Large-Angle Elastic Scattering of 1.33-MeV Photons from Lead and Uranium

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The differential cross sections for the elastic scattering of 1.33-MeV photons from lead and uranium have been remeasured at angles ranging from 60° to 135°, using a lithium-drifted germanium detector. Dispersion relations are used to fix the relative phases among nuclear Thomson, Rayleigh, and Delbrück scattering. These relative phases and the calculations of the amplitudes for the individual scattering processes permit the theoretical cross sections to be determined. A discrepancy exists between the theoretical calculations and the experimental results. Previous suggestions for removing this discrepancy include incoherence between scattering from the nucleus and from the electrons, and a 180° phase shift between the parallel-polarization components of the nuclear Thomson and Rayleigh amplitudes. These suggestions are discussed and another possibility, destructive interference at large angles between Rayleigh scattering from the K shell and from the L shell, is presented.

INTRODUCTION

THERE has been much interest in determining the Delbrück contribution to the elastic scattering of photons by atoms. In the intermediate state of this scattering process an electron-positron pair is formed in the static Coulomb field surrounding the nucleus. This pair subsequently annihilates, producing a photon of almost the same energy as the incident photon. The Delbrück scattering amplitude is complex, the real part corresponding to virtual pairs and the imaginary part corresponding to real pairs in the intermediate state. The primary interest is in establishing the existence of the real part of the scattering amplitude, thus confirming the contribution of virtual pairs to physical processes. Although this contribution has already been observed (see the review of Kane *et al.*[1]) further evidence from a very different type of experiment is always desirable.

In addition to Delbrück scattering, nuclear Thomson, Rayleigh, and nuclear resonance scattering might contribute significantly to the elastic scattering of photons with energies below 3 MeV. To search for the presence of Delbrück scattering, the differential elastic cross section is measured as a function of scattering angle and compared with theoretical calculations of the coherent sum of Thomson, Rayleigh, and nuclear resonance scattering. A significant difference between experimental and theoretical angular distributions then constitutes evidence for a contribution from Delbrück scattering. In this manner the existence of the imaginary part of the Delbrück scattering amplitude was definitely established (see Jackson *et al.*[2] and earlier references contained therein). Since the real and imaginary scattering amplitudes are connected by a dispersion relationship, evidence for the existence of the latter provides indirect evidence for the existence of the former.

Our main interest is the degree of agreement between theory and experiment at photon energies of 1.33 and 2.62 MeV. At these energies the contribution from nuclear resonance scattering is small and can be estimated.[2,3] The nuclear Thomson amplitudes are accurately known. Accurate K-shell Rayleigh amplitudes have been calculated by Brown *et al.*[4] and Cornille *et al.*[5] for a mercury scatterer and photon energies up to 2.62 MeV. Ehlotzky *et al.*[6] have calculated the Delbrück scattering amplitudes up to 17 MeV. Of course, to obtain the theoretical differential cross sections, the relative phases among these amplitudes must be known. Determination of these relative phases is discussed in the next section.

Before the calculations of Ehlotzky *et al.*[6] were published, the discrepancy between the measurements of Bernstein *et al.*[7] and Eberhard *et al.*[8] and the sum of the theoretical Rayleigh and nuclear Thomson amplitudes was taken as evidence for the existence of Delbrück scattering[7,9] at a photon energy of 2.62 MeV. Now that the Delbrück amplitudes are available, these measurements can be compared with the sum of Rayleigh, nuclear Thomson, and Delbrück scattering. Figure 1 shows this comparison. Clearly there is a discrepancy. Possible reasons for this discrepancy include inaccurate or incomplete calculations and systematic errors in the experiments.

In the present work, the high energy-resolution

[1] P. P. Kane and G. Basavaraju, Rev. Mod. Phys. **39**, 52 (1967).
[2] H. E. Jackson and K. J. Wetzel, Phys. Rev. Letters **22**, 1008 (1969).
[3] J. S. Levinger, Phys. Rev. **84**, 523 (1951).
[4] G. E. Brown and D. F. Mayers, Proc. Roy. Soc. (London) **A242**, 89 (1957).
[5] H. Cornille and M. Chapdelaine, Nuovo Cimento **14**, 1386 (1959).
[6] F. Ehlotzky and G. C. Sheppey, Nuovo Cimento **33**, 1185 (1964).
[7] A. M. Bernstein and A. K. Mann, Phys. Rev. **110**, 805 (1958)
[8] P. Eberhard, L. Goldzahl, and E. Hara, J. Phys. Radium **19**, 658 (1958).
[9] L. Goldzahl, P. Eberhard, H. Cornille, and M. Chapdelaine, Compt. Rend. **249**, 401 (1959).

capabilities of a lithium-drifted germanium semiconductor detector were used to decrease the uncertainty in the measurements and to reduce the likelihood that large systematic errors would remain undetected. An energy of 1.33 MeV was selected because ^{60}Co sources are inexpensive, because much experimental work has already been done at this energy (see Standing *et al.*[10] and references contained therein), and because agreement between theory and experiment can be checked at an energy where the Delbrück contribution is small. Recently, Dixon *et al.*[11] have reported measurements using photons from a ^{60}Co source, a lead scatterer, and a lithium-drifted germanium detector. These authors have emphasized that a discrepancy exists between theoretical calculations and their experimental results

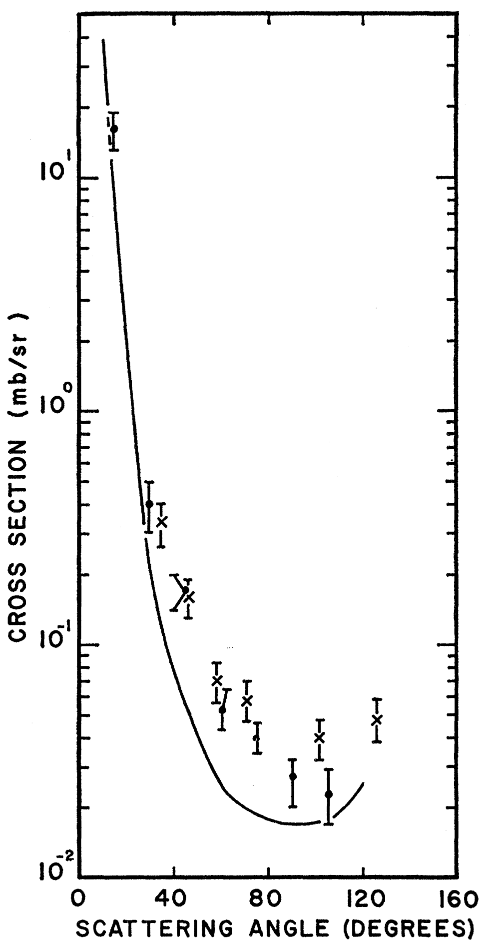

FIG. 1. The curve is the coherent sum of nuclear Thomson, Rayleigh, and Delbrück scattering of 2.62-MeV photons from lead. The dots are the results from Ref. 7 and the crosses are the results from Ref. 8.

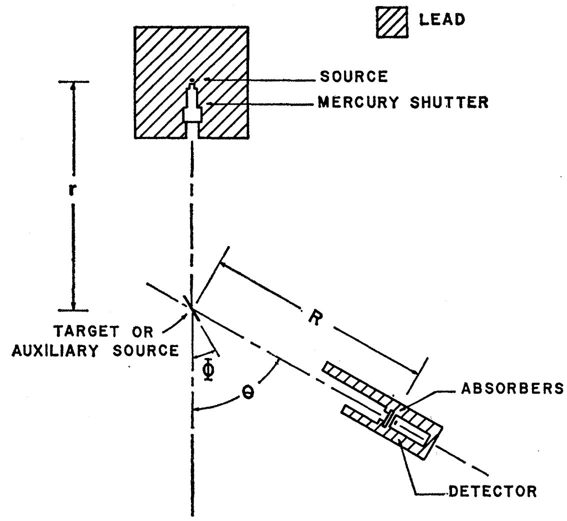

FIG. 2. A cross section of the apparatus (median plane) as seen from above. The source is approximately 100 Ci of ^{60}Co. A lithium-drifted germanium detector of 12-cm^3 active volume is used to detect the scattered photons.

and have discussed possible reasons for this discrepancy. The results reported in this paper agree well with those of Dixon *et al.*[11] for a lead scatterer and show that a similar discrepancy exists for a uranium scatterer. This discrepancy will be discussed after the experimental results have been presented.

THEORY

In calculating the complex amplitudes for individual elastic scattering processes, an over-all sign is not important. However, to obtain the total differential elastic scattering cross section, the relative phases of these amplitudes must be known. Since the basis on which relative phases have been assigned is often not explicit, and hence difficult to assess, the reasons for the present choices will be discussed. Dispersion relations are used to establish the relative phases in the forward direction, the relative phases at the other angles then being fixed by the calculations of the individual scattering amplitudes.

The dispersion relation will be written in the form (see Erber,[12] and Gell-Mann *et al.*[13])

$$a(\nu)=a(0)+\frac{\nu^2}{2\pi^2}\int_0^\infty \frac{\sigma_T}{(\nu')^2-\nu^2}\,d\nu',$$

where a represents the real part of the forward-scattering amplitude, ν the photon frequency, and σ_T the total cross section. If this relation is applied to scattering from a bare nucleus, then $q(0)$ is the nuclear

[10] K. G. Standing and J. V. Jovanovich, Can. J. Phys. **40**, 622 (1962).

[11] W. R. Dixon and R. S. Storey, Can. J. Phys. **46**, 1153 (1968).

[12] T. Erber, Ann. Phys. (N.Y.) **6**, 319 (1959).

[13] M. Gell-Mann, M. L. Goldberger, and W. E. Thirring, Phys. Rev. **95**, 1612 (1954).

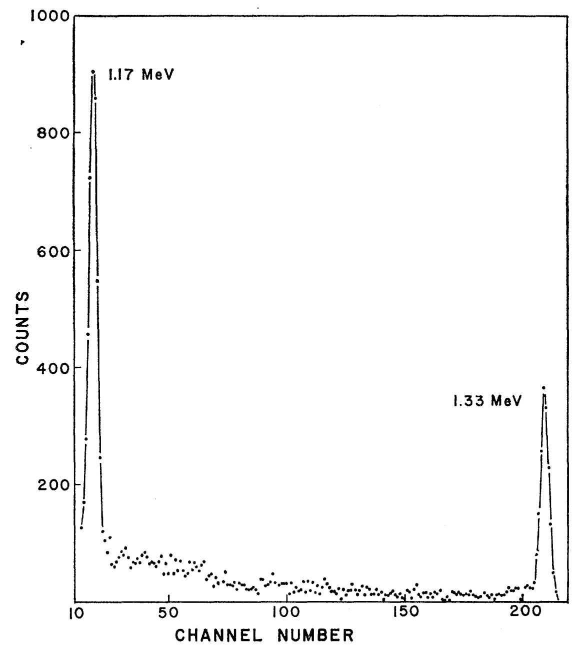

FIG. 3. The spectrum of ^{60}Co photons elastically scattered through an angle of 60° from a uranium target. A biased amplifier was used to select and expand the region containing the 1.17-MeV and 1.33-MeV photons. These counts were obtained in a run of 2400 "live minutes" followed by a run to subtract room background. A 7.2-g/cm² lead absorber was in front of the detector.

Thomson scattering amplitude[13] and hence

$$a^N(\nu)=-\frac{(Ze)^2}{M_N}+\frac{\nu^2}{2\pi^2}\int_0^\infty\frac{\sigma_{\mathrm{PP}}}{(\nu')^2-\nu^2}d\nu'+\frac{\nu^2}{2\pi^2}\int_0^\infty\frac{\sigma_{\mathrm{S}}{}^N}{(\nu')^2-\nu^2}d\nu',$$

where σ_{PP} is the pair-production cross section and σ_{S} the scattering cross section. Rohrlich *et al.*[14] perform the first integration and show, by comparing the result of this integration with that obtained using conventional Feynman techniques, that it is the real part of the Delbrück forward-scattering amplitude, a^D. Evaluation of the integral at an energy of 1.33 MeV yields a positive result and hence, at a scattering angle of 0°, Delbrück scattering is out of phase with nuclear Thomson scattering. Of course, this conclusion does not depend on the value of the final integral.

TABLE I. Differential cross sections for elastic scattering of 1.33-MeV photons from uranium (mb/sr).

Scattering angle (deg)	Present results	Goldzahl *et al.*[a]	Bernstein *et al.*[b]
50		1.23±20%	
60	0.430±0.025		0.81
70		0.453±20%	
75	0.270±0.017		0.38
90	0.226±0.014	0.245±20%	0.37
105	0.211±0.013		
115		0.231±20%	
120	0.205±0.012		
135	0.191±0.011		

[a] L. Goldzahl and P. Eberhard, J. Phys. Radium **18**, 33 (1957).
[b] A. M. Bernstein and A. K. Mann, Phys. Rev. **110**, 805 (1958).

[14] F. Rohrlich and R. L. Gluckstern, Phys. Rev. **86**, 1 (1952).

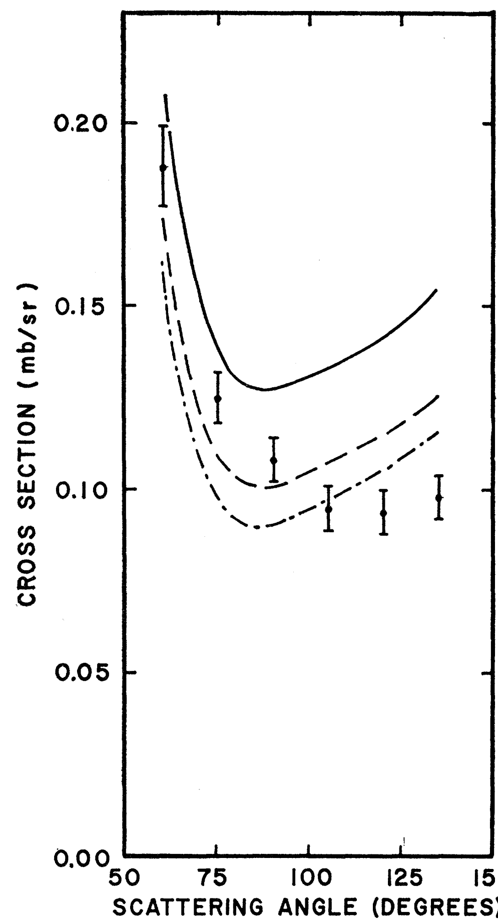

FIG. 4. The dots are the present results from a lead target scattering 1.33-MeV photons. The dot- and dash-curve is the coherent sum of nuclear Thomson scattering and Rayleigh scattering from the K shell. The addition of a Delbrück contribution gives the dash curve, while the addition of both Delbrück scattering and a spin-flip contribution from L-shell Rayleigh scattering gives the solid curve.

Application of dispersion relations to an atom for the case of ν corresponding to an energy on the order of 1 MeV gives

$$a^A(\nu)=a^A(0)+\frac{\nu^2}{2\pi^2}\int_0^\infty\frac{\sigma_{\mathrm{PE}}+\sigma_{\mathrm{PP}}-\sigma_{\mathrm{PP}B}+\sigma_{\mathrm{S}}{}^A}{(\nu')^2-\nu^2}d\nu',$$

where σ_{PE} is the photoelectric cross section and $\sigma_{\mathrm{PP}B}$ is that part of the pair production cross section in which the electron produced goes into a quantum state occupied by one of the atomic electrons (see Levinger

TABLE II. Differential cross sections for elastic scattering of 1.33-MeV photons from lead (mb/sr).

Scattering angle (deg)	Present results	Dixon *et al.*[a]	Standing *et al.*[b]	Bernstein *et al.*[c]	Goldzahl *et al.*[d]
50					$0.580\pm20\%$
60	0.190 ± 0.011	0.185 ± 0.013	0.206 ± 0.011	0.47	
70					$0.139\pm20\%$
75	0.125 ± 0.007	0.118 ± 0.009	0.136 ± 0.004	0.24	
90	0.108 ± 0.006	0.113 ± 0.007	0.111 ± 0.003	0.16	$0.145\pm20\%$
105	0.095 ± 0.006	0.099 ± 0.006	0.105 ± 0.011	0.12	
115					$0.104\pm20\%$
120	0.094 ± 0.006	0.093 ± 0.006	0.105 ± 0.011		
135	0.098 ± 0.006				$0.113\pm20\%$
150		0.099 ± 0.012	0.117 ± 0.007		

[a] W. R. Dixon and R. S. Storey, Can. J. Phys. **46**, 1153 (1968).
[b] K. G. Standing and J. V. Jovanovich, Can. J. Phys. **40**, 622 (1962).
[c] A. M. Bernstein and A. K. Mann, Phys. Rev. **110**, 805 (1958).
[d] L. Goldzahl and P. Eberhard, J. Phys. Radium **18**, 33 (1957).

et al.[15]). According to Erber[12] $a^A(0)=0$, and so

$$a^A(\nu)=a^D+\frac{\nu^2}{2\pi^2}\int_0^\infty \frac{\sigma_{\mathrm{PE}}-\sigma_{\mathrm{PPB}}}{(\nu')^2-\nu^2}\,d\nu' + \frac{\nu^2}{2\pi^2}\int_0^\infty \frac{\sigma_{\mathrm{S}}{}^A}{(\nu')^2-\nu^2}\,d\nu'.$$

Levinger *et al.*[15] have evaluated the first integral approximating σ_{PE} by the photoelectric effect from the K-shell electrons ($\sigma_{\mathrm{PE}K}$) and, by comparing their results with the calculations of Brown *et al.*,[4] have shown that it is the Rayleigh amplitude for scattering from K-shell electrons. Therefore,

$$a^A(\nu)=a^D+a^{RK}+\frac{\nu^2}{2\pi^2}\int_0^\infty \frac{\sigma_{\mathrm{S}}{}^A}{(\nu')^2-\nu^2}\,d\nu'.$$

Evaluation of the integral yields a negative a^{RK} at a photon energy of 1.33 MeV. Hence, at a scattering angle of 0°, the real parts of the Rayleigh K-shell and Delbrück amplitudes have opposite signs.

The optical theorem applied to the atom gives

$$b(\nu)=(\nu/4\pi)[(\sigma_{\mathrm{PE}}-\sigma_{\mathrm{PPB}})+\sigma_{\mathrm{PP}}+\sigma_{\mathrm{S}}],$$

where b represents the imaginary part of the forward-scattering amplitude. Again Levinger *et al.*[15] show that

$$(\nu/4\pi)(\sigma_{\mathrm{PE}}-\sigma_{\mathrm{PPB}})\approx(\nu/4\pi)(\sigma_{\mathrm{PE}K}-\sigma_{\mathrm{PPB}})=b^{RK},$$

and according to Rohrlich *et al.*,[14]

$$(\nu/4\pi)\sigma_{\mathrm{PP}}=b^D,$$

so the imaginary parts of the Rayleigh K-shell and Delbrück forward-scattering amplitudes have the same sign.

One check on these signs is afforded by the calculations of the complex scattering amplitudes for Delbrück and Rayleigh scattering, since these calculations should give the correct relative sign between the real and imaginary parts. From the above discussion both a^D and b^D have the same sign in the forward direction, and this agrees with the calculations of Ehlotzky *et al.*[6]

[15] J. S. Levinger and M. L. Rustgi, Phys. Rev. **103**, 439 (1956).

Again a^{RK} and b^{RK} have opposite signs, which agrees with the relative signs given by Brown *et al.*[4]

For the case of photons, the scattering amplitudes have two polarization components. If the choice is made for components parallel and perpendicular to the scattering plane, then in the forward direction the two components must be identical and the above phase considerations apply independently to either polarization state. Since the amplitudes for one choice of polarization states can be written in terms of the amplitudes for any other choice of polarization states, this fixes the relative phases regardless of the choice of base states.

APPARATUS AND PROCEDURE

Figure 2 is a schematic diagram of the experimental arrangement. The lead or uranium target is exposed to a beam of γ rays by forcing mercury, which normally blocks the beam, into a reservoir. The intensity of the ^{60}Co source was about 111 Ci. A lithium-drifted germanium detector of 12-cm^3 sensitive volume was used to detect the γ rays. Figure 3 shows a typical photon spectrum.

The lead and uranium targets were 14.0 cm by 16.5 cm and thicknesses of 0.178 and 0.0665 cm, respectively. To minimize the spread in the scattering angle due to the finite size of the targets, the angle Φ was chosen to satisfy the relationship

$$(\sin\Phi)/\sin(\Theta-\Phi)=r/R,$$

where Θ represents the scattering angle, r the source to target distance and R the target to detector distance. To check the position of the target in the beam, an x-ray plate was located behind the target and the source opened for a short period of time. The developed plate showed the "shadow" cast by the target.

To avoid measuring the efficiency of the detector or calculating the solid angle subtended by the detector at the target, a second measurement was performed using an auxiliary source. To construct this source a piece of cardboard, with the same length and width as the targets', was painted uniformly with a liquid containing ^{60}Co. This auxiliary source was placed in the target

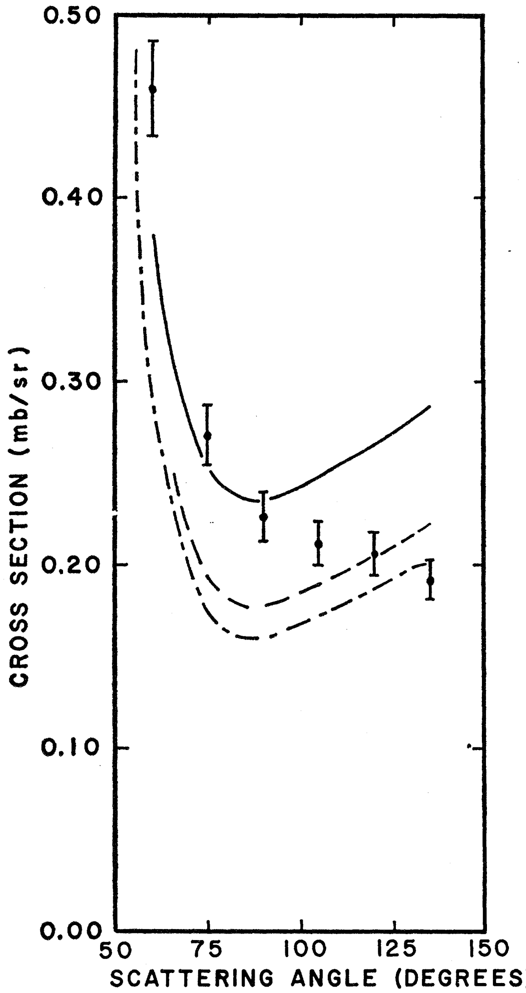

FIG. 5. The dots are the present results from a uranium target scattering 1.33-MeV photons. The dot-dash curve is the coherent sum of nuclear Thomson scattering and Rayleigh scattering from the K shell. The addition of a Delbrück contribution gives the dash curve, while the addition of both Delbrück scattering and a spin-flip contribution from L-shell Rayleigh scattering gives the solid curve.

position and the counting rate determined. In the notation of Standing *et al.*,[10]

$$(d\sigma/d\Omega)_{\mathrm{el}} = (n_a/n_b)(b/a)(r^2/N).$$

Since the techniques for determining the first two ratios, as well as r and N, are straightforward,[10,16] they will not be discussed in the present paper. Corrections were made for the absorption of both the incident beam and the elastically scattered γ rays by the target and for the variation of r due to the finite size of the target.

RESULTS

The present results for uranium and lead, as well as those of other workers, are given in Tables I and II, respectively. The results of Bernstein *et al.*[7] were obtained from the graph in their paper. They quote a probable error of 10% which must be combined with the statistical errors in their elastic scattering count rates. The uncertainties in the results of Standing *et al.*[10] do not include the error in the source ratio, b/a, which they estimate as $\pm 5\%$. The uncertainties quoted for the present results are standard deviations.

The present results for uranium are in good agreement with those of Goldzahl *et al.*[17] and somewhat lower than those of Bernstein *et al.*[7] In the case of lead, the results of Dixon *et al.*[11] are in excellent agreement with the results presented in this paper. The results of Standing *et al.*[10] are, at all angles, slightly higher than the present results, but agreement is still good.

DISCUSSION

Theoretical calculations of the cross sections for 1.33-MeV photons scattered by lead and uranium require

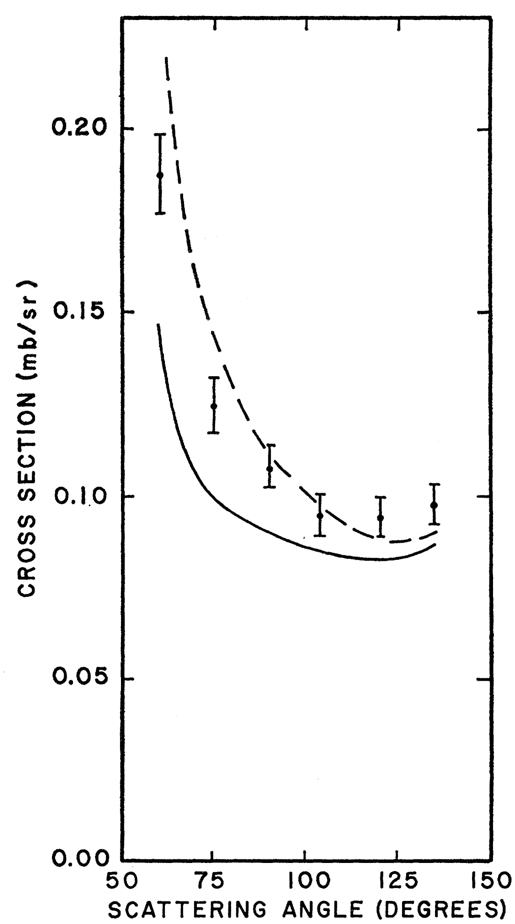

FIG. 6. The dots are the present results for a lead target scattering 1.33-MeV photons. The solid curve, which includes a contribution from Delbrück and L-shell scattering, is obtained by assuming that Rayleigh scattering is incoherent with nuclear scattering. The dash curve, which also includes Delbrück and L-shell contributions, is obtained by arbitrarily changing the sign of the parallel-polarization amplitude for Rayleigh scattering.

[16] V. A. N. Murty, V. Lakshminarayana, and S. Jnanananda, Nucl. Phys. **62**, 296 (1965).

[17] L. Goldzahl and P. Eberhard, J. Phys. Radium **18**, 33 (1957).

the extrapolation of the Rayleigh K-shell amplitudes, which were calculated by Brown and Mayers[4] for a mercury scatterer and a photon energy of 1.31 MeV. The Rayleigh K-shell form factor, derived by Bethe,[18] was used to extrapolate the amplitudes to the slightly higher energy. Extrapolation to other targets was done using Z dependence which is a function of the change in momentum of the photon caused by the scattering, as suggested by Anand and Sood.[19] If form factors were used to extrapolate to higher Z, the cross sections would increase, and the discrepancy between theory and experiment would also increase.

The Rayleigh calculations are for K-shell electrons only, but the contribution from L-shell electrons cannot be neglected. Following the suggestion of Brown and

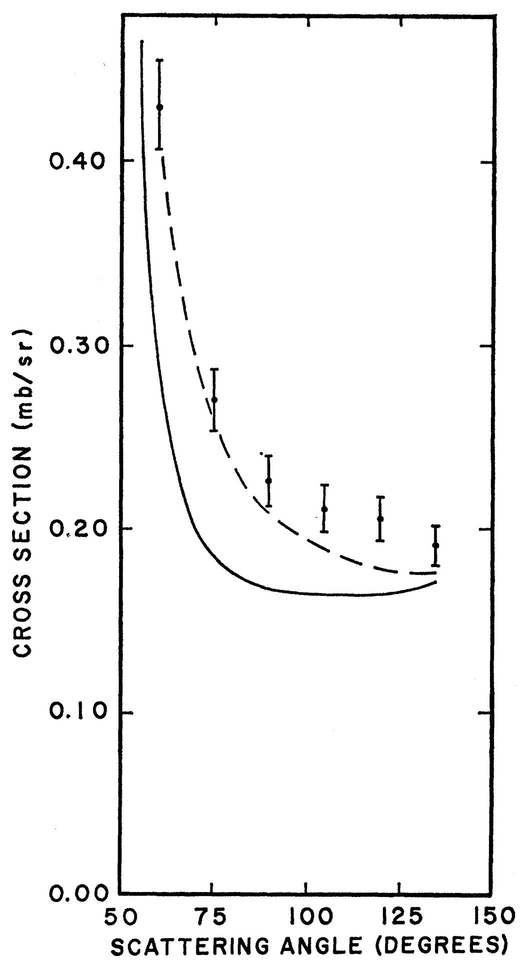

FIG. 7. The dots are the present results for a uranium target scattering 1.33-MeV photons. The solid curve, which includes a contribution from Delbrück and L-shell scattering, is obtained by assuming that Rayleigh scattering is incoherent with nuclear scattering. The dash curve, which also includes Delbrück and L-shell contributions, is obtained by arbitrarily changing the sign of the parallel-polarization amplitude for Rayleigh scattering.

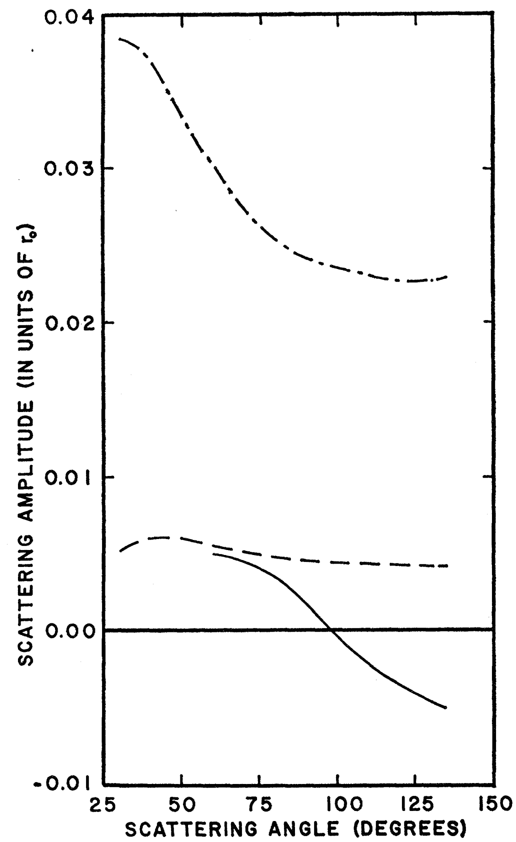

FIG. 8. The curves are Rayleigh spin-flip amplitudes, in units of the classical electron radius (r_0), for a lead scatter and a photon energy of 1.33 MeV. The dot-dash curve is the K-shell spin-flip amplitude. The dash curve is the L-shell spin-flip amplitude obtained from the K-shell amplitude by using form factors. The solid curve is the spin-flip L-shell amplitude necessary to secure agreement between the theoretical calculations and experimental results.

Mayers[4] only a spin-flip L-shell amplitude was added. The L-shell contribution (a^L) was estimated using the form factors (F) derived from Woodward[20]:

$$a^L(SF)=a^K(SF)(F^L/F^K),$$

where a^K represents the Rayleigh K-shell amplitudes extrapolated in Z and energy. Over the range of scattering covered in this paper, adding an L-shell contribution to the non-spin-flip amplitudes would not change the cross sections appreciably.

In Figure 4 the present data for a lead target are compared with some theoretical calculations. The dot-dash curve includes only the nuclear Thomson and Rayleigh K-shell amplitudes. The dash curve shows the increase in the cross section when the Delbrück amplitudes are included. Finally, the solid curve also contains a spin-flip L-shell contribution. In Fig. 5, the present results for a uranium scatterer are com-

[18] See J. S. Levinger, Phys. Rev. **87**, 656 (1952).
[19] S. Anand and B. S. Sood, Nucl. Phys. **73**, 368 (1965).
[20] J. B. Woodward, thesis, University of Birmingham, 1953 (unpublished).

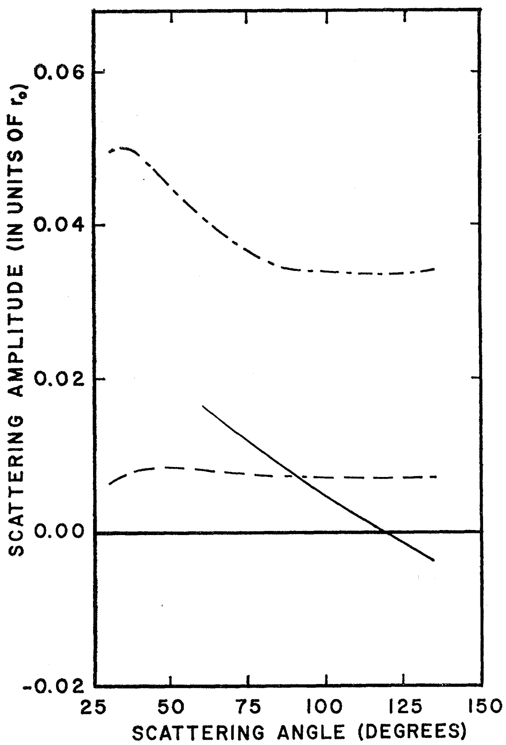

FIG. 9. The curves are Rayleigh spin-flip amplitudes, in units of the classical electron radius (r_0), for a uranium scatterer and a photon energy of 1.33 MeV. The dot-dash curve is the K-shell spin-flip amplitude. The dash curve is the L-shell spin-flip amplitude obtained from the K-shell amplitude by using form factors. The solid curve is the spin-flip L-shell amplitude necessary to secure agreement between the theoretical calculations and experimental results.

pared with similar theoretical calculations. For both targets there is clearly a discrepancy between theory and experiment, a discrepancy which does not seem attributable to the uncertainty in the L-shell contribution as calculated using form factors. Dixon and Storey[11] have pointed out that substantially better agreement can be obtained in either of two ways. One is to assume that for angles greater than 90°, scattering from the electrons is incoherent with the nuclear scattering, while for smaller scattering angles there is complete coherence. In Figs. 6 and 7 the solid line is the theoretical cross section, including Delbrück and spin-flip L-shell amplitudes, obtained by assuming such incoherence. The fit is reasonably good at back angles for lead, but too low for uranium. However, the extrapolation of the Rayleigh scattering amplitudes from mercury to uranium is quite uncertain and, in fact, the solid curve would be above the experimental points if form factors were used to extrapolate these amplitudes. The present authors are unaware of any detailed theoretical studies exploring the validity of this suggested transition from coherence to incoherence.

The second observation of Dixon and Storey is that if the relative sign between the nuclear Thomson and Rayleigh amplitudes be changed for the parallel components of polarization while there is no corresponding change for the perpendicular components, substantially better agreement will be obtained. The theoretical curves which result upon changing the signs of the Rayleigh parallel-polarization components are shown dotted in Figs. 5 and 6. Again agreement at the back angles is very much improved. Remaining discrepancies could convincingly be attributed to inaccurate L-shell corrections and uncertainty in the extrapolated Rayleigh amplitudes. In the section of this paper devoted to theory, arguments to determine the relative phases of nuclear Thomson and Rayleigh scattering were presented. If these arguments are correct this empirical explanation is not tenable.

Since the Rayleigh L-shell amplitudes are presumably the only significant ones not accurately calculated, it is worthwhile to inquire what they must be to secure agreement between theory and experiment, assuming that the non-spin-flip L-shell amplitudes can be neglected. The solid curve in Fig. 8 gives the Rayleigh L-shell spin-flip amplitude necessary to obtain agreement between theory and experiment for a lead target. Also on Fig. 7 are the Rayleigh K-shell spin-flip amplitude (dot-dash curve) and the L-shell amplitude obtained by using the form factors (dash curve) as described above. Figure 9 presents the same set of curves for a uranium scatterer. Again these are subject to uncertainties because of the extrapolation from mercury to uranium. The magnitudes of the L-shell contributions necessary to secure agreement are certainly reasonable. However, at the back angles, they interfere destructively with the Rayleigh K-shell spin-flip amplitudes. If this is indeed possible it would seem to be the most straightforward explanation for the discrepancy between theory and experiment. The authors cannot give a theoretical justification for this hypothesis although they are not aware of any arguments which make it untenable. Obviously, accurate calculations of the L-shell amplitudes would greatly aid interpretation of experiments in the region around 1 MeV.

ACKNOWLEDGMENTS

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