Investigations of the First Excited State of ^{4}He via the Reaction ^{7}Li(p, α)*

WEN-KUAN LIN, F. SCHEIBLING,† AND R. W. KAVANAGH
California Institute of Technology, Pasadena, California 91109

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The α-particle energy spectra from bombardment of ^{7}Li with 9.1-MeV protons have been obtained for $2.5°\leq\theta_\alpha\leq 120°$. The high-energy portions of the α spectra are interpreted as due to the ^1S, $p+{}^3$H final-state interaction through the first excited state of ^{4}He at 20.06 MeV. The factored-wave-function method is used to deduce the resonance parameters of this state. Consistency in the use of this method is obtained by a plane-wave Born-approximation calculation based on triton-transfer mechanisms to account for the forward peaking in the angular distribution. Coincidence measurements between α particles and other charged reaction products give additional evidence for the 0^+ assignment to the state and indicate the importance of the $\alpha+{}^3$H and $\alpha+{}^1$H final-state interactions at still higher excitation energy of the ^{4}He system.

I. INTRODUCTION

THE first excited state of ^{4}He, proposed by Werntz[1] in interpreting the neutron energy spectra[2,3] from the reaction ^{3}H(d, n), has been seen in inelastic electron,[4] proton,[5] and α-particle[6] scattering. Its effects in the reactions ^{3}H(p, p), ^{3}H(p, n), ^{3}He(n, n), ^{3}H(d, n), and ^{3}He(d, p) have been investigated by many authors; the results have been reviewed recently by Meyerhof and Tombrello.[7] In terms of s-wave $p+{}^3$H phase shifts, the properties of the ^{4}He system above the $p+{}^3$H threshold (19.814 MeV) are characterized by a rapid rise in ^1S phase with energy up to the $n+{}^3$He threshold (20.578 MeV), with only slow change thereafter. The ^3S phase is approximately described by the scattering from a hard sphere of radius 4 F.[8]

α-particle energy spectra from the reaction ^{7}Li(p, α) at 43.7-MeV proton energy have been reported by Cerny *et al.*[9] In addition to the peak due to the first excited state of ^{4}He at 20.1 MeV, a second peak has been found at 21.4 MeV in agreement with the work of Parker *et al.*[10] The present work was undertaken to reexamine this reaction with improved energy resolution and particle identification. It has been found possible to deduce from the α-particle energy spectra a set of resonance parameters for the first excited state and thus the ^1S, $p+{}^3$H phase shifts to compare with those given in the literature from other experiments.

II. EXPERIMENTAL METHOD

Metallic lithium, enriched to 99.99% mass 7, was evaporated onto gold-foil backings ($\sim$80 μg/cm^2) *in situ*. A magnetically analyzed 9.1-MeV proton beam from the ONR-CIT tandem accelerator was collimated to a 1.5-mm square before striking the target. α-particle energies were measured in a 180° double-focusing magnetic spectrometer of 61-cm radius. The horizontal and vertical acceptance angles of the spectrometer were set at 1° and 4°, respectively. Particles traversing the spectrometer were detected by a surface-barrier counter located in the focal plane; a slit just in front of the counter defined an energy window of $\Delta E=E/90$. Nickel foil of appropriate thickness was placed in front of the counter to separate out the particles (e.g., protons) which are of the same energy and of the same Z^2/M as the α particles. In the target chamber, there was an additional surface-barrier counter, 2.75 cm from the target rod, in the plane defined by the beam axis and the center of the spectrometer entrance slits. Except in the coincidence measurements to be described, this counter was fixed at 145° to the beam in order to monitor the yields from the ^{7}Li(p, α_0) reaction. The ^{7}Li(p, α) differential cross section is then expressed in terms of the spectrometer yield Y_{Sp} and the monitor-counter yield Y_{Mon} as

$$d^2\sigma/dE_\alpha d\Omega_\alpha = 2.643(90/E_\alpha)(Y_{\text{Sp}}/Y_{\text{Mon}}) \times 0.86 \text{ mb/sr MeV},$$

where the first constant is the ratio of solid angle of the monitor counter to that of the spectrometer, and $E_\alpha/90$ is the energy window of the spectrometer. The last constant is the ^{7}Li(p, α_0) differential cross section

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† Present address: Centre de Recherches Nucléaires, Strasbourg, France.

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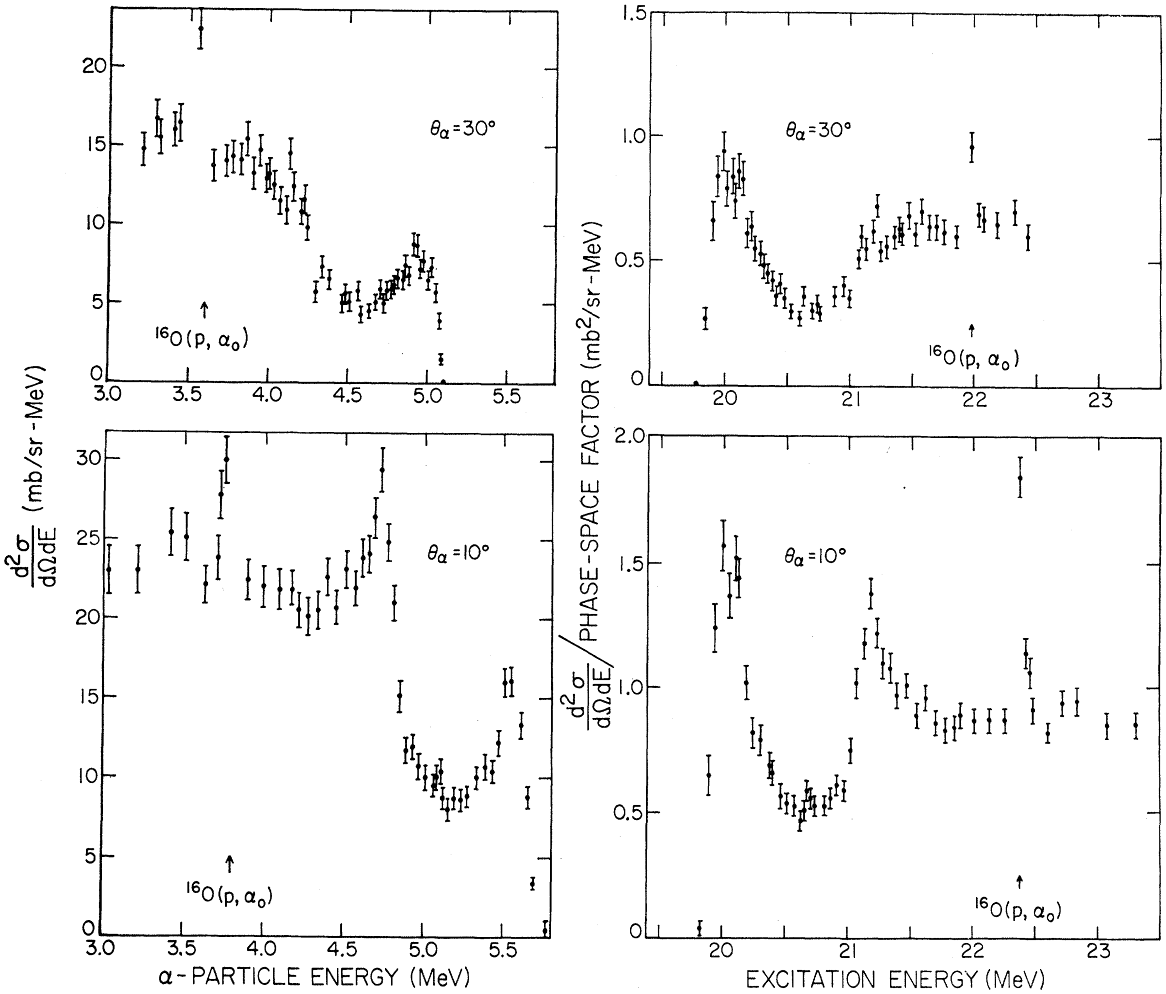

FIG. 1. The α-particle energy spectra from ^{7}Li(p, α) at E_p=9.1 MeV, θ_α=10° and 30°. The only contaminant reaction was ^{16}O(p, α_0). The phase-space factor, which is proportional to $[E_\alpha(E_x-19.814)]^{1/2}$, was taken out, and the result is shown as a function of E_x in the right-hand side of the figure. The enhancement in the transition probability was interpreted as due to the strong ^1S, $p+{}^3H$ final-state interaction through the first excited state of ^{4}He. The second excited state appears at E_x=21.2 MeV.

at 145°. It was measured to be 0.86±0.09 mb/sr with a gold-backed LiF target, whose thickness was determined with the spectrometer by measuring the energy loss of α particles reflected off the gold backing. Figure 1 shows the α-particle energy spectra obtained at 10° and 30°. The first excited state of ^{4}He is seen at 20.06 MeV, and the second excited state appears as a peak near 21.2 MeV. At larger angles, this peak becomes less obvious and is masked by the contributions from other final-state interacting pairs. Limiting the interest to the study of the first excited state, only the high-energy parts of the spectra were taken in the later stage of the experiment. Figure 2 shows the spectra obtained at laboratory angles from 10° to 120°.

For a three-body reaction 1+2→3+4+5, only three of the five independent kinematic variables were determined in the setup just described (i.e., from the momentum of particle 3), and the relative energies of the pairs (3+4) and (3+5) were not fixed. To remove these ambiguities, coincidence measurements were made to define the direction along which particle 4 was emitted. The α particles were detected in the spectrometer at 30° in coincidence with the other charged reaction products observed with the counter in the target chamber near −110°. The counting rate for the counter in the target chamber was low enough that coincidence circuitry with a resolving time of 1 μsec was satisfactory. Figures 3 and 4 show some of the coincidence spectra taken at E_α=5.00 MeV and E_α=4.10 MeV which correspond to E_x=20.01 MeV and E_x=21.27 MeV, respectively. For the latter, the ^{4}He system was excited above its $n+{}^3$He threshold, so that among the reaction products there were neutrons

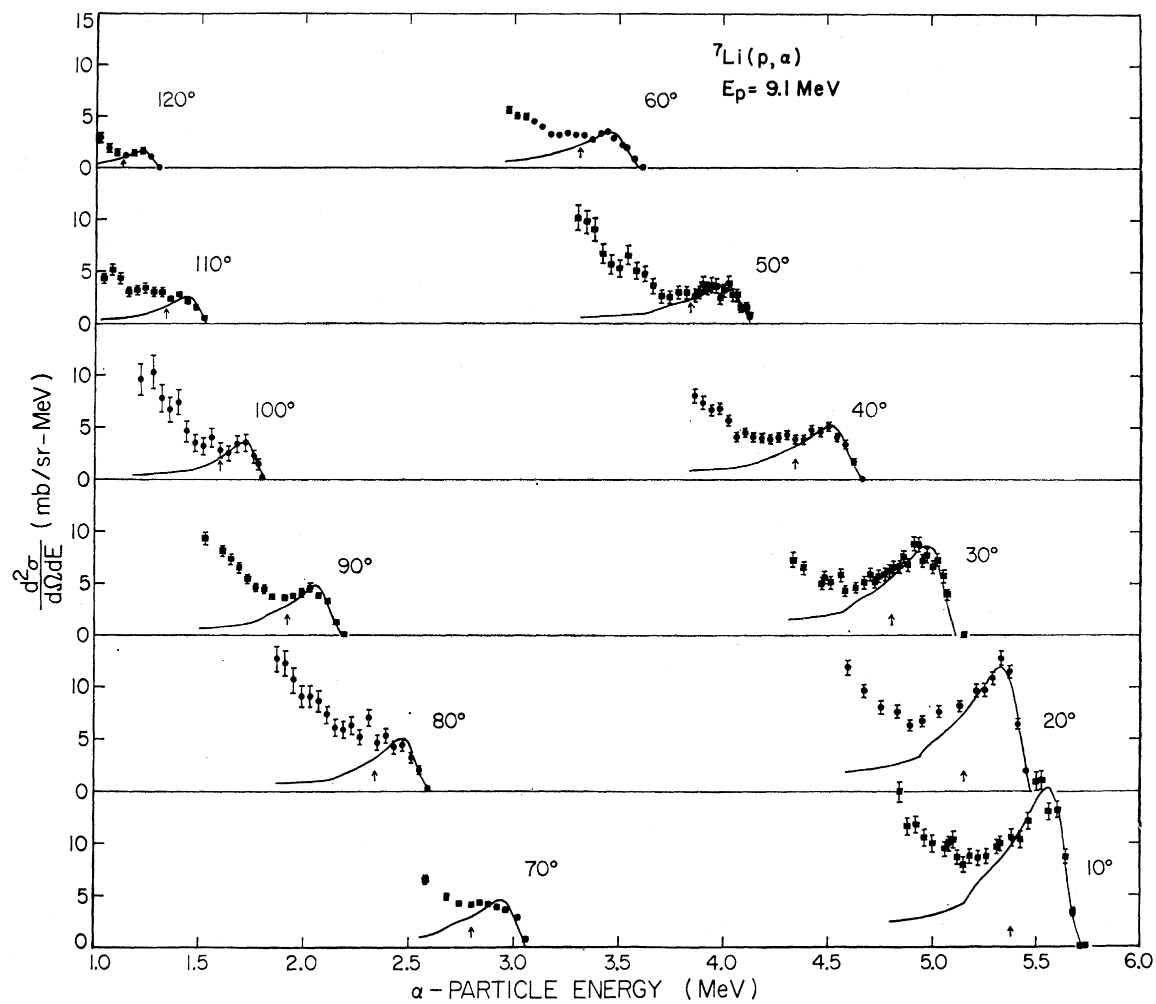

FIG. 2. The α-particle energy spectra from ^{7}Li(p, α) at $E_p=9.1$ MeV and $10°\leq\theta_\alpha\leq120°$. The high-energy portions of the α spectra are shown here together with the spectral shapes calculated from the expression (5) with the resonance parameters (a, E_{00}, $\gamma_{p00}{}^2$, $\gamma_{n00}{}^2$) = (3.0, 20.35, 5.53, 2.88). The least-squares fits were made using data points for α-particle energies greater than those indicated by arrows, which correspond to $E_x=20.30$ MeV.

and ^{3}He's. The counts at high channel numbers, well removed from the kinematic loci, are randoms due to the elastic scatterings from the target backing. After each of the coincidence runs, a singles spectrum was taken to facilitate correction for random coincidences, which were usually less than 10%, but rose as high as 50% in a few instances. As indicated in Figs. 3 and 4, the coincidence counts summed from the channels defined by the parentheses, normalized to 1000 α-particle counts in the spectrometer, are plotted out in angular correlations shown in Figs. 5 and 6.

The angular correlation for $E_x=20.01$ MeV can be interpreted as if the recoil ^{4}He system were decaying isotropically in its c.m. system, i.e., for a triton-transfer reaction mechanism, the p-wave amplitude is found to be less than 15% of the s-wave amplitude. This supports the 0^+ spin-parity assignment for the first excited state of ^{4}He, and indicates that only the $p+{}^3$H final-state interaction is important. For $E_x=$ 21.27 MeV, however, the protons were found to be strongly correlated with the α particles along a pair of directions, consistent with the undetected triton interacting strongly with the α particle through the 4.63-MeV excited state of ^{7}Li. Similarly, the $\alpha+p$ final-state interaction via the ground state of ^{5}Li and possibly also the $\alpha+n$ final-state interaction through that of ^{5}He were seen, respectively, in the α-^{3}H and α-^{3}He angular correlations.

III. RESONANCE PARAMETERS

By virtue of the short-range nature of nuclear interactions, the particles in the final state of a reaction

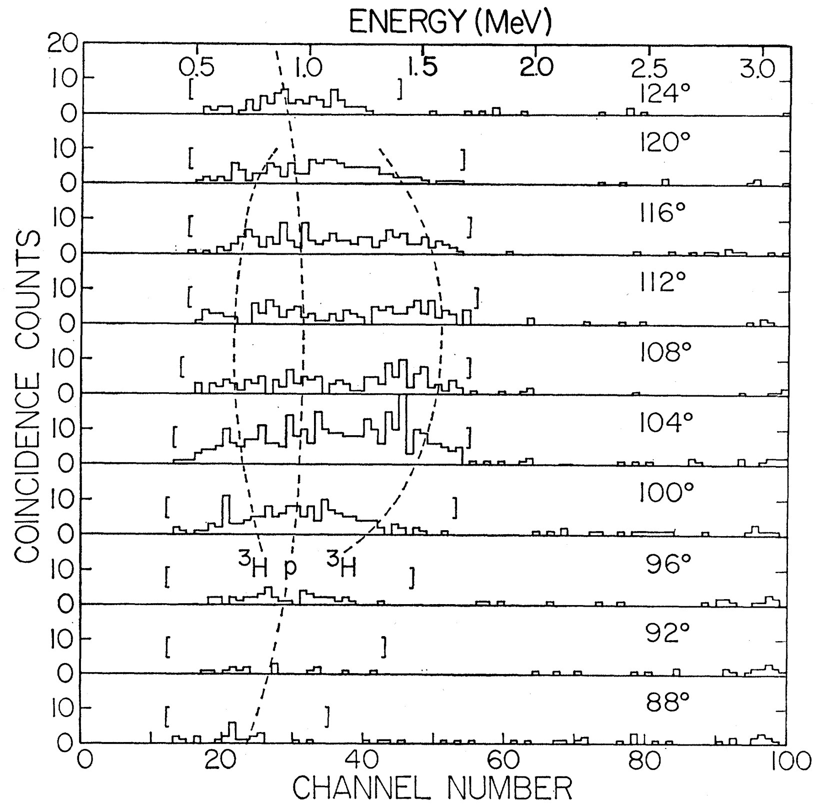

FIG. 3. Coincidence spectra from ^{7}Li(p,α) at $E_p=9.1$ MeV. The 5.0-MeV α particles were detected at 30° in the magnetic spectrometer, whose resolutions were set at $\delta E/E=1.11\%$, $\delta\theta=1°$, and $\delta\Phi=4°$. The acceptance angles for the triton or proton counter in the target chamber, set at the negative angles indicated, were 3.8° and 15.5° along the θ and Φ directions. The dashed lines are kinematically predicted loci where protons and tritons are expected. The parentheses define the regions where the coincidence counts were summed.

are produced in the neighborhood of one another. The reaction probability is proportional to the squared modulus of the wave functions of the particles formed when they are in the reaction zone,[11] i.e., at a distance a apart which is of the order of the range of the nuclear forces. To determine the dependence of the transition rate of a three-body reaction on the characteristics of the relative motion of a particular pair of particles only, it is sufficient to consider only the wave function of this motion. The transition matrix element is then approximated by a form generally referred to as the factored-wave-function method, as

$$T_{fi}{}^c=\text{const}\,[\psi_{\mathbf{k}_c}(\mathbf{r})]_{r=a},\tag{1}$$

where $\hbar\mathbf{k}_c$ is the relative momentum of the pair; c, taken to be either p or n, respectively, stands for the reaction $^7\text{Li}+p\to\alpha+p+{}^3\text{H}$ or $^7\text{Li}+p\to\alpha+n+{}^3\text{He}$. The actual form of $\psi_{\mathbf{k}_c}(\mathbf{r})$ at $r\sim a$ is unknown but will be estimated by continuing inward the solution in the external region. This is permissible only when the relative energy of the pair is small. Since the pair is created in an energy continuum moving in a definite direction $\mathbf{k}_c$, the solution in the external region is given by

$$\psi_{\mathbf{k}_c}(\mathbf{r})=(2\pi i/k_cr)\exp(-i\sigma_0)\times\sum_{lm}(O_{cl}-U_{ccsl}{}^*I_{cl})\,i^lY_l{}^{m*}(\hat{k}_c)\,Y_l{}^m(\hat{r}),\tag{2}$$

where O_{cl} and I_{cl} are the outgoing and incoming wave solutions of the radial wave equation in the external region, and $\sigma_l=\arg\Gamma(l+1+i\eta)$ is the l-wave Coulomb phase shift.

Without including the spin-orbit force in the final-state interaction, each partial wave (s,l), in accordance with Meyerhof and McElearney,[12] is described by a 2×2 scattering matrix. Its diagonal element U_{ccsl} can be expressed in terms of the phase shift δ_{csl} and the scattering-matrix amplitude D_{sl} as $U_{ccsl}=D_{sl}\exp(2i\delta_{csl})$. The radial wave function in expression (2) evaluated at $r=a$ then becomes

$$O_{cl}(k_ca)-U_{ccsl}{}^*I_{cl}(k_ca)=\exp(-i\delta_{csl})\,|O_{cl}(k_ca)|\times[(1-D_{sl})\cos(\delta_{csl}-\omega_{cl}+\varphi_{cl})+i(1+D_{sl})\times\sin(\delta_{csl}-\omega_{cl}+\varphi_{cl})],\tag{3}$$

where $\omega_{cl}=\delta_{c,p}(\sigma_l-\sigma_0)$ and $\varphi_{cl}=\omega_{cl}+\arg O_{cl}(k_ca)$. If

[11] L. D. Landau and E. M. Lifshitz, in *Quantum Mechanics* (Addison-Wesley Publishing Co., Reading, Mass., 1965), 2nd ed., pp. 562–565.

[12] W. E. Meyerhof and J. N. McElearney, Nucl. Phys. **74,** 533 (1965).

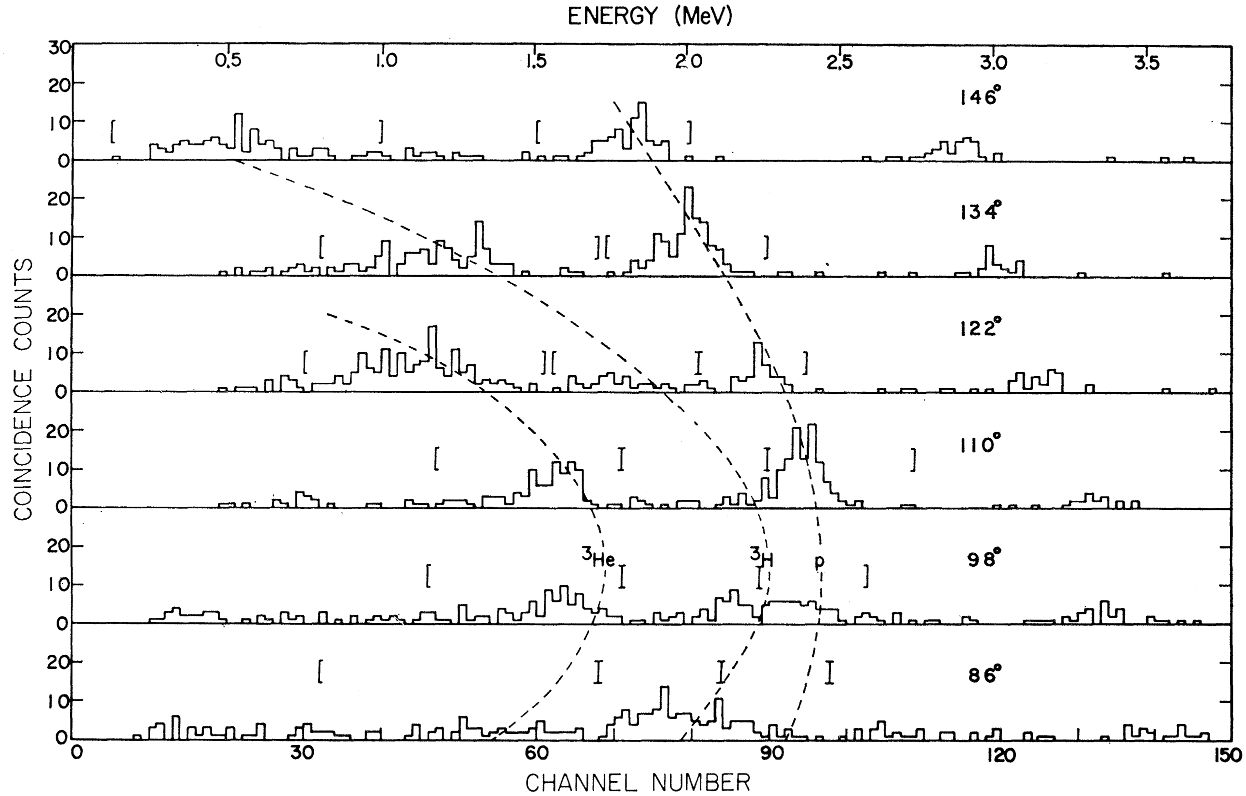

FIG. 4. Coincidence spectra from $^7Li(p, \alpha)$ at $E_p=9.1$ MeV. The 4.1-MeV α particles were detected at 30° in the magnetic spectrometer with $\delta E/E=1.11\%$, $\delta\theta=1°$, and $\delta\Phi=4°$. The acceptance angles for the counter in the target chamber at the negative angles indicated were 7.6° and 15.5°, respectively, along the θ and Φ directions. The dashed lines are the kinematically predicted loci where protons, tritons, and ^{3}He's are expected. The square brackets define the regions where the coincidence counts were summed.

there is one resonance level in the partial wave (s, l), with resonance energy E_{sl} and proton and neutron reduced widths γ_{psl}^2 and γ_{nsl}^2, the phase shifts and the scattering-matrix amplitude are given explicitly, in the neighborhood of this resonance, by

$$\delta_{psl}=\omega_{pl}-\varphi_{pl}+\tfrac{1}{2}\tan^{-1}\Gamma_p\epsilon/[\epsilon^2+\tfrac{1}{4}(\Gamma_n^2-\Gamma_p^2)],$$

$$\delta_{nsl}=-\varphi_{nl}+\tfrac{1}{2}\tan^{-1}\Gamma_n\epsilon/[\epsilon^2+\tfrac{1}{4}(\Gamma_p^2-\Gamma_n^2)], \qquad (4)$$

and

$$D_{sl}=\{1-[\Gamma_p\Gamma_n/\epsilon^2+\tfrac{1}{4}(\Gamma_p+\Gamma_n)^2]\}^{1/2}.$$

Here $\epsilon=E_{sl}+\Delta_{psl}+\Delta_{nsl}-E_x$, $\Gamma_c=2\gamma_{csl}^2P_{cl}$, and $\Delta_{csl}=\gamma_{csl}^2(B_{csl}-S_{cl})$. E_x is the ^{4}He excitation energy. The penetrability and shift function P_{cl} and S_{cl} are defined by Lane and Thomas.[13] In accordance with the convention used by Werntz[1,14] and by Meyerhof and

[13] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

[14] C. Werntz, Phys. Rev. **133**, B19 (1964).

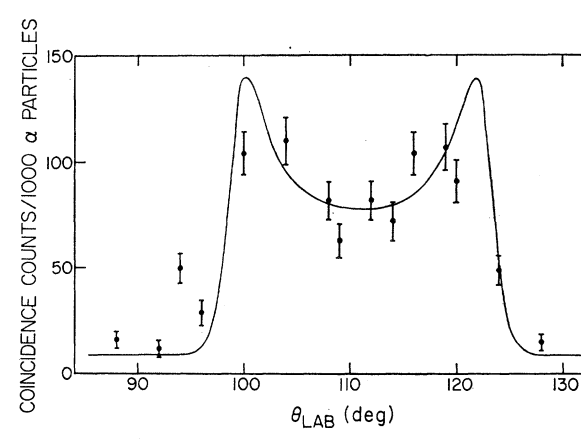

FIG. 5. α-^{1}H and α-^{3}H angular correlations. Because the kinematic $(\partial E/\partial\theta)\Delta\theta$ is large, the sum of the unresolved α-^{1}H and α-^{3}H coincidence counts, from the coincidence spectra shown in Fig. 3, has been plotted against the angular position of the counter in the target chamber. The errors include the statistical error and the uncertainty in random coincidence corrections. The curve is the predicted sum of the angular correlations, if an isotropic decay of excited ^{4}He in its c.m. system is assumed.

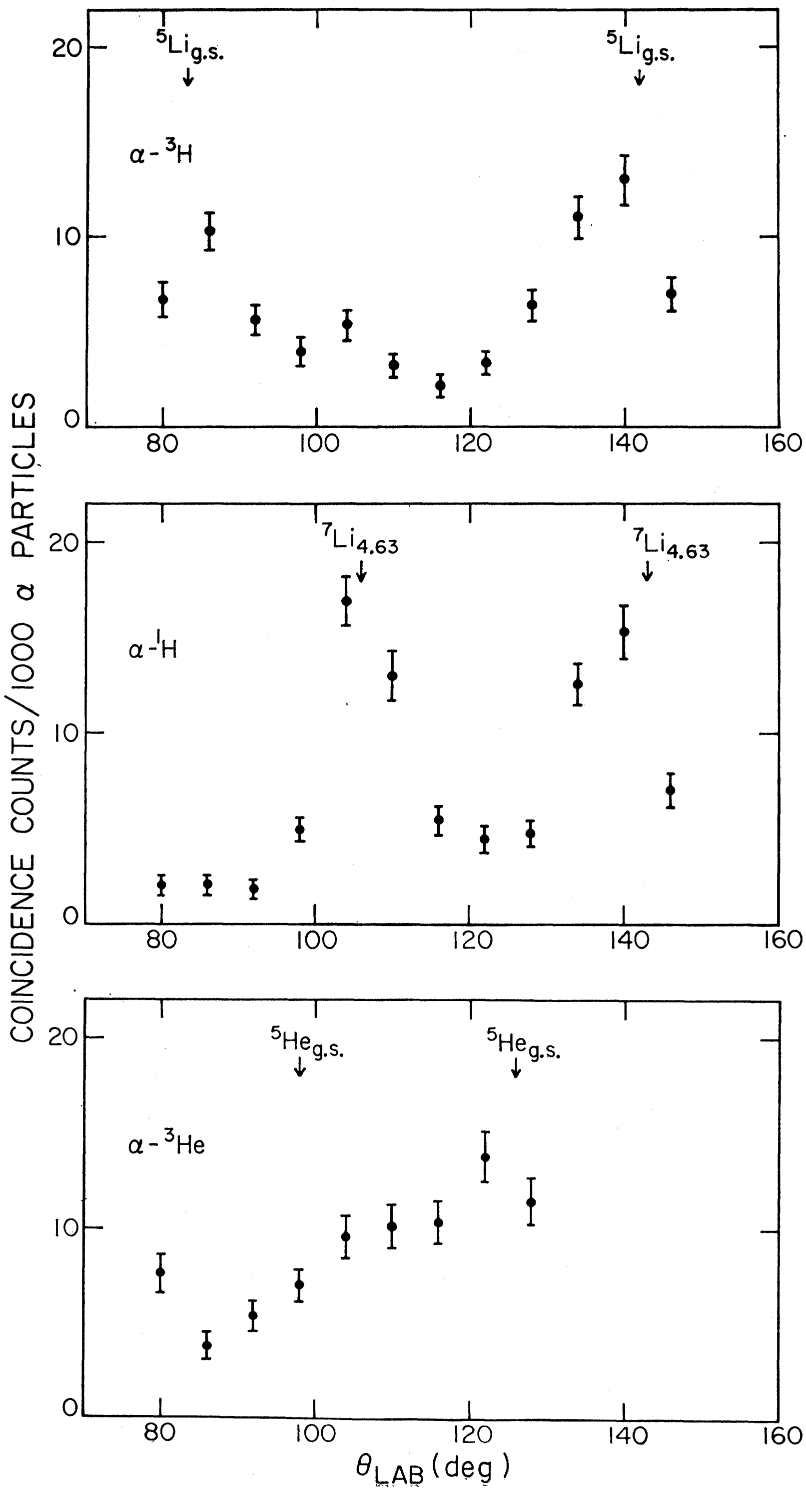

FIG. 6. α-^{1}H, α-^{3}H, and α-^{3}He angular correlations, calculated from the coincidence spectra shown in Fig. 4, corrected for randoms. In this case, the kinematic lines are farther apart, and it is possible to separate different groups of particles. For some angles, however, the errors include the ambiguity in this separation. The arrows indicate the positions where the $\alpha+{}^3$H, $\alpha+{}^1$H, and $\alpha+n$ final-state interactions through the relevant compound states are expected to be important.

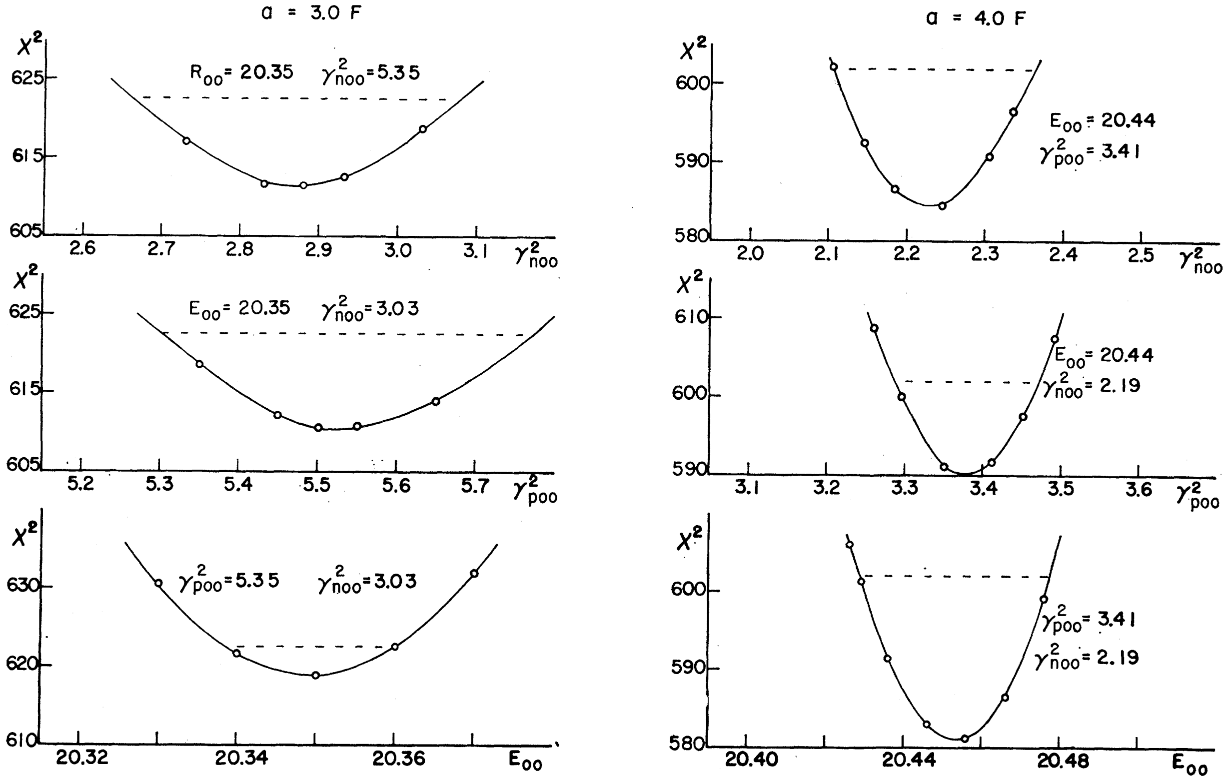

FIG. 7. Sensitivity of χ^2 to resonance parameters in the least-squares fits. The χ^2, in the least-squares fits of the spectral shapes to the α-particle energy spectra, are calculated in the neighborhood of the two sets of solutions for $(a, E_{00}, \gamma_{p00}{}^2\ \gamma_{n00}{}^2)$. The dashed lines indicate the levels to which χ^2 increases from its minimum $\chi_{\min}{}^2$ to $\chi_{\min}{}^2\ (1+2/f)$, where $f=229-(26+3)$ is the total number of degrees of freedom in the fittings, and may be used to assign the error for each of the resonance parameters. The resonance energies and the reduced widths are expressed in units of MeV.

McElearney, the boundary-condition constants B_{csl} are chosen such that $\delta_{psl}(E_{sl})=\pi/2$ and $\Delta_{nsl}(E_{sl})=0$.

To determine expression (1), it is sufficient to consider the s wave alone, since at low energies the scattering matrices for partial waves with $l\neq 0$ are relatively small. As mentioned previously, the ^3S phase shift is describable by a hard-sphere energy dependence, and its contribution to expression (3) is also small. In the following analysis, only a resonance in singlet s wave is assumed. $|T_{fi}{}^c|^2$ is then reduced to

$$|T_{fi}{}^c|^2=\text{const}\times\left[(k_ca)^{-1}\frac{D_{00}{}^2+1-2D_{00}\cos 2(\delta_{c00}+\varphi_{c0})}{P_{c0}}\right],$$

and the $^7\text{Li}(p,\alpha)$ differential cross section is given by the sum of the contributions from the $p+{}^3\text{H}$ and $n+{}^3\text{He}$ channels, as

$$d^2\sigma/dE_\alpha d\Omega_\alpha=\text{const}\sqrt{E_\alpha}\,[k_p|T_{fi}{}^p|^2+k_n|T_{fi}{}^n|^2],\qquad(5)$$

where $k_p=[(2\mu/\hbar^2)(E_x-19.814)]^{1/2}$ and $k_n=[(2\mu/\hbar^2)\times(E_x-20.578)]^{1/2}$, and μ is the reduced mass of the final-state interacting pair of particles. Below the $n+{}^3\text{He}$ threshold, $|T_{fi}{}^n|^2=0$ and $D_{00}=1$, and expression (5) includes essentially a factor like

$$\rho_{sl}(ka)\equiv\sin^2(\delta_{sl}-\omega_l+\varphi_l)/P_l(ka),$$

which is usually referred to as the generalized density-of-states function, as discussed by Barker and Treacy[15] and Phillips *et al.*[16]

Let the experimental differential cross section at $\theta_\alpha=\theta_i$ and $E_\alpha=E_j$ be σ_{ij}, and the corresponding value predicted by expression (5) be $\bar{\sigma}_{ij}$. Since σ_{ij} is equal to the actual number of counts N_{ij} times a normalization constant, the root-mean-square error for σ_{ij} is assigned as $\Delta\sigma_{ij}=\sigma_{ij}/(N_{ij})^{1/2}$. The fitting procedure involves a search for the minimum of the expression

$$\chi^2=\sum_{i=1}^{M}\sum_{j=1}^{N_i}\left(\frac{\sigma_{ij}-\bar{\sigma}_{ij}}{\Delta\sigma_{ij}}\right)^2,$$

where M is the number of spectra and N_i is the

[15] F. C. Barker and P. B. Treacy, Nucl. Phys. **38**, 33 (1962).

[16] G. C. Phillips, T. A. Griffy, and L. C. Biedenharn, Nucl. Phys. **21**, 327 (1960).

number of data points included in the spectrum at $\theta_\alpha=\theta_i$. The constant, appearing in expression (5) and depending on θ_i, and the resonance parameters (E_{00}, γ_{p00}^2, γ_{n00}^2) are adjusted to minimize the χ^2. Since expression (5) is not expected to be valid for high ^{4}He excitation energies, a cutoff at 20.30 MeV was introduced. This limited the number of data points to

$$\sum_{i=1}^{M=26} N_i=229$$

in a total of 26 spectra. The range of nuclear forces a, chosen to be identical for both the $p+{}^3$H and $n+{}^3$He channels, was fixed either at 3.0 or at 4.0 F in the fittings. By iterating the solutions,[17] χ^2 was found to be minimum at

a (F)	E_{00} (MeV)	γ_{p00}^2 (MeV)	γ_{n00}^2 (MeV)
3.0	20.35	5.53	2.88
4.0	20.45	3.38	2.23.

The sensitivity of χ^2 to those resonance parameters is shown in Fig. 7, and the ^1S, $p+{}^3$H phase shifts calculated from expression (4) are shown in Fig. 8. The curves in Fig. 2 are the predicted spectral shapes calculated from expression (5) for $a=3.0$ F. The fits at $E_x=20.06$ MeV for all the spectra are quite satisfactory; the data points corresponding to this excitation energy are taken, without assuming any background contribution, as the angular distribution of the α-particle group leading to the first excited state of ^{4}He. The result is shown in Fig. 9, and is discussed in Sec. IV.

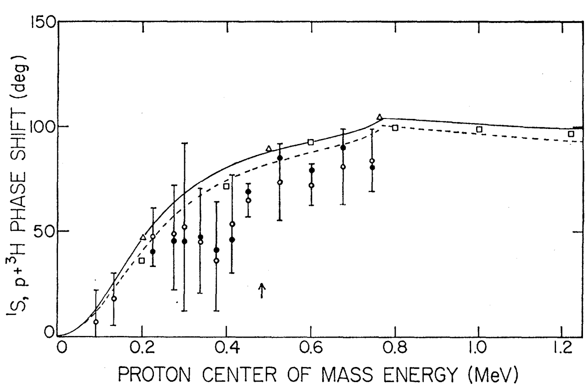

FIG. 8. The ^1S, $p+{}^3$H phase shifts. The solid and dotted curves are the phase shifts calculated, respectively, from the resonance parameters (3.0, 20.35, 5.53, 2.88) and (4.0, 20.45, 3.38, 2.23). Triangles, squares, and circles give the phase shifts of Werntz, Meyerhof *et al.*, and Kurepin, respectively. The arrow corresponds to an excitation energy of 20.30 MeV below which the least-squares fits were made (see caption for Fig. 2).

[17] L. Janossy, *Theory and Practice of the Evaluation of Measurements* (Clarendon Press, Oxford, 1964), pp. 258–259.

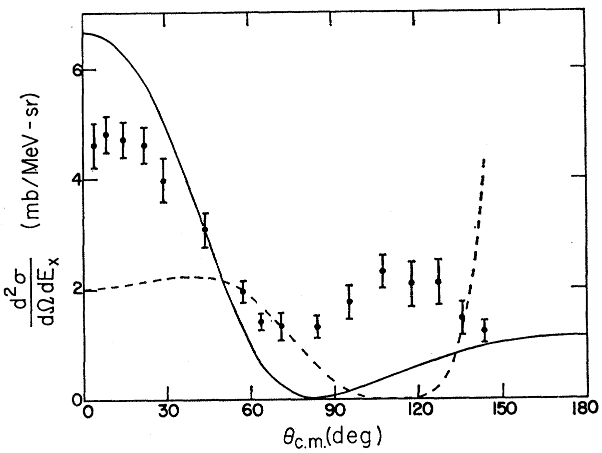

FIG. 9. The angular distribution of the α-particle group leading to the first excited state of ^{4}He($E_x=20.06$ MeV) at $E_p=9.1$ MeV. The dashed curve shows the contribution of the α-particle knockout process. The cutoff introduced in the $\mathbf{r}_{\alpha t}$ and $\mathbf{r}_{pt}$ integrals are, respectively, $R_c=5.2$ F and $r_c=3.0$ F. The solid curve includes both the α-particle knockout and the triton-pickup processes. Estimations of their relative amplitudes are explained in text.

IV. TRITON-TRANSFER MECHANISMS

The angular distribution of the α-particle group leading to the first excited state of ^{4}He shown in Fig. 9 has a forward peak. In this section, calculations are made to account for this prominent feature. For calculating the transition matrix element in the plane-wave Born approximation, the wave functions in the initial and final states of the reaction are written as

$$\Psi_i=\varphi_4(\mathbf{r}_1-\mathbf{R}_1)\varphi_{34}[\mathbf{R}_2-\tfrac{1}{4}(\mathbf{r}_1+3\mathbf{R}_1)]\times\exp\{i\mathbf{k}_p\cdot[\mathbf{r}_2-\tfrac{1}{7}(\mathbf{r}_1+3\mathbf{R}_1+3\mathbf{R}_2)]\}$$

and

$$\Psi_f=\chi_4(\mathbf{r}_1-\mathbf{R}_1)\chi_4'(\mathbf{r}_2-\mathbf{R}_2)\times\exp\{i\mathbf{k}_\alpha\cdot[\tfrac{1}{4}(\mathbf{r}_1+3\mathbf{R}_1)-\tfrac{1}{4}(\mathbf{r}_2+3\mathbf{R}_2)]\}+\chi_4(\mathbf{r}_2-\mathbf{R}_2)\chi_4'(\mathbf{r}_1-\mathbf{R}_1)\times\exp\{i\mathbf{k}_\alpha\cdot[\tfrac{1}{4}(\mathbf{r}_2+3\mathbf{R}_2)-\tfrac{1}{4}(\mathbf{r}_1+3\mathbf{R}_1)]\},$$

where φ_4 describes the relative motion of a $p+{}^3$H two-particle subsystem in the ^{7}Li nucleus, and φ_{34} describes that of the other triton with respect to the c.m. of the two-particle subsystem. χ_4 and χ_4' are, respectively, the wave functions of the detected α particle and the recoil ^{4}He system. The vector $\mathbf{r}_i(\mathbf{R}_j)$ designates the position of ith proton (jth triton), and $\hbar\mathbf{k}_p$ ($\hbar\mathbf{k}_\alpha$) is the initial-state proton (final-state α particle) momentum in the c.m. system. Neglecting the interactions of the unbound pair in the final state, the interaction responsible for the triton-transfer reactions contains only $V_{pt}(\mathbf{r}_2-\mathbf{R}_2)$. The transition matrix element is

$$T_{fi}=(\Psi_f,\ V_{pt}\Psi_i)\equiv T_{\rm di}+T_{\rm ex}, \qquad (6)$$

where

$$T_{\rm di}=\int d\mathbf{r}\,\chi_4{}^*(\mathbf{r})\varphi_4(\mathbf{r})\int d\mathbf{r}_{pt}\chi_4{}'(\mathbf{r}_{pt})V_{pt}(\mathbf{r}_{pt})\times\exp(i\mathbf{q}_p\cdot\mathbf{r}_{pt})\int d\mathbf{r}_{\alpha t}\varphi_{34}(\mathbf{r}_{\alpha t})\exp(-i\mathbf{q}_\alpha\cdot\mathbf{r}_{\alpha t})$$

and

$$T_{\rm ex}=\int d\mathbf{r}\,\chi_4{}'^*(\mathbf{r})\varphi_4(\mathbf{r})\int d\mathbf{r}_{pt}\chi_4(\mathbf{r}_{pt})V_{pt}(\mathbf{r}_{pt})\times\exp(i\mathbf{q}_p{}'\cdot\mathbf{r}_{pt})\int d\mathbf{r}_{\alpha t}\varphi_{34}(\mathbf{r}_{\alpha t})\exp(-i\mathbf{q}_\alpha{}'\cdot\mathbf{r}_{\alpha t}).$$

Here the new set of integration variables are defined as $\mathbf{r}=\mathbf{r}_1-\mathbf{R}_1$, $\mathbf{r}_{pt}=\mathbf{r}_2-\mathbf{R}_2$, and $\mathbf{r}_{\alpha t}=\mathbf{R}_2-\frac{1}{4}(\mathbf{r}_1+3\mathbf{R}_1)$, and the momentum transfers $\hbar\mathbf{q}_p$ and $\hbar\mathbf{q}_\alpha$ are given as $\mathbf{q}_p=\mathbf{k}_p+\frac{1}{4}\mathbf{k}_\alpha$ and $\mathbf{q}_\alpha=(4/7)\mathbf{k}_p+\mathbf{k}_\alpha$. The corresponding vectors $\mathbf{q}_p{}'$ and $\mathbf{q}_\alpha{}'$ in the expression for $T_{\rm ex}$ are obtained from $\mathbf{q}_p$ and $\mathbf{q}_\alpha$ with $\mathbf{k}_\alpha$ replaced by $-\mathbf{k}_\alpha$.

Since the recoil ^{4}He system is in its first excited state, $\chi_4{}'$ is orthogonal to χ_4. If one takes the model of ^{7}Li nucleus as a ^{3}H$+\alpha$ two-particle system seriously, i.e., $\varphi_4=\chi_4$, then only the direct term $T_{\rm di}$ contributes to the reaction. To estimate $T_{\rm di}$, cutoff radii R_c and r_c were introduced for the $\mathbf{r}_{\alpha t}$ and $\mathbf{r}_{pt}$ integrals, respectively. The logarithmic derivative of the radial wave function at cutoff radius was fixed either by the binding energy of the ^{7}Li nucleus or by the ^1S, $p+{}^3$H phase shift obtained in Sec. III for φ_{34} or $\chi_4{}'$, respectively. The dashed curve in Fig. 9 shows the result of this calculation. This direct term involves the process that the incident proton knocks out an α particle and forms an excited state of ^{4}He with the triton in forward directions, and thus the predicted α-particle angular distribution peaks at the backward angles.

To allow for the contribution from an exchange term $T_{\rm ex}$, one has to drop the orthogonality between $\chi_4{}'$ and φ_4, i.e., to allow that the $p+{}^3$H two-particle subsystem in ^{7}Li may have a finite probability of being excited to the first excited state of ^{4}He. For convenience in computing the relative amplitudes of the direct and exchange processes, the wave functions involved were assumed as

$$\varphi_4(\mathbf{x})=(2\beta^2/\pi)^{3/4}\exp(-\beta^2x^2),$$

$$\chi_4(\mathbf{x})=(2\gamma^2/\pi)^{3/4}\exp(-\gamma^2x^2),$$

$$\varphi_{34}(\mathbf{x})=(4\pi)^{1/2}(2\alpha^2/\pi)^{3/4}\exp(-\alpha^2x^2)Y_1{}^m(\hat{x}),$$

and

$$\chi_4{}'(\mathbf{x})=(i/2k_px)\exp(-i\sigma_0)f_0(k_px).$$

Here $f_0(k_pr)$ is the radial wave function of the first excited state of ^{4}He. It becomes the ^1S component of expression (2) for $r\gtrsim a$. The parameter α was chosen to be 0.28 F^{-1} to reproduce one of the published reduced widths of the ground state of ^{7}Li.[18] The inverse-square decay length β^2 of the $p+{}^3$H subsystem in ^{7}Li is not known; it is arbitrarily taken as that of a free α particle $\gamma^2=0.21$ F^{-2} from electron scattering data. As was argued by Werntz,[1] most of the contribution to those integrals involving $f_0(k_pr)$ comes from the region $r\lesssim a$, and for low energies the shape of $f_0(k_pr)$ in this region is energy-independent. $f_0(k_pr)$ is set to $O_{p0}(k_pa)-D_{00}\exp(-2i\delta_{p00})I_{p0}(k_pa)$ and is factored out of the integrals. This factor appears in both $T_{\rm di}$ and $T_{\rm ex}$, and the expression (6) thus predicts the same spectral shape as the expression (1) does. It was found that a cutoff radius R_c is also needed for the $\mathbf{r}_{\alpha t}$ integral. As in the previous approximation, for a cutoff radius[19] of 5.2 F, the calculated angular distribution is shown by the solid curve in Fig. 9.

V. CONCLUSION

Nuclear reactions with three outgoing particles are very complicated in comparison with the reactions where there are only two particles in both the initial and final state. We have shown[20] that in the reaction ^{7}Li$(p,\ \alpha)$ there is a particular phase-space region in which the $p+{}^3$H final-state interaction is dominant. From the α-particle energy spectra and the angular correlation measured at $E_x=20.01$ MeV, the energy and spin-parity characteristics of the final-state interacting pair of particles are found to be in good agreement with those obtained from other two-body reactions, e.g., the ^{3}H$(p,\ p)$ elastic scattering. The factored-wave-function method is useful in understanding the spectral shape, and the importance of the exchange effect in the triton-transfer reaction is reflected in a prominent forward peak in the α-particle angular distribution.

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[18] T. A. Tombrello and P. D. Parker, Phys. Rev. **131**, 2582 (1963).

[19] This cutoff radius gives the best fit to the ^{7}Li$(p,\ \alpha_0)$ angular distribution at 9.1-MeV incident energy by the plane-wave Born-approximation triton-pick-up mechanism discussed by D. R. Maxson [Phys. Rev. **128**, 1321 (1962)].

[20] The details of this work are given in the Ph.D. thesis of W. K. Lin, California Institute of Technology, Pasadena, 1969 (unpublished).