can be concluded from the correlation coefficient $A_2(303e_K^- \text{-}81\gamma) = +0.003 \pm 0.008$.

In addition, the conversion coefficients of the L_{I}, L_{II}, L_{III}, and the M_{I} shell of the 81-keV transition were determined by directly measuring $\alpha_{L+M+N}(81)$ in a coincidence experiment and comparing the result to the measured relative subshell intensities of Siegbahn *et al.*[22] These coefficients are in good agreement with the calculated subshell coefficients of Pauli.[3]

It is interesting to recall that one of the possible causes of enhancement of the penetration parameter λ is a strong retardation of the $M1$ γ-ray transition. We have reevaluated the retardation factors for the 81- and 161-keV transitions using the present values: $\delta(81) \simeq -0.152$, $\alpha_K \simeq 1.46$, $\delta(161) \simeq 0.58$, and $\alpha_K(161) \simeq 0.21$. The results are $R(M1, 81) \simeq 420$ and $R(M1, 161) \simeq 230$. The retardation factor $R(M1, 303) \simeq 97$ is given in Ref. 7. It is clear that in this case, some process other than retardation of the $M1$ γ transition plays a dominant role in the penetration effect, causing $\lambda(161)$ to be significantly larger than $\lambda(81)$. The authors suggest that this case could be useful in testing the accuracy of nuclear models in the nondeformed nuclide region and that a theoretical calculation of all three quantities $\lambda(81)$, $\lambda(161)$, and $\lambda(303)$ might be very helpful in furthering the understanding of penetration effects and the mechanism of their enhancement.

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Beta-Gamma Angular Correlations and Shape-Factor Measurements in ^{186}Re and ^{188}Re

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The A_2 coefficients in the directional correlation between the nonunique first forbidden 0.935-MeV β group and the 0.137-MeV γ ray in the decay of ^{186}Re, and in the directional correlation between the nonunique first forbidden 1.980-MeV β group and the 0.155-MeV γ ray in the decay of ^{188}Re, have been measured as a function of energy. An investigation of perturbation caused by a magnetic hfs-type interaction yielded a negative result. Both of these β spectra were found to have the "allowed" shape over the region examined.

I. INTRODUCTION

The β-γ angular correlation coefficients for the 0.935-MeV β, 0.137-MeV γ cascade in ^{186}Re and the 1.980-MeV β, 0.155-MeV γ cascade in ^{188}Re have been previously measured by a number of authors.

In particular, the A_2 coefficients in ^{186}Re have been measured with good statistical accuracy by Novey *et al.*[1] The 1.980-MeV β, 0.155-MeV γ directional correlation coefficients in ^{188}Re have been determined by Wyley *et al.*,[2] but their results are not very useful for theoretical calculations because of the relatively large statistical errors.

The question of attenuation of the β-γ angular correlation pattern, because of the half-life ($\sim$0.5 nsec) of the intermediate state of ^{186}Os has been raised by Novey.[1] If the value of A_2 is significantly attenuated in the ^{186}Re correlation, it will also be attenuated in the ^{188}Re case. There has been no further mention of this in the literature, and it is still an open question.

Different results have been reported by a number of authors for the β-spectrum shape factors for these decays. Porter *et al.*[3] reported an energy-dependent shape factor for the 0.935-MeV ^{186}Re β spectrum. Koertz[4] and also Bashandy[5] reported an allowed shape but neither author indicated the accuracy with which the energy dependence of the shape factor may be deduced. Johns *et al.*[6] also reported an allowed shape for this

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[1] T. B. Novey, M. S. Freedman, F. T. Porter, and F. Wagner, Jr., Phys. Rev. **103**, 942 (1956).

[2] L. D. Wyley, C. H. Braden, and H. Dulaney, Phys. Rev. **129**, 315 (1963).

[3] F. T. Porter, M. S. Freedman, T. B. Novey, and F. Wagner Jr., Phys. Rev. **103**, 921 (1956).

[4] L. Koertz, Phys. Rev. **95**, 1358 (1954).

[5] E. Bashandy and M. S. El-Nesr, Nuovo Cimento **24**, 1169 (1963).

[6] M. W. Johns, C. C. McMullen, I. R. Williams, and S. V. Nablo, Can. J. Phys. **34**, 69 (1956).

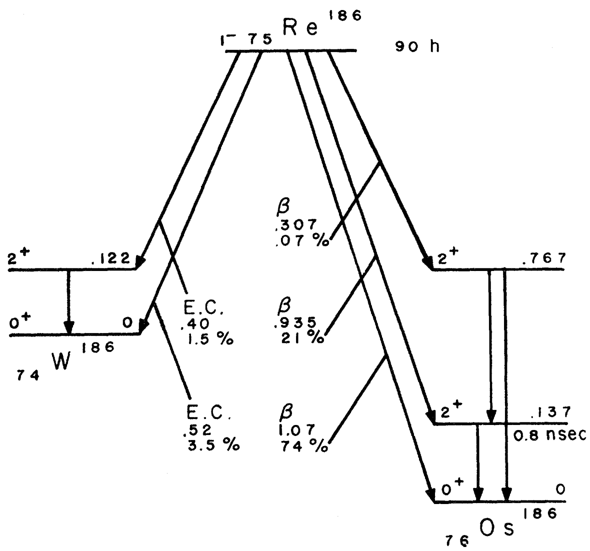

FIG. 1. The decay of ^{186}Re.

spectrum, but their method involved subtracting out the 0.935-MeV β spectrum from the composite β spectrum. This method is not as precise as the coincidence method. In particular, the shape of the β spectrum from the decay to the ground state must be known. Andre and Liaud[7] have reported an energy-dependent shape factor.

Conflicting results have also been obtained for the 1.980-MeV β spectrum in ^{188}Re. According to Johns *et al.*[6] the spectrum exhibits deviations from the allowed shape. Bashandy *et al.*[5] reported an allowed shape for the 1.980-MeV β spectrum but again they did not indicate the accuracy with which the energy dependence of the shape factor may be deduced. Andre and Liaud[7] reported an energy-dependent shape factor for this decay also. It is important that the correlation coefficients and the energy dependence of the shape factor be known to an accuracy of a few percent to provide a meaningful test of nuclear models. Also, since in these decays only the nuclear matrix elements of ranks 1 and 2 appear, there exists the opportunity of obtaining a good value for the ratio y/x and so providing a test for the conserved vector current (CVC) theory.[8]

II. APPARATUS

The apparatus used for the β-γ angular-correlation measurements, consisting of a Gerholm lens[9] and two γ detectors was described previously.[10] A digital spectrum stabilizer was added to the equipment to stabilize the gains of the two γ detectors. The long term drift in gain of the γ detector system was within $\pm 0.1\%$. This stabilizer was used in all measurements.

The data were punched out on paper tape and thereafter transferred to I.B.M. cards. All analyses were subsequently carried out with the aid of digital computers.

The shape of the photopeak due to the 0.137-MeV γ ray of ^{186}Os was examined for both detectors as a function of angle. The ratios of the count rates in three energy intervals ΔE straddling the photopeak for the 90° and 180° positions for counter 1 and the 180° and 270° positions for counter 2 were determined and they were found to be constant to 0.2%. It was assumed that the shape of the 0.155-MeV photopeak in ^{188}Os was also independent of angle.

If the shape of the γ spectrum is independent of angle, the most important source of error in the determination of A_2 involves the chance rate correction and the normalization procedures.

To test this, the β-γ angular correlation in ^{60}Co was determined, with the γ detectors moved off center so as to simulate a 7% anisotropy. The resolving times were determined by obtaining the chance rates when the β pulses were delayed. The true to chance rate was 7:1. The data were corrected for chance and normalized to the single-γ-ray rates. The values of A_2 obtained were

$$A_2 = +0.005 \pm 0.007 (\gamma_1 \text{ probe}),$$

$$A_2 = -0.0003 \pm 0.006 (\gamma_2 \text{ probe}).$$

III. MEASUREMENTS

A. β-γ Angular Correlation in ^{186}Re and ^{188}Re

The decay scheme of ^{186}Re is shown in Fig. 1.[11] The directional correlation between the first forbidden

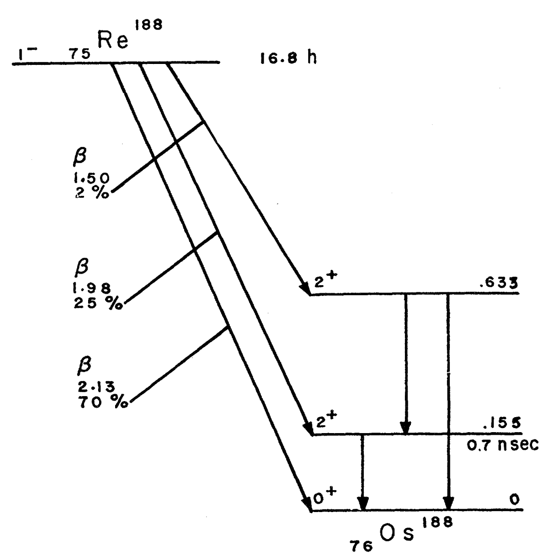

FIG. 2. The decay of ^{188}Re.

[7] S. Andre and P. Liaud, Nucl. Phys. **A121**, 337 (1968).
[8] J. I. Fujita, Phys. Rev. **126**, 202 (1962); J. Eichler, Z. Physik **171**, 463 (1963).
[9] T. R. Gerholm, R. Othaz, and M. S. El-Nesr, Arkiv Fysik **21**, 253 (1962).
[10] E. E. Habib, H. Ogata, and W. Armstrong, Can. J. Phys. **44**, 1157 (1966).
[11] Nucl. Data **B1**, No. 2 (1966).

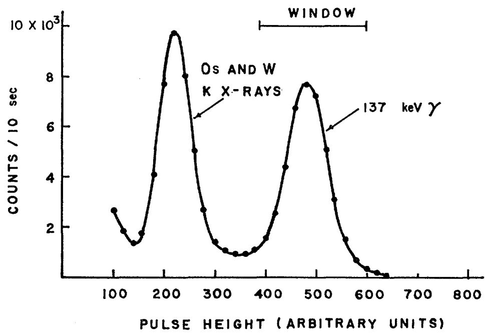

FIG. 3. The γ-ray spectrum of ^{186}Re.

0.935-MeV β group and the 0.137-MeV γ ray was measured as a function of β energy above 0.300 MeV.

The decay scheme of ^{188}Re is shown in Fig. 2.[11] The directional correlation between the 1.980-MeV β group and the 0.155-MeV γ ray was measured as a function of β energy above 1.0 MeV. For both of these cascades the spin sequence is $1^-(\beta)2^+(\gamma)0^+$. The correlation function will be of the form

$$W(\theta)=1+A_2P_2(\cos\theta)$$

if contributions from the third- and higher-order forbidden terms are neglected.

The enriched isotopes ^{185}Re ($96.7\%\,^{185}$Re$+3.3\%\,^{187}$Re) and ^{187}Re ($99.2\%\,^{187}$Re$+0.8\%\,^{185}$Re) were obtained from the Oak Ridge National Laboratory.

The samples, sealed in quartz vials, were irradiated in the reactor at McMaster University at a flux of about 10^{13} neutrons per cm^2 per second. The vials were crushed under nitric acid and the resulting solution was evaporated to dryness. The residue was deposited by vacuum evaporation on aluminum foil (0.0003 in. thick) mounted on a source ring. The sources were circular deposits 5 mm in diameter, and 20–50 μg/cm^2 thick.

Figure 3 shows the windows accepted by the γ detectors in the ^{186}Re runs. Figure 4 shows the window accepted in the ^{188}Re runs.

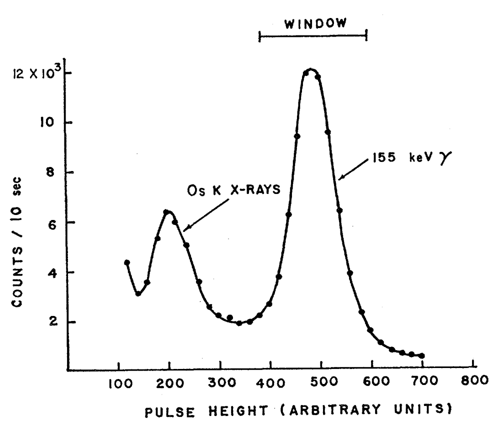

FIG. 4. The γ-ray spectrum of ^{188}Re.

TABLE I. The values of A_2 obtained for the β-γ angular correlation in ^{186}Re.

Energy (W)	A_2 γ_1 counter	A_2 γ_2 counter	A_2 average
1.60	0.038 ± 0.0054	0.045 ± 0.0052	0.042 ± 0.004
1.80	0.059 ± 0.0052	0.059 ± 0.0046	0.059 ± 0.004
2.00	0.077 ± 0.0054	0.082 ± 0.0048	0.079 ± 0.004
2.20	0.090 ± 0.0054	0.080 ± 0.0048	0.085 ± 0.004
2.40	0.098 ± 0.0060	0.099 ± 0.006	0.098 ± 0.004
2.60	0.093 ± 0.010	0.093 ± 0.009	0.093 ± 0.006

Several sources were run in each case. True and chance rates were recorded alternately. The resolving time calculated from the chance runs remained constant to within 6% over all the runs.

The positions of the counters were changed every 4–30 min, depending on the source strength. The coincidence rates for each run were corrected for chance, using the resolving times obtained before and after the run. The data were normalized to the single-channel γ rates, and the anisotropy computed for each successive pair of readings. The weighted mean values were then calculated using weighting factors equal to the reciprocal of the relative standard deviation squared. The geometrical attenuation factor for the β-ray spectrometer was determined by the method described by Gerholm[9] and the values for the NaI(Tl) scintillation spectrometers were obtained from the tables of West.[12] The correction for γ scattering was negligible. The values of A_2 obtained are shown in Tables I and II.

B. Investigation of Attenuation of A_2 Coefficients

Two attenuation mechanisms are possible. One arises from an interaction between the electric quadrupole moment of the nucleus and the electric field gradient of the lattice. This interaction is weak and it is unlikely to produce observable effects in the time interval of 0.5 nsec. The other type of attenuation arises from a hfs interaction between the magnetic dipole moment of the nucleus and the magnetic field of the extranuclear electrons. A strong field may arise as a result of a

TABLE II. The values of A_2 obtained for the β-γ angular correlation in ^{188}Re.

Energy relativistic units (W)	A_2 γ_1 counter	A_2 γ_2 counter	A_2 average
3.01	0.135 ± 0.010	0.129 ± 0.0054	0.133 ± 0.005
3.22	0.159 ± 0.0082	0.145 ± 0.0050	0.151 ± 0.004
3.42	0.159 ± 0.0011	0.162 ± 0.0072	0.161 ± 0.006
3.61	0.182 ± 0.0076	0.172 ± 0.0057	0.176 ± 0.004
3.81	0.183 ± 0.010	0.182 ± 0.0071	0.183 ± 0.006
4.01	0.203 ± 0.0071	0.199 ± 0.0063	0.201 ± 0.004
4.21	0.213 ± 0.0095	0.219 ± 0.0085	0.216 ± 0.006

[12] H. I. West, University of California Radiation Laboratory Report No. UCRL-5451, 1959 (unpublished).

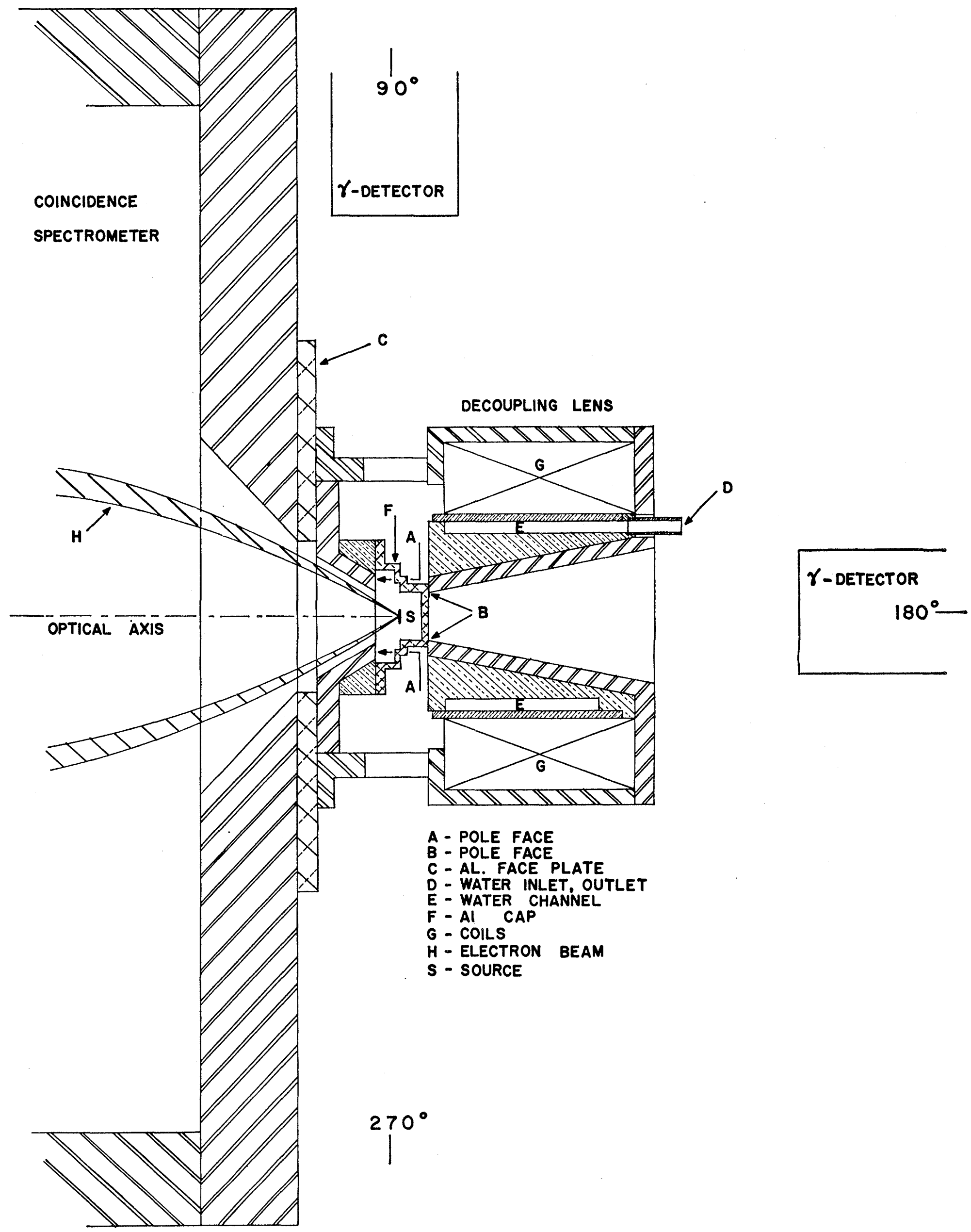

FIG. 5. The decoupling lens.

shaking off process which may occur before the daughter nucleus becomes neutral. The time taken for this process to occur, and hence the magnitude of the attenuation depends on the time taken for electrons to migrate from the source backing into the source. This depends on both the source material and the type of backing used. It is possible to decouple the nucleus from the magnetic field of the electrons by means of an external magnetic field applied in the direction of the β spectrometer axis.

A decoupling lens (Fig. 5), which produced a uniform field of about 3000 G in the region of the source, was constructed. The power was provided by a current regulated supply.

The effect of the decoupling lens field on the action of the β-ray spectrometer was investigated using a ^{137}Cs source. The shift in energy with the field on, was minor and completely negligible. The A_2 attenuation factor of the modified β-ray spectrometer was remeasured and was found to be unchanged. There was, however, γ-ray scattering from the structure which caused small errors in the absolute determination of

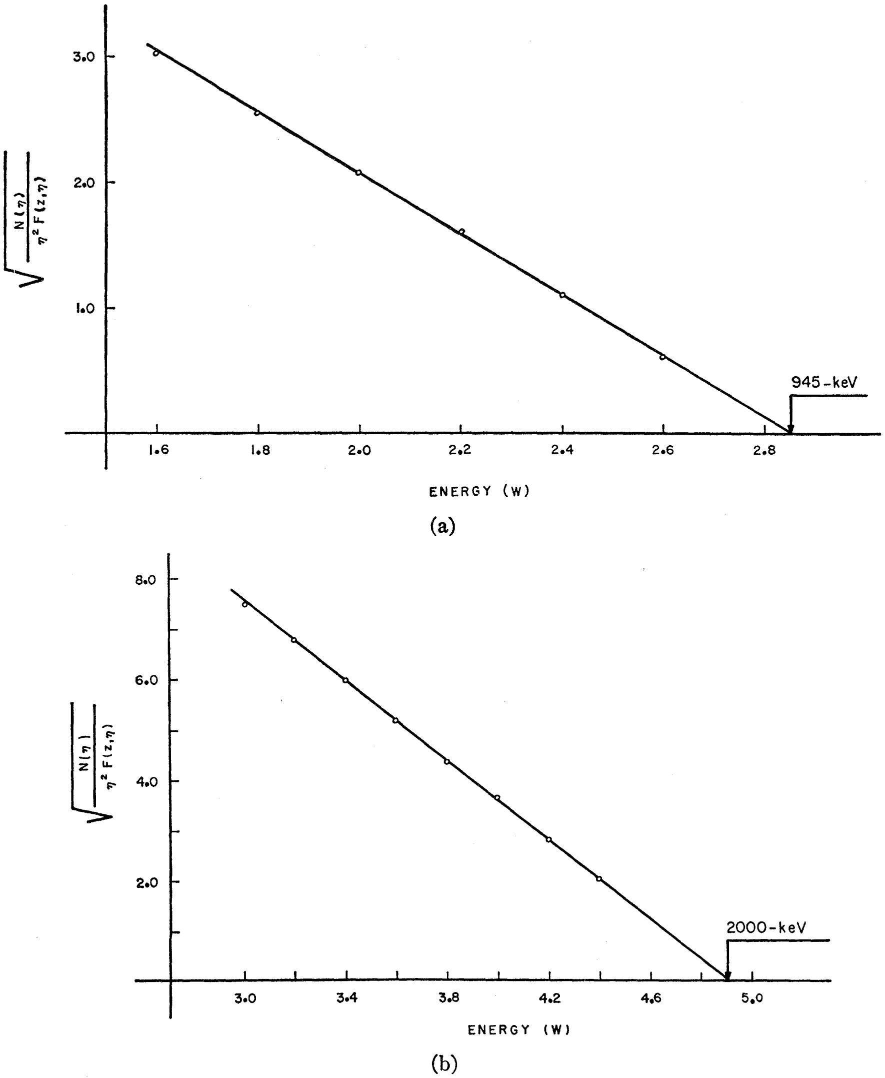

FIG. 6. (a) The Fermi plot of the β group in coincidence with the 0.137-MeV γ ray in the decay of ^{186}Re. (b) The Fermi plot of the β group in coincidence with the 0.150-MeV γ ray in the decay of ^{188}Re.

A_2, but this was unimportant. The value of A_2 was determined with the field on and off in this setup, for ^{186}Re at a β-ray energy of 0.511 MeV ($W=2.0$).

The results are

$$A_2=0.067\pm0.003, \quad \text{field on}$$

$$A_2=0.064\pm0.003, \quad \text{field off.}$$

Because of the similarity of the two decays, it was assumed that similar results would be obtained for ^{188}Re.

It was therefore concluded that any attenuation present would cause errors substantially less than the statistical errors quoted in Tables I and II.

TABLE III. A tabulation of end point energies for ^{186}Re and ^{188}Re.

^{186}Re (keV)	^{188}Re (keV)	Reference
934.3±1.3		3
927±2	1961±2	6
937±14	1982±31	5
	1998±5	13
945±5	2004±22	This investigation
927±3	1958±5	7

C. Shape Factor Determination

Smaller sources, 3 mm in diameter, were used in this determination. The resolution of the β spectrometer was set at 2% and the transmission obtained was 2% of 4π. The axes of the γ spectrometers were set at 135° and 225° with respect to the axis of the β spectrometer.

TABLE IV. Nuclear matrix elements.

	^{186}Re	^{188}Re
Y	2.70±0.10	2.65±0.10
u	0.010±0.005	0.067±0.005
x	0.145±0.015	170±0.020

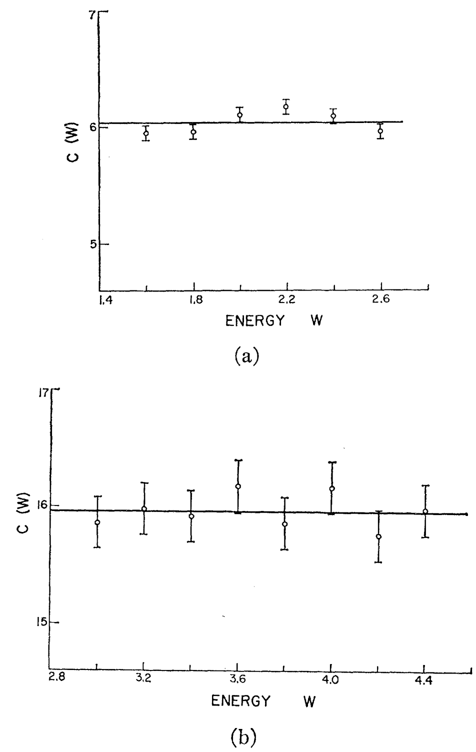

FIG. 7. (a) The shape correction factor in arbitrary units ^{186}Re. (b) The shape correction factor in arbitrary units ^{188}Re.

The β-γ coincidence rate was recorded at several β-ray energies in the region $W=1.6$–2.6 for ^{186}Re and $W=3.0$–4.4 for ^{188}Re.

The detection efficiency of the β detector varied slightly with β energy. The response of the β detector was obtained by simultaneously recording the β detector output in a kicksorter, gated by the triple-coincidence pulses. The tail of the spectrum was cut off by the single-channel analyser in the β side channel. It was assumed that this tail was flat to zero energy, and so the efficiency of the β detector was given by the ratio of the counts recorded to the total counts extrapolated to zero pulse height. The efficiency varied by about 3% over the range $W=1.6$–2.6 for ^{186}Re and 2% over the range $W=3.0$–4.4 for ^{188}Re. The efficiency of the coincidence circuit was virtually 100% at all energies.

The β-γ coincidence rate was corrected for chance, and efficiency, and normalized to the γ single rate. The drifts in the γ channels were negligible because of the action of the stabilizers. The data were also corrected for the energy dependence of the β-γ rate at 135° (and 225°). This correction was small. The Fermi plots gave straight lines with end points at 0.945 ± 0.005 MeV for ^{186}Re and 2.000 ± 0.022 MeV for ^{188}Re, respectively (Fig. 6). The published values for the end-point energies given by various authors are shown in Table III.[13]

The shape correction factor was constant within $\pm1.5\%$ over the range investigated for both decays (Fig. 7).

IV. DISCUSSION

It is a bit disturbing that there should be such large discrepancies in the published values for the end points of these spectra. The linearity of the Gerholm lens has been determined to be within 0.2% over the range 40 keV–3.6 MeV.[14] From observations made on the two unique transitions in ^{74}As in our laboratory, and from observations on the internal conversion lines in Thorium, ^{198}Au and ^{137}Cs, the lens is linear to within $\approx0.07\%$ in the range 150–1.5 MeV.

An error in the end-point energy can cause an energy dependence in the shape factor to appear, but in our analysis the end points are directly determined from the Fermi plots. The spectrum is therefore treated independently of the ground-state transition. We would like to point out, however, that if one neglects the efficiency corrections, and the energy dependence of the angular distribution of the γ ray, energy dependence of the shape factor can easily be obtained.

A. Nuclear Matrix Elements

A discussion of the nuclear matrix elements will be covered fully in a separate paper, but, for the sake of completeness, we would like to quote some of the results obtained. The theoretical expressions for the β-γ angular-correlation coefficient, and the shape-correction factors were taken from Kotani.[15]

A range of matrix elements which satisfy simultaneously the angular-correlation coefficient, the shape-correction factor, and the Fujita relation based on the CVC theory are shown in Table IV. The goodness of fit is 95% for both cases.

If one accepts the CVC theory, these results may be used for testing nuclear models. However, it must be realized that the results by themselves do not constitute a satisfactory experimental verification of the CVC theory but only that the CVC prediction is consistent with the experimentally-obtained results.

ACKNOWLEDGMENTS

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[13] S. B. Burson and E. B. Shera, Phys. Rev. **136**, 1 (1964).
[14] T. R. Gerholm, Arkiv Fysik **11**, 55 (1956); and private communication.
[15] T. Kotani, Phys. Rev. **114**, 795 (1959).