reactions considered ($E_d\approx 20$ MeV, $A\approx 50$). Hence, for small $R$ where $\bar{V}(R)\approx -90$ MeV, the replacement (11) should be a good approximation even for $E<20$ MeV. Well outside the nucleus the inequality required would appear to be $E\gg 10$ MeV. For this region, an obvious source of error in the use of Eq. (13) is that the outgoing waves associated with the $\chi(\epsilon_k, \mathbf{R})$ are treated as all having the same wave number as the elastic channel. Note, however, that we are only concerned with the use of our approximation in the stripping matrix element so that these errors are not likely to produce large errors in stripping cross sections even when the inequality $E\gg 10$ MeV is only moderately well satisfied [see point (b) above]. Note also that, at least for small-angle stripping, the smoothness of the neutron form factor $\phi(n)$ strongly favors terms in $\psi^{(+)}$ with a wave number similar to that of the elastic channel.

It is one of the main purposes of this paper to give some justification for the adiabatic approximation from a source other than stripping. We shall attempt this in the following sections, but first we remark that it is straightforward to insert a finite-range correction into the formalism developed so far. We shall do this in terms of a simple generalization of the approximate treatment[20] found to be successful in the DWBA theory.[21] It is clear that since by assumption the important breakup corrections are associated with wavelengths very similar to those of the elastic component, we can assume that a similar approximate finite-range treatment will apply.

The usual treatment[20] uses the approximation[22]

$$\langle\mathbf{K}\mid V_{np}\mid\phi_0\rangle\approx D_0[1-(K/\beta)^2], \quad (16)$$

where

$$D_0=\langle\mathbf{K}=0\mid V_{np}\mid\phi_0\rangle = \int d\mathbf{r}\ V_{np}\phi_0(r), \quad (17)$$

and $1/\beta$ is a measure of the range of $V_{np}$.

For $K<\beta$, we also have, similarly,

$$\langle\mathbf{K}\mid V_{np}\mid\phi^{(+)}(\epsilon_k)\rangle = \langle\mathbf{K}=0\mid V_{np}\mid\phi^{(+)}(\epsilon_k)\rangle[1-(K/\beta')^2], \quad (18)$$

and we now need the additional assumption that $\beta'=\beta$. This requires that the important $\epsilon_k$ in the expansion (8) should lie within the region of validity of the effective range expansion (shape-independent approximation). However, since we shall still want to assume that only relative $S$ waves have a non-negligible overlap with $V_{np}$ in the stripping matrix element (1), this is not an essentially new approximation.

It is now straightforward to show that the generaliza-

[20] P. J. A. Buttle and L. J. B. Goldfarb, Proc. Phys. Soc. (London) **A83**, 701 (1964); G. Bencze and J. Zimanyi, Phys. Letters **9**, 246 (1964); F. G. Perey and D. Saxon, *ibid.* **10**, 107 (1964).

[21] J. K. Dickens, R. M. Drisko, F. G. Perey, and G. R. Satchler, Phys. Letters **15**, 337 (1965).

[22] We neglect tensor forces.

tion of Eqs. (13) and (14) is

$$T(d,p)=D_0\int d\mathbf{R}\,[\chi^{(-)}(\mathbf{R})]^* \times\{1-[(\tfrac{1}{2}\mathbf{K}_d-\mathbf{K}_p)/\beta]^2\}\bar{\chi}(\mathbf{R}), \quad (19)$$

where[23]

$$[E+\epsilon_0-T_R-V_C(R)-\bar{V}(R)]\bar{\chi}(\mathbf{R})=0, \quad (20a)$$

$$\bar{V}(R)=\langle\mathbf{K}=0\mid(V_n+V_p)V_{np}\mid\phi_0\rangle/D_0, \quad (20b)$$

$$\bar{\chi}(\mathbf{R})=\langle\mathbf{K}=0\mid V_{np}\mid\psi^{(+)}\rangle/D_0 \xrightarrow{R\to\infty}\exp(i\mathbf{k}_d\cdot\mathbf{R})+(\text{outgoing waves}) \quad (20c)$$

and $D_0$ is given in Eq. (17).

In Eq. (19), $\mathbf{K}_d$ and $\mathbf{K}_p$ are momentum operators which act on the "deuteron" and proton distorted waves and can be handled in the usual manner.[20]

### C. Discussion of the Method

The numerical work involved in the evaluation of (19) clearly differs little from that involved in a standard DWBA calculation.[24] However, the distorted wave $\bar{\chi}$ has quite a different interpretation from the elastic scattering wave function that appears in the DWBA. The function $\bar{\chi}$ contains outgoing waves associated with breakup into low-energy relative $^3S$ states as well as elastic scattering. Thus, the potential $\bar{V}(R)$ [Eq. (20b)] does not generate elastic deuteron scattering. Our present belief is that $\bar{V}$ is to be generated according to Eq. (20b) with $V_n$ and $V_p$ taken from analysis of nucleon elastic scattering from the target at $E_d/2$.

We remark that in the limit that $V_n$ and $V_p$ are real potentials, so is $\bar{V}$, whereas, of course, the deuteron optical potential would still be complex if the breakup and/or the stripping channels were open. The physical reason for this is that the adiabatic treatment of the $^3S$ states implies that if the neutron and proton appear at the nuclear surface with a separation less than the range of $V_{np}$ then they do not drift apart to beyond the range of $V_{np}$ during the time it takes them to cross the nucleus. Therefore, the flux associated with $\psi^{(+)}(\mathbf{r},\mathbf{R})$ and $r<$ range of $V_{np}$ is conserved.

It is to be noted that our use of the adiabatic approximation has enabled us to bypass a discussion of the questions raised under (ii) in Sec. II B concerning the nature of the deuteron optical potential. The remainder of this paper will be concerned with this question. The results of stripping calculations performed with our modified prescription will be published elsewhere. Preliminary results are very encouraging.[25,26]

Finally, in this section, we add some remarks concerning our use of the term "adiabatic." Our approximation can be roughly stated to be the assumption that, as far as small separations in relative $^3S$ states are concerned, the internal motion in the $n$-$p$ system is "slow" compared to the motion of its center of mass. This is to be contrasted with the "adiabatic" treatments used in Refs. 12, 13, and 27 which assume, roughly speaking, that the opposite is the case.[28] The use of the term "adiabatic" which appears to be closest to the sense in which we use it here is that of Berezhnoi and Inopin.[29] There is also some affinity between the "adiabatic" approximation used in our work and the

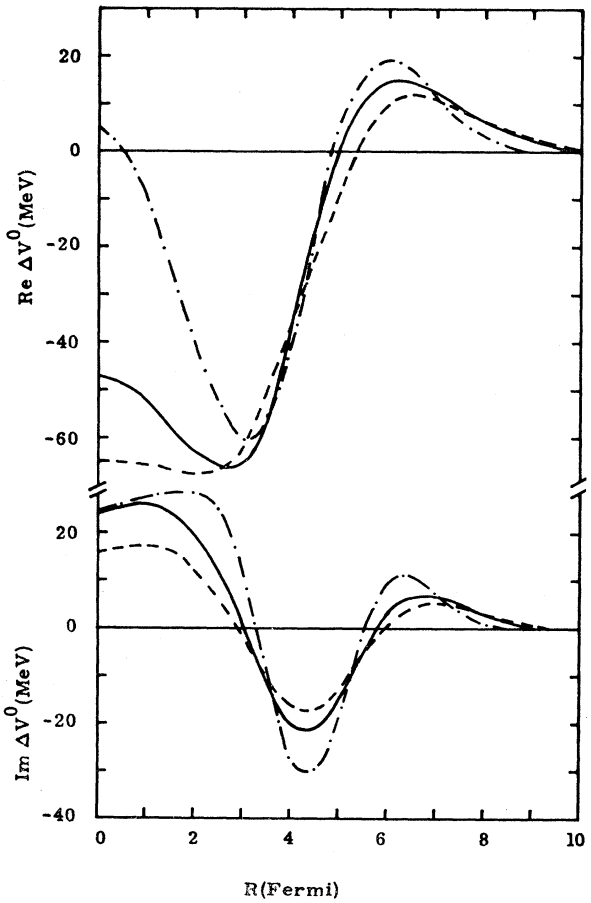


FIG. 1. The matrix elements $\Delta V^0(\epsilon_k, R)$ as functions of $R$ for $\epsilon_k=2.5$ MeV (dashed curve), $\epsilon_k=5$ MeV (solid curve), and $\epsilon_k=10$ MeV (dashed-dot curve). The nucleon optical potentials used correspond to $Ca^{40}$ and $E_d=21.4$ MeV.

[23] In this paper, we use the bra-ket notation to denote inner products in the space of functions of $\mathbf{r}$ only.

[24] A similar prescription was proposed earlier from a different point of view by L. R. Dodd and K. R. Greider, Phys. Rev. **146**, 675 (1966).

[25] R. C. Johnson, J. D. Harvey, and F. D. Santos, Science Research Council, Report No. RHEL/R. 170, 1968, p. 97 (unpublished); R. C. Johnson and J. D. Harvey (unpublished).

[26] G. R. Satchler (private communication). We are grateful to Dr. Satchler for communicating his results to us and for most useful discussions.

[27] J. Testoni and L. C. Gomes, Nucl. Phys. **89**, 288 (1966).

[28] We do not mean to imply that we consider the "adiabatic" treatment of Coulomb breakup given, e.g., in Ref. 12, to be a bad approximation. For a discussion of this point see, e.g., S. T. Butler, *Nuclear Stripping Reactions* (John Wiley & Sons, Inc., New York, 1957), p. 62 (footnote).

[29] Yu. A. Berezhnoi and E. V. Inopin, Yadern. Fiz. **6**, 1197 (1967) [English transl.: Soviet J. Nucl. Phys. **6**, 872 (1968)]. We should like to thank Dr. M. B. Hooper for drawing our attention to this work.

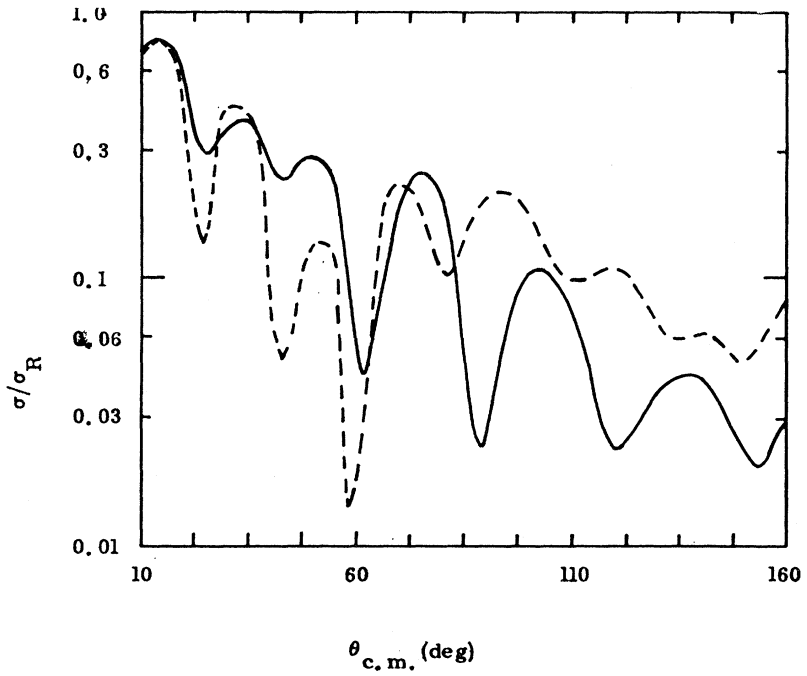


FIG. 2. Differential cross sections for $d$+Ni at $E_d$= 21.6 MeV. The solid curve is generated by $V_{PH}$ and the dashed curve by $V_{00}$ as discussed in the text.

"sudden" approximation of Tanifuji[30] and Butler *et al.*[31] It is to be noted, however, that, in contradistinction to the latter authors, we place emphasis on an approximation to the three-body wave function inside the range of $V_{np}$ only.

## III. ELASTIC DEUTERON SCATTERING

### A. Preliminaries

The application of our adiabatic approximation to the treatment of elastic deuteron scattering is not as straightforward as the case of stripping. The basic difference is, of course, that the elastic component of the three-body wave function $\psi^{(+)}(p, n)$ involves neutron-proton separations of the order of the size of the deuteron rather than of the order of the much smaller range of $V_{np}$. The implications of this can be seen by inserting the expansion (8) into the Schrödinger equation (2). Taking the inner product of both sides of the equation with $|\phi_0\rangle$, we obtain

$$[E+\epsilon_0-T_R-V_C(R)-V_{00}(R)]\chi_0(\mathbf{R})$$
$$=\int d\mathbf{k}\,\langle\phi_0|V_N|\phi^{(+)}(\epsilon_k)\rangle\chi(\epsilon_k,\mathbf{R})+\langle\phi_0|V_N|\psi'\rangle, \quad (21)$$

where we have defined

$$V_N(\mathbf{R},\mathbf{r})=V_n(\mathbf{R}+\tfrac{1}{2}\mathbf{r})+V_p(\mathbf{R}-\tfrac{1}{2}\mathbf{r}), \quad (22a)$$
$$V_{00}(\mathbf{R})=\langle\phi_0|V_N|\phi_0\rangle. \quad (22b)$$

For sufficiently small $\epsilon_k$ the contributions to the right-hand side from states other than relative $S$ state will be negligible. However, because of the loosely bound structure of the deuteron, higher relative partial waves have a strong overlap with the vector $V_N|\phi_0\rangle$ at somewhat lower energies $\epsilon_k$ than was the case when the overlap with $V_{np}$ was being considered in Sec. III. There are, nevertheless, a number of reasons why we believe it is a good first approximation to ignore the second term on the right-hand side of Eq. (21).

There have been a number of estimates of the contribution to the deuteron optical potential from breakup in the nuclear field.[27,32] Both these authors use second-order perturbation theory to obtain a deuteron optical potential $V_d$ in the form

$$\langle\mathbf{R}|V_d|\mathbf{R}'\rangle=V_{00}(\mathbf{R})\delta(\mathbf{R}-\mathbf{R}')$$
$$+\int d\mathbf{k}\,\langle\phi_0|V_N(\mathbf{R})|\phi^{(+)}(\mathbf{k})\rangle$$
$$\times\langle\mathbf{R}|(E-T_R-\epsilon_k+i\epsilon)^{-1}|\mathbf{R}'\rangle$$
$$\times\langle\phi^{(+)}(\mathbf{k})|V_N(\mathbf{R}')|\phi_0\rangle. \quad (23)$$

An important feature of this expression is that the breakup contribution gives rise to a strongly nonlocal correction to $V_d$. All estimates that we are aware of make some effort to approximate this term by an equivalent local potential. There are well-tried ways of doing this for a certain class of nonlocal potentials of not too large a range of nonlocality.[15] Unfortunately, the potential (23) is not of this type: for $(E_d-\epsilon_0-\epsilon_k)$ not small, the Green function in (23) is a highly oscillating function of $|\mathbf{R}-\mathbf{R}'|$. On the other hand, both

[30] M. Tanifuji, Nucl. Phys. **58**, 81 (1964).
[31] S. T. Butler, R. G. L. Hewitt, B. H. J. McKellar, and R. M. May, Ann. Phys. (N.Y.) **43**, 282 (1967).
[32] G. Baumgärtner, Z. Physik **204**, 17 (1967).

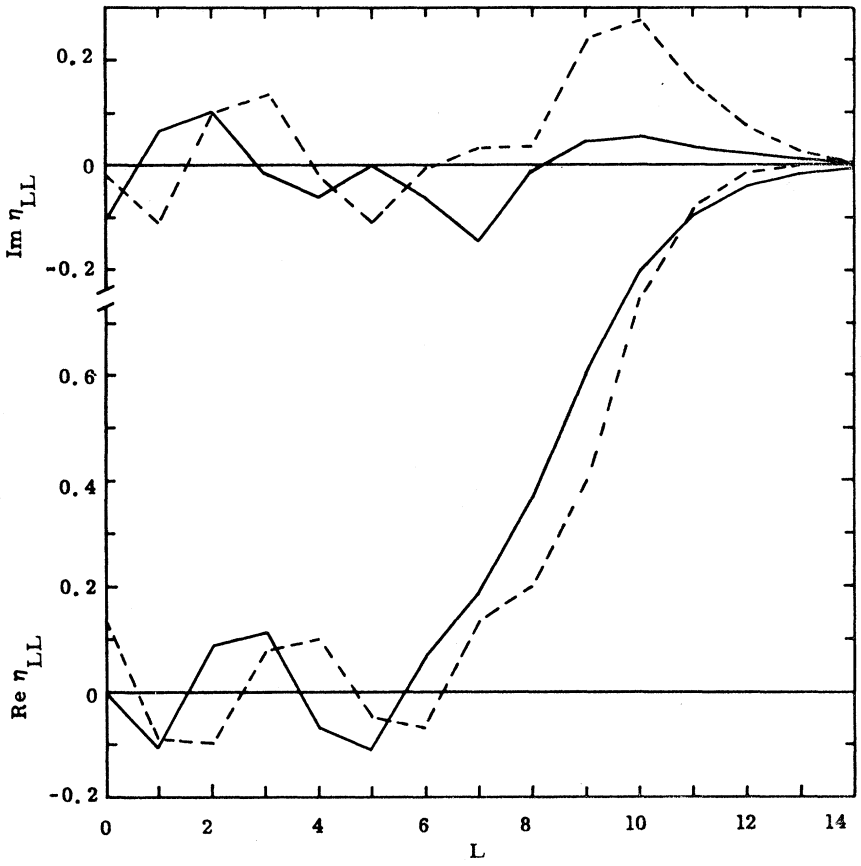


FIG. 3. The reflection coefficients $\eta_{LL}$ generated by $V_{PH}$ (solid curve) and $V_{00}$ (dashed curve) $d$+Ni at $E_d$=21.6 MeV.

the estimates given in Refs. 27 and 32 lead to the conclusion that the breakup corrections are negligibly small.[33] A crude treatment of the nonlocality of $V_d$ is presumably more accurate for the intermediate states of high $\epsilon_k$ in (23), i.e., for the contribution from non-$S$-states of relative motion. We shall therefore appeal to the results of Refs. 27 and 32 as a justification of the neglect of the contribution from all except relative $S$ states on the right-hand side of (21). We shall, however, treat the contribution from $S$ states more carefully than hitherto. In particular, we will not attempt to find an equivalent local optical potential, and our results will contain contributions to all orders in the interaction $V_N$.[34]

[33] The approach used in this paper and in the work of Refs. 27 and 32 should be carefully distinguished from the approach of Sano [M. Sano, in *Proceedings of the International Conference on Direct Reactions and Nuclear Reaction Mechanisms*, edited by E. Clémentel and C. Villi (Gordon and Breach, London, 1965), pp. 204–207], who obtains large second-order breakup corrections. Sano's calculation treats breakup in the ground state of the target on the same footing as breakup in excited states of the target. The contributions from the latter type of intermediate state are included in the effective interaction $V_e(n, p)$ [Eq. (2)] in our approach. This point has been emphasized by Mukherjee (Ref. 10). See also Sec. IV C.

[34] A preliminary version of the present treatment was given by R. C. Johnson, in *International Conference on Nuclear Physics*, edited by R. L. Becker (Academic Press Inc., New York, 1967), pp. 140–143.

### B. Treatment of $S$-Wave Breakup

When the contribution from $\psi'$ in Eq. (21) is neglected, we obtain

$$(E+\epsilon_0-T_R-V_C-V_{00})\chi_0(\mathbf{R}) = \int d\mathbf{k}\ \langle\phi_0 \mid V_N \mid \phi^{(+)}(\epsilon_k)\rangle\chi(\epsilon_k, \mathbf{R}). \quad (24)$$

We shall also require the equation obtained by taking the inner product of both sides of Eq. (2) with $\mid\phi^{(+)}(\epsilon_k)\rangle$. We obtain

$$(E-\epsilon_k-T_R-V_C)\chi(\epsilon_k, \mathbf{R}) = \langle\phi^{(+)}(\epsilon_k) \mid V_N \mid \phi_0\rangle\chi_0(\mathbf{R}) + \int d\mathbf{k}'\langle\phi^{(+)}(\epsilon_k) \mid V_N \mid \phi^{(+)}(\mathbf{k}')\rangle\langle\phi^{(+)}(\mathbf{k}') \mid \psi^{(+)}\rangle. \quad (25)$$

It is well known that a straightforward "coupled-channel" approach to the solution of Eqs. (24) and (25) is very difficult.[35] In our approach, we shall make systematic use of the adiabatic approximation discussed earlier to obtain an approximate solution. The basic ingredients of our analysis are as follows:

(i) The adiabatic approximation implies that very large values of $\epsilon_k$ do not play an important role in the integral on the right-hand side of Eq. (24).

[35] See, e.g., R. G. Newton, *Scattering Theory of Waves and Particles* (McGraw-Hill Book Co., New York, 1966), Chap. 17, pp. 550–557, for a discussion of the three-body problem from this point of view.

(ii) For the restricted range of values of $\epsilon_k$ of interest the dependence of the matrix elements $\langle\phi_0 \mid V_N \mid \phi^{(+)}(\epsilon_k)\rangle$ on $\epsilon_k$ can be factored out.

Figure 1 shows the typical behavior of the quantity

$$\Delta V^0(\epsilon_k, R) = D_0[\langle \mathbf{K}=0 \mid V_{np} \mid \phi^{(+)}(\epsilon_k)\rangle]^{-1} \times (1+\epsilon_k/\epsilon_0)\langle\phi_0 \mid V_N \mid \phi^{(+)}(\epsilon_k)\rangle, \quad (26)$$

as a function of $\epsilon_k$ and $R$. We have used the potential $V_{np}$ given by Yamaguchi[36] to compute the two-body wave functions $\phi_0$ and $\phi^{(+)}(\epsilon_k)$ and we have used the nucleon optical potentials mentioned earlier.

It is clear from Fig. 1 that to high accuracy we can write

$$(E+\epsilon_0-T_R-V_C-V_{00})\chi_0(\mathbf{R}) = \Delta V^0(\bar{\epsilon}_k, R)\int d\mathbf{k}\, g(k)\chi(\epsilon_k, \mathbf{R}), \quad (27)$$

where

$$g(k) = [D_0(1+\epsilon_k/\epsilon_0)]^{-1}\langle \mathbf{K}=0 \mid V_{np} \mid \phi^{(+)}(\epsilon_k)\rangle. \quad (28)$$

For $\epsilon_k \gg 10$ MeV, the form (26) does not give a good account of the dependence of $\langle\phi_0 \mid V_N \mid \phi(\epsilon_k)\rangle$ on $\epsilon_k$. Hence provided such high energies do not play an important role (adiabatic assumption) our approximation (27) should be accurate and insensitive to the precise value of $\bar{\epsilon}_k$ in the range $0<\bar{\epsilon}_k<10$ MeV.

(iii) In order to obtain a closed system of equations, we shall assume that the dominant dependence of $\chi(\epsilon_k, \mathbf{R})$ on $\epsilon_k$ is determined by dropping the second term in the right-hand side in Eq. (25), i.e., we take the energy dependence of $\chi(\epsilon_k, \mathbf{R})$ from

$$\chi(\epsilon_k) = (E-\epsilon_k-T_R-V_C+i\epsilon)^{-1}\langle\phi^{(+)}(\epsilon_k) \mid V_N \mid \phi_0\rangle\chi_0. \quad (29)$$

A sufficient condition for the validity of this step is clearly that the dominant component of $\psi^{(+)}(p, n)$ is the elastic component. We shall, however, only make use of this argument in order to extract the energy dependence of $\chi(\epsilon_k, \mathbf{R})$ and we shall make no further use of perturbation-theory ideas. [On the basis of our calculations, it is indeed found[37] that the elastic component of $\psi^{(+)}(p, n)$ is dominant.]

We shall use the information contained in Eq. (29) in order to simplify the right-hand side of (27). We shall therefore need an accurate description of $\chi(\epsilon_k, R)$ only in the region $\Delta V^0 \neq 0$ and, hence, for reasons similar to those discussed after Eq. (15) above, we can replace the energy $\epsilon_k$ in the Green function on the right-hand side of (24) by $\bar{\epsilon}_k$ with small error.

With these approximations it is now straightforward to show that Eq. (24) becomes

$$(E+\epsilon_0-T_R-V_C-V_{00})\chi_0 = \Delta V(\bar{\epsilon}_k)\langle \mathbf{K}=0 \mid V_{np}Q_0^s \mid \psi^{(+)}\rangle/D_0, \quad (30)$$

where

$$\Delta V(\bar{\epsilon}_k) = \Delta V^0(\bar{\epsilon}_k)\left(\int d\mathbf{k} \mid g(k) \mid^2\right) \times \left(\int d\mathbf{k} \mid g(k) \mid^2(1+\epsilon_k/\epsilon_0)\right)^{-1}, \quad (31)$$

and

$$Q_0^s = \int d\mathbf{k} \mid \phi^{(+)}(\epsilon_k)\rangle\langle\phi^{(+)}(\epsilon_k) \mid. \quad (32)$$

We have

$$\langle \mathbf{K}=0 \mid V_{np} \mid \phi^{(+)}(\epsilon_k)\rangle \propto \sin\delta(k)/k \quad (33)$$

and

$$k\cot\delta(k) = -a^{-1}+\tfrac{1}{2}r_0k^2, \quad (34)$$

where $\delta(k)$, $a$, and $r_0$ are, respectively, triplet $n$-$p$ phase shift, scattering length, and effective range. From (28), (31), (33), and (34) we obtain ($h^2\gamma^2/M=\epsilon_0$)

$$\Delta V(\bar{\epsilon}_k) = \gamma a(1+\gamma r_0)[2(1+\gamma a)+\gamma^2 a r_0]^{-1}\Delta V^0(\bar{\epsilon}_k) = 0.349\Delta V^0(\bar{\epsilon}_k), \quad (35)$$

where we have taken

$$a=5.38 \text{ F}, \qquad r_0=1.71 \text{ F}, \qquad \gamma=0.2316 \text{ F}^{-1}.$$

Equation (30) together with Eqs. (20) of Sec. II B provide a set of coupled differential equations from which $\chi_0$ and hence the elastic scattering can be calculated.

It is convenient to rewrite these equations slightly. From Eq. (20c) and the definition (32), we have

$$\bar{\chi} = \chi_0+\chi_1, \quad (36)$$

where we have defined

$$\chi_1(\mathbf{R}) = D_0^{-1}\langle \mathbf{K}=0 \mid V_{np}Q_0^s \mid \psi^{(+)}\rangle. \quad (37)$$

Equations (30) and (20a) can now be rewritten ($E_d = E+\epsilon_0$)

$$(E_d-T_R-V_C-V_{00})\chi_0 = \Delta V(\bar{\epsilon}_k)\chi_1, \quad (38a)$$

$$(E_d-T_R-V_C-[\bar{V}-\Delta V(\bar{\epsilon}_k)])\chi_1 = (\bar{V}-V_{00})\chi_0. \quad (38b)$$

These equations are to be solved with the boundary conditions[37]

$$\chi_0(R) \xrightarrow[R\to\infty]{} \exp(i\mathbf{k}_d\cdot\mathbf{R}) + (\text{outgoing waves}),$$

$$\chi_1(R) \xrightarrow[R\to\infty]{} (\text{outgoing waves}),$$

in addition to a regularity condition at $R=0$ and appropriate modifications due to Coulomb forces. The potentials appearing in (38a) and (38b) are defined in Eqs. (22), (35), (26), and (20b). In the calculations reported in the next sections, we have used the approximation

$$\bar{V}(R) = V_n(R)+V_p(R). \quad (39)$$

The formal solution of Eqs. (38) defines a deuteron optical potential by

$$(E+\epsilon_0-T_R-V_C-V_d)\chi_0 = 0, \quad (40)$$

where

$$\langle \mathbf{R} \mid V_d \mid \mathbf{R}'\rangle = V_{00}(\mathbf{R})\delta(\mathbf{R}-\mathbf{R}') + \Delta V(\bar{\epsilon}_k, \mathbf{R})\langle \mathbf{R} \mid (E_d-T_R-V_C - [\bar{V}-\Delta V(\bar{\epsilon}_k)+i\epsilon])^{-1} \mid \mathbf{R}'\rangle[\bar{V}(\mathbf{R}')-V_{00}(\mathbf{R}')]. \quad (41)$$

[36] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).
[37] P. J. R. Soper, Ph.D. thesis, University of Surrey, 1968 (unpublished).

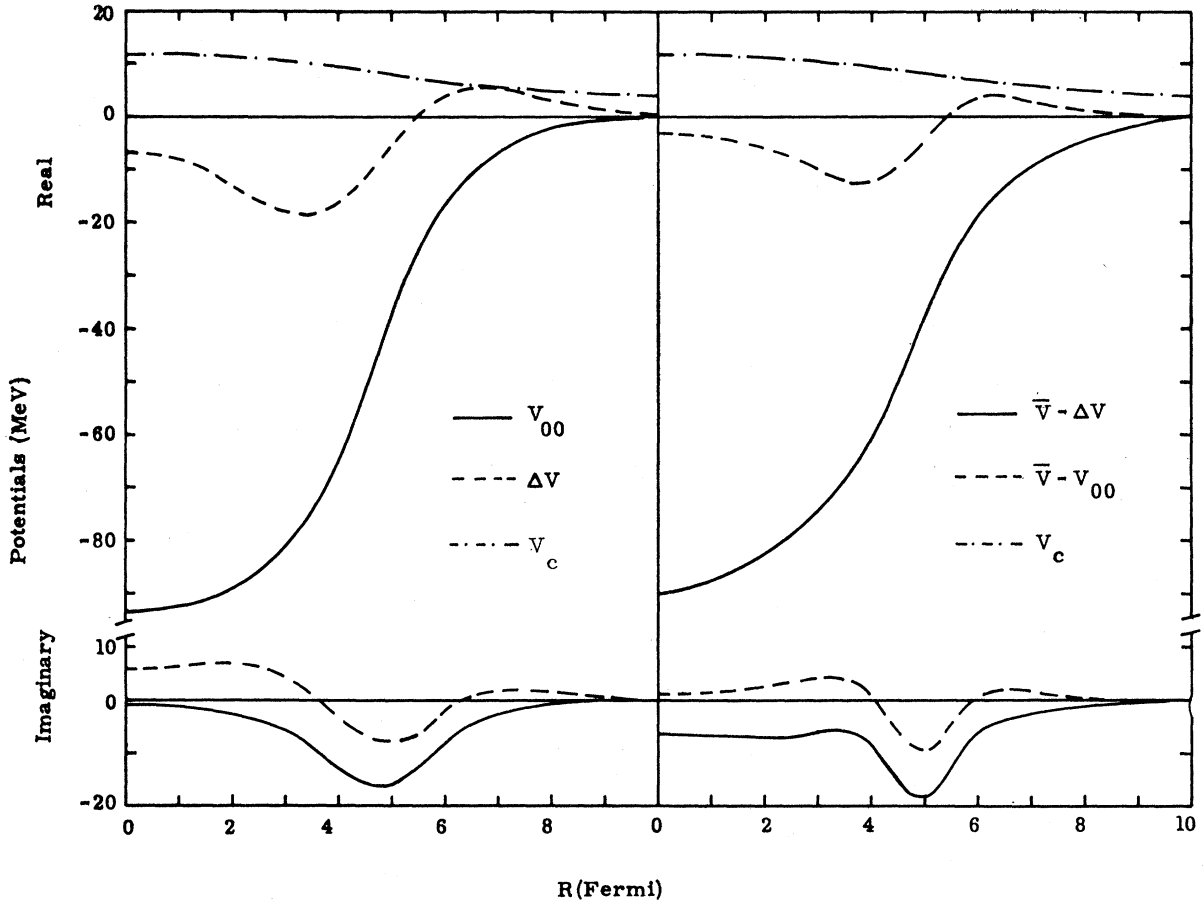


FIG. 4. Potentials used in coupled equations [Eqs. (38)] with $\bar{\epsilon}_k=5$ Mev for $d$+Ni and $E_d=21.6$ MeV.

This is a nonlocal energy-dependent potential and we know of no reliable prescription by which it can be replaced by a local potential with predicted parameters. It is, however, a simple matter to obtain the elastic scattering by solving the coupled equations (38).

## IV. RESULTS OF CALCULATIONS

### A. Surface Partial Waves

In order to test the theory outlined in Sec. III, we have selected the reactions Ni($d$,$d$)Ni at $E_d=21.6$ MeV.[38] For nuclei in this mass region, the observed elastic scattering varies smoothly with mass number[39] and in this energy region effects due to isolated states in the compound nucleus are expected to play a minor role. We are here well above the Coulomb barrier so that Coulomb breakup effects are not expected to be important. By the same token the angular distributions show considerable structure and provide a good test of the theory. The energy region considered is of considerable interest for stripping theory.[1,40] It is for this reason that we have not considered higher energies, although it is to be expected that the accuracy of the adiabatic treatment would increase with energy, in general.

In our calculations we use "average" parameters for the nucleon optical potentials as discussed at the end of Sec. II A. We, therefore, cannot expect to reproduce effects arising from the detailed structure of particular nuclei. For example, the $Ca^{40}$ data at 21.6 MeV [39] show a marked difference from trends in heavier nuclei and we indeed find that our model is unable to reproduce these data.[37] Rawitscher has pointed out[41] that the stripping cross sections for $Ca^{40}(d, p)Ca^{41}$ leading to low-lying levels of $Ca^{41}$ are unusually large. This may be an example of the breakdown of our assumption that relative $S$ states of the neutron and proton are dominant for processes involving the ground state of the target.

We first consider the cross sections produced when breakup effects are neglected and hence the elastic scattering is generated by the Watanabe[42] potential

[38] J. L. Yntema, Phys. Rev. **113**, 261 (1959).
[39] See, e.g., the compilation of experimental data in Ref. 45.
[40] J. L. Yntema and H. Ohnuma, Phys. Rev. Letters **19**, 1341 (1967).
[41] G. H. Rawitscher, Phys. Rev. **163**, 1223 (1967); Phys. Rev. Letters **20**, 673 (1968).
[42] S. Watanabe, Nucl. Phys. **8**, 484 (1958).

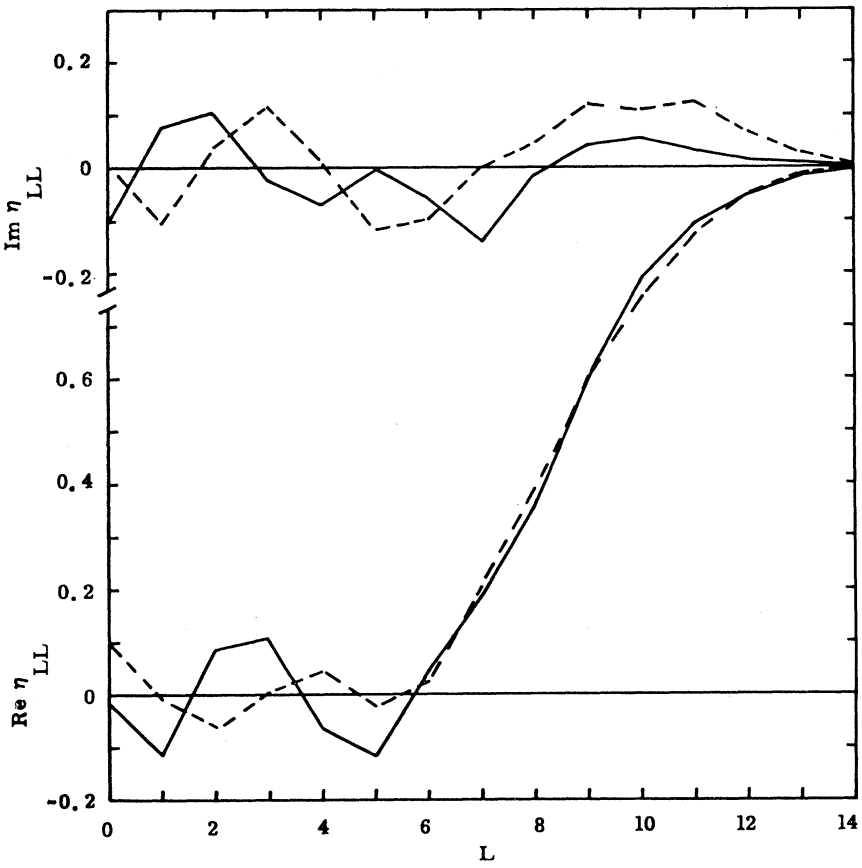


FIG. 5. Reflection coefficients $\eta_{LL}$ generated by $V_{PH}$ (solid curve) and the coupled equations with $\bar{\epsilon}_k=5$ MeV (dashed curve) for $d$+Ni at $E_d=21.6$ MeV.

$V_{00}(\mathbf{R})$ [see Eqs. (38)]. The properties of this potential have been considered many times,[42,32,43] and a careful comparison with the data was made most recently by Perey and Satchler.[44]

Figure 2 is a comparison between the predictions of $V_{00}$ and the predictions of the optical potential $V_{PH}$ of Perey and Perey[45] for Ni.[46] (The latter fits the data very accurately.) In the forward hemisphere, the agreement is remarkably good. Although the relative heights of the maxima and the depths of the minima are not given accurately by $V_{00}$, their angular positions are essentially correct. Beyond 90°, however, the agreement is poor.

In order to examine further the features of $V_{00}$ which produce these discrepancies it is instructive to compare the reflection coefficients $\eta_{LJ}[=\exp(2i\delta_{LJ})]$, as a function of the angular momentum $L$ of the deuteron center of mass, produced by $V_{00}$ and $V_{PH}$. This comparison is shown in Fig. 3 for $L=J$. We emphasize several qualitative aspects of these curves.

(i) An outstanding feature is that the region in angular momentum over which $\mathrm{Re}(\eta_{LL})$ passes from very small values to values close to unity ("the surface region") is much larger for $V_{PH}$ than for $V_{00}$.

(ii) In the same region of angular momentum, $\mathrm{Im}(\eta_{LL})$ has a large bump in the $V_{00}$ case which is considerably suppressed in the curve corresponding to $V_{PH}$.

Both of the features (i) and (ii) are symptomatic of the fact that the imaginary part of $V_{00}$ does not account adequately for the way deuterons are absorbed in the nuclear surface and they are associated[44,37] with the deep minima observed in the $V_{00}$ cross section and the lack of structure at large angles (Fig. 2).

(iii) Both potentials produce reflection coefficients which oscillate with similar amplitudes as a function of $L$ for $L\lesssim 6$ ("the inner region"). As we shall see below, the phase of these oscillations has an important effect on the cross section. We note here that $V_{00}$ and $V_{PH}$ produce oscillations differing in phase by about 90°.

We now turn to the results of calculations using the coupled equations (38).[47] (Figure 4 shows the qualitative features of various potentials which appear in

[43] See, e.g., J. R. Rook, Nucl. Phys. **61**, 219 (1965); F. J. Bloore, *ibid.* **68**, 298 (1965); E. Coffou and L. J. B. Goldfarb, *ibid.* **A94**, 241 (1967); J. Raynal, Ph.D. thesis, University of Paris, 1965 (unpublished); J. Raynal, Phys. Letters **29B**, 93 (1969).

[44] F. G. Perey and G. R. Satchler, Nucl. Phys. **A97**, 515 (1967).

[45] C. M. Perey and F. G. Perey, Phys. Rev. **152**, 923 (1966).

[46] To signify that a quantity is calculated from $V_{PH}$ we always use the abbreviation *PH*.

[47] These calculations were performed with the factor 0.349 appearing in Eq. (35) replaced by 0.31.

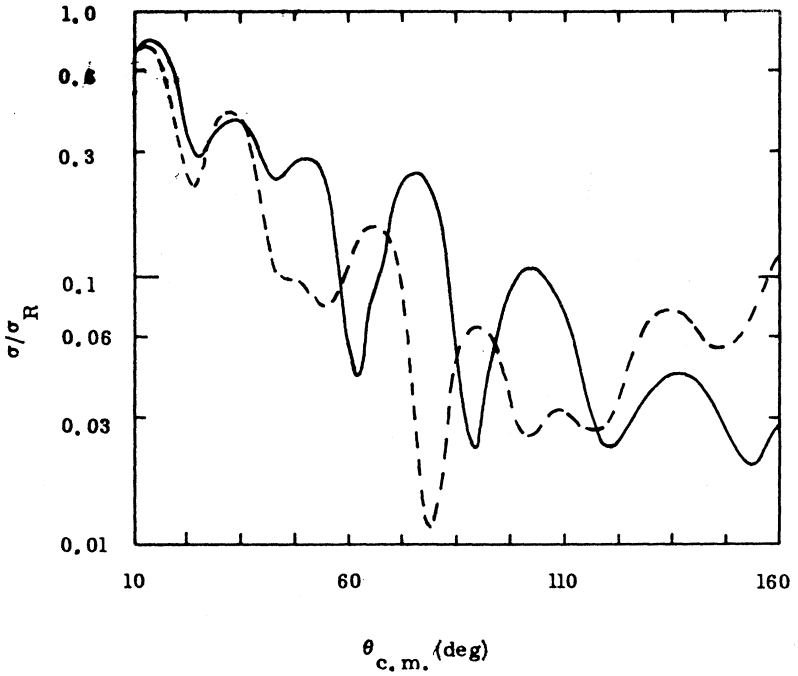


FIG. 6. Differential cross sections for $d$+Ni at $E_d=21.6$ MeV. The solid curve is generated by $V_{PH}$ and the dashed curve by the coupled equations (with $\bar{\epsilon}_k=5$ MeV).

the equations.) The reflection coefficients produced by these equations are compared with those produced by $V_{PH}$ in Fig. 5. It is immediately clear that the two curves agree very well in the surface region and the discrepancies mentioned under (i) and (ii) above are largely removed.

We note, however, that the coupling to $S$-wave breakup channels has a very small effect on partial waves in the inner region. As we shall see below, this is almost certainly the reason for the poor agreement with the experimental cross section shown in Fig. 6.

The insensitivity of these results to the precise energy $\bar{\epsilon}_k$ at which the coupling potential $\Delta V(\bar{\epsilon}_k, R)$ [Eq. (35)] that appears in the coupled equations is evaluated is shown in Fig. 7. This provides considerable justification for our treatment of the continuum of $S$-wave breakup channels given in Sec. III B.

It is well known that the optical potential deter-

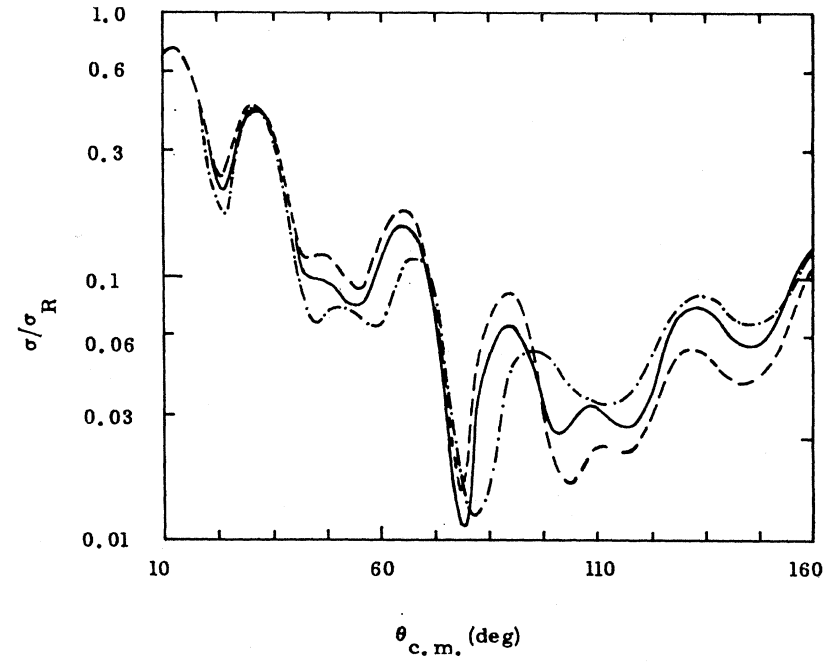


FIG. 7. Differential cross sections for $d$+Ni at $E_d=21.6$ MeV generated by the coupled equations with $\bar{\epsilon}_k=2.5$ MeV (dashed curve), $\bar{\epsilon}_k=5$ MeV (solid curve), and $\bar{\epsilon}_k=10$ MeV (dashed-dot curve).

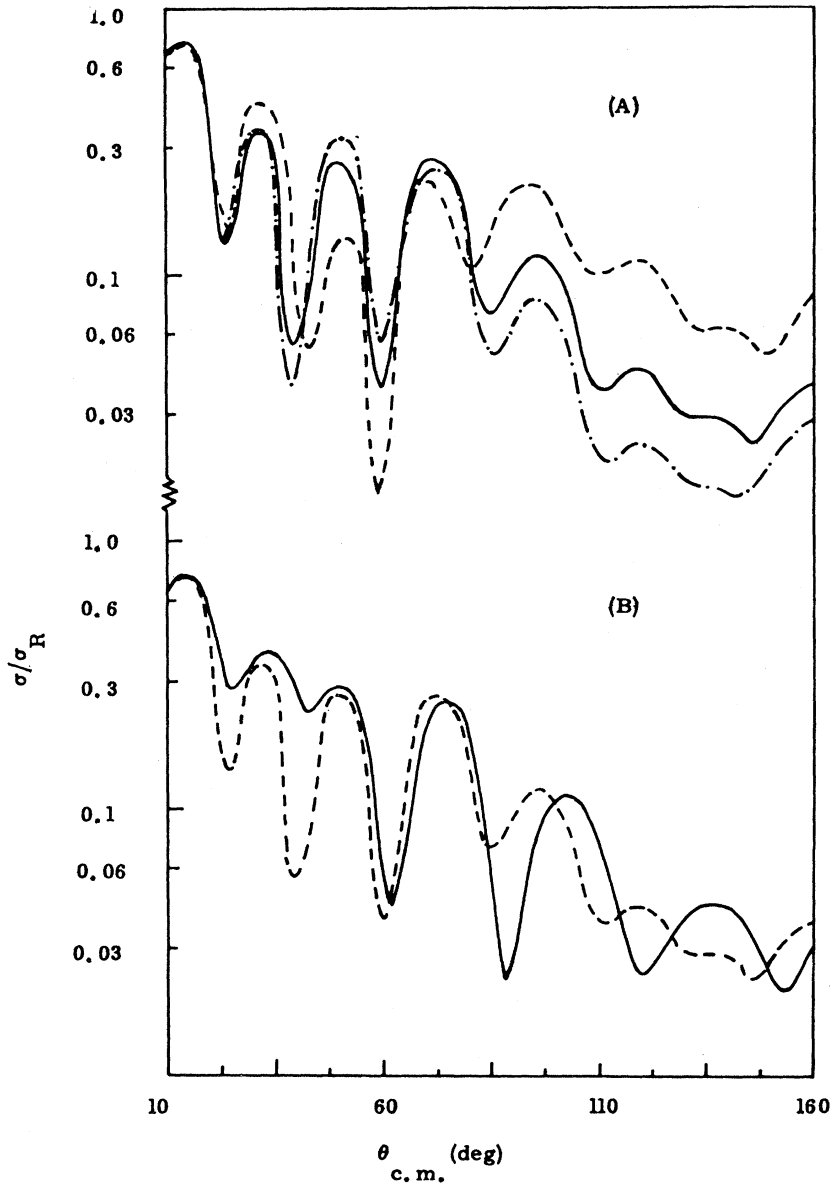


FIG. 8. (A) Changes in the differential cross sections (for $d$+Ni at $E_d=21.6$ MeV) generated by $V_{00}$ (no coupling) when the real part of $V_{00}$ is multiplied by 1.0 (dashed curve), 0.9 (solid curve), and 0.86 (dashed-dot curve). (B) Comparison between the cross section generated by $V_{PH}$ (solid curve) and the cross section obtained when Re $V_{00}$ is multiplied by 0.9 (dashed curve).

mined from fits to experimental data is not unique. There is some evidence, however, that the different families of potentials are associated with the same set of phase shifts and that, therefore, definite physical significance can be attached to the latter.[48] It is for this reason that we believe that the good agreement with the $\eta_{LJ}{}^{PH}$ that we obtain for the surface partial waves provides strong support for our treatment of breakup. The surface partial waves are, of course, the ones of most relevance to stripping. Note also that the wave function to be used in the stripping matrix element, according to the development in Sec. II B, plays an important part in the elastic scattering calculation. It is clear, therefore, that the calculations reported here give some justification for the adiabatic treatment of the stripping matrix element discussed in Sec. II.

### B. Inner Partial Waves

The situation as far as the small partial waves are concerned is less straightforward. We have seen in Sec. IV A that coupling to low-energy $S$-wave breakup channels has a small effect on these partial waves. This, of course, is physically reasonable and inspection of Fig. 4 makes this clear.

We may be guided here by the fact that a simple modification of the calculation produces a marked improvement in the situation. In Fig. 8, we indicate how the scattering produced by $V_{00}$ (neglecting the coupling to breakup channels) is modified when the real part of $V_{00}$ is multiplied by the factors shown. It is clear

[48] See, e.g., Ref. 17.

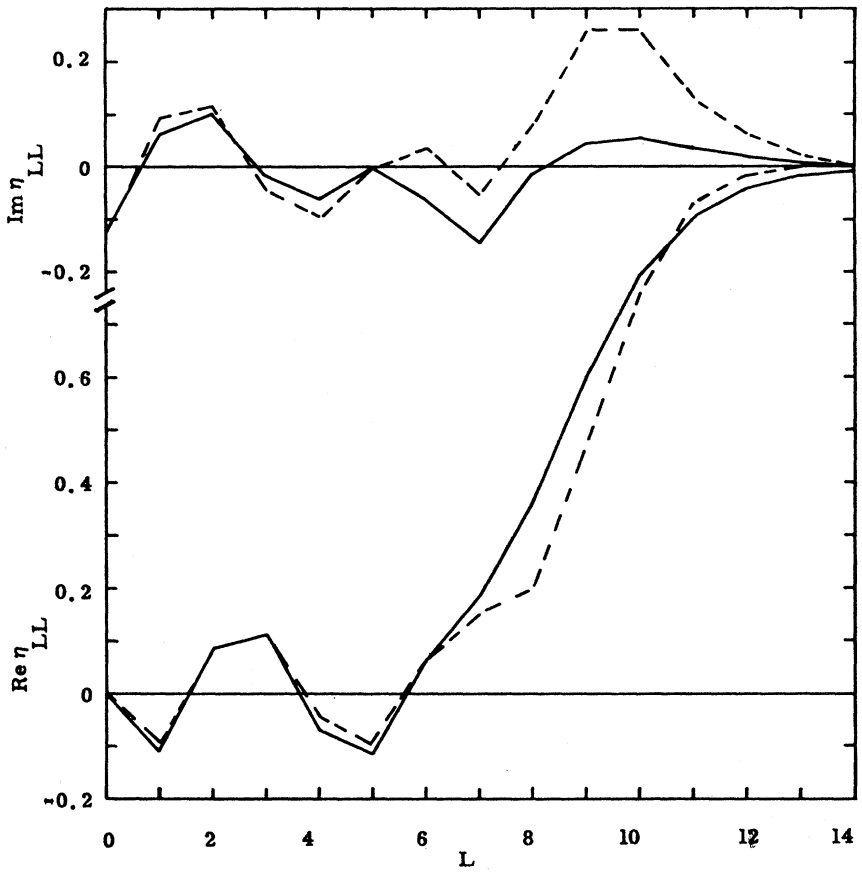


FIG. 9. Reflection coefficients $\eta_{LL}$ for $d+$Ni at $E_d=21.6$ MeV, generated by $V_{PH}$ (solid curve) and the potential 0.9 Re $(V_{00})+i$ Im$(V_{00})$ (dashed curve).

that the relative heights of the maxima in the forward hemisphere and the magnitude of the average cross section at large angles is very sensitive to small changes in the depth of $V_{00}$. With a factor of 0.9 the fit to the data is considerably improved. More significant, from our point of view, is that this improvement is achieved mainly by a modification of the contribution from the partial waves in the inner region. This is shown in Fig. 9 where it can be seen that with this factor the $\eta_{LL}$ for $L \lesssim 6$ are in close agreement with the reflection coefficients generated by $V_{PH}$, while at the same time the $\eta_{LL}$ for $L$ in the surface region are essentially unchanged.

Figure 10 shows that when the coupling to breakup channels is added in the agreement between the predicted $\eta_{LL}$ and the $\eta_{LL}{}^{PH}$ is now very good for all $L$. It is shown in Fig. 11 that the agreement between cross sections is also now very much improved. A comparison with Fig. 8 shows that the effect of the coupling is to fill in the minima at forward angles and to produce the right type of structure at large angles, as is to be expected from an improved treatment of the absorption.[44]

A similar modification of the real part of $V_{00}$ was found to be necessary by Perey and Satchler.[44] They found that the potential $V_{00}$ gives the real part of a deuteron optical potential which yields good agreement with elastic scattering data with no change in shape but with a 10–20% change in strength, when the parameters of the imaginary part of the potential are optimized. Their analysis did not determine the sign of this correction, however. In our case we have kept the absorption fixed and as given by the imaginary parts of the nucleon optical potentials and the coupling to breakup channels. Although an extensive search on the effect of keeping the real part of $V_{00}$ more attractive was not made, the trends shown in Fig. 8 strongly suggest that the correction to our model is repulsive. It is interesting to consider the most likely source of this correction.

In the first place, our treatment of the three-body Hamiltonian could be inaccurate. But precisely because it is the low partial waves that are in question here, we feel that this is unlikely.[49] In any case, all calculations we are aware of which start from a three-body Hamiltonian give a small *attractive* correction from breakup to the deuteron optical potential.[12,32,27]

A much more likely possibility is that we are here

[49] It can be shown (Ref. 37) that the improvement in the fit to experiment that is obtained by adjusting the reflection coefficients produced by the coupled equations to equal the $\eta_{LL}{}^{PH}$ for $L>7$ is much smaller than the improvement achieved by altering the real part of $V_{00}$ by 10%.

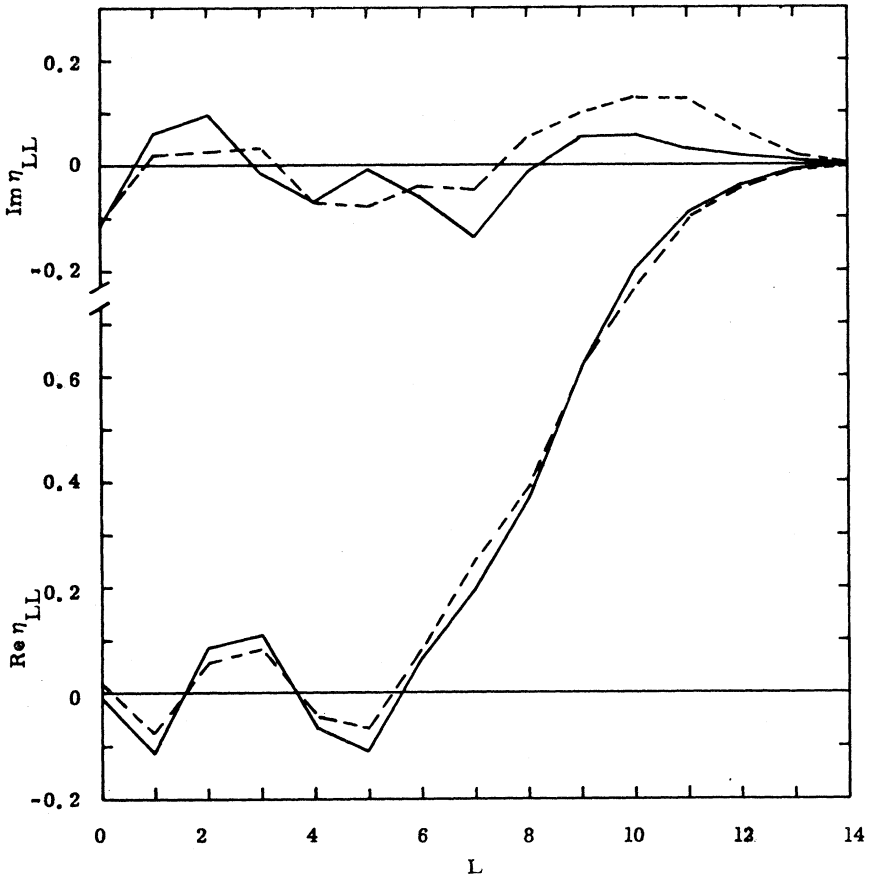


FIG. 10. Reflection coefficients $\eta_{LL}$ for $d$+Ni at $E_d$=21.6 MeV. The solid curve is generated by $V_{PH}$ and the dashed curve is generated by the coupled equation (38) with $V_{00}$ replaced by 0.9 Re $(V_{00})+i$ Im$(V_{00})$.

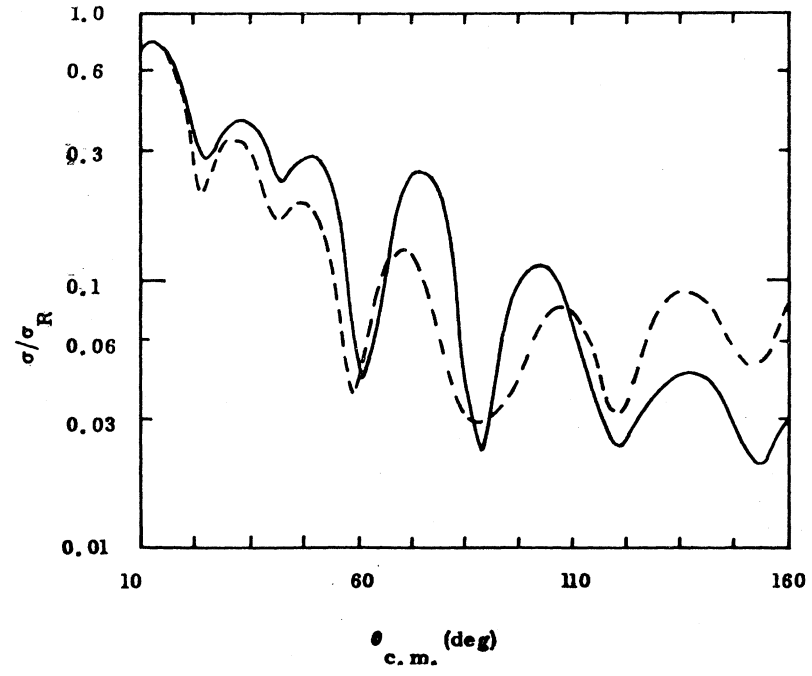


FIG. 11. Differential cross sections $d$+Ni at $E_d$=21.6 MeV generated by $V_{PH}$ (solid curve) and the coupled equations with $V_{00}$ modified as in Figs. 9 and 10.

probing the inadequacies of the three-body model Hamiltonian (effective interaction) we have assumed. Thus, for example, in order to perform calculations in our model it is necessary to come to a decision concerning the nucleon optical potentials to be used. A limited investigation[37] performed with alternative nucleon optical potentials[50] has shown that our results are not significantly altered by this ambiguity. There is, however, an uncertainty in the energy at which the nucleon optical potentials are to be evaluated. We have chosen local potentials at the energy $E_d/2$. If we assume that the energy dependence of the observed local nucleon optical potential is entirely associated with nonlocality of range small compared to the size of the deuteron, it can be shown that this is a very accurate prescription.[37] This may be a reasonable assumption for the real part of the effective interaction which according to Austern and Richards[11] is dominated by the neutron and proton Hartree-Fock potentials.[51] However, Austern and Richards[11] have shown that there are expected to be definite corrections to the imaginary part of the effective interaction we have assumed. These corrections would remedy an obvious defect in the effective interaction we have assumed: It cannot generate that portion of the three-body wave function which describes stripping into bound states because the nucleon optical potentials have fixed imaginary parts. It is to be noted, however, that as far as stripping is concerned the only use we make of the effective interaction is to generate a three-body wave function to be substituted in the right-hand side of matrix element (1). The problem we have just mentioned becomes acute only when the behavior of $\psi^{(+)}(p, n)$ for $|\mathbf{r}|$ far outside the range of $V_{np}$ is considered.

A third correction to the effective interaction, which is certainly present even to first order in the nucleon-nucleon interaction, is the effect due to the Pauli exclusion principle discussed in Refs. 9 and 52 and estimated by Baumgärtner[32] in a nuclear matter approximation. We have repeated Baumgärtner's calculation with the Yamaguchi $n$-$p$ interaction[36] and with a simple Coulomb energy correction. We find that the resulting real correction to $V_{00}$ is repulsive and very sensitive to the Fermi momentum, the range of the $n$-$p$ force and the deuteron energy. For a 21-MeV deuteron, we obtain results in the range $+3$ to $+15$ MeV. This effect is certainly of the order of magnitude and sign required. We hope to return to a discussion of this effect on another occasion.

Other higher-order corrections to the effective interaction have been discussed by Mukherjee.[10]

[50] L. Rosen, J. G. Beery, A. S. Goldhaber, and E. H. Auerbach, Ann. Phys. (N.Y.) **34**, 96 (1965); F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. **182**, 1190 (1969).

[51] It should be mentioned here, however, that if nucleon optical potentials evaluated at $E_d$, rather than $E_d/2$, are used to evaluate $V_{00}$ then the commonly assumed energy dependence of the nucleon optical potential gives a repulsive coorection of 10 MeV to $V_{00}$.

[52] D. J. Thouless, Nucl. Phys. **75**, 128 (1966).

### C. Final Discussion

We have shown that for the reaction Ni$(d,d)$Ni at $E_d=21.6$ MeV the scattering and absorption of deuterons with angular momenta corresponding classically to the surface region of the target is accounted for very well in terms of our model. It should be noted that in this model by far the biggest contribution to the reaction cross section is associated with the imaginary parts of the nucleon optical potentials that we have assumed are present in the effective interaction. The reaction cross section produced by $V_{00}$ alone is 1735 mb. The reaction cross section produced by the coupled equations is slightly *less* (1693 mb) and in close agreement with the reaction cross section generated by $V_{PH}$ (1689 mb). This implies that the major contribution to the deuteron reaction cross section arises from inelastic deuteron scattering, stripping and deuteron break-up and other channels involving excited states of the target. The contributions from these sources appear to be accounted for very well in terms of the imaginary parts of the nucleon optical potentials. In the model presented here, the main role of breakup channels associated with the ground state of the target is to redistribute the absorption among the surface partial waves by producing more absorption in the higher partial waves and less absorption in the lower surface partial waves (see Figs. 3 and 6), i.e., by producing more absorption at large distances.

It is interesting to note that the elastic wave functions generated by the coupled equations are very similar to those generated by $V_{PH}$ for all partial waves. We obtain no strong suppression of the wave functions at short distances, in contradistinction to the work of Ref. 41. Of course, according to the development of Sec. II, the elastic wave function is not relevant to the stripping calculation. It has been found[25,26] that when stripping calculations are performed using our modified prescription results are obtained which are similar to those obtained by suppressing the interior contribution in a standard DWBA by large factors.[1,40]

Finally, we remark that the correction to the deuteron optical potential due to $S$-wave breakup given in Eq. (41) has some interesting properties. If the coupling potentials shown in Fig. 4 are approximated by square barriers of height $V$ and width $b$, then it is not difficult to show that the effect of the coupling term in (41) is to add to the real part of $V_d$ an equivalent local term of order of magnitude

$$(M/\hbar^2)b^2V^2 \sin 2K_0R_0$$

in the nuclear surface. Here $R_0$ is a measure of the nuclear radius and $K_0$ is the local wave number in the potential $V_{00}$. The feature we wish to emphasize is that this is a nonmonotonic function of the nuclear radius. For $V=5$ MeV, $b=2$ F, and $K_0=2$ F$^{-1}$ (see Fig. 4)

the correction gives rise to oscillations as a function of $A^{1/3}$ in the depth of $V_d$ in the nuclear surface which are similar in amplitude and wavelength to those found by Perey and Satchler[44] in their accurate analysis of the elastic data. Further investigations of this point are planned.

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# $^{28}$Si Nucleus in the Projected Hartree-Fock Model*

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The structure of the $^{28}$Si nucleus is studied using a variational procedure. The method of angular-momentum projection from a deformed intrinsic state is applied, and each $J^\pi$ state is projected from a determinant which is variationally "best" for that state. This more general variational procedure includes important vibrational correlations, which in $^{28}$Si have hexadecapole character. The level spacings in the energy spectrum improve considerably compared to those in the Hartree-Fock method followed by angular momentum projection, and this leads to a much better agreement with the experimental spectrum. The $E2$ transition probabilities, except for the $6^+ \rightarrow 4^+$ transition, are also in good agreement with the experiments.

## 1. INTRODUCTION

A NUMBER of calculations[1–3] have been done in recent years to study the low-lying states in $^{28}$Si. All the calculations which consider $^{28}$Si to be deformed and obtain the energy spectrum in either the $SU(3)$ model or the Hartree-Fock (HF) model have the shortcoming that the energy spectrum is too dense by about a factor of 2. Das Gupta and Harvey[2] discuss the possible reasons for this compression and suggest that for the oblate HF solution there are residual correlations ($\beta$ vibration) which are of importance. They then show that the states in $^{28}$Si associated with the oblate minimum can in fact be explained in terms of the rotation-vibration collective model, and suggest that this additional degree of freedom should be included in a many-body calculation.

Recently, Rowe[4] has discussed several methods for describing "vibrational" correlations in finite nuclei. A comparison of these methods by Parikh and Rowe[5] in the model of Lipkin *et al.*[6] has shown that the projected Hartree-Fock (PHF) approximation in which the variation is carried out after projection, gives good results for a whole range of situations. This goes from the case where the nucleus is vibrational to the other extreme where it has static deformation and includes the transitional region in between. Similar results in

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