Information on Short-Range Nucleon-Nucleon Correlations and Woods-Saxon Nuclear Potential from High-Energy Elastic Electron Scattering

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(Received 18 November 1969)

A method has been developed which allows the effect of the short-range dynamical correlations on the nuclear-charge form factors generated in arbitrary potential wells to be taken into account. Recent high-energy experimental data on elastic electron scattering by ^{6}Li, ^{12}C, and ^{16}O have been analyzed with this method. The single-particle radial wave functions have been generated in a state-dependent Woods-Saxon well whose parameters have been chosen to fit the eigenenergies to the proton separation energies, as obtained from $(p, 2p)$ reactions. It is shown that only by the inclusion of correlations is it possible to reproduce theoretically the experimental charge form factor of ^{6}Li and ^{16}O nuclei. It is also pointed out that the inclusion of correlations leads to the use of potential wells which are deeper and narrower than usually assumed. The radial wave functions of the harmonic oscillator have also been used. It is shown that these wave functions also give a good reproduction of the experimental data, provided that correlations are taken into account. The effect of the center-of-mass motion on the form factors calculated in the finite well is also considered.

1. INTRODUCTION

RECENTLY, the effect on the charge form factor of several p-shell nuclei of the short-range correlations due to the repulsive part of the two-body interaction has been analyzed in detail[1] with an independent-particle model (IPM) generated in the harmonic-oscillator (HO) well.[2–7] In Ref. 1, it has been shown that the high-momentum parts ($q \gtrsim 3$ F^{-1}) of the form factors calculated with and without correlations behave in completely different ways, which indicates that electron scattering at high momentum transfer could give useful information on the short-range correlations. Actually, recent experimental data from the Stanford Group[8] on elastic electron scattering by ^{16}O and ^{6}Li not only show strong deviations from the predictions made by the HO IPM, but, at the same time, are in qualitative agreement with the predictions of Ref. 1. However, it has also been shown[9] that the use of the Woods-Saxon (WS) well instead of the HO, appreciably changes the charge form factor in just that (high-momentum) region where the effect of correlations is most important. An attempt[10] to clarify whether the high-energy electron elastic scattering by light nuclei can be interpreted, without correlations, by an IPM generated in a realistic WS well, indicated that no such well can be found reproducing the high-momentum part of the charge form factor of ^{16}O. Since, however, the WS IPM can qualitatively predict the position of the second diffraction minimum found in recent experiments on this nucleus,[8] while the HO cannot, it seemed to us very important to examine the effect of correlations using an IPM generated in a realistic finite well. To this end we present a method which makes it possible to consider the effect of correlations in any type of potential well. An application to ^{6}Li, ^{12}C, and ^{16}O is given which uses both the HO and the WS wells. In Sec. 2, the method which takes the correlations in any type of potential well into account is described. Numerical calculations for ^{6}Li, ^{12}C, and ^{16}O are presented in Sec. 3. Finally, in Sec. 4, the effect of the center-of-mass (c.m.) motion in the WS well is analyzed.

2. CORRELATED CHARGE FORM FACTOR

Using a Jastrow-type wave function

$$\Psi(\mathbf{r}_1 \cdots \mathbf{r}_A) = (A)^{-1/2} \det\{\varphi_\alpha(\mathbf{r})\} \prod_{i<j}^{A} F(r_{ij}), \quad (1)$$

the monopole part of the form factor can be calculated by means of the Iwamoto Yamada cluster expansion,[11] the first terms of which describing only two-particle

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[1] C. Ciofi degli Atti, Nucl. Phys. **A129**, 350 (1969); Nuovo Cimento **55B**, 570 (1968); Phys. Rev. **175**, 1256 (1968).

[2] Using HO wave functions, the effect of short-range correlations on the charge form factors has also been studied in Refs. 3–7 and 23. For further references see Ref. 6.

[3] F. C. Khanna, Phys. Rev. Letters **20**, 871 (1968).

[4] G. Ripka (private communications).

[5] T. Stovall and D. Vinciguerra, Lettere al Nuovo Cimento **1**, 100 (1969).

[6] C. Ciofi degli Atti and M. Grypeos, Lettere al Nuovo Cimento **2**, 587 (1969).

[7] M. Grypeos, University of Surrey report (unpublished).

[8] J. S. McCarthy and I. Sick (private communications); R. R. Whitney (private communications).

[9] T. W. Donnelly and G. E. Walker, Phys. Rev. Letters **22**, 1121 (1969).

[10] C. Ciofi degli Atti and N. M. Kabachnik, in Proceedings of the Third International Conference on High-Energy Physics and Nuclear Structure, Columbia University, New York, 1969 (unpublished).

[11] F. Iwamoto and M. Yamada, Prog. Theoret. Phys. (Kyoto) **17**, 543 (1967).

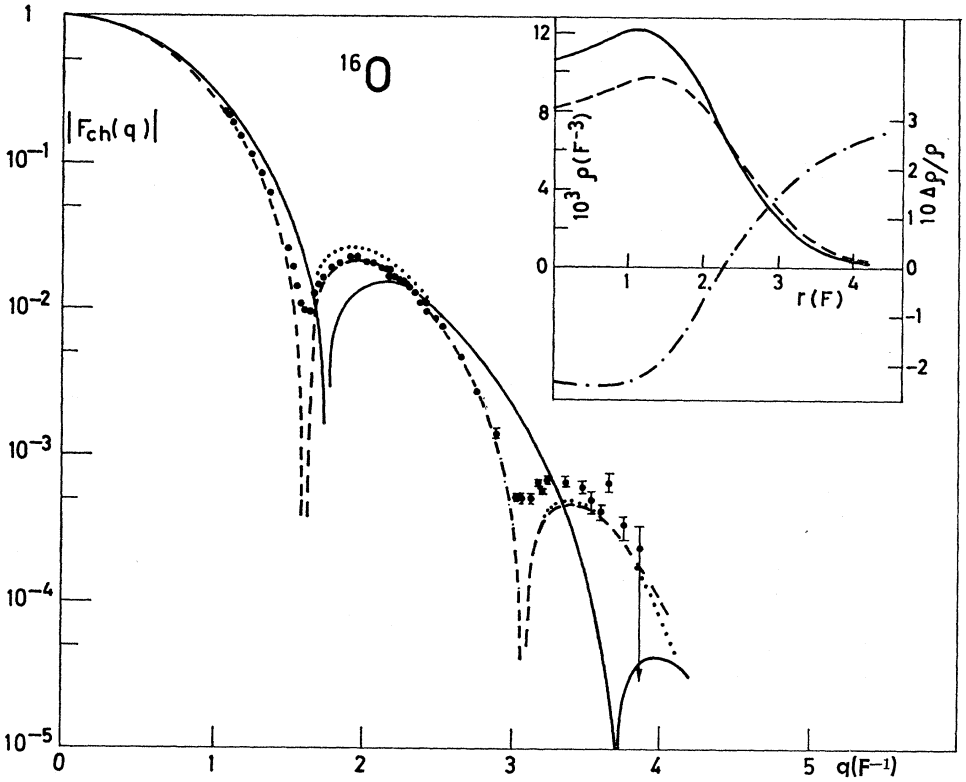

FIG. 1. The charge form factor of the ^{16}O nucleus. The experimental points represent the 375- and 750-MeV data (Ref. 8). The full line was obtained, without correlations ($b=0$), using the WS well parameters listed in Table I, while the dashed line was obtained with the same well parameters taking into account the correlations ($b=0.9$ F). The results for the HO wave functions, with oscillator length $a=1.64$ F and correlation parameter $b=0.9$ F, are given by the dotted line (where this line is not shown, it practically coincides with the dashed line). The point-charge densities, calculated with (dashed line) and without (continous line) correlations using the WS well, are shown in the insert. The relative difference between the correlated and uncorrelated densities, $\Delta\rho/\rho$, is represented by the dot-dashed line. The scale for this curve is given on the right-hand side. The densities calculated with HO wave functions do not practically differ from those calculated in the WS well.

correlations, can be written in the following form[12]:

$$\tilde{F}(q)=f_N(q)f_{DF}(q)\,(1/A)\{\sum_{\alpha}\langle\alpha\,|\,j_0(qr_1)\,|\,\alpha\rangle + \sum_{\alpha\beta}[\langle\alpha\beta\,|\,j_0(qr_1)h(r_{12})\,|\,\alpha\beta-\beta\alpha\rangle - \langle\alpha\,|\,j_0(qr_1)\,|\,\alpha\rangle\langle\alpha\beta\,|\,h(r_{12})\,|\,\alpha\beta-\beta\alpha\rangle]\}, \quad (2)$$

where α and β are single-particle occupied nucleon states, and $h(r_{12})=F(r_{12})^2-1$. In expression (2), $f_{DF}(q)$ represents the so-called Darwin-Foldy term[13]

$$f_{DF}(q)=1-q^2/8M^2, \qquad (M \text{ is the nucleon mass})$$

while $f_N(q)$ is the nucleon form factor[14]

$$f_N(q)=\frac{2.50}{1+q^2/15.7}-\frac{1.6}{1+q^2/26.7}+0.1.$$

In the case of HO orbitals, expression (2) can be easily calculated with the Talmi-Moshinsky transformation and by expressing the Bessel function through the two-nucleon relative and c.m. coordinates, as indicated in Ref. 1.[15] For single-particle states other than those of the HO, this procedure can no longer be used, but a method analogous to the tensor expansion of a two-body operator[16] can be developed. Namely, instead of going into the relative and c.m. coordinates, we expand the correlation function in the following way:

$$h(r_{12})=\sum_{k=0}^{\infty}a_k(r_1,r_2)\,P_k(\cos\theta), \qquad \cos\theta=\mathbf{r}_1\cdot\mathbf{r}_2/r_1r_2 \quad (3)$$

where

$$a_k(r_1,r_2)=\tfrac{1}{2}(2k+1)\int h(r_{12})\,P_k(\cos\theta)\,d(\cos\theta). \quad (4)$$

In this way, the two-body matrix element in (2) can be expressed through linear combinations of the following modified Slater integrals, whose integrands no longer

[12] For a critical review of the different methods for taking into account correlations see Ref. 6.

[13] K. W. McVoy and L. van Hove, Phys. Rev. **108**, 482 (1957); T. de Forest, Jr., and J. D. Walecka, Advan. Phys. **15**, 1 (1966).

[14] T. Jenssens, R. Hofstadter, E. B. Hughes, and Y. M. Yearian, Phys. Rev. **142**, 1922 (1966).

[15] In the expression (6) of Ref. 1 the phase should read $(-)^{k+\nu}$ and not $(-)^{\nu}$. However, $k=0$ if correlations are taken into account only in s state of relative motion (as in Ref. 1).

[16] A. de Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press, Inc., New York, 1963), p. 208.

depend on the angle θ:

$$F_{n_1l_1n_2l_2}{}^{(k)}=\int \mid R_{n_1l_1}(r_1)\,R_{n_2l_2}(r_2)\mid^2 j_0(qr_1)\,a_k(r_1,r_2)\,r_1^2dr_1r_2^2dr_2, \tag{5}$$

$$G_{n_1l_1n_2l_2}{}^{(k)}=\int R_{n_1l_1}(r_1)\,R_{n_2l_2}(r_2)\,R_{n_1l_1}(r_2)\,R_{n_2l_2}(r_1)j_0(qr_1)\,a_k(r_1,r_2)\,r_1^2dr_1r_2^2dr_2. \tag{6}$$

Using standard techniques we have, in the case of the $1p$-shell nuclei,

$$\tilde{F}(q)=f_N(q)f_{\mathrm{DF}}(q)\,(1/Z)$$
$$\times\{Z_sF_{0s}(q)+Z_pF_{0p}(q)-Z_s[(A_s-1)G_s(q)+A_pG_{sp}(q)-F_{0s}(q)\tilde{B}_1]-Z_p[(A_p-1)G_p(q)+A_sG_{ps}(q)-F_{0p}(q)\tilde{B}_2]\} \tag{7}$$

where

$$F_{0s}=4\pi\int j_0(qr)\,R_{1s}(r)^2r^2dr, \qquad F_{0p}=4\pi\int j_0(qr)\,R_{1p}(r)^2r^2dr, \tag{8}$$

$$\tilde{B}_1=(A_s-1)\,g_s+A_pg_{sp}, \qquad \tilde{B}_2=(A_p-1)\,g_p+A_sg_{sp}, \tag{9}$$

$$G_s(q)=F_{1s1s}{}^{(0)}(q), \qquad g_s=G_s(0), \tag{10}$$

$$G_p(q)=F_{1p1p}{}^{(0)}(q)-(2/55)\,F_{1p1p}{}^{(2)}(q), \qquad g_p=G_p(0), \tag{11}$$

$$G_{sp}(q)=F_{1s1p}{}^{(0)}(q)-\tfrac{1}{12}G_{1s1p}{}^{(1)}(q), \qquad g_{sp}=G_{sp}(0)=G_{ps}(0), \tag{12}$$

$$G_{ps}(q)=F_{1p1s}{}^{(0)}(q)-\tfrac{1}{12}G_{1s1p}{}^{(1)}(q). \tag{13}$$

Choosing a Gaussian-type correlation function $F(r)=[1-\exp(-r^2/b^2)]^{1/2}$, widely used in this type of calculation, [1,3–7,10] we have

$$h(r_{12})=-\exp(-r^2/b^2)=-\exp[-(r_1^2+r_2^2-2r_1r_2\cos\theta)/b^2]$$

and expression (4) can be easily integrated analytically over θ.

In this case, the following expressions for F and G are obtained:

$$F_{l_1l_2}{}^{(0)}=\tfrac{1}{2}b^2\int j_0(qr_1)\mid R_{1l_1}(r_1)\,R_{1l_2}(r_2)\mid^2\exp[-(r_1^2+r_2^2)/b^2]\,shA\,r_1dr_1r_2dr_2, \tag{14}$$

$$F_{l_1l_2}{}^{(2)}=\tfrac{5}{2}b^2\int j_0(qr_1)\mid R_{1l_1}(r_1)\,R_{1l_2}(r_2)\mid^2\exp[-(r_1^2+r_2^2)/b^2]\{chA-A^{-1}shA\}\,r_1dr_1r_2dr_2, \tag{15}$$

$$G_{l_1l_2}{}^{(1)}=G_{l_2l_1}{}^{(1)}=\tfrac{3}{4}b^4\int j_0(qr_1)\,R_{1l_1}(r_1)R_{1l_2}(r_2)\,R_{1l_1}(r_2)\,R_{1l_2}(r_1)\,\exp[-(r_1^2+r_2^2)/b^2]\{[(A^2+3)/A]shA-3chA\}\,dr_1dr_2 \tag{16}$$

with $A\equiv 2r_1r_2/b^2$. Note that in obtaining the above expressions no assumptions have been made on the state of the two-particle relative motion. Consequently, it can easily be shown that if HO radial wave functions are used in Eqs. (14)–(16), the expressions (10)–(13) coincide with the analogous expressions obtained by means of the Talmi-Moshinsky method, provided that correlations are introduced into all the states of relative motion.

In particular, if only terms of the order $(1+2a^2/b^2)^{-3/2}$ are retained (these are the terms corresponding to correlations only in relative s state and a is the oscillator length), the expressions (10)–(13) coincide with the analogous expressions given in Ref. 1.

The method described can very easily be extended to heavier nuclei.

TABLE I. The parameters of the WS wells used in the calculations and the theoretical ($\epsilon_{\mathrm{theor}}$) and experimental ($\epsilon_{\mathrm{expt}}$) proton separation energies (energies in MeV and radii in F).

Nucleus	Level	V_0	r_0	V_{so}	a	$\epsilon_{\mathrm{theor}}$	ϵ_{expt}[a]
^{6}Li	$1s_{1/2}$	67.2	1.275		0.65	25.7	22.7±2
	$1p_{3/2}$	61.2	1.320	8.3	0.65	5.5	4.9±1
^{16}O	$1s_{1/2}$	75	1.27		0.65	45.6	44±2
	$1p_{3/2}$	60	1.27	13	0.65	19.9	19±1
	$1p_{1/2}$	60	1.27	13	0.65	12.1	12.4±1

[a] Reference 17.

3. NUMERICAL CALCULATIONS FOR ^{6}Li, ^{12}C, AND ^{16}O

An application to ^{6}Li, ^{12}C, and ^{16}O has been carried out using radial eigenfunctions of the WS potential well

$$V(r)=-V_0\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}-V_{so}\left(\frac{h}{m_\pi c}\right)^2r^{-1}\frac{d}{dr}$$
$$\times\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}\mathbf{l}\cdot\boldsymbol{\sigma}+V_C, \tag{17}$$

where V_0 and V_{so} are the central and spin-orbit potential strengths, $R=r_0(A-1)^{1/3}$, and V_C is the Coulomb potential which was taken to be due to an equivalent uniform sphere of radius R.

The values of the various parameters which appear in Eq. (17) and which have been used in the calculations are listed in Table I. HO wave functions have been

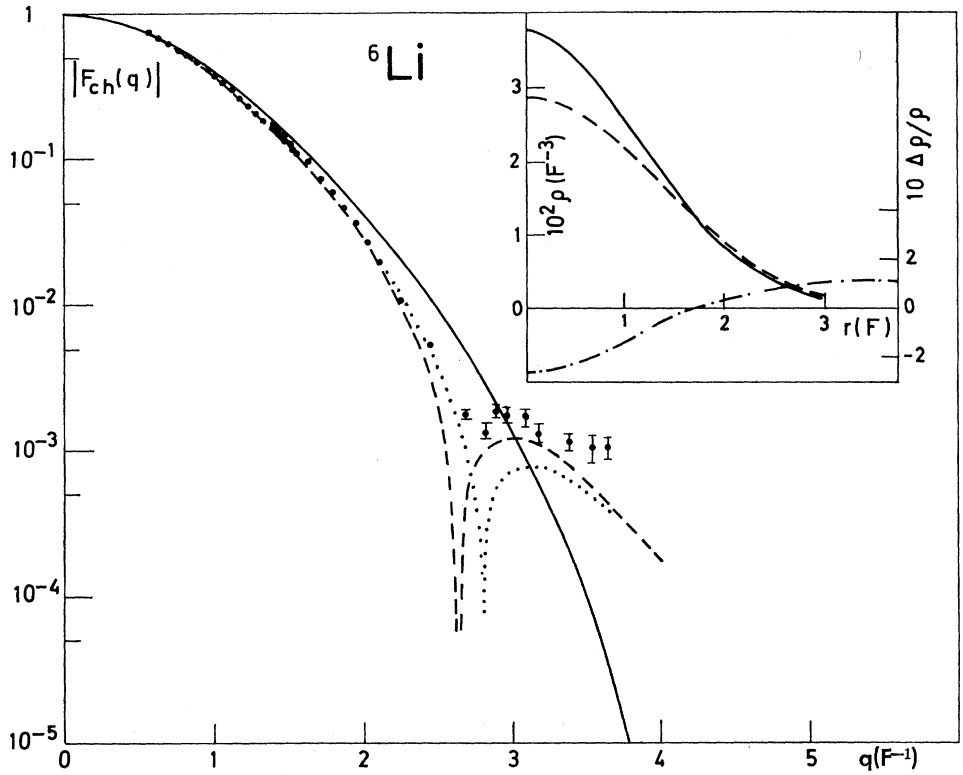

FIG. 2. The same as in Fig. 1 but for ^{6}Li. The experimental points are the 200- and 500-MeV data (Ref. 8). The HO parameters are in this case, $a_s=1.49$ F and $a_p=2.07$ F.

considered too, because in Ref. 1 only correlations in the s state of relative motion were taken into account. The results of the calculations for ^{6}Li and ^{16}O are shown in Figs. 1 and 2. In the inserts in these figures the effect of correlations on the point-charge density is shown.

Discussing the results of the calculations, we should like first of all to stress the fact, that we did not succeed in finding a WS well which could explain, without correlations, the experimental data for ^{6}Li and ^{16}O. All the wells we considered gave a good fit to the low-momentum part of the form factor ($q\lesssim 2.5$ F^{-1}) but gave too small a high-momentum part (by about an order of magnitude). A well which brings the second theoretical maximum in ^{16}O closer to the experimental points can be chosen[9] but then the fit to data at low momentum transfer is destroyed. When a simple state-independent well is used, the situation is particularly unpleasant for ^{16}O, because no well can be found which simultaneously gives a good fit to the low-momentum part of the form factor and to the $1s$ proton separation energy as obtained from $(p, 2p)$ reactions.[17] Since it has been shown[18] that the experimental values of the separation energies for both light and heavy nuclei can be obtained by means of a state-dependent IPM, we decided to use a model of this type. The state-dependent parameters given in Ref. 18 can predict the correct behavior of the form factor up to $q\lesssim 2.5$ F^{-1} but for higher momentum transfer they cannot explain the experimental data (see Figs. 3 and 4). If, using these parameters, which can be considered the best-fitting parameters when the correlations are not taken into account, we introduce the correlations, the first maximum in ^{6}Li and the second one in ^{16}O can be raised, but, at the same time, the fit at lower momentum transfer is destroyed since the inclusion of the correlations shifts the form factor inward. The shift, which clearly appears in Figs. 1 and 2, is simply due to the fact that, as shown in the inserts of the figures, the correlations make the nucleus more diffuse and consequently the form factor oscillates with higher frequency. For this reason, in order to obtain a good fit both at low- and at high momentum transfer, we reduced the radii and increased the depths of the wells of Ref. 18 by about 10%. With the resulting parameters, listed in Table I, the extremely good fit in Figs. 1 and 2 (dashed lines) was obtained.[19] A search of the best-fitting parameters was not performed, because of the enormous computational time that this would have required.

In the case of HO wells without correlations, not even a qualitative interpretation of the high-momentum-

[17] H. Tyrén *et al.*, Nucl. Phys. **79**, 321 (1966).
[18] L. R. B. Elton and A. Swift, Nucl. Phys. **A94**, 52 (1967).

[19] The graphical results for ^{12}C are not presented since they do not qualitatively differ from those for ^{16}O. Namely, using the well parameters of Ref. 18 the second diffraction minimum is located at $q=4$ F^{-1}, while introducing the correlations the minimum is shifted to $q=3.5$ F^{-1}. It must be stressed, however, that the experimental data (Ref. 8) extended up to $q\sim 3.6$ F^{-1} do not reveal, at the moment, further diffraction minima except that at $q\sim 1.8$ F^{-1}.

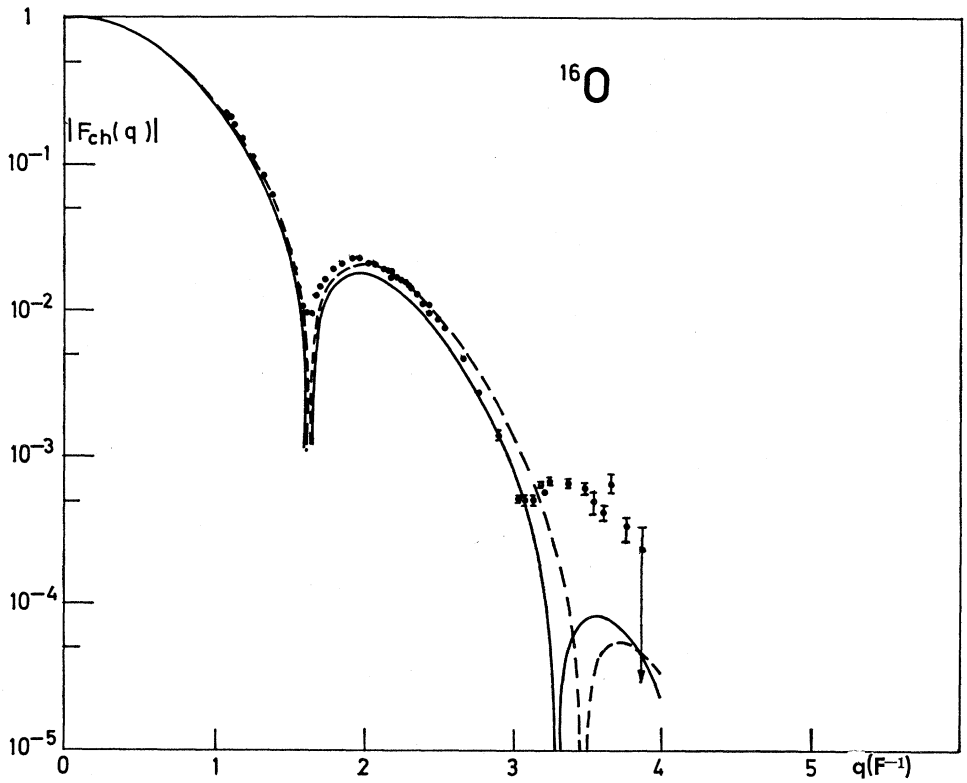

FIG. 3. The charge form factor of the ^{16}O nucleus calculated, without correlations, with the WS well parameters taken from Elton and Swift (Ref. 18). In the case of the continuous line the spurious c.m. motion has been neglected, while in the case of the dashed line it has been taken into account by means of the Gartenhaus-Schwartz transformation.

transfer data can be given (for more details see Refs. 1, 10). However, if correlations are introduced, a fit as good as that in the case of the WS well can be obtained. Note that in this case too, the values of the well parameters [the oscillator length $a=\hbar/(M\hbar\omega)^{1/2}$] are about 10% less than the corresponding values obtained many years ago[20] by using the simple IPM without correlations.

The sensitivity of the form factor to the correlation parameter b is rather high. An increase (decrease) of the parameter b by about 10% increases (decreases) the first maximum in ^6Li and the second maximum in ^{16}O by about 30%. Consequently, by using a larger correlation parameter than the one used in the present calculations, one could obtain a better fit at high momentum transfer. However, it must be stressed that the use of a large correlation parameter is very dangerous, because this leads to the introduction of long-range correlations and, consequently, the higher-order terms in the cluster expansion of the form factor cannot be neglected. The value $b=0.9$ F, though not very small, is still acceptable since the correlated and uncorrelated wave functions describing the s state of relative motion differ by only 2% for two-nucleon separations larger than about 1.5 F and, consequently, the correlations can be considered short range enough. However, it must be realized that, as pointed out in Ref. 6, the matrix elements which appear in the cluster expansion of the form factor are functions of the momentum transfer, so that it may happen that at large momentum transfer, higher-order contributions may become important, even for sufficiently short-range correlations. For this reason a calculation of at least three-particle correlation terms would be highly desirable. It should be remembered, furthermore, that at high momentum transfer higher-order contributions to the first Born approximation are probably not negligible.

Concerning the effect of the correlations on the charge density it should be emphasized that the correlations depress the density at the center, and increase it at the surface, the two effects being of the same order. It should also be mentioned that the correlations (with $b=0.9$ F) lead to a 5–7% increase of the rms radii. This does not mean, of course, that the values of the radii obtained without correlations[18,20] have to be increased by this quantity, because when correlations are introduced, the well radii must be, at the same time, decreased.

4. GARTENHAUS-SCHWARTZ TRANSFORMATION FOR A FINITE WELL

In the described WS calculation the spurious c.m. motion has not been eliminated. In order to estimate the importance of this effect, we have used the Gartenhaus-

[20] R. Hofstadter, Rev. Mod. Phys. **28**, 214 (1956); L. R. B. Elton, in *Nuclear Sizes* (Oxford University Press, New York, 1961).

Schwartz[21] transformation to calculate the uncorrelated charge form factors. Since correlations are not expected to affect the spurious c.m. motion, because they act on the relative coordinates only, it is reasonable to think that the c.m. motion effects are essentially the same on the uncorrelated and the correlated form factors.

If the Gartenhaus-Schwartz transformation is used, the form factor has the form

$$F(q)=Z^{-1}\langle\Psi\,|\sum_{k=1}^{A}\epsilon_k\exp i\mathbf{q}\cdot(\mathbf{r}_k-A^{-1}\sum_{l=1}^{A}\mathbf{r}_l)\,|\,\Psi\rangle. \quad (18)$$

From (18), using for Ψ Slater determinants, we obtain

$$F(q)=\tfrac{1}{9}(M_sM_{p0}+M_{sp}^2)[2\tilde{M}_sM_{p0}M_s^2-2\tilde{M}_{sp}M_{sp}M_s^2+\tilde{M}_{p0}M_s^3+\tilde{M}_sM_sM_{sp}^2]$$
$$+(2/9)M_s^3M_{p1}(2\tilde{M}_sM_{p1}+\tilde{M}_{p1}M_s) \quad (19)$$

for ^{6}Li,

$$F(q)=(2/9)(M_sM_{p0}+M_{sp}^2)^3\{M_{p1}^4[\tilde{M}_sM_{p0}+\tilde{M}_{p0}M_s-2\tilde{M}_{sp}M_{sp}]+M_{p1}^3[\tilde{M}_{p1}M_sM_{p0}+\tilde{M}_{p1}M_{sp}^2]\}$$
$$+\tfrac{1}{9}M_s^3M_{p1}^7[\tilde{M}_sM_{p1}+2\tilde{M}_{p1}M_s] \quad (20)$$

for ^{12}C, and

$$F(q)=\tfrac{1}{4}(M_sM_{p0}+M_{sp}^2)^3\{M_{p1}^8[\tilde{M}_sM_{p0}+\tilde{M}_{p0}M_s-2\tilde{M}_{sp}M_{sp}]+2M_{p1}^7[\tilde{M}_{p1}M_sM_{p0}+\tilde{M}_{p1}M_{sp}^2]\} \quad (21)$$

for ^{16}O, where

$$M_s=\langle 1s\,||\,j_0\,||\,1s\rangle, \qquad M_{sp}=\sqrt{3}\langle 1s\,||\,j_1\,||\,1p\rangle, \quad (22)$$

$$M_{p0}=\langle 1p\,||\,j_0\,||\,1p\rangle-2\langle 1p\,||\,j_2\,||\,1p\rangle, \qquad M_{p1}=\langle 1p\,||\,j_0\,||\,1p\rangle+\langle 1p\,||\,j_2\,||\,1p\rangle, \quad (23)$$

$$\tilde{M}\equiv M[(A-1)/A\cdot q], \qquad M\equiv M(A^{-1}q), \qquad \langle 1l\,||\,j_\lambda\,||\,1l'\rangle=\int R_{1l}(r)j_\lambda(qr)R_{1l'}(r)r^2dr. \quad (24)$$

The above expressions are valid for any type of central well.[22,23] (See *Note added in proof* below.) In particular, it is easy to check that in the case of HO orbitals, if Eqs. (19)–(21) are used, the well-known correction $\exp(a^2q^2/4A)$[24] is obtained. The effect of the spurious c.m. motion on the ^{6}Li and ^{16}O uncorrelated form factors is shown in Figs. 3 and 4. It can be seen that the c.m. motion effect is quite different in the HO and the WS wells. In fact, in the former only the amplitude of the form factor is affected (increased), while in the second not only the locations of the minima are shifted,[9] but, furthermore, the amplitude of the form factor is influenced in opposite ways depending on the momentum transfer, being increased at low momentum transfers, and decreased at higher ones. Fortunately, the c.m. motion effect is not so large in the nuclei considered, so that the values of the well parameters listed in Table I (obtained neglecting this effect) can be considered quite well founded.

5. FINAL REMARKS

In closing, the following remarks seem to us to be appropriate:

(1) If correlations are not introduced, the elastic electron scattering experiments for ^{6}Li, ^{12}C and ^{16}O can be explained equally well, up to $q\sim 2$ F^{-1}, by the HO (see Refs. 1, 10) and the WS (see Figs. 3 and 4 of this work) IPM models, provided one uses state-dependent oscillator lengths for the ^{6}Li nucleus. At higher momentum transfers, however, the two models give quite different predictions, because in the case of HO well the form factor has only one minimum, while in the case of the WS well, it has several.[9]

(2) Correlations must be included for the experimental data to be reproduced. No WS well can be found which, without correlations, can reproduce the amplitude of the first maximum in ^{6}Li and of the second maximum in ^{16}O. If correlations are introduced, both the WS and the HO wells can give a satisfactory interpretation of the experimental data, provided potential wells deeper and narrower than the ones obtained without correlations from the analysis of the low-energy electron scattering data, are used.[25]

[21] S. Gartenhaus and C. Schwartz, Phys. Rev. **108**, 482 (1957).

[22] The Gartenhaus-Schwartz transformation has already been used in Ref. 23 in the calculation of the charge form factors of the ^{6}Li, ^{12}C, and ^{16}O nuclei in the case of different oscillator lengths for s and p shells.

[23] A. Malecki and P. Picchi, Lettere al Nuovo Cimento **1**, 823 (1969).

[24] L. J. Tassie and F. C. Barker, Phys. Rev. **111**, 940 (1958).

[25] When this work was almost completed two papers appeared, in which the effect of the correlations on the charge form factor calculated with WS well has been considered: W. J. Gerace and D. A. Sparrow, Phys. Letters **30B**, 71 (1969); S. T. Tuan, L. E. Wright, and M. G. Huber, Phys. Rev. Letters **23**, 174 (1969). In the first paper as in ours, the Iwamoto-Yamada cluster expansion with a Gaussian correlation function has been used. The nuclear potential has on the countrary been described by means of a state-independent WS well, whose parameters were chosen from optical-model analyses and from a fit to the experimental outermost separation energies as obtained from (γ, p) and (γ, n) reactions. The method used to take into account correlations in the finite well has not been described; however, it must be noted that the qualitative effect of the correlations was found to be the same as in our work. In the second paper, a different approach to the choice of the correlation function has been put forward and the Gaussian-type correlation function has been criticized. Though we share the opinion that one has to be careful with the possible long-range correlations induced by the Gaussian correlation function, we would like to stress that in our calculations the correlated and uncorrelated relative wave functions are practically the same at large two-nucleon separations (see Sec. 3). By performing the Fourier transform of the relative wave functions (uncorrelated and correlated), it can be easily shown that for relative mementa $q_{\rm rel}<1$ F^{-1} they differ by only 1–3%. Thus, if the correlation parameter b is sufficiently small, the Gaussian correlation function can ensure the modification of the two-body wave function only at high relative momenta.

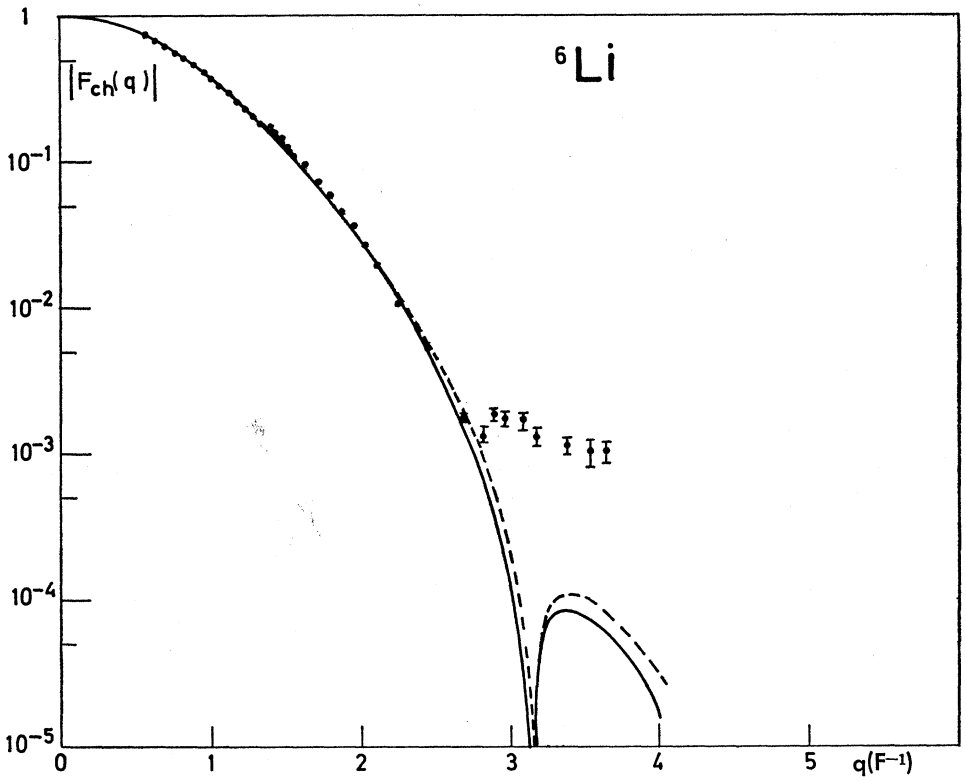

FIG. 4. The same as in Fig. 3 but for ^{6}Li.

(3) A satisfactory interpretation of both the separation energies as obtained from $(p, 2p)$ reactions[17] and the elastic electron scattering data,[8] seems to be possible only by using, at the same time, state-dependent potential wells and short-range correlations. This indicates that further details on the average nuclear potential and on the short-range correlations could probably be obtained by a unified analysis of different type of high-energy nuclear reactions.

Summarizing, it should be emphasized that the elastic electron scattering at high momentum transfers seems to give a strong indication of the presence of short-range dynamical correlations in nuclei. It should be remembered, however, that when analyzing the experimental data those phenomena, whose effects on the single-particle densities, and consequently on the form factors, are similar to the effect of correlations, must be considered. Among these only the effects of configuration mixing has at the moment been estimated,[9] showing that it is too small to account for the deviation from the IPM found in electron scattering experiments.

Note added in proof. Dr. Palumbo has pointed out to us that Eq. (18), with $\Psi \equiv \Psi(\varrho)$ describing the intrinsic motion, is none other than the general expression of the matrix element written in the intrinsic system of frame. If the shell-model wave function $\Psi(\mathbf{r})$ is factorable into the intrinsic $\Psi(\varrho)$, and center-of-mass $\mathbf{X}(\mathbf{R})$ parts, then it is easy to show that in Eq. (18) the intrinsic wave function $\Psi(\varrho)$ can be replaced by the shell-model wave function $\Psi(\mathbf{r})$ (Slater determinant), and the matrix element can be calculated, obtaining Eqs. (19)–(21). However, this factorization is possible only for HO [J. P. Elliot and T. H. R. Skyrme, Proc. Roy. Soc. (London) **A232,** 561 (1955)], so that for WS wave functions is not completely correct to use the Garthenaus-Schwartz transformation [i.e. Eq. (18) with $\Psi \equiv \Psi(\mathbf{r})$]; nevertheless, it is usually assumed that, because of the similarity with the HO wave functions, the factorization still holds for the WS wave functions as well. A general method for handling the center-of-mass motions is given by F. Palumbo, Nucl. Phys. **A99,** 100 (1967).

ACKNOWLEDGMENTS

One of the authors (C.D.A.) is deeply indebted to Dr. J. S. McCarthy, Dr. I. Sick, and Dr. R. R. Whitney of the Stanford High Energy Laboratory for sending the experimental data prior to publication and for useful correspondence; he also acknowledges many discussions with Professor G. Ripka. The other author (N.M.K.) is grateful to Professor U. Amaldi and Professor G. Cortellessa for kind hospitality at the Physics Laboratory of the Istituto Superiore di Sanità. Both authors are grateful to Professor U. Amaldi for reading the manuscript of this paper and for useful remarks.