^{3}He Polarization Measurements and Phase Shifts for p-^{3}He Elastic Scattering*

D. H. McSherry and S. D. Baker
Bonner Nuclear Laboratories, Rice University, Houston, Texas 77001

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This paper reports further measurements for the elastic scattering of protons by polarized ^{3}He in the 4–11-MeV range. A phase-shift analysis based on cross-section, polarization, and spin-correlation data is presented, and the precision with which some of the parameters are determined is discussed.

I. INTRODUCTION

RECENT experimental studies of the elastic scattering of protons by ^{3}He in the 4–11-MeV range have contributed to an improved determination of the scattering phase shifts. These studies include measurements of the cross section,[1,2] proton polarization,[2,3] ^{3}He polarization,[4] and the spin-correlation parameter A_{xz}.[5] Phase-shift analyses using cross-section and polarization data have been reported by Tombrello[6] and by Morrow and Haeberli.[3] Further investigation of the phase shifts including the spin-correlation data is given in Ref. 5.

This paper reports ^{3}He polarization measurements which supplement those of Ref. 4. These data are useful not only as further constraints on the phase-shift parameters but also as a direct measure of the analyzing power of the scattering for ^{3}He polarization, which may be relevant to double-scattering experiments involving ^{3}He.

We also report a phase-shift analysis for five energies between 4.00 and 10.77 MeV. The emphasis in this work was to determine the precision of the phase shifts in the solution region already found rather than to search for new solution regions. We especially wished to study the constraints placed by the data on the values of the $^3P_0$ phase shift and the $\epsilon(1^-)$ mixing parameter which were the least well determined in prior searches.[3,5,6] For this reason, the solution space for these and other selected phase shifts and mixing parameters was studied in detail at 4.0 and 8.8 MeV.

II. EXPERIMENT

The ^{3}He target nuclei were polarized by means of optical pumping; the apparatus used in this process is the same as that described by Baker *et al.*,[4] except that left-right scattering asymmetries were measured at 54° and 109° (70.3° and 127.4° in the center-of-mass system) and the scattering angle was defined by a 2.2-mm×6.2-mm slit formed in the glass work of the cell and a 1.5-mm×6.2-mm slit immediately before the charged-particle detector. The slits were 38 mm apart. This collimation resulted in an angular resolution of approximately 3° (full width at half-maximum) for the data reported here.

The target polarization was monitored by optical measurements as described in Ref. 4; the definitions of the parameters given below are also contained in that reference. The ratio of optical signals, $\delta I/I$, ranged from 0.60 to 0.53 during the experiment, and the values of ρ, a, b, and c are the same as in Ref. 4. The value of f was chosen to be 0.9±0.1 on the basis of comparison of ^{3}He polarization data in ^{3}He-^{4}He elastic scattering[7] with predictions of Barnard *et al.*[8] All the data reported or summarized in the present paper assume this value of f. Since a number of measurements of f ranging from 0.6 to 1.0 have been reported by various workers,[9] the data given here may be in error by a systematic factor. This error may be taken into account by multiplying the values in Tables I and II by a single factor between 0.77 and 1.04. In any case, the effect of such a correction was found to have a very small effect on the determination of the phase-shift parameters for p-^{3}He elastic scattering (see Sec. IV).

III. DATA

Data were taken at eight energies between 3.86 and 10.94 MeV at c.m. scattering angles of 70.3° and 127.4°. The left-right scattering asymmetries were calculated from the data as described in Ref. 4. These experimental asymmetries, corrected for target polarization, are equivalent to the ^{3}He recoil polarization in an unpolarized beam and target experiment. An estimate of

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[1] T. B. Clegg, A. C. L. Barnard, J. B. Swint, and J. L. Weil, Nucl. Phys. **50**, 621 (1964).
[2] D. G. McDonald, W. Haeberli, and L. W. Morrow, Phys. Rev. **133**, B1178 (1964).
[3] L. W. Morrow and W. Haeberli, Nucl. Phys. **A126**, 225 (1969).
[4] S. D. Baker, D. H. McSherry, and D. O. Findley, Phys. Rev. **178**, 1616 (1969).
[5] D. H. McSherry, S. D. Baker, G. R. Plattner, and T. B. Clegg, Nucl. Phys. **A126**, 233 (1969).
[6] T. A. Tombrello, Phys. Rev. **138**, B40 (1965).
[7] D. M. Hardy, M. A. thesis, Rice University, 1968 (unpublished). These measurements were made at 87° c.m. in the region of the lowest $\frac{7}{2}^-$ resonance in ^{7}Be, where the polarization appears to be relatively insensitive to the splitting of the P-wave phase shifts.
[8] A. C. L. Barnard, C. M. Jones, and G. C. Phillips, Nucl. Phys. **50**, 629 (1964).
[9] See footnote 21 of Ref. 4.

TABLE I. ^{3}He recoil polarizations and errors. A possible systematic correction to these values is discussed in the text. Data at c.m. angles 58° and 110° were published previously in Ref. 4 for a different calculation of target polarization. The incident proton energy has an uncertainty of ±0.05 MeV.

Incident proton energy (MeV)	Center-of-mass scattering angle 58°	70.3°	110°	127.4°
3.86	0.040±0.015	0.087±0.019	0.113±0.031	0.056±0.018
4.38	0.023±0.015		0.104±0.033	
4.89	0.002±0.015	0.016±0.016	0.121±0.024	0.094±0.018
5.90	−0.008±0.015	−0.010±0.014	0.144±0.020	0.181±0.019
6.91	−0.078±0.015	−0.040±0.014	0.124±0.020	0.198±0.019
7.92	−0.056±0.015	−0.072±0.015	0.112±0.021	0.165±0.016
8.93	−0.082±0.015	−0.080±0.011	0.068±0.028	0.200±0.023
9.93	−0.078±0.015	−0.127±0.016	0.082±0.030	0.202±0.023
10.94	−0.024±0.015	−0.132±0.019	0.054±0.043	0.240±0.030

the errors due to nonstatistical fluctuations was obtained from the values of P_3P_0.[4] These quantities are determined by combining the data in such a way that all instrumental and polarization asymmetries should be canceled. For each set of data at a particular angle, an additional error was added in quadrature to the statistical error in each P_3P_0, so that χ^2 for the P_3P_0 distribution about zero attained a 50% probability level. This additional error was then folded into the statistical errors for the experimental asymmetries.

The ^{3}He recoil polarization for c.m. scattering angles 70.3° and 127.4° is given in Table I. Table I also lists previously published ^{3}He polarization data[4] for c.m. scattering angles 58° and 110° which have been recalculated to conform to the choice of $f=0.9$ in determining target polarization (see Sec. II). The measurements of the spin-correlation parameter A_{xz},[5] also recalculated for $f=0.9$, are given in Table II. The "minimum error" for A_{xz} includes statistical error only; the "maximum error" includes possible systematic error as explained in Ref. 5.

The ^{3}He polarization data are displayed in Fig. 1. Solid lines through the points are fits to the ^{3}He polarization from the phase-shift analysis discussed in the next section. Proton polarization curves are also shown in Fig. 1 to indicate the similar shapes of the angular distributions.

TABLE II. The A_{xz} values and errors of Ref. 5, recalculated to conform to the same method of determining target polarization as used for the data in Table I. The incident proton energy was 8.8±0.2 MeV. $\theta_{c.m.}$ is the center-of-mass scattering angle, $\Delta A_{xz,min}$ is the error due to statistics, and $\Delta A_{xz,max}$ is an error estimate which includes systematic error as explained in Ref. 5.

$\theta_{c.m.}$	A_{xz}	$\Delta A_{xz,min}$	$\Delta A_{xz,max}$
77°	0.135	±0.058	±0.127
136°	−0.064	±0.084	±0.141

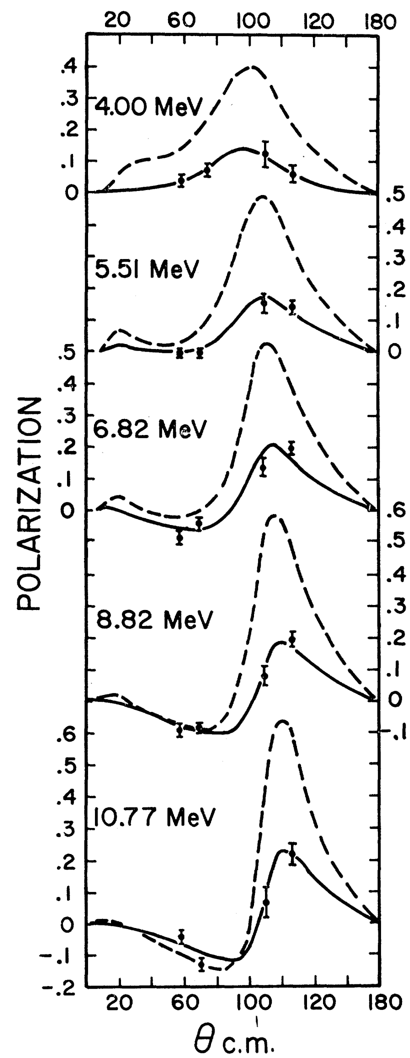

FIG. 1. ^{3}He polarization data. Solid lines through the points are fits to the ^{3}He polarization data from the phase-shift analysis. Dashed lines are calculated proton-polarization curves.

TABLE III. Phase shifts and mixing parameters. See Fig. 3 for plots of the likelihood functions for values marked with an asterisk.

	Incident proton energy (MeV)				
	4.00	5.51	6.82	8.82	10.77
$^1S_0$	−47.7±6.7	−59.1	−66.6	−78.2±8.3	−90.0
$^3S_1$	−52.2	−60.8	−67.6	−78.2±1.4	−87.2
$^1P_1$	34.5	41.2	45.7	49.5	49.0
$^3P_0$	10.1±7.8	25.0	27.5	34.0±5.1	43.3
$^3P_1$	22.7	23.7	21.9	22.9±4.2	23.9
$^3P_2$	35.4	50.9	57.7	62.6	65.5
$\epsilon(1^-)$	72.1*	75.0	78.0	77.7*	76.8
$^1D_2$	−4.6	−6.6	−9.9	−12.7	−15.1
$^3D_1$	1.8	−0.3	−1.6	−2.7	−2.4
$^3D_2$	−1.0	−0.9	0.5	1.9	1.8
$^3D_3$	0.0	0.2	−0.5	−0.4	0.8
$\epsilon(2^+)$	−3.7	2.8	−0.6	1.6	2.0
ϵ_T	3.5	1.0	3.2	8.5±3.5	8.9
No. of data points	35	56	56	59	53
$\chi_{\min}^2$	11.3	30.0	43.0	26.1	33.4

IV. PHASE-SHIFT ANALYSIS

Parameters for the phase-shift analysis include singlet and triplet S-, P-, and D-wave phase shifts: $^1S_0$, $^3S_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$, $^1D_2$, $^3D_1$, $^3D_2$, and $^3D_3$. Mixing parameters[10] include two channel-spin-mixing parameters, $\epsilon(1^-)$ for $^1P_1$ and $^3P_1$ mixing and $\epsilon(2^+)$ for $^1D_2$ and $^3D_2$ mixing, and a tensor coupling parameter ϵ_T for $^3S_1$ and $^3D_1$ mixing. Calculations were done at 4.00, 5.51, 6.82, 8.82, 10.77 MeV since at these energies cross-section,[1,2] proton-polarization,[2,3] and spin-correlation[5] data were available. ^{3}He polarization values for these energies were taken from smooth curves drawn through the data given in Table I. The "minimum error" in A_{xz} was used in all calculations except where noted.

The search routine used was a simple grid search in which each parameter was individually varied in preset steps until a minimum value of χ^2 was found. Other more sophisticated search routines were used from time to time but it appeared that very near the optimal values of the parameters the simple program worked just as well.

The values of the phase shifts and estimates of the precision with which selected parameters are specified by the data are given in Table III and are discussed below. Special attention was directed to obtaining the phase shifts for two energies. The lower energy 4.0 MeV was chosen to see whether the additional ^{3}He polarization data would improve the precision of the phase shifts at the energy for which previous phase-shift sets showed large differences in the values of $^3P_0$ and $\epsilon(1^-)$. The higher energy 8.8 MeV was chosen because of the existence of A_{xz} data at that energy.

The following procedure was adopted: The value o. one parameter was fixed and all the remaining parameters were adjusted until a minimum χ^2 was obtained- The parameter was then set to a new value and a new search was made. Continuing in this manner, a plot of χ^2 versus the parameter was generated which indicated the possible solution region for that parameter.

The results of this procedure for two phase-shift parameters at 4.0 MeV are summarized in the two curves in the upper half of Fig. 2. The solid curve indicates the results of a sweep through $\epsilon(1^-)$ as described in the previous paragraph. Each point on the solid curve is plotted to show the value of $\epsilon(1^-)$, which was fixed, and the value of $^3P_0$, which resulted from the search in all the parameters except $\epsilon(1^-)$. The values of χ^2 associated with each point are given next to the points. The dashed curve summarizes a sweep through values of $^3P_0$, again showing values of χ^2 and the values of $\epsilon(1^-)$ obtained in the searches in all parameters except $^3P_0$. The open circles are local minima obtained by letting all the parameters vary starting with values very close to those found during the process of generating the curves. When $\epsilon(1^-)$ is set very close to 0° or 90°, χ^2 becomes very large, since for $\epsilon(1^-)$ equal to 0° or 90° the calculated proton and ^{3}He polarizations are equal, whereas the measured polarizations are quite different. The values of $^3P_0$ and $\epsilon(1^-)$ for solutions I and III of Ref. 3 are shown by the positions of the numerals I and III.

It is interesting to note that the χ^2 near III is much

[10] J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24**, 258 (1952). Elements of the scattering matrix $S_{l',s';l,s}^J$ for the case of mixing were calculated from Eq. (4.19) of this reference, where the eigenphase shift corresponding to a smaller l or s was taken as δ_1.

higher than that near I. Moreover, there is a smooth decrease of χ^2 between III and I along the line produced by sweeping the values of $^3P_0$. It may also be noted that between the two local minima on the $\epsilon(1^-)$ line, the values of χ^2 do not rise very high. In addition, the value of $^3P_0$ remains nearly constant at the region-I value as $\epsilon(1^-)$ is changed. From these considerations it appears that the region-I values of $^3P_0$ and $\epsilon(1^-)$ are favored by the data, although $\epsilon(1^-)$ is very porly defined.

A similar procedure has been followed for the 8.8-MeV data and is illustrated in the lower half of Fig. 2. The principal difference is that the values in region III do not lie on either curve. Moreover, the χ^2 at region III is much higher than at the only minimum found,

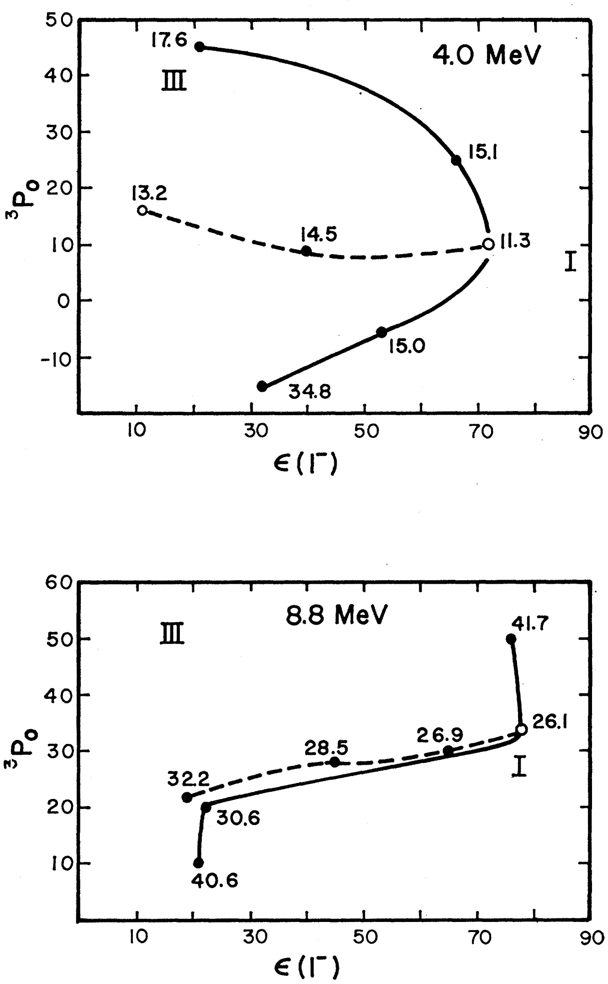

FIG. 2. Correlation curves for $\epsilon(1^-)$ and $^3P_0$ at 4.0 and 8.8 MeV. The solid line indicates a sweep through $\epsilon(1^-)$ values and the dotted line indicates a sweep through $^3P_0$ values. The numbers are minimum χ^2's allowing all other parameters to vary as explained in the text. Open circles are local minima obtained by searching in all parameters. The values of $^3P_0$ and $\epsilon(1^-)$ for solutions I and III of Ref. 3 are shown by the positions of the numerals I and III.

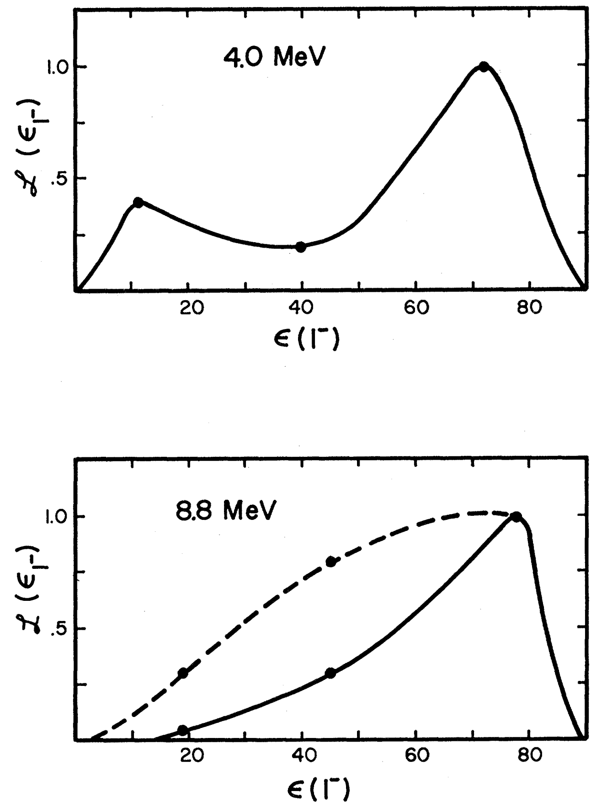

FIG. 3. Relative likelihood functions for $\epsilon(1^-)$ at 4.0 and 8.8 MeV. The solid line represents searches using $\Delta A_{xz,\min}$. The dashed curve corresponds to $\Delta A_{xz,\max}$.

and a search initiated at III goes to the minimum shown. Again it appears that the lower value of $^3P_0$ and a broad allowable range of $\epsilon(1^-)$ (closer to 90° than to 0°) are indicated by the data. If the A_{xz} data are left out of the search routine, the higher values of $\epsilon(1^-)$ are no longer favored over the lower ones, although the value of $^3P_0$ remains near that of solution I.

To make a quantitative estimate of the precision with which the data specify a given phase-shift parameter δ_k, we have calculated the relative likelihood function[11]

$$L(\delta_{ki})=\tfrac{1}{2}\exp[(\chi_{\min}^2-\chi_{ki}^2)],$$

where χ_{ki}^2 is the minimum χ^2 for δ_k set at the value δ_{ki} and $\chi_{\min}^2$ is the minimum χ^2 found in the entire range of δ_k.

The likelihood functions for $^3P_0$ at 4.0 and 8.8 MeV are approximately Gaussian. Hence a standard deviation Δ_k, calculated from the half-width at half-height, is assigned as the estimate of precision for the $^3P_0$'s. The likelihood functions for $\epsilon(1^-)$ at 4.0 and 8.8 MeV are not Gaussian, however, as is shown in Fig. 3. Hence,

[11] J. Orear, University of California Radiation Laboratory Report No. UCRL-8417, 1958 (unpublished).

no estimates of precision are given for $\epsilon(1^-)$, although it is clear from Fig. 3 that certain $\epsilon(1^-)$ values are more probable than others.

Similar calculations of χ^2 dependence on the parameters δ_k and resultant likelihood functions $L(\delta_k)$ were also done for $^1S_0$, $^3S_1$, $^3P_1$, and ϵ_T at 8.8 MeV and for $^1S_0$ at 4.0 MeV. The $^1S_0$ phase shift was studied since it appeared to be strongly correlated with the other phase shifts, varying by relatively large amounts for small changes in $\epsilon(1^-)$, $^3P_0$, and ϵ_T at certain points. Searches were done for the $^3S_1$ and $^3P_1$ phase shifts in order to make selective checks on parameters which appeared to be well defined. Since the likelihood functions were all approximately Gaussian, the precision estimates in Table III are the standard deviations of the likelihood functions for each parameter.

We wish to emphasize that the precision estimates Δ_k obtained in this manner should be viewed as an indication of how sensitive the various phase shifts are to the available data, rather than as a measure of the absolute accuracy with which the phase shifts have been determined. As can be seen from Table III, the minimum χ^2 obtained is generally considerably smaller than the number of degrees of freedom. These low values of χ^2 occur because the errors quoted for the experimental cross-section and proton-polarization data include estimates of systematic errors in the measurements. As a result, a straightforward assignment of confidence limits on the parameters on the basis of the absolute values of χ^2 is not possible, and the use of the relative likelihood function is chosen instead. Other systematic effects on the values of the phase shifts are discussed later in this section.

The fact that $^3S_1$ is well defined by the data is of interest because of its importance in the choice of the stepping interval in the search program. If the stepping interval is comparable to the precision with which $^3S_1$ is determined by the data, then it is possible that the program will not arrive at the optimum values of $^3S_1$, nor of the other parameters, because of their correlation with $^3S_1$. This was borne out in earlier searches made on these data with 0.5° steps, for which χ^2's were considerably higher than those found in subsequent searches with 0.1° step size. The χ^2's found with 0.5° steps did not decrease unifoımly when the step size was decreased, with the result that the final minima, using 0.1° steps, were shifted or broadened. In addition, one apparent local minimum in $\epsilon(1^-)$ at 4.0 MeV was eliminated when a 0.1° step size was used.

The phase shifts at the energies other than 4.0 and 8.8 MeV were found with the search program starting at the region-I values. This admittedly biases the results toward region I, but in light of the results at 4.0 and 8.8 MeV described above, we expect that this is not serious. In fact, in the case of $^3P_0$, the values have all moved away from region I and toward the center of the solution region found by Morrow and Haeberli.[3]

There are several factors which may introduce systematic errors in the values quoted in Table III. First, the normalization of the target polarization in the experiments involving polarized ^{3}He may be in error. The effect of this was briefly considered by carrying out a search at 8.8 MeV, with the optical pumping parameter f set at 0.6 instead of 0.9 (see Sec. II). None of the phase shifts was changed by more than 1°. Second, the effects of higher partial waves were ignored in most of the searches. However, in one search at 8.8 MeV in which F waves were included, χ^2 was not significantly improved and the other phase shifts and mixing parameters were changed by less than 0.8°. Moreover, the F-wave phases were all less than 1°. Third, the effect of inelastic processes was ignored.

It is interesting to note that a small, but apparently significant, positive value of ϵ_T has been found at all energies. Unless this result is an artifact produced by systematic errors in the data or the factors described above, it indicates mixing of S and D waves in the 1^+ channel.

Although the present study has specified most of the p-^{3}He scattering parameters fairly well, further experiments appear to be necessary to define $\epsilon(1^-)$ more accurately. Calculated values of proton and ^{3}He polarizations show little dependence on $\epsilon(1^-)$; therefore, further measurements of these quantities may not decrease the $\epsilon(1^-)$ solution region. Since the spin-corelation parameters A_{xx}, A_{yy}, and A_{xz} do exhibit large variations as a function of $\epsilon(1^-)$ at certain scattering angles, further measurements in the 4–11-MeV energy region may serve to increase the precision with which $\epsilon(1^-)$ and other phase-shift parameters can be specified. Before taking additional spin-correlation data, however, the effect that any new results would have on the phase shifts should be carefully checked.

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