

Variation in Nuclear-Matter Binding Energies with Phase-Shift-Equivalent Two-Body Potentials*

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For any given two-body Hamiltonian, there exists a large class of unitarily equivalent Hamiltonians that lead to the same scattering phase shifts at all energies. The purpose of this paper is to exhibit typical saturation curves for reasonable equivalent potentials. The binding energy per particle changes by several MeV in either direction, and the saturation minimum shifts to higher or lower density as the binding increases or decreases. Softening the potential increases the binding. The separation approximation for the reaction matrix provides qualitative insight into these effects. Our exact calculations start with simple local s -wave potentials with either a hard core or a Yukawa core. The binding energy per particle is calculated in the Brueckner approximation with self-consistent single-particle energies below the Fermi level. For our examples we use unitary transformations that differ from the identity by a short-range operator of rank 2 and transformations induced by distortions of the radial scale. The latter class of transformations alters the core radius and produces potential terms that are linear in the square of the momentum.

I. INTRODUCTION

RECENT work indicates that the Brueckner approximation to the binding energy of nuclear matter should be a good approximation.¹⁻³ Numerical computations in that approximation give only about 11 MeV per particle.⁴ On the assumption that all many-body correction terms are indeed insufficient to raise this number to the empirical value of 16 MeV, the question arises: What conclusion, if any, can be drawn from such a disagreement? If we adopt a different two-body force (but one that is still consistent with all our empirical and theoretical understanding of the nucleon-nucleon interaction), by how much can the calculated equilibrium properties of nuclear matter be changed? This question is studied in the present paper.

Nucleon-nucleon potentials are determined by the requirement that they reproduce the two-nucleon scattering data and the properties of the deuteron. At large and intermediate distances, further reasonable restrictions on the potentials follow from field-theoretic considerations.⁵⁻⁸ But the s -wave interaction at short distances is not restricted by any properties other than the two-body data. In particular, it need not be local. These specifications still leave us with a large class of two-body Hamiltonians, each of which is as good a

phenomenological Hamiltonian as any other. For many-body problems these Hamiltonians are, of course, not equivalent. In particular, the binding energy per particle in infinite nuclear matter may vary substantially.

Our purpose is to test the size of the effects to be expected by detailed computations for simple potentials and conveniently parametrized transformations. Some qualitative insight with the separation approximation is obtained in Sec. III. The numerical procedures and the results of the exact calculations are described in Sec. IV.

Similar results have been obtained by Green,⁹ by Lomon,¹⁰ and by Miller *et al.*¹¹ Green obtained four similar, velocity-dependent, central S -wave potentials⁹ that give nearly the same 1S_0 phase shift as the "standard hard-core potential" (SHCP) of Moszkowski and Scott¹² up to a lab energy of 250 MeV. Green found that the velocity-dependent potentials all gave similar results in nuclear matter. The two main features were the following: (1) At a given Fermi momentum k_F , i.e., at a given density, the velocity-dependent potentials gave several MeV more binding than the SHCP. (2) The velocity-dependent potentials gave no saturation out to $k_F = 1.8 \text{ F}^{-1}$, while the SHCP saturates at 1.4 F^{-1} . These results were attributed by Moszkowski¹³ to the fact that the velocity-dependent potentials have much smaller wound integrals than the SHCP. This idea was supported by the detailed calculations of Preston and Bhaduri.¹⁴ Lomon computes the binding energy of nuclear matter for phase-shift-equivalent

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¹ B. D. Day, *Rev. Mod. Phys.* **39**, 719 (1967).

² R. Rajaraman and H. A. Bethe, *Rev. Mod. Phys.* **39**, 745 (1967).

³ B. H. Brandow, *Phys. Rev.* **152**, 863 (1966).

⁴ D. W. Sprung, in *Proceedings of the International Conference on Atomic Masses*, University of Manitoba, Winnipeg, 1967 (unpublished); P. C. Bhargava and D. W. Sprung, *Ann. Phys. (N.Y.)* **42**, 222 (1967).

⁵ R. A. Bryan and B. L. Scott, *Phys. Rev.* **135**, B434 (1964); **164**, 1215 (1967).

⁶ L. Ingber, *Phys. Rev.* **174**, 1250 (1968).

⁷ T. Ueda and A. E. S. Green, *Phys. Rev.* **174**, 1350 (1968).

⁸ F. Coester, in *Quanta: A Collection of Papers Dedicated to Gregor Wentzel*, edited by P. G. O. Freund, C. J. Goebel, and Y. Manbu (University of Chicago Press, Chicago, 1970).

⁹ A. M. Green, *Nucl. Phys.* **33**, 218 (1962); *Phys. Letters* **1**, 136 (1962); **3**, 60 (1962).

¹⁰ E. Lomon, *Bull. Am. Phys. Soc.* **14**, 493 (1969).

¹¹ M. Miller, M. Sher, P. Signell, N. Yoder, and D. Marker, *Phys. Letters* **30B**, 157 (1969).

¹² S. A. Moszkowski and B. L. Scott, *Ann. Phys. (N.Y.)* **11**, 65 (1960).

¹³ S. A. Moszkowski, *Phys. Rev.* **129**, 1901 (1963).

¹⁴ M. A. Preston and R. K. Bhaduri, *Phys. Letters* **6**, 193 (1963); *Can. J. Phys.* **42**, 696 (1964).

boundary-condition models. In his case the implied Hamiltonians are not unitarily related since the point spectrum is different in different models. The interpretation of his results hinges on the interpretation of this point spectrum. Miller *et al.*¹¹ start with a realistic hard-core potential and properly include the Coulomb potential in the proton-proton potential. The 1S_0 potential is then changed to a phase-shift-equivalent 1S_0 potential without hard core, and the resulting change in binding is calculated at a single density.

II. EQUIVALENT HAMILTONIANS

Two Hamiltonians \tilde{H} and H that have the same spectrum are unitarily equivalent, i.e.,

$$\tilde{H} = UHU^\dagger. \quad (1)$$

If they also produce the same phase shifts, then for every state ψ in the Hilbert space we have

$$\lim_{t \rightarrow \pm\infty} \|(U-1) \exp(-iH_0 t) \psi\| = 0, \quad (2)$$

where H_0 is the kinetic energy. Conversely, if the unitary operator U satisfies Eq. (2), then \tilde{H} and H produce the same phase shifts.¹⁵⁻¹⁷ Equation (2) is satisfied if $U-1$ is completely continuous; the simplest examples are those in which $U-1$ is of finite rank. It is then easy to choose the operator U such that for all states ψ with $\|\psi\| = 1$, $(U-1)\psi$ is arbitrarily small outside some fixed radius. We can thus make sure that the potential is modified only for short interparticle distances.

Another useful class of unitary transformations is induced by distortions of the radial scale. Consider the Hilbert space of radial wave functions $\psi(R)$ with $0 \leq a \leq R < \infty$, for which

$$\|\psi\|^2 = \int_a^\infty dR |\psi(R)|^2. \quad (3)$$

Let R be a function of r such that

$$dR/dr = \mu^{-1/2}(r) > 0, \quad (4)$$

$R-r \rightarrow 0$ for large r , and $R(b) = a$ for some $b \geq 0$. The relation

$$\tilde{\psi}(r) = [\mu(r)]^{-1/4} \psi[R(r)] \quad (5)$$

defines a mapping of the functions $\psi(R)$, $a \leq R < \infty$ onto the functions $\tilde{\psi}(r)$, $b \leq r < \infty$. With the norm

$$\|\tilde{\psi}\|^2 = \int_b^\infty dr |\tilde{\psi}(r)|^2,$$

that mapping

$$\tilde{\psi} = U\psi \quad (6)$$

is unitary. The condition (2) is satisfied if $R(r)$ is a smooth function and $R-r$ vanishes at least as fast as

$1/r$ for large r . For our purposes we shall assume that $R-r$ vanishes exponentially for large r . If the partial-wave Hamiltonian is of the form

$$H = -\frac{1}{2} \left(\omega(R) \frac{d^2}{dR^2} + \frac{d^2}{dR^2} \omega(R) \right) + \frac{L(L+1)}{R^2} + V_L(R), \quad (7)$$

then the transformed Hamiltonian $\tilde{H} = UHU^\dagger$ is defined by the condition that for every ψ in the domain of H one has

$$UH\psi = \tilde{H}\tilde{\psi}. \quad (8)$$

From Eqs. (5) and (7) it follows that

$$\begin{aligned} \tilde{H} = UHU^\dagger = & -\frac{1}{2} \left(\mu\omega \frac{d^2}{dr^2} + \frac{d^2}{dr^2} \mu\omega \right) \\ & + \frac{1}{4}\omega \left[\frac{d^2\mu}{dr^2} + \frac{1}{4}\mu^{-1} \left(\frac{d\mu}{dr} \right)^2 \right] + \frac{L(L+1)}{R^2(r)} + V_L(R(r)). \end{aligned} \quad (9)$$

The Hamiltonians \tilde{H} and H have repulsive hard cores of radius b and a , respectively, an L -dependent local potential, and a velocity-dependent potential proportional to the square of the momentum. Special cases of this transformation have been considered previously.^{11,16-18} Local hard-core potentials have often been accepted as more real than others. Unitary transformations are then model operators¹⁹ introduced for mathematical convenience. They are designed to produce equivalent many-body Hamiltonians and thus necessarily generate many-body potentials.¹⁹ It is then required that the transformed potential be soft enough to allow Hartree-Fock approximations, and that induced many-body forces are negligible.

Here we assume a different point of view. For reasonably smooth functions $\omega(R)$ and $V_L(R)$ with reasonable values of $a \geq 0$, each equivalent Hamiltonian of the form (7) is as good a phenomenological two-body Hamiltonian as any other. Then, as usual, the many-body Hamiltonian is in each case the sum of kinetic energies plus the sum of all two-body interactions. These many-body Hamiltonians are then, of course, not equivalent.

Another scheme for generating equivalent two-body interactions has been proposed by Baranger *et al.*²⁰ It is based on the observation that, in the absence of bound states, instead of fitting a two-body potential V to the scattering data it may be advantageous to fit the kernel $\langle \mathbf{k}' | T | \mathbf{k} \rangle$ of the operator

$$T = V\Omega_-,$$

where Ω_- is the Møller operator. For $|\mathbf{k}'| = |\mathbf{k}|$, the kernel $\langle \mathbf{k}' | T | \mathbf{k} \rangle$ is determined by the phase shifts.

¹⁸ M. Ristig and S. Kistler, *Z. Physik* **215**, 419 (1968).

¹⁹ F. Villars, in *Proceedings of the International School of Physics "Enrico Fermi"—Course 23, 1961*, edited by V. F. Weisskopf (Academic Press Inc., New York, 1963).

²⁰ M. Baranger, B. Giraud, S. K. Mukhopadhyay, and P. U. Sauer, *Nucl. Phys.* **A138**, 1 (1969).

¹⁵ H. Ekstein, *Phys. Rev.* **117**, 1590 (1960).

¹⁶ G. A. Baker, *Phys. Rev.* **128**, 1485 (1962).

¹⁷ P. Mittelstaedt, *Acta Phys. Hung.* **19**, 303 (1965).

Knowledge of T determines the Møller operator Ω_- , i.e., the scattering states

$$\psi_{\mathbf{k}}^{(+)} = \Omega_- |\mathbf{k}\rangle = |\mathbf{k}\rangle + (k^2/2m - H_0 + i\epsilon)^{-1} T |\mathbf{k}\rangle.$$

Orthonormality and completeness of the scattering states gives

$$\Omega_-^\dagger \Omega_- = \Omega_- \Omega_-^\dagger = 1,$$

which restricts the off-diagonal elements of T but does not determine them uniquely. Once T has been chosen, the potential V is determined by

$$V = T\Omega_-^{-1} = T\Omega_-^\dagger.$$

For practical purposes there are two main disadvantages: (1) the requirement that there be no bound states, and (2) the difficulty in translating requirements on the tail of the potential into restrictions on $(\mathbf{k}' | T | \mathbf{k})$. We have not used this method in our calculations, but there is no doubt that the results would follow the same qualitative pattern.

III. QUALITATIVE ANALYSIS

In this section, we extend the ideas of Moszkowski^{12,13} to gain a qualitative understanding of the results to be expected for different equivalent potentials. The binding energy of nuclear matter is given by

$$\varepsilon(\rho) = T(\rho) + \frac{1}{2} \bar{U}(\rho), \quad (10)$$

where ρ is the density, $T(\rho) = 0.3k_F^2$ is the kinetic energy per particle, and $\bar{U}(\rho)$ is the average potential energy

$$\bar{U}(\rho) = 3k_F^{-3} \int_0^{k_F} dp p^2 U(p). \quad (11)$$

For Serber forces the single-particle potential energy is given in the Brueckner approximation by

$$U(p) = \frac{3}{2} \int d\mathbf{p}' (\mathbf{k} | \bar{G}(\mathbf{P}) | \mathbf{k}), \quad (12)$$

where $\mathbf{k} = \frac{1}{2}(\mathbf{p} - \mathbf{p}')$, $\mathbf{P} = (\mathbf{p} + \mathbf{p}')$, and \bar{G} is the reaction matrix averaged over spin and isotopic spin.

A qualitative insight into the density dependence of ε may be conveniently obtained from the separation approximation for G .¹² Following Bethe *et al.*,²¹ we put

$$G \approx G_s + V_l, \quad (13)$$

where V_l is the long-range part of the potential and G_s is the reaction matrix obtained from the short-range potential V_s by

$$G_s = V_s - V_s Q e^{-1} G_s. \quad (14)$$

For G_s the reference-spectrum approximation G_s^R is defined by

$$G_s^R = V_s - V_s e^{-1} G_s^R. \quad (15)$$

It is easy to estimate the error

$$\begin{aligned} (\mathbf{k} | G_s - G_s^R | \mathbf{k}) &= (\mathbf{k} | G_s^R (1 - Q) e^{-1} G_s | \mathbf{k}) \\ &\approx \frac{1}{8} (2\pi)^3 \rho (\mathbf{k} | G_s^R e^{-1} | \mathbf{k}) (\mathbf{k} | G_s | \mathbf{k}). \end{aligned} \quad (16)$$

The quantity $(\mathbf{k} | G_s^R e^{-1} | \mathbf{k})$ is the Fourier transform of the defect function $\chi_{\mathbf{k}}(\mathbf{x})$ that satisfies the differential equation

$$(-\nabla^2 + V_s + \gamma^2) \chi_{\mathbf{k}}(\mathbf{x}) = (\mathbf{x} | V_s | \mathbf{k}). \quad (17)$$

We have therefore the estimate

$$|(\mathbf{k} | G_s^R e^{-1} | \mathbf{k})| \leq (2\pi)^{-3/2} \int d\mathbf{x} |\chi_{\mathbf{k}}(\mathbf{x})|. \quad (18)$$

For the SHCP,¹² the relative error is about 4%.²¹ In all cases of interest, the function $\chi_{\mathbf{k}}(\mathbf{x})$ is limited in magnitude and range such that the error is small.

From Eqs. (12) and (13) it follows that

$$U(p) = U_s(p) + U_l(p). \quad (19)$$

The density dependence of $\bar{U}_s(\rho)$ may be approximated as follows. Consider the identity

$$\begin{aligned} \bar{U}_s(\rho) &= -[\frac{4}{3}\pi k_F^3]^{-1} \int d\mathbf{p} \int d\mathbf{p}' \\ &\times [U(\mathbf{p}) + U(\mathbf{p}')] (\mathbf{k} | e^{-1} G_s^R | \mathbf{k}), \end{aligned} \quad (20)$$

and note that the last factor in the integrand is approximately constant.²¹ Thus

$$\bar{U}_s(\rho) \approx -\rho \bar{U}(\rho) I, \quad (21)$$

where

$$I = \frac{1}{2} (2\pi)^3 (\mathbf{k}_0 | e^{-1} G_s^R | \mathbf{k}_0) \quad (22)$$

is approximately constant. Since

$$\bar{U}(\rho) = \bar{U}_s(\rho) + \bar{U}_l(\rho), \quad (23)$$

it follows that

$$\bar{U}(\rho) = \bar{U}_l(\rho) / (1 + \rho I). \quad (24)$$

Let $\varepsilon(\rho)$ and $\varepsilon_0(\rho)$ be the binding energies from two equivalent Hamiltonians and consider only transformations that do not produce significant changes in U_l . We may then eliminate $\bar{U}_l(\rho)$ from the equations for ε and ε_0 and find

$$\varepsilon(\rho) - \varepsilon_0(\rho) = [\varepsilon_0(\rho) - T(\rho)] \rho (I_0 - I) / (1 + \rho I). \quad (25)$$

Thus, to the extent that our approximations are valid, the saturation curves are a one-parameter set depending only on I . Using Eq. (25), we have plotted in Fig. 1 the family of saturation curves obtainable from potentials that are equivalent to the SHCP. The curve labeled $I = I_0$ is the result of an exact self-consistent Brueckner calculation. The other curves, corresponding to different values of I , are calculated from Eq. (25).

Since I is the Fourier transform of the defect function $\chi_{\mathbf{k}}(\mathbf{x})$, we expect increased binding for softer potentials. This expectation is borne out by the exact calculations described in the next section.

²¹ H. A. Bethe, B. H. Brandow, and A. G. Petschek, Phys. Rev. **129**, 225 (1963), Sec. 10.

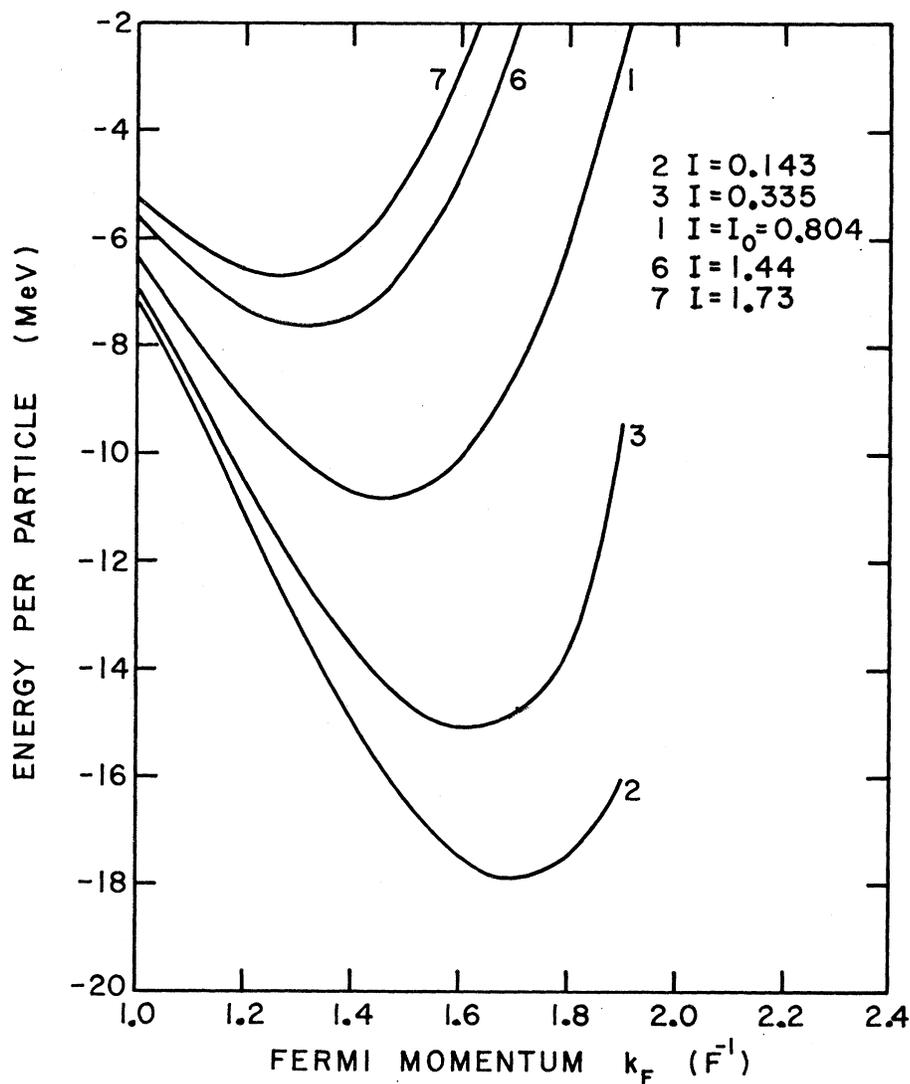


FIG. 1. Approximate saturation curves computed with Eq. (25). For $\epsilon_0(\rho)$ we used the exact result for the untransformed SHCP (S waves only) shown in curve 1 of Fig. 3. For I_0 we use the approximation $I_0 = 4\pi a^3$ (suggested in Ref. 21). The values of $I \neq I_0$ are chosen to give agreement at $k_F = 1.4 \text{ F}^{-1}$ with some of the exact calculations shown in Fig. 3.

IV. NUMERICAL RESULTS

We start with a simple spin-dependent Yukawa-core potential

$$V_s(r) = V_{0s} [4 \exp(-2\mu_s r) - \exp(-\mu_s r)] / \mu_s r, \quad (26)$$

where $s=0, 1$ denotes the spin, $V_{00} = 2307 \text{ MeV}$, $V_{01} = 3991 \text{ MeV}$, $\mu_0 = 2.07 \text{ F}^{-1}$, $\mu_1 = 2.31 \text{ F}^{-1}$. For this potential which we assume acts on s states only,²² the transformation to the momentum representation is easily done in closed form.

A unitary transformation for which $U-1$ is of rank 2 is a rotation in a two-dimensional subspace. It may be parametrized such that

$$\begin{aligned} \langle r' | U-1 | r \rangle = - \{ (1 - \cos\theta) [g_1(r')g_1(r) + g_2(r')g_2(r)] \\ + \sin\theta [g_1(r')g_2(r) - g_2(r')g_1(r)] \}, \quad (27) \end{aligned}$$

where $-\pi \leq \theta \leq \pi$, and the two functions $g_1(r)$ and $g_2(r)$ form an orthonormal set, i.e.,

$$\frac{1}{2} \int_0^\infty dr g_i(r) g_j(r) = \delta_{ij}. \quad (28)$$

Specifically, we choose

$$\begin{aligned} g_1(r) &= \alpha^{3/2} r \exp(-\frac{1}{2}\alpha r), \\ g_2(r) &= (3\alpha^3)^{1/2} r (1 - \frac{1}{3}\alpha r) \exp(-\frac{1}{2}\alpha r). \quad (29) \end{aligned}$$

Their Fourier transforms

$$\tilde{g}_i(k) = \left(\frac{2}{\pi}\right)^{1/2} \int_0^\infty dr \sin kr g_i(r) \quad (30)$$

are easily obtained analytically. Thus the momentum representation of the new potential $\tilde{V} \equiv \tilde{H} - T$ may also be readily obtained in closed form.

We have computed the binding energy per particle as a function of the density by solving the integral

²² F. Coester and E. Yen, Nuovo Cimento **30**, 674 (1963).

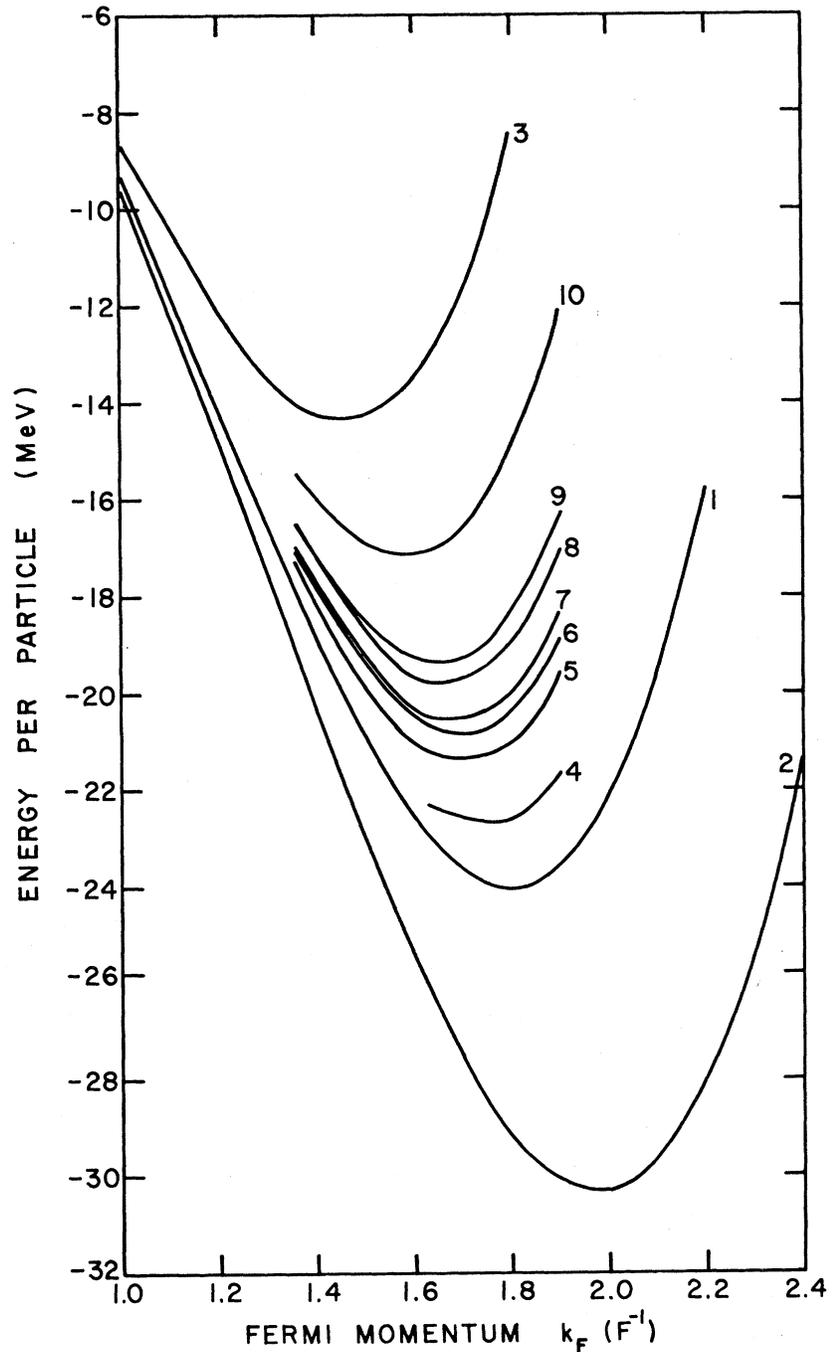


FIG. 2. Saturation curves for the Yukawa-core potential (YCP) (S waves only). The effects of transformations of rank 2 and radial distortions are illustrated. Untransformed YCP: curve 1. Radial distortions: curve 2. $s=0.2$, $\alpha=0.5$, $\beta=0.05$: curve 3. $s=-1.2$, $\alpha=0.6$, $\beta=0.4$. Rank-2 transformations: curve 4. $\alpha=32.64$, $\cos\theta=0.2$ ($\theta<0$): curve 5. $\alpha=32.64$, $\cos\theta=0.2$: curve 6. $\alpha=21.76$, $\cos\theta=0.9$: curve 7. $\alpha=32.64$, $\cos\theta=-0.2$: curve 8. $\alpha=32.64$, $\cos\theta=-1.0$: curve 9. $\alpha=21.76$, $\cos\theta=0.8$: curve 10. $\alpha=21.76$, $\cos\theta=0.6$. Except where indicated, $\theta\geq 0$. For the transformations of rank 2, the binding is always decreased.

equation for the reaction matrix

$$G = V - VQe^{-1}G \quad (31)$$

in the momentum representation. We have used Gaussian quadratures after mapping the infinite-momentum interval into a fine range by an appropriate rational function. The single-particle energy above the Fermi level is purely kinetic; below the Fermi level the single-particle energies are computed self-consistently.

The projection operator Q is treated exactly. Since we consider only s -wave interactions, only the angle average of the operator Q occurs in the exact expression.

As a check on the numerical accuracy, we verified in all cases that the phase shifts are independent of α and θ . Computations were done for $\alpha=21.76 \text{ F}^{-1}$, 32.64 F^{-1} , and 43.52 F^{-1} , and several angles θ . For the largest value of α , the range of the transformation is so short that there is no significant effect on the saturation

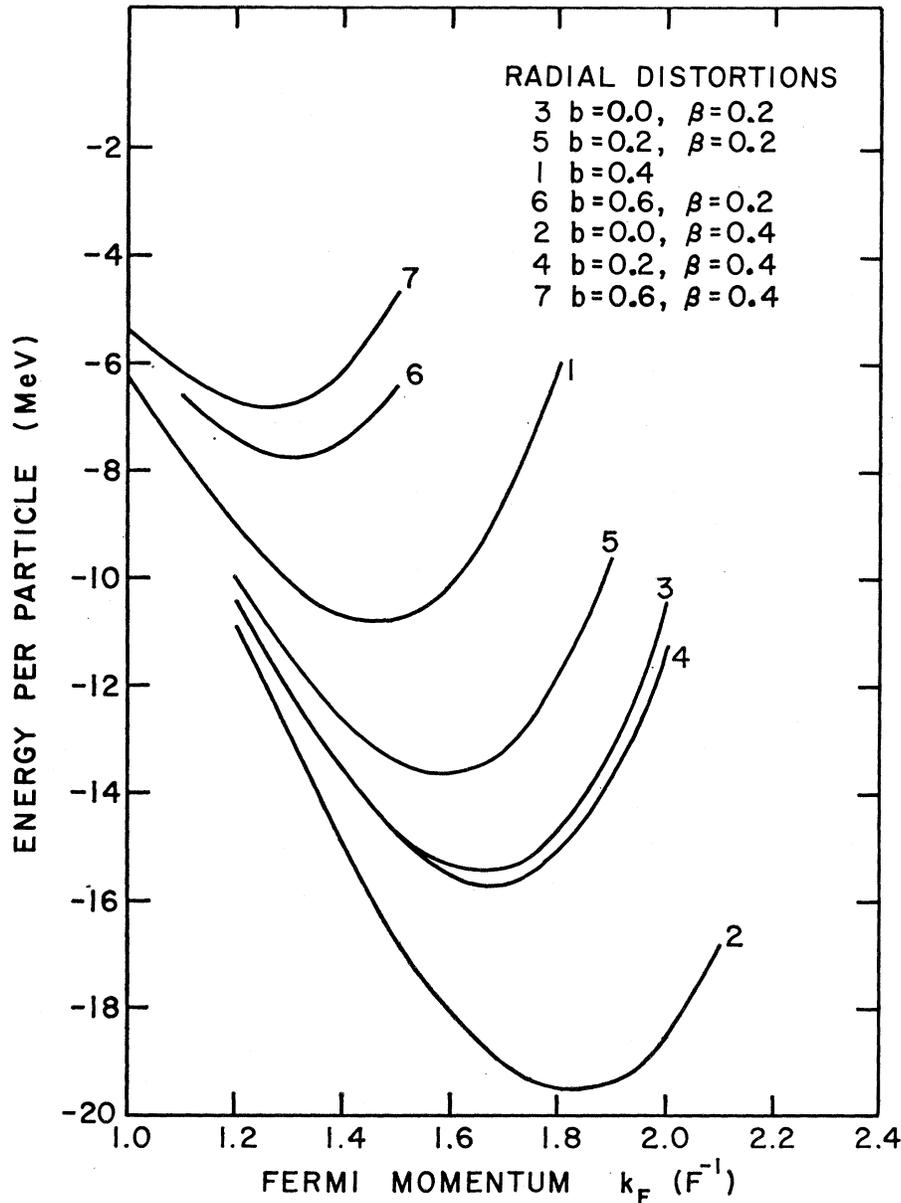


FIG. 3. Saturation curves for the SHCP. The effects of unitary transformations induced by radial distortions are illustrated.

curves. Typical results for other values of the parameters are given in Fig. 2.

We have also considered the unitary transformations induced by the radial distortions

$$R = r + s [\exp(-r/\alpha) - \exp(-r/\beta)] \quad (32)$$

for $s = 0.2 F$, $\alpha = 0.5 F$, $\beta = 0.05 F$ and for $s = -1.2 F$, $\alpha = 0.6 F$, $\beta = 0.4 F$. For the transformed potentials the momentum representation is not available in closed form. In that case we compute the reaction matrix by solving the equation²³

$$G = G^R - G^R(Qe^{-1} - e_0^{-1})G, \quad (33)$$

where the reference-spectrum reaction matrix G^R is obtained from the expression

$$(k' | G^R | k) = (4/\pi) \int dr \sin k' r V(r) \psi_k(r) \quad (34)$$

after solving the differential equation

$$(H + \gamma_0^2) \psi_k(r) = (k^2 + \gamma_0^2) \sin kr \quad (35)$$

for $\psi_k(r)$. The quantity γ_0 is a fixed parameter. Thus $e = k^2 + \gamma^2$ and $e_0 = k^2 + \gamma_0^2$ are usually different. Equation (33) is exact for all values of γ_0 . The generalization to velocity-dependent potentials is straightforward. Again we have checked the invariance of the phase shifts. The saturation curves are shown in Fig. 2. We see that the

²³ H. S. Köhler, Nucl. Phys. A128, 273 (1969).

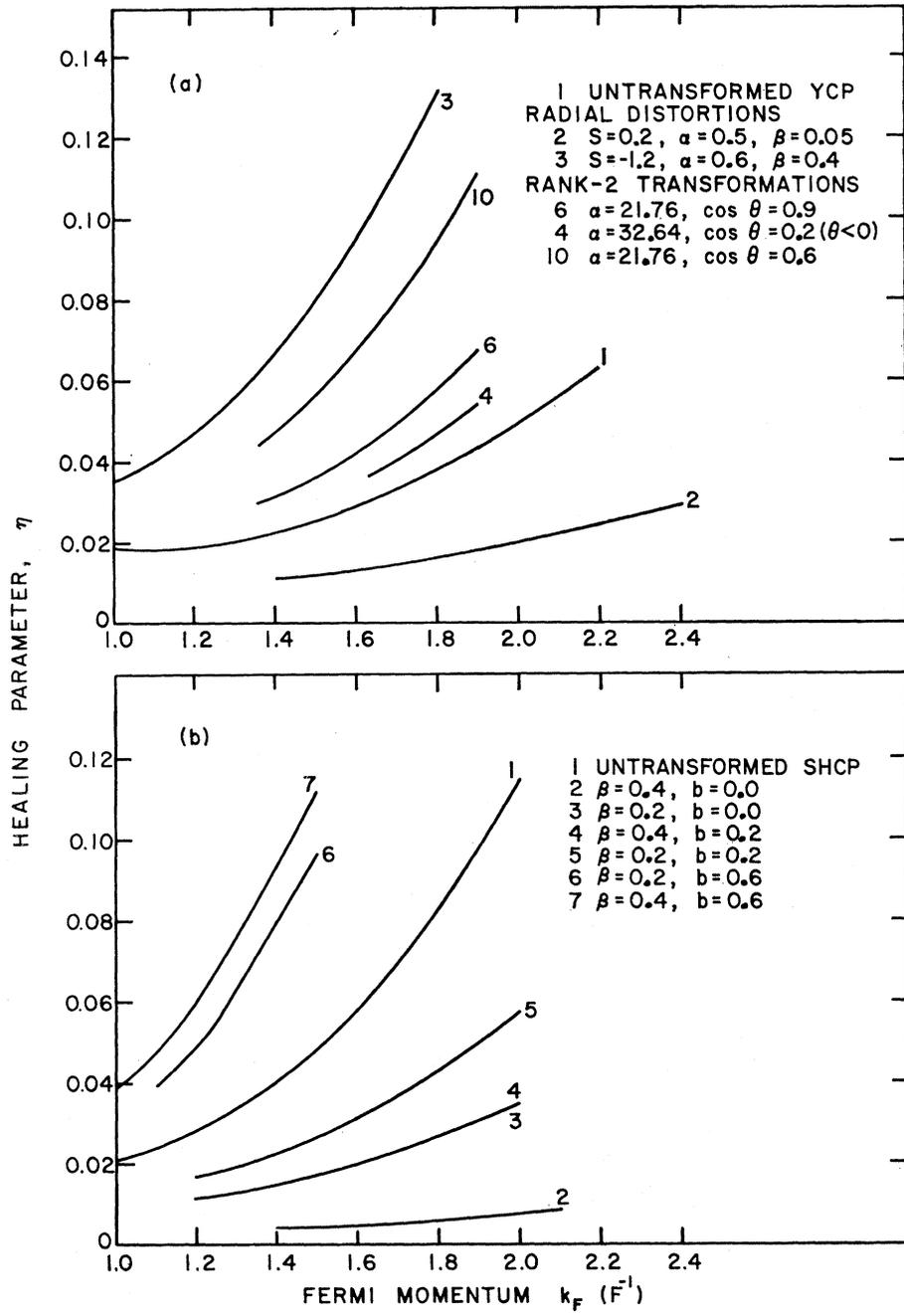


FIG. 4. Healing parameter η for (a) the Yukawa-core potential, and (b) the SHCP. In (b), curves 3 and 4 are indistinguishable.

binding may be either increased or decreased, depending on the choice of a transformation.

As a simple example with hard core, we take the S -wave part of the SHCP given by

$$V(r) = +\infty \quad \text{for } r < a$$

$$= -V_0 \exp[-\mu_0(r-a)] \quad \text{for } r > a, \quad (36)$$

where $a=0.4 \text{ F}$, $\mu_0=2.083 \text{ F}^{-1}$, and $V_0=260.16 \text{ MeV}$.

The transformations are induced by the radial distortions¹⁶

$$R-a = r-b + 2\beta \ln \frac{1+(1+se^{-r/\beta})^{1/2}}{1+(1+se^{-b/\beta})^{1/2}}. \quad (37)$$

The reaction matrices G^R and G were obtained as before. The results are shown in Fig. 3.

As a by-product we have also computed the healing

parameter η , defined²⁴ by

$$\eta = 3[2(2\pi)^3\rho]^{-1} \int d\mathbf{k}' \int d\mathbf{k} \int d\mathbf{P} |(\mathbf{k}' | Qe^{-1}\bar{G}(\mathbf{P}) | \mathbf{k})|^2. \quad (38)$$

The results are given in Fig. 4.

V. CONCLUSIONS

We have seen that equivalent two-body Hamiltonians can give widely different results for the binding energy and equilibrium density of nuclear matter. However, these large changes in energy and density tend to be correlated in an interesting way: An increase in binding energy is accompanied by an increase in equilibrium

density. Our rough calculation in the separation approximation shows why this is true. The saturation curves form a one-parameter family. The parameter characterizes the distortion of the wave function. Thus the softer of two equivalent potentials, which produces less distortion in the wave function, will give a larger binding energy and density. Exact values of the healing parameter η , which is a measure of the distortion in the wave function, have been calculated. These values support the idea that smaller distortion implies larger binding energy and density.

The wide range of results obtainable from equivalent two-body Hamiltonians suggests that nuclear-matter calculations might help to pin down the nature of the nucleon-nucleon potential. Theoretical error bounds on the higher-order corrections to the Brueckner approximation would be essential for that purpose. It is important to emphasize additional theoretical specifications for acceptable potentials and to scrutinize the justification of these specifications.

²⁴ F. Coester and H. Kümmel, Nucl. Phys. **17**, 477 (1960). Brandow's parameter κ (defined in Ref. 3) is related to η by $\kappa = 2\eta$. See also F. Coester, in *Lectures in Theoretical Physics*, edited by K. T. Mahanthappa (Gordon and Breach, Science Publishers, Inc., New York, 1969), Vol. XI.

Nonconservation of Isospin in the $^{14}\text{N}(d, d')^{14}\text{N}$ Reaction*

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The $^{14}\text{N}(d, d')^{14}\text{N}$ reaction to the first excited state of ^{14}N (2.31 MeV, 0^+ , 1) was investigated for isospin nonconservation. Angular distributions were taken for nine incident deuteron energies between 5 and 10 MeV. The symmetries observed in the angular distributions indicate a predominantly compound-nuclear reaction mechanism. The observed violation arises most probably from "Coulomb mixing" in the ^{16}O compound nucleus. The measured cross-section ratio of the first to the second (3.95 MeV, 1^+ , 0) excited state of ^{14}N varied from 3 to 1% for deuteron energy increasing from 6 to 10 MeV. The region of excitation of ^{16}O between 26.0 and 31.0 MeV was investigated by measuring the excitation function for inelastic deuteron scattering to the second excited state of ^{14}N for deuteron energies between 5.9 and 12.2 MeV at a laboratory angle of 60° . Gross structure was observed at excitation energies of 27.2 and 29.6 MeV in ^{16}O . The presented data are compared with photoabsorption data for this range of excitation energy.

I. INTRODUCTION

THIS work was undertaken to study the effects of isospin nonconservation in the $^{14}\text{N}(d, d')^{14}\text{N}$ reaction. Previous deuteron inelastic scattering experiments¹⁻³ involving light nuclei showed no direct evidence of the reaction proceeding to states whose formation is forbidden by isospin conservation. Comparison with the yields to nearby states for which isospin is

conserved typically gave upper limits on the order of a few percent or less.

Preliminary investigations of the $^{14}\text{N}(d, d')^{14}\text{N}$ reaction indicated the feasibility of directly observing the inelastic deuterons that populate the first excited state at 2.31 MeV ($J^\pi = 0^+$, $T = 1$). The primary goal of this study was to find the shape of the angular distribution of the 2.31-MeV level and to measure the cross section as a function of incident deuteron energy. Interest in the energy dependence of the angular distribution of this isospin-nonconserving reaction was stimulated by work done on the $^{12}\text{C}(d, \alpha)^{10}\text{B}$ (1.74 MeV, 0^+ , 1) reaction^{4,5} and more recently on the $^{16}\text{O}(d, \alpha)^{14}\text{N}$ (2.31

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