measurement. It differs from other potentials mostly in the larger radius and in the narrowness of the surface absorption. We tried to reproduce our data by slightly increasing the diffuseness a_2 of the imaginary potential and decreasing the depth W, keeping the product a_2W constant in order not to disturb the agreement reached by Moldauer.[14] Within these restrictions, a possible new set would be $W=9-0.2E$ MeV and $a_2=0.78$ fm and all other parameters as given by Engelbrecht and Fiedeldey (set B in Table II). An equally good fit is obtained for an optical potential without volume absorption, but $W=10-0.2E$ MeV and $a_2=0.70$ fm (set A in Table II).

In Fig. 1, the data points represent the measured values of σ_{res}. The solid curves are the sums of transmission coefficients calculated from set B (Table II), starting with the T_{lj} given at each curve. Higher partial waves with $l\geq 5$ can be omitted since these contribute less than 3%. The measured transition rates from the decay of the 4^+ resonance are normalized to the calculation at $E_n=1.43$ MeV for the transition $4^+\rightarrow\frac{5}{2}^-$ and those from the decay of the 2^+ resonance at $E_n=3.11$ MeV for $2^+\rightarrow\frac{1}{2}^-$. Since all parameter sets give nearly the same absolute values for the odd-l transmission coefficients, this normalization was the most reasonable way to determine the constant in the above formula. The agreement between the measured and calculated transition strength is very good for the transitions to all final states independent of its parity. Only the calculated $T_{p1/2}$ seems to be somewhat too large.

For the purpose of comparison, the sum $T_{s1/2}+T_{d3/2}+\cdots$, calculated from Perey and Buck[11] (dashed curve) and with the local equivalent parameters of Engelbrecht and Fiedeldey[13] (dotted curve) are included in Fig. 1. The obvious difference of the theoretical and experimental values is out of the experimental error. Similarly, systematic discrepancies arise for the other curves with even-l coefficients.

Our method of analysis is based on the assumption of a statistical neutron decay of isobaric analog resonances. If it were not to hold in this case, we would expect differences between the two resonances studied because the wave functions of the isobaric analog states are different.[15] Yet the analysis given here, and the further evaluation including all measured neutron transitions,[10] give no indication of any departure from the statistical decay.

The neutron decay of isobaric analog resonances is a precise method to determine transmission coefficients. Thereby one obtains additional restrictions to the parameters of the optical model.

[15] J. D. Fox, Argonne National Laboratory Report No. ANL-6878, 1964, p. 231 (unpublished).

PHYSICAL REVIEW C VOLUME 1, NUMBER 2 FEBRUARY 1970

Inelastic Triton Scattering from ^{92,94,96}Zr†

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(Received 8 September 1969)

The inelastic scattering of 20-MeV tritons from ^{92}Zr, ^{94}Zr, and ^{96}Zr has been studied. The principal objective was to obtain an effective interaction strength to be used in a microscopic description of the excitation of neutron states. Only the lower levels of ^{92}Zr and ^{94}Zr were studied, but because of the paucity of available data on ^{96}Zr, this isotope is studied in a more complete way, with 28 levels and 17 spins assigned. The shell-model results indicate a much smaller effective interaction strength for the excitation of neutron states than noted earlier for proton states in triton inelastic scattering.

I. INTRODUCTION

INELASTIC scattering has proven to be an effective mechanism for obtaining spectroscopic information about nuclear states. The most general use has been in determining the angular momentum of excited states of the target nucleus. By application of a collective-model interpretation to the data, it is possible to also obtain the collective-model parameters which describe many of the levels excited in an inelastic process. In addition, when the target nuclei are near closed shells, it is also possible to interpret some of the observed nuclear states by a shell-model scheme which considers only a few nearby orbitals. Both of these techniques generally rely on a distorted-wave (DW) analysis of the data. If the incident particle is strongly absorbed by the target nucleus, the DW approach is somewhat simplified and less ambiguous than for particles which penetrate deeply into the nucleus. This simplification is generally reflected in a better determination of the angular momentum (l) transfer due to the more diffractionlike structure of the differential cross section and also smaller dependence upon the parameters which describe the distorted waves, i.e., the optical-model parameters for the incoming and outgoing particle.

A previous experiment with the inelastic scattering

† Work performed under the auspices of the U.S. Atomic Energy Commission.

of tritons from ^{90}Zr has verified the above discussion.[1] Because of its nuclear spin of $\frac{1}{2}$ and its isospin of $+\frac{1}{2}$, it is possible to make close comparisons of the levels excited by the triton with those excited by protons and neutrons. In the study of ^{90}Zr, 30 levels were observed up to an excitation energy of 6.56 MeV. Of these, 14 could be assigned definite l transfers with an additional seven suggested. These assignments were based on DW calculations involving both a collective-model form factor and a microscopic or shell-model form factor with both techniques yielding almost equivalent angular distributions. Comparisons of these results to previous measurements involving inelastic scattering of protons, ^{3}He ions, and ^{4}He ions were good.

For the case of ^{90}Zr$(t, t')^{90}$Zr*, a detailed comparison was made with the inelastic proton results of Gray *et al.*[2] The principal effort here was to compare with the microscopic shell-model calculations of Johnson *et al.*[3] Based on rather simple shell-model configurations for the low-lying states for the two reactions, a comparison was made of the effective interaction strengths for exciting such levels. In comparing the triton strength to the proton strength (and without considering the effect of core polarizations[4]), it was found that the triton strength was slightly over three times that of the proton. This result was found to be in rather striking agreement with the ratio of the optical-model real well depths generally assumed for both proton and triton. However, this result was valid only for the low-lying levels which were assumed to be primarily proton configurations. When somewhat higher excited levels were considered which could be due to particle-hole states formed by promoting a neutron across the closed shell, the resulting effective interaction was more comparable to that required to excite these states by proton inelastic scattering. That the strength for exciting neutron states is less than that for exciting proton states can be determined from a study of the inelastic scattering to low-lying (and presumably neutron) states in the other zirconium isotopes. This has been pointed out by Stautberg and Kraushaar[5] in their analysis of ^{92,94}Zr experiments. By considering the low-lying 2^+ and 4^+ states which are observed as being principally due to the recoupling of the $(2d_{5/2})_0{}^{2,4}$ neutrons in the ground states of ^{92,94}Zr, they find that essentially the same effective interaction is needed to describe the excitation of both proton and neutron states by the inelastic scattering of protons. Therefore, the difference in the effective interaction for exciting proton and neutron states in ^{90}Zr needs to be explored further by triton inelastic scattering from the low-lying (neutron) states of ^{92,94}Zr. The principal goal in the present work is thus to study only the low-lying levels because of their expected more simple character. The spectroscopy of these levels is rather well known from both inelastic scattering experiments[5] and (t, p) reaction studies.[6]

In the case of ^{96}Zr, the level spectroscopy is uncertain. Inelastic proton scattering experiments[7] have been able to make four definite spin assignments and ten level positions. Also, there is disagreement between these results and previous (d, d') measurements.[8] For this reason, the present study of ^{96}Zr is as exhaustive as the data permit in order to establish the energies and spins of these levels for as large an excitation energy range as possible. The shell-model descriptions of the ^{96}Zr states are also especially interesting because the closure of the $2d_{5/2}$ shell produces a sudden change in the expected configuration of the low-lying states.

II. EXPERIMENTAL TECHNIQUE

The data presented here were obtained by using a 20-MeV beam of tritons from the Los Alamos three-stage Van de Graaff. Beams of up to 1.2 μA in magnitude were employed during these exposures. The targets were rolled zirconium of about 500 μg/cm^2 and were fabricated by the Oak Ridge Isotopes Division of the Oak Ridge National Laboratory. The ^{92,94}Zr targets were isotopically enriched to greater than 95% purity; however, the ^{96}Zr was of 85% purity which complicated the identification of the ^{96}Zr levels. The elastic and inelastic tritons produced by the incident triton beam on these targets were detected by solid-state detectors.

The solid-state detector telescope consisted of a 500-μ surface-barrier silicon detector and a 3-mm lithium-drifted silicon detector. The telescope was operated in a 20-in. scattering chamber and could be rotated around the target remotely from the accelerator control room. The typical resolution of this arrangement was 35 keV for this experiment. The triton spectra were stored in a 512-channel array in the core of an SDS-930 computer. The electronic gain was arranged so that each channel corresponded to 18 keV of excitation. Thus, it was possible to store data up to an excitation energy of 9 MeV, although a conservative positioning of the elastic peak in the upper portion of the spectrum usually limited this to 7 MeV. A typical spectrum for ^{96}Zr is shown in Fig. 1. A detailed explanation of the counter telescope and computer particle-identification system is given elsewhere.[1,9]

Extraction of peak areas and positions for the counter

[1] E. R. Flynn, A. G. Blair, and D. D. Armstrong, Phys. Rev. **170**, 1142 (1968).
[2] W. S. Gray, R. A. Kenefick, J. J. Kraushaar, and G. R. Satchler, Phys. Rev. **142**, 735 (1966).
[3] M. B. Johnson, L. W. Owen, and G. R. Satchler, Phys. Rev. **142**, 748 (1966); **154**, 1206 (1967).
[4] W. G. Love and G. R. Satchler, Nucl. Phys. **A101**, 424 (1967).
[5] M. M. Stautberg and J. J. Kraushaar, Phys. Rev. **151**, 969 (1966).
[6] J. G. Beery, Ph.D. thesis, University of New Mexico, 1968 (unpublished).
[7] M. M. Stautberg, R. R. Johnson, J. J. Kraushaar, and B. W. Ridley, Nucl. Phys. **A104**, 67 (1967).
[8] R. K. Jolly, E. K. Lin, and B. L. Cohen, Phys. Rev. **128**, 2292 (1962).
[9] D. D. Armstrong, J. G. Beery, E. R. Flynn, W. S. Hall, P. W. Keaton, Jr., and M. P. Kellogg, Nucl. Instr. Methods **70**, 69 (1969).

TABLE I. Optical-model parameters used in DW calculations.

	V (MeV)	W (MeV)	r_r (F)	r_i (F)	a_r (F)	a_i (F)
^{92}Zr	159.1	25.0	1.24	1.401	0.670	0.809
^{94}Zr	153.4	18.8	1.24	1.489	0.681	0.787
^{96}Zr	154.0	18.6	1.24	1.391	0.672	0.990

data was accomplished by means of least-squares computer programs. Although several such programs were used during the course of the data analysis, the bulk of the data was analyzed using a program due to Rickey.[10] This routine utilizes a skewed-Gaussian shape which it fits to a selected peak in order to adjust parameters. It then locates all of the other peaks in the spectrum and obtains their areas and locations by using the selected peak as a standard. The absolute cross sections were obtained by using the areas frcm the elastic peaks at small angles and normalizing them to the predictions of an optical-model code due to Perey.[11] The predictions at small angles are quite insensitive to the optical-model parameters, for medium to heavy nuclei, and for incident energies on the order of that employed here. The accuracy of cross sections is estimated to be $\pm 10\%$.

III. ELASTIC SCATTERING ANALYSIS

It was necessary to perform an optical-model analysis of the elastic scattering results in order to obtain the necessary parameters for the DW calculations. The initial guesses for the optical-model search program[11] were provided by the results for ^{90}Zr given in Ref. 1. The potential form used there was of the Woods-Saxon type:

$$U = V_C(r) - [Vf(r) + iWg(r)], \tag{1}$$

where

$$f(r) = [1+\exp\{(r_r A^{1/3} - r)/a_r\}]^{-1}, \tag{2}$$

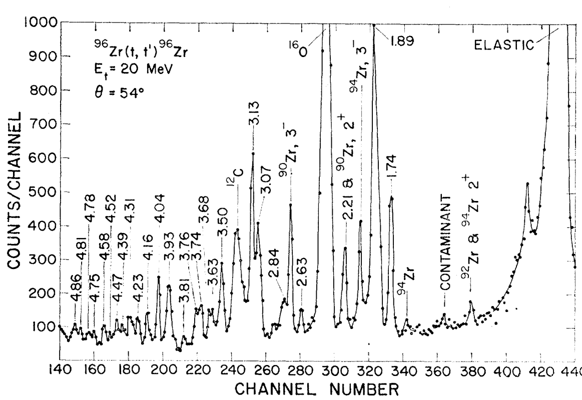

FIG. 1. Typical ^{96}Zr spectrum.

[10] F. Rickey (private communication).
[11] F. G. Perey, Phys. Rev. **131**, 745 (1963).

and

$$g(r) = [1+\exp\{(r_i A^{1/3} - r)/a_i\}]^{-1}, \tag{3}$$

where V_C is the usual spherical Coulomb potential, r_r and r_i are the real and imaginary radius parameters, and a_r and a_i are the diffuseness parameters. As in Ref. 1, the real radius was fixed at 1.24 F and the real potential depth was assumed to be about three times the depth for a free nucleon, about 150 MeV. The final values of all parameters after performing the search and minimizing χ^2 are given in Table I, and the comparison to the data is shown in Fig. 2.

In a recent survey of triton elastic scattering,[12] two potentials which describe the triton elastic scattering results from ^{40}Ca through ^{208}Pb were discussed. One of these potentials was based on the assumption that $r_r = 1.24$ F as is done above and the other used $r_r =$

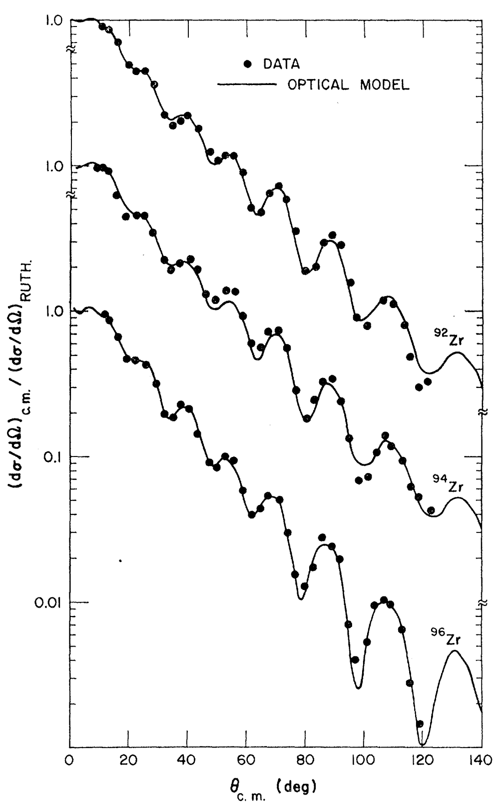

FIG. 2. Optical-model fits to the elastic scattering.

[12] E. R. Flynn, D. D. Armstrong, J. G. Beery, and A. G. Blair, Phys. Rev. (to be published).

1.16 F. Experience in two-nucleon transfer reactions, namely, the (t, p) reaction, had indicated that the 1.16-F radius was needed to fit these data well. In particular, in the region of Zr, several studies have been made[6,13] and both obtain the best DW description of the data when using this radius. However, when this smaller real radius potential, utilizing the parameters given in Ref. 12, was used here, no distinction could be made of the results from those discussed above. This result is not unexpected since the inelastic scattering of tritons involves strongly absorbed particles in both the entrance and exit channels of the reaction, whereas the (t, p) reaction does not. The inelastically scattered triton thus probes only a small region of the nuclear surface and only a few partial waves enter into the reaction. This results in a relative insensitivity to the optical-model parameters which describe the elastic scattering as long as they indeed do describe it. It is this quality which tends to make the interpretation of inelastic scattering of tritons somewhat less ambiguous than inelastic nucleon scattering. For consistency with Ref. 1, the parameters used throughout this study will be those given in Table I.

IV. INELASTIC SCATTERING THEORY

The most successful procedure for interpreting inelastic scattering data in the energy range of interest here is the DW approximation. In particular, for the case of strongly absorbed particles, the major features of the predicted angular distribution are determined by the description given to the entrance and exit channels, i.e., the optical-model parameters, and the transferred angular momentum. The predicted magnitude of the cross section then depends on the nuclear form factor chosen. The transition amplitude for the DW approximation is written as

$$T_{if}=\int d\mathbf{r}\, \chi_f^{(-)*}(\mathbf{k}_f,\ \mathbf{r}_f)\langle\phi_f \mid v \mid \phi_i\rangle\chi_i^{(+)}(\mathbf{k}_i,\ \mathbf{r}_i), \quad (4)$$

where the $\chi(\mathbf{k}, \mathbf{r})$ are the distorted waves determined by the elastic scattering potential. Here, as usual, an adiabatic assumption is made so that the same parameters describe both χ_f and χ_i.

As in Ref. 1, two approaches are used for the nuclear form factor $\langle\phi_f \mid v \mid \phi_i\rangle$ describing the transition between the initial nuclear state ϕ_i and the final nuclear state ϕ_f brought about by the interaction v. These are the collective-model and the shell-model types of form factors.

A. Collective Model

In its usual form for DW calculations, the form factor for the vibrational excitation of a single phonon in the nucleus is given by the first term in a Taylor expansion of the optical potential about the equilibrium radius. The expression is thus[14]

$$\langle\phi_f \mid v \mid \phi_i\rangle=\beta_L\{R_r[dV(r)/dr] + iR_i[dW(r)/dr]\}i^L Y_L^M, \quad (5)$$

where V and W are the optical-model potentials defined above and β_L is the deformation parameter for an excitation of multipolarity L. The importance of deforming both real and imaginary wells for strongly absorbed particles has been pointed out previously for ^{3}He particles[15] and tritons.[1]

B. Shell Model

The shell-model form factors used in the present analyses were generated by the code ATHENA of Johnson and Owen[16] and are described in Refs. 2 and 3. The cross section expected from this calculation is given by the equation

$$d\sigma/d\Omega=(2J+1)(V_\alpha+V_\beta\tau_i\cdot\tau_t)^2M_L^2\sigma_L(\theta), \quad (6)$$

where the reduced angular matrix elements M_L are defined in Ref. 3. The effective interaction strengths are given by V_α and V_β, the latter giving the isospin contribution with a sign given by the proper choice of isospins τ_i and τ_t. The DW cross section is represented here by $\sigma_L(\theta)$ and is of the form (4) with a radial form factor given by[16]

$$I_L(r)=\int U_2(r_i)U_1(r_i)(\alpha r_{it})^{-1}\exp(-\alpha r_{it})r_i^2 dr_i. \quad (7)$$

Here the U's are the nuclear radial wave functions for the active particles, $r_{it}=\mid \mathbf{r}_i-\mathbf{r}_t \mid$, the radius between the active nucleon in the target and the triton, and α is the range parameter of the Yukawa interaction. Reference 1 investigated the form of the interaction and found that the simple Yukawa with $\alpha=1.0$ F^{-1} yielded the best description of the observed differential cross sections. It was also found that spin-flip terms were most likely less than 10% of the nonspin-flip strength and these have been ignored in the present analysis.

A difficulty in all of these analyses in the zirconium region is the mixed proton configurations of the ground state. The principal components of this mixture are generally considered as

$$\mid i\rangle=a\mid p_{1/2}{}^2\rangle+b\mid g_{9/2}{}^2\rangle, \quad (8)$$

where $\mid i\rangle$ is the initial ground state. The final states are considered pure, and for simplicity the neutron final states are considered to depend only on a. The values of a and b are expected to vary, however, as the number of neutrons is increased. Preedom *et al.*[17] have

[13] A. G. Blair, J. G. Beery, and E. R. Flynn, Phys. Rev. Letters **22**, 470 (1969).

[14] R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).

[15] E. R. Flynn and R. H. Bassel, Phys. Rev. Letters **15**, 168 (1965).

[16] M. B. Johnson and L. W. Owen (unpublished).

[17] B. M. Preedom, E. Newman, and J. C. Hiebert, Phys. Rev. **166**, 1156 (1968).

TABLE II. Deformation parameters and single-particle estimates for the lowest 2^+ and 3^- states of the zirconium isotopes.

Isotope	E_x (MeV) 2^+	E_x (MeV) 3^-	β_L 2^+	β_L 3^-	$B(E,L\to 0)/B_{\mathrm{s.p.}}$ 2^+	$B(E,L\to 0)/B_{\mathrm{s.p.}}$ 3^-
90	2.18	2.75	0.07	0.12	3	9
92	0.93	2.34	0.11	0.14	8	13
94	0.92	2.05	0.08	0.15	4	15
96	1.74	1.89	0.06	0.16	2	17
98	1.74	1.81	···	···	···	···

measured these values by means of the (d, ^{3}He) reaction, and these values are used in the present work. The values of a they obtain are given in the captions of the tables. For ^{92,94}Zr, these must be considered as uncertain to between 10–20%. For ^{96}Zr, the value of a is even less certain since in the work of Ref. 17 no $g_{9/2}$ pickup strength was seen, and the result was inferred from the amount of $p_{1/2}$ strength present.

V. COLLECTIVE MODEL

The lowest-lying 2^+ states in the three zirconium isotopes considered here are shown in Fig. 3 where they are compared with DW calculations. In Fig. 4, the lowest 3^- states are shown. Table II summarizes the values of β_2 and β_3 and their corresponding single-particle estimates obtained for all of the zirconium isotopes including the results from Ref. 1 and the positions of the 2^+ and 3^- for ^{98}Zr from Ref. 13. These

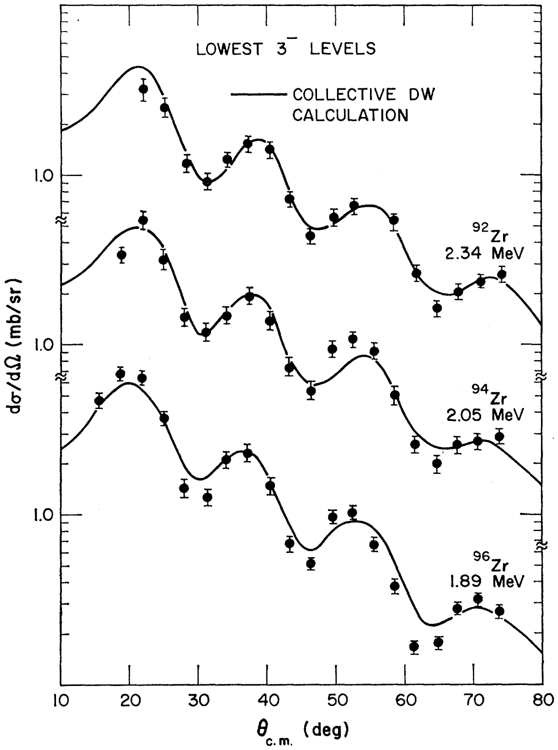

FIG. 4. Lowest 3^- states in ^{92,94,96}Zr compared to the DW calculation using a collective form factor.

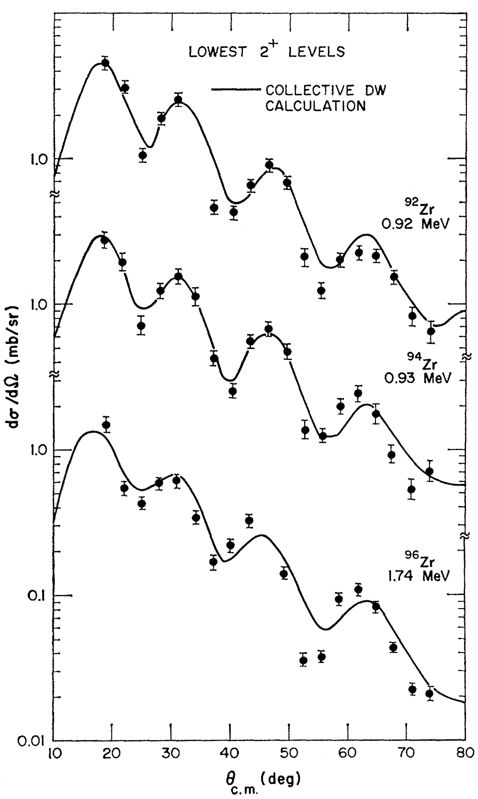

FIG. 3. Lowest 2^+ states in ^{92,94,96}Zr compared to the DW calculation using a collective form factor.

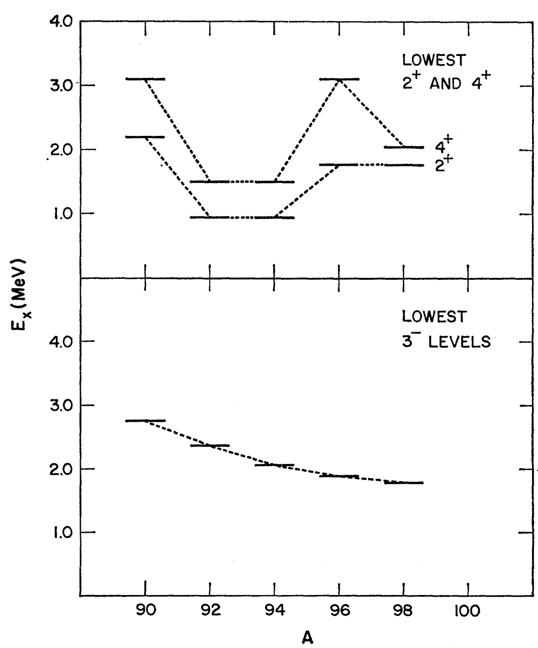

FIG. 5. Excitation energy of the lowest 2^+, 3^-, and 4^+ states versus mass number for the even isotopes of Zr.

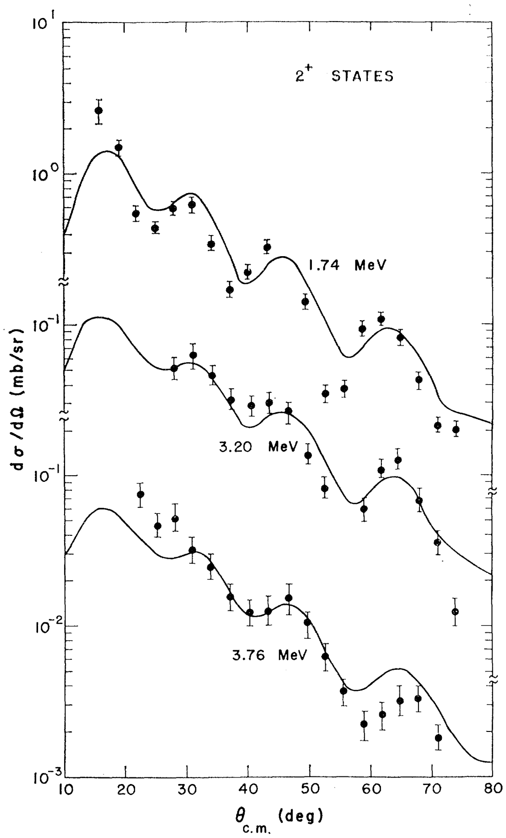

FIG. 6. Comparison of the DW calculations for a collective-model form factor to the data for $l=2$ assignments in ^{96}Zr.

calculations were done with the Oak Ridge code JULIE[18] and the code DWUCK from the University of Colorado.[19] Corrections were made for Coulomb excitation in the manner described in Ref. 14.

The excitation energies of the lowest 2^+, 4^+, and 3^- states are shown in Fig. 5 as a function of mass number for all of the zirconium isotopes. It is obvious from the behavior of the 2^+ and 4^+ states that the shell configuration peculiar to each isotope is having a profound effect upon the microscopic structure of these states. The addition of particles into the $2d_{5/2}$ shell which begins at ^{92}Zr for the cases considered here produces a marked lowering of these states by over 1 MeV and also an increase in the value of β_2 as seen in Table II. When the $2d_{5/2}$ shell is filled at ^{96}Zr and can then no longer couple to 2^+ or 4^+ states because of the Pauli principle, the levels are raised by approximately $\frac{3}{4}$ MeV. This behavior indicates that the major configuration of these levels is probably $2d_{5/2}{}^2$ for the ^{92,94}Zr cases, and suggests that a shell-model form factor of rather simple configuration should be meaningful. This approach is discussed below. On the other hand, the octupole state is behaving quite smoothly as a function of mass number as is also the deformation parameter β_3. These characteristics are typical of a strong collective state. This approach of the 3^- level to the ground state along with the corresponding increase in β_3 shown in Table II indicates an increasing number of particle-hole components as more neutrons are added. Shell-model calculations for such states must necessarily involve a large number of particle-hole configurations to be correct.

In the case of ^{96}Zr, only a few l values are known and to establish further the level scheme a more exhaustive DW treatment was made. The results of the comparison of DW calculations to the data are shown in Figs. 6–10. A summary of all of the β_l's obtained for ^{96}Zr is given in Table III and the proposed level scheme in Fig. 11, where it is compared with previous results. The levels at 2.33 and 2.75 MeV seen in the (p, p') data would appear to be contaminant levels from other Zr isotopes in the target.[20] These would be the strongest contaminant levels since they are the 3^- states (see Table II). In addition, the state seen at 2.03 MeV in the (d, d') experiment could also belong to this cate-

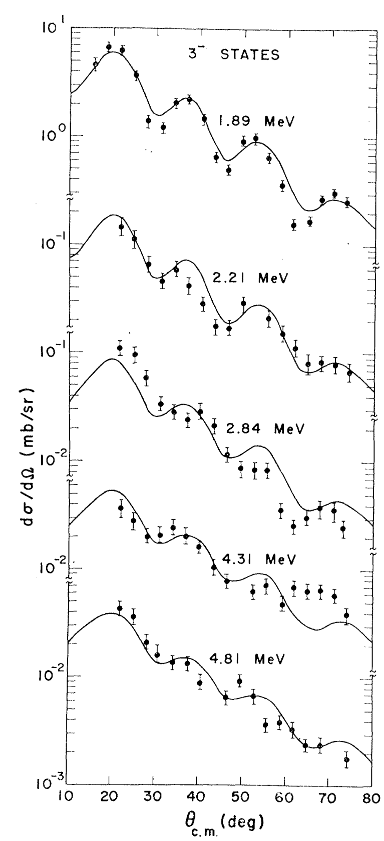

FIG. 7. Comparison of the DW calculations for a collective-model form factor to the data for $l=3$ assignments in ^{96}Zr.

[18] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240 (unpublished); R. M. Drisko (private communication).

[19] Distorted-wave program of P. D. Kunz, University of Colorado.

[20] B. W. Ridley (private communication) confirms this.

gory since subtraction of the 3^- impurity of ^{94}Zr in the present experiment revealed no remaining strength at this energy position.

An extensive search was made for a low-lying 0^+ state in ^{96}Zr. Such a state has been observed in all of the other Zr isotopes, and for these cases is considered to be primarily a proton rearrangement state. However, the state is observed weakly in all of the zirconium inelastic triton spectra. Unfortunately, because of the necessity of subtracting levels from other zirconium

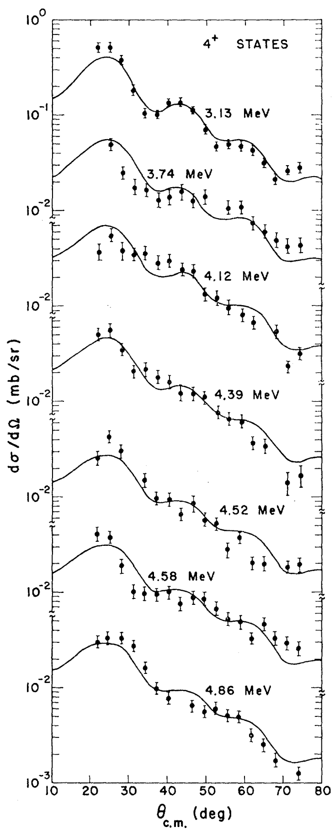

FIG. 8. Comparison of the DW calculations for a collective-model form factor to the data for $l=4$ assignments in ^{96}Zr.

isotopes (such as the 1.49-MeV level of ^{92}Zr and 1.47-MeV level of ^{94}Zr), the ability to observe a small level in this energy region is reduced. The 0^+ level noted in Fig. 11 as seen in the (t, p) reaction[6] at 1.59 MeV appears to be of different character than the low-lying 0^+ states in the other zirconium isotopes although it may be degenerate with such a level. The intensity of this level in the two-nucleon transfer reaction indicates it is principally a 2-particle–2-hole state whose principal configuration should be of the type $(2d_{5/2})_0^{-2}(3s_{1/2})_0^2$. Such a state could not be directly populated in an inelastic reaction.

The level schemes for ^{92}Zr and ^{94}Zr are given in Fig. 12. This should not be considered as an exhaustive

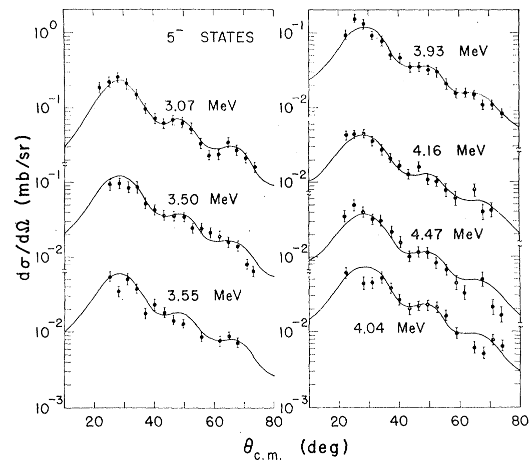

FIG. 9. Comparison of the DW calculations for a collective-model form factor to the data for $l=5$ assignments in ^{96}Zr.

study of these nuclei, since only the principal low-lying levels are included. The l assignments are made on the basis of DW calculations using a collective form factor. Several assignments differ from those suggested by the work of Stautberg *et al.*[5] It was pointed out in Ref. 1 that differences between proton and triton inelastic

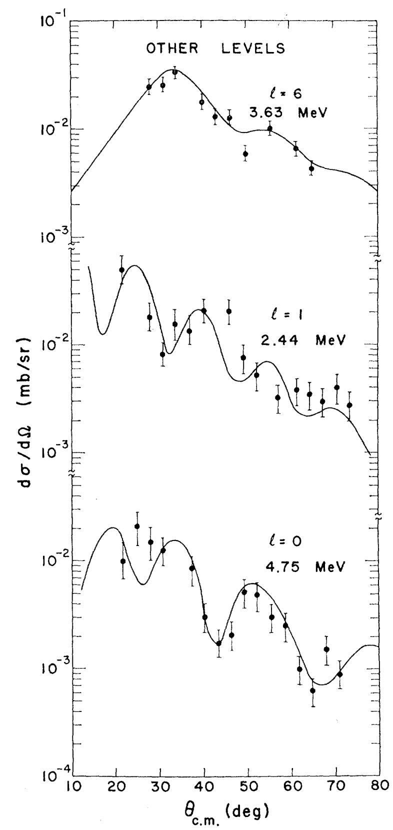

FIG. 10. Comparison of the DW calculations for a collective-model form factor to the data for other possible l assignments in ^{96}Zr.

TABLE III. Deformation parameters and angular momentum values for the levels of ^{96}Zr measured in the present experiment.

E_x (MeV)	L	βR_i (F)	β
1.74	2	0.382	0.060
1.89	3	1.050	0.165
2.21	(3)	0.184	0.029
2.44	(1)	0.102	0.016
2.84	3	0.133	0.021
3.07	5	0.293	0.046
3.13	4	0.344	0.054
3.20	2	0.140	0.022
3.50	5	0.223	0.035
3.55	5	0.153	0.024
3.63	(6)	0.127	0.020
3.74	4	0.134	0.021
3.76	2	0.108	0.017
3.93	5	0.223	0.035
4.04	(5)	0.178	0.028
4.12	(4)	0.146	0.023
4.16	5	0.134	0.021
4.31	(3)	0.121	0.019
4.39	4	0.121	0.019
4.47	5	0.127	0.020
4.52	(4)	0.108	0.017
4.58	4	0.102	0.016
4.81	3	0.108	0.017
4.86	4	0.102	0.016

scattering occur, with certain levels having relatively greater cross sections for triton scattering than for proton scattering. This difference permits a less ambiguous choice for spin assignments for some levels excited by inelastic triton scattering.

VI. SHELL-MODEL ANALYSIS OF THE DATA

As discussed previously, particles in the $2d_{5/2}$ neutron shell should produce low-lying 2^+ and 4^+ states in both ^{92}Zr and ^{94}Zr. The 2^+ level at slightly over 900 keV in both isotopes is likely to be predominantly of this configuration. Figure 5 supports this conclusion because of the large energy shift in the lowest 2^+ state observed when this shell is expected to be empty at ^{90}Zr and when it is full (thus coupling only to 0^+) at ^{96}Zr. DW calculations using the form factor given in Eq. (7) and considering a recoupling of the $(2d_{5/2})_0{}^2$ configuration of the ground state to a $(2d_{5/2})_2{}^2$ configuration were performed and the results shown in Fig. 13 for both isotopes. The effective interaction strengths, $V_0 = V_\alpha - V_\beta$, for excitation of a neutron state, are shown in Table IV. These values must be considered to be an upper limit. Although this configuration is certainly the dominant one for these 2^+ states, a certain amount of collective strength is to be expected from the quadrupole interaction. Indeed the lowering of the 2^+ position in these isotopes as noted in Fig. 5 should enhance such admixtures due to an increased amount of correlations. This is also reflected in the increased amount of single-particle strength as shown in Table II. The lowest 2^+ level in ^{90}Zr showed a substantial enhancement in the value of V_0 over that obtained for the other proton levels. A factor of 1.9 was observed in the ratio of the assumed $(\pi g_{9/2})_2{}^2$ state to the average proton strength which would infer that the true interaction strength for recoupling $(2d_{5/2})_0{}^2$ neutrons to an angular momentum of 2 is a factor of 2 less than that given in Table IV.

On the other hand, the 4^+ levels observed near 1.5 MeV in excitation may be expected to be of relatively pure $(2d_{5/2})_4{}^2$ configuration. There are no low-lying 4^+ states in either ^{90}Zr or ^{96}Zr and this would tend to substantiate this configuration assignment since this shell is expected to be empty in ^{90}Zr and full in ^{96}Zr. Thus, these levels might be expected to give the most accurate estimate of V_0 for neutrons in this region. The DW calculations for each of these levels is shown in Fig. 13 and the values of V_0 are given in Table IV as

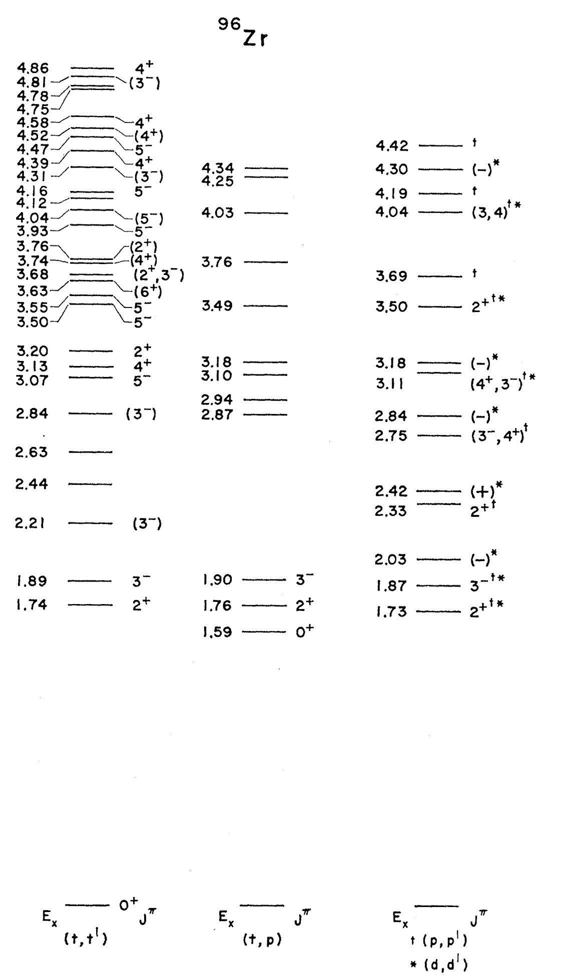

FIG. 11. Level diagram for ^{96}Zr. The assigned values of J^π assume no possible spin flip. The (t, p) results are from Ref. 6, and the (p, p') and the (d, d') from Ref. 7.

320 MeV for ^{92}Zr and 290 MeV for ^{94}Zr. As expected, they are approximately one-half of the value obtained for the 2^+ states.

It was pointed out in Ref. 7 that an interesting relationship exists between the angular matrix elements for particle-hole states in the various zirconium isotopes based on promoting a $d_{5/2}$ particle to one of the empty shells above. The situation is similar to the spectroscopic factors observed in pickup reaction where these increase as the number of available particles increase. Thus, for the present case (assuming that the orbits above the $d_{5/2}$ shell are always empty), the expected cross sections to states of $(2d_{5/2}^{-1}nl_j)$ character are in the ratios 1:2:3 for ^{92}Zr, ^{94}Zr, and ^{96}Zr, respectively. The amplitudes for these levels are given by the M_l's of Eq. (6) and defined in Ref. 3. Such states should be the lowest neutron levels of ^{96}Zr and easily identified because of the comparatively large cross section suggested by the above ratio. The $(d_{5/2}^{-1}s_{1/2})_2$ configuration should be the lowest of these, based on the suggested neutron single-particle scheme of Cohen.[21] The level most likely to be of this type is the state at 1.74 MeV in ^{96}Zr which is at the correct energy for the $d_{5/2}$-$s_{1/2}$ separation.[21] The DW calculation for this assumption is shown in Fig. 14. The effective interaction value of $V_0=242$ MeV is reasonably consistent with the values obtained for the $(2d_{5/2})_4{}^2$ states as shown in Table IV. States of this same character are more difficult to find with much certainty in ^{92,94}Zr;

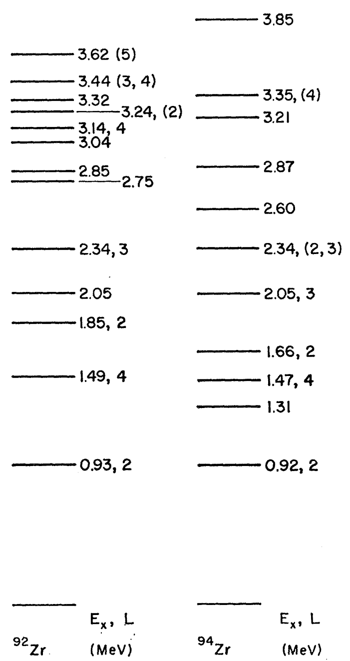

FIG. 12. Low-lying levels in ^{92,94}Zr. The region above 4.75 MeV in excitation was not examined. See also Ref. 5. The L values given here are from the present analysis.

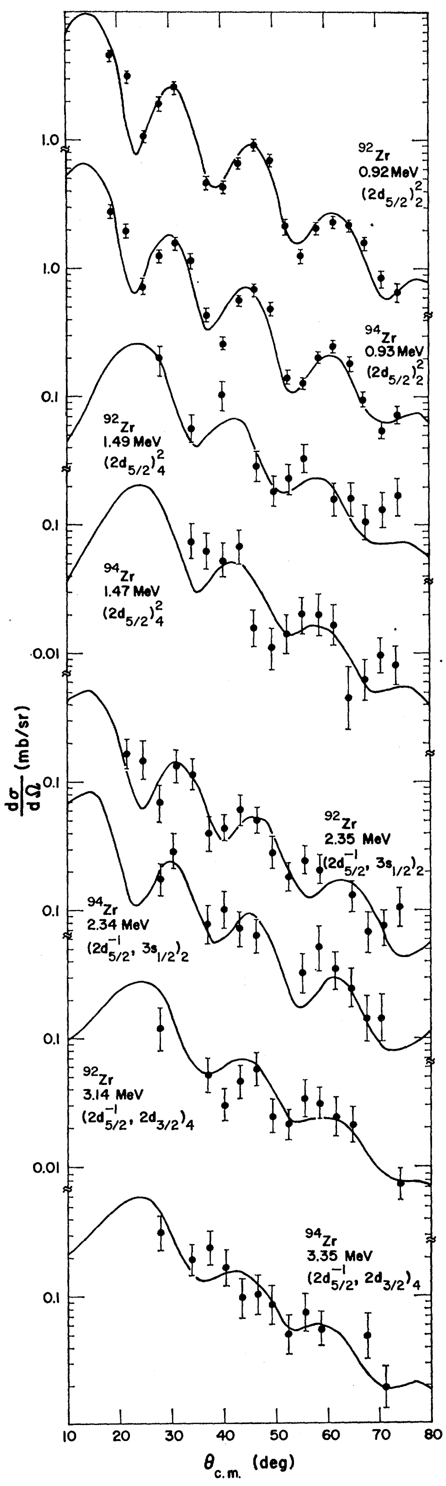

FIG. 13. Distorted-wave calculations for a microscopic-model form factor for the possible shell-model configurations of ^{92}Zr and ^{94}Zr.

[21] B. L. Cohen, Phys. Rev. **130**, 227 (1963).

TABLE IV. Effective interaction strengths for the low-lying states of ^{92}Zr, ^{94}Zr, and ^{96}Zr. Here $V_0 = V_\alpha + (\tau_i \cdot \tau_t) V_\beta$ and $|a_{92}| = 0.74$, $|a_{94}| = 0.81$, and $|a_{96}| = 0.93$.

Configuration	^{92}Zr E_x (MeV)	^{92}Zr V_0 (MeV)	^{94}Zr E_x (MeV)	^{94}Zr V_0 (MeV)	^{96}Zr E_x (MeV)	^{96}Zr V_0 (MeV)
$(2d_{5/2})_2{}^2$	0.93	700	0.92	620	...	...
$(2d_{5/2})_4{}^2$	1.49	320	1.47	290	...	...
$(2d_{5/2}{}^{-1}3s_{1/2})_2$	2.85	220	2.34	210	1.74	240
$(2d_{5/2}{}^{-1}2d_{3/2})_2$	...	...	...	...	3.20	190
$(2d_{5/2}{}^{-1}2d_{3/2})_4$	3.14	290	3.35	350	3.13	220
$(2d_{5/2}{}^{-1}1g_{7/2})_2$	...	...	...	...	3.76	302
$(2d_{5/2}{}^{-1}1g_{7/2})_4$	...	...	...	...	3.74	212
$(2d_{5/2}{}^{-1}1g_{7/2})_6$	...	...	...	...	3.63	150
$(\pi g_{9/2})_2{}^2$	1.85	710	1.66	760	...	...

however, likely candidates are the 2^+ level at 2.83 MeV in ^{92}Zr and at 2.34 MeV in ^{94}Zr. These assignments differ from that of Ref. 5. However, they were most representative of the expected value of V_0 and in the region of excitation expected from theoretical calculations.[22] The values of 220 and 210 MeV shown in Table IV are certainly in line with the value of 242 MeV found for ^{96}Zr and show agreement with the cross-section ratios of 1:2:3 for the three isotopes.

Somewhat higher in excitation should lie levels based on the promotion of a $2d_{5/2}$ particle to a $2d_{3/2}$ orbital, i.e., $(2d_{5/2}{}^{-1}, 2d_{3/2})_J$ states. Assignment of such levels for $J=2$ in ^{92}Zr and ^{94}Zr is difficult because of the multitude of 2^+ states in this region. However, the level at 3.20 MeV in ^{96}Zr is a strong candidate for this configuration and the resulting value of $V_0 = 190$ MeV is in support of this supposition. The fit to this level is also shown in Fig. 14. The $J=4$ levels should be somewhat more certain, however, since they are less common. The lowest $L=4$ transitions seen in these three isotopes are the 3.14-MeV states in ^{92}Zr, the 3.35-MeV level in ^{94}Zr, and the 3.13-MeV state in ^{96}Zr. The values of V_0 assuming the $(d_{5/2}{}^{-1}, d_{3/2})_4$ configuration are, respectively, 290, 350, and 220 MeV. These are all of the right order found for the $(2d_{5/2})_4{}^2$ states and all three are at approximately the same excitation energy supporting the assigned configuration. The 3.13-MeV state has a rather large cross section and its $L=4$ assignment was suggested by Stautberg *et al.*[7] on this basis. They were unable to resolve this state and the nearby 5^- at 3.07 MeV; however, the present work supports their tentative 4^+ assignment. The reasonable behavior of the effective interaction strength V_0 for these three levels is encouraging in view of the factor of 3 between their cross sections.

The closure of the $2d_{5/2}$ shell at ^{96}Zr simplifies the low portion of this isotope's spectrum as shown in Fig. 11. For this reason it is possible to consider neutron levels based on raising a $2d_{5/2}$ neutron to a $1g_{7/2}$ orbital giving states of $J=2$, 4, and 6. Possible levels corresponding to this configuration are the $L=2$ state at 3.76 MeV, the $L=4$ state at 3.74 MeV, and the $L=6$ state at 3.63 MeV which give values of $V_0=302$, 212, and 150 MeV, respectively. These again are approximately of the values seen for the $(2d_{5/2})_4{}^2$ levels discussed above.

A number of levels belonging to proton excitation are to be expected in this energy range examined here. The most prominent of these should be those based on the $(g_{9/2})_J{}^2$ and the $(p_{1/2}g_{9/2})_5$ configurations. These are discussed at length for triton inelastic scattering from ^{90}Zr in Ref. 1. The $(\pi g_{9/2})_2{}^2$ states can possibly be assigned to the 1.85-MeV level of ^{92}Zr and the 1.66-MeV level of ^{94}Zr which yield V_0 strengths of 710 and 760 MeV, respectively, when DW calculations are performed for this assumption. Since the observed cross section depends directly on the value of b in Eq. (8), this level would be smaller in ^{96}Zr based on Preedom's[17] measurement of the amount of $(g_{9/2})_0{}^2$ strength in the ground state or ^{96}Zr. The position of the $(g_{9/2})_2{}^2$ level would also vary depending on this value. No definite assignment is thus made in ^{96}Zr for this strength. Certainly, many of the $l=3$ and 5 states seen in ^{96}Zr must be due to proton particle-hole pairs involving the almost empty shell of the $g_{9/2}$ orbital and the filled shells of the $f_{5/2}$ and $p_{3/2}$. Such configurations would produce primarily odd L states.

VII. DISCUSSION

A number of levels of the three isotopes of zirconium containing $2d_{5/2}$ neutrons have been interpreted as being principally due to excitations involving these particles. With the exception of the lowest 2^+ levels of ^{92}Zr and ^{94}Zr a reasonably consistent value of the effective interaction V_0 for exciting neutron states was obtained. The two exceptions can easily be explained by the addi-

[22] H. J. Martin, Jr., M. B. Sampson, and R. L. Preston, Phys. Rev. **135**, 942 (1963).

tion of quadrupole components in their wave function due to their relatively low excitation energy and the corresponding increase in ground-state correlations. These results are summarized in Table IV along with several suspected proton states belonging to the $(1g_{9/2})_J^2$ configuration.

The average strength of the assumed neutron levels in ^{90}Zr as given in Ref. 1 was 240 MeV. The average of all of the values given in Table IV (with the exception of the $(2d_{5/2})_2{}^2$ levels) is 250 MeV, agreeing quite closely with the previous result. Even the enhancement noted in the $(2d_{5/2})_2{}^2$ levels from this value is comparable to that observed for the lowest 2^+ state in ^{90}Zr which was believed to be principally a proton excitation state. Thus, by this type of microscopic shell-model analysis, the effective interaction strength for exciting neutron states would appear to be of the order 240–250 MeV. This is to be compared to the

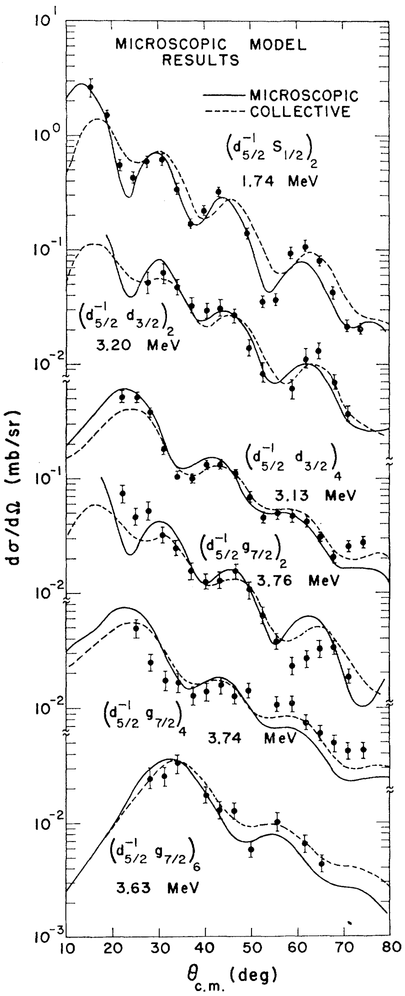

FIG. 14. Distorted-wave calculations for a microscopic-model form factor for the possible shell-model configurations of ^{96}Zr states.

average of 660 MeV for exciting proton states. Expanding the expression for V_0 into the form in which it occurs in Eq. (6), namely, $V_0=V_\alpha+V_\beta(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_t)$, permits the evaluation of V_α and V_β. By using the value of $V_0=660$ MeV for $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_t=-1$ and the present value of 250 MeV for $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_t=+1$, the expression $V_0=455$-$205(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_t)$ is obtained. This result is rather surprising in view of the fact that a similar experiment using protons as a projectile found no isospin dependence.[5] However, it has been noted that an apparently strong isospin term exists in the elastic scattering[12] of tritons which appears to be three times the constant term, whereas in proton scattering the terms are of equal magnitude. This isospin term was contained in the imaginary optical-model potential and thus would be reflected in the reaction channels. If in the proton case the isospin term is of one-third the strength, it would be difficult to observe. Another explanation involves the strong absorption which is characteristic of the triton and makes it more sensitive to the tail of the wave function. If the tail of the bound-state wave function is incorrect, the effect of this would be emphasized in the calculations for triton inelastic scattering over proton inelastic scattering. This latter question can only be answered by further checking using alternative shell-model form factor calculations and more complete wave functions.

The DW calculations have been used to extend the spectroscopic information on ^{96}Zr considerably. The level scheme shown in Fig. 11 contains a number of new levels and many new spin assignments. Also a few areas of disagreement have been removed in the existing data. The lowest levels in this spectrum have been interpreted as being principally neutron excited states as shown in Table IV and these results indicate rather simple configurations. The complete absence of a low-lying 0^+ state suggests that the level seen in Ref. 6 at 1.59 MeV is primarily a 2-particle–2-hole state referring to ^{96}Zr as the core.

Finally, the systematic behavior of the lowest 3^- states shown in Fig. 5 is taken as evidence for this state being a highly collective octupole state which is only slightly perturbed by the addition of more neutrons. On the other hand, the large jump in excitation energy experienced by the lowest 2^+ states as new neutron orbitals are being filled indicates that these levels are not typical of collective quadrupole states but are dominated by one configuration.

ACKNOWLEDGMENTS

We are grateful to the crew of the Los Alamos three-stage Van de Graaff for providing the excellent triton beam. The help of J. Levin and M. Kellogg in the use of the computer program is also gratefully acknowledged. The advice and encouragement of A. G. Blair are appreciated.