Internal-Conversion Penetration Effects in $M1$ Transitions of ^{133}Cs†

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Internal-conversion-coefficient and directional correlation measurements were used to determine the penetration parameter λ for the 81-, 161-, and 303-keV transitions of ^{133}Cs. The following internal-conversion coefficients were measured: $\alpha_K(356)=0.020\pm0.001$, $\alpha_K(303)=0.039\pm0.002$, $\alpha_K(161)=0.21\pm0.02$, $\alpha_K(81)=1.46\pm0.05$, and $\alpha_{L+M+N}(81)=0.314\pm0.020$. Existing L-, M-, and N-shell relative intensities of the 81-keV transition were used with $\alpha_{L+M+N}(81)$ to deduce the following subshell coefficients: $\alpha_{L_I}(81)=0.196\pm0.020$, $\alpha_{L_{II}}(81)=0.029\pm0.004$, $\alpha_{L_{III}}(81)=0.022\pm0.004$, and $\alpha_{M_I}(81)=0.044\pm0.009$. The $K/(L+M)$ conversion ratios of the 276- and 161-keV transitions were measured as 4.75 ± 0.15 and 3.75 ± 0.25, respectively. The following directional correlation functions were measured: $W(356\gamma-81\gamma)=1+(0.0355\pm0.0037)P_2(\cos\theta)+(0.000\pm0.008)P_4(\cos\theta)$, $W(276\gamma-161\gamma)=1-(0.420\pm0.009)P_2(\cos\theta)-(0.028\pm0.010)P_4(\cos\theta)$, $W(276\gamma-161e_K^-)=1-(0.256\pm0.020)P_2(\cos\theta)-(0.090\pm0.025)P_4(\cos\theta)$. The following directional correlation coefficients were also measured: $A_2(276e_K^- -161\gamma)=-0.7\pm0.1$ and $A_2(303e_K^- -81\gamma)=0.003\pm0.008$. Using the above body of data, the following penetration parameters were determined: $\lambda(81)=2.8\pm2.3$, $\lambda(161)=41\pm6$, and $\lambda(303)=-2.0_{-3.1}^{+3.3}$. The retardation factors for the $M1$ components of the 81- and 161-keV transitions were reevaluated as $\sim$420 and $\sim$230, respectively.

I. INTRODUCTION

Internal-conversion coefficients (ICC's) and directional correlation measurements, involving conversion electrons, are frequently very helpful in the assignments of spins and parities to nuclear levels. There are cases, however, in which nuclear-structure effects on the conversion process become important and the usefulness of the data for these assignments is doubtful. In such cases, the internal-conversion coefficients and directional correlation particle parameters convey valuable information about the nucleus. The dependence of the conversion coefficients and correlation particle parameters on the nuclear wave functions can (in most cases where accurate measurements are possible) provide an independent test of the accuracy of nuclear models.

These nuclear-structure effects arise because the initial and final electron wave functions overlap with the nuclear wave functions. The so-called "penetration terms" of the internal-conversion matrix elements can easily be separated. A review of this subject is given by Church and Weneser.[1] The magnetic internal-conversion coefficients and particle parameters are given by Pauli[2,3] in the very convenient forms shown below:

$$\beta_L(\lambda)=\beta_L^0(1+b_1\lambda+b_2\lambda^2), \tag{1}$$

$$B_2(\lambda)=\gamma(1+d_1\lambda+d_2\lambda^2)/(1+b_1\lambda+b_2\lambda^2). \tag{2}$$

The constants b_1, b_2, d_1, d_2, γ, and β_L^0 depend on electron integrals and are not dependent on the details of the nucleus. These constants vary significantly with atomic shell. The penetration parameter λ contains all of the nuclear-structure information and, to a good approximation, is shell-independent. Following the arguments of Ref. 1, the nuclear-structure parameter λ can be written in the following form:

$$\lambda=\int d\tau_n\,\mathbf{j}_n\cdot\mathbf{A}_L^*\int_0^{r_n}d\tau_e\,\mathbf{j}_e\cdot\left(\mathbf{B}_L-\frac{h_L^{(1)}(r_n)}{j_L(r_n)}\mathbf{A}_L\right)\Big/\int d\tau_n\,\mathbf{j}_n\cdot\mathbf{A}_L^*\int_0^{R}d\tau_e\,\mathbf{j}_e\cdot\left(\mathbf{B}_L-\frac{h^{(1)}(R)}{j_L(R)}\mathbf{A}_L\right), \tag{3}$$

where $\mathbf{j}_n$ and $\mathbf{j}_e$ are the nuclear- and electron-transition currents, respectively, $\mathbf{A}_L$ and $\mathbf{B}_L$ are the magnetic solutions to the vector Helmholtz equation, $h_L^{(1)}$ and j_L are spherical Hankel and Bessel functions, respectively, R is the nuclear radius, and r_n is a nuclear coordinate.

The best-known example of penetration in the internal conversion of a magnetic dipole transition is the case of the 482-keV transition in ^{181}Ta. λ for this transition was reported as 210 ± 30 by Gerholm *et al.*[4] based on the $482e_K^- -133\gamma$ directional correlation measured by Grabowski *et al.*[5] A theoretical calculation of $\lambda\simeq600$ for this transition was made by Church and Weneser[1]

† Work partially supported by National Science Foundation under Grant No. GP 7690.

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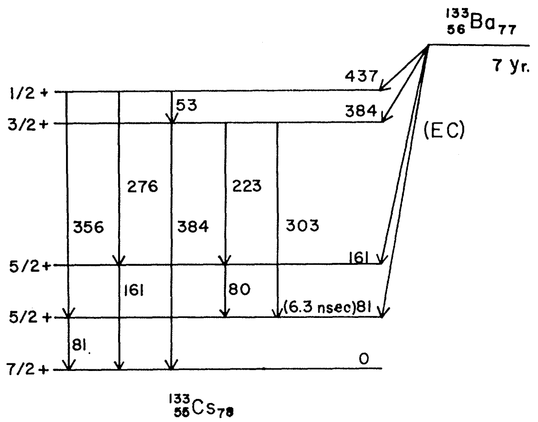

FIG. 1. Decay scheme of ^{133}Ba.

in which the 482-keV level was interpreted in terms of rotational bands in the Bohr-Mottelson unified model. A value of $\lambda(482) = 190 \pm 25$ was determined from the L-subshell measurements of Seltzer and Hager.[6] Agreement between the experimental values and the theoretical prediction of λ is not very good in this case.

The present work is an effort to measure, as accurately as possible, the penetration parameter λ of several transitions in the same nucleus in which shell-model wave functions might be a good zero-order approximation. ^{133}Cs was chosen for several reasons: First, the L-subshell ratios of the 161-keV transition have been found to be anomalous by Hennecke *et al.*[7] and, second, all of the low-lying levels involved in the decay of ^{133}Ba, with the exception of the 161-keV transition, can be interpreted, in terms of the shell model. The decay scheme is shown in Fig. 1. The calculations of Kisslinger and Sorensen[8] predict the ground state with spin $\frac{5}{2}$ which does not agree with experiment, but predict the 437-keV $\frac{1}{2}^+$ state which agrees with experiment. The spin of the 437-keV level was questioned[9]; however, this value of the spin was later verified by directional correlation[10] and $K/(L+M)$ shell-ratio measurements.[11] The calculations of Ref. 8 predict $\frac{7}{2}$ and $\frac{1}{2}$ levels crossing at approximately 190 keV, whereas the only nearby observed level is the $(\frac{5}{2}+)$ 161-keV level. The possibility that the spin of this level is $\frac{3}{2}$ was eliminated by the fact that the 276γ-161γ directional correlation has a nonzero value of the A_4 (see Table I[12–16]). This is supported by the $K/(L+M)$ and $A_2(276e_K^-$-$161\gamma)$ of the present and earlier work.[7,17]

The three transitions of interest and the earlier single-particle shell-model interpretations of the nuclear levels involved[12,13] are as follows: the 81-keV transition between the $(d_{5/2+})$ 81-keV level to the $(g_{7/2+})$ ground state, the 303-keV transition between the $(d_{3/2+})$ 384-keV level to the 81-keV level and the 161-keV transition between the 161-keV level with spin $\frac{5}{2}+$ (which is not easily interpreted in terms of shell-model states) and the ground state.

II. 81-keV TRANSITION

The 81-keV transition is known to be a retarded, predominantly M1 transition. The retardation factor has been given by Thun *et al.*[18] as $R(M1) \simeq 390$. $R(M1)$ is the ratio of the measured half-life and the corresponding Weisskopf estimate. Although there is general agreement in previous measurements of the 356γ-81γ directional correlation,[12–14,18–20] a careful remeasurement was undertaken, in the present work, in order to investigate the disagreements between the values of $|\delta| = 0.173 \pm 0.004$, $|\delta| = 0.164 \pm 0.006$, and $|\delta| = 0.181_{-0.026}^{+0.023}$ from L-subshell measurements,[7,21,22] and the average δ from γ-γ directional correlation measurements[7] $\delta = -0.153_{-0.002}^{+0.003}$, where the sign has been changed to be consistant with the convention adopted in the present work. Although the differences here are not large, the internal-conversion data seem to give consistently higher values of $|\delta|$. A careful check on the previous results is desirable, since errors in $\delta(81)$ will propagate through the analysis of the internal-conversion data of the present work.

TABLE I. Measurements of the 276γ-161γ directional correlation.

Reference	A_2	A_4
12	-0.442 ± 0.009	-0.040 ± 0.012
13	-0.328 ± 0.009	-0.067 ± 0.016
14	-0.421 ± 0.011	-0.016 ± 0.013
15	-0.442 ± 0.011	-0.026 ± 0.014
16	-0.450 ± 0.040	-0.014 ± 0.012
Present	-0.420 ± 0.009	-0.028 ± 0.010

[6] E. Seltzer and R. Hager, Ref. 33, p. 309.
[7] Hans J. Hennecke, J. C. Manthuruthil, O. Bergman, and C. R. Cothern, Phys. Rev. **159**, 955 (1967).
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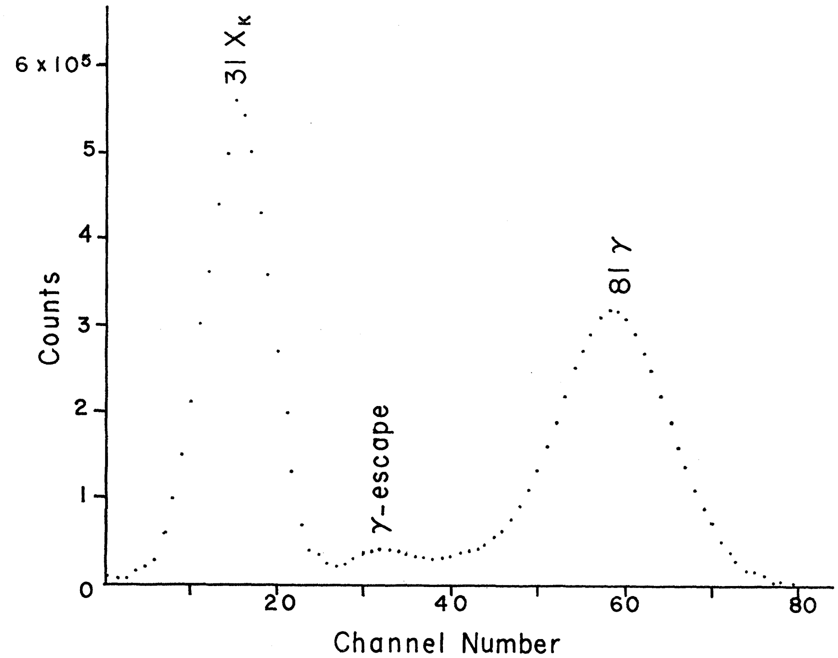

FIG. 2. 81-keV γ ray and K x ray observed in the decay of ^{133}Xe.

A source of liquid $^{133}BaCl_2$ in HCl solution was used in this measurement. The source was approximately 20 μC and was contained in a 1-mm-i.d. glass capillary tube with a wall thickness of approximately 0.025 in. and approximately 2.5 mm in length. The 356- and 81-keV γ rays were detected in 2×2-in. and $1\frac{1}{2}\times\frac{1}{4}$-in. NaI(Tl) detectors, respectively. The crystals were shielded by 1 mm of cadmium sheet to attenuate the 31-keV x ray and were placed 7 cm from the source axis. The strength of this cascade and the energies of the γ rays preclude any serious difficulties due to scattering between the crystals. The photomultiplier tubes were powered by high-stability rate-gain-stabilized voltage supplies. The data were collected over several hundred data cycles at each relative counter position in which the angles between detector axes were 90°, 135°, 180°, 225°, and 270°. The effect of adding the gain stabilizers was immediately evident in the improved accuracy and reproducibility of the data. The correlation coefficients were found to be $A_2=+0.0355\pm0.0037$ and $A_4=0.000\pm0.008$. The error bars are computed from the mean-square deviation of the many data runs which constitute totals of more than 10^5 coincidences at each position. These results are consistent with the mixing ratio $\delta=-0.152\pm0.006$, in good agreement with the average value from $\gamma\gamma(\theta)$ data.[7] The small but consistent differences in $|\delta|$ from internal-conversion data and $|\delta|$ from $\gamma\gamma(\theta)$ data could possibly be attributed to small penetration effects and was the main reason for our reinvestigation of this transition.

TABLE II. Experimental α_K values for the 81-keV transition with ranges of λ implied when compared to Pauli's caluclations.

Reference	α_K	Range of λ
24	1.77±0.15	(−17.5, −4.4)
25	1.75±0.05	(−12.1, −7.8)
26	1.47±0.05	(−1.0, +4.4)
9	1.44±0.03	(+2.3, +5.0)
23	1.39±0.06	(+3.2, +8.8)
18	1.36±0.05	(+5.0, +9.7)
27	1.35±0.05	(+5.5, +10.3)
28	1.34±0.08	(4.4, +12.3)
Present	1.46±0.05	(+0.5, +5.1)

The disagreement in previously reported values $\alpha_K(81)$ and the corresponding implied ranges of $\lambda(81)$ are considerable as seen in Table II.[9,18,23–28] For this reason, a remeasurement of $\alpha_K(81)$ was undertaken by determining the relative intensities of the K x rays and the 81-keV γ rays in the decay of ^{133}Xe. The ^{133}Xe source was in gaseous form in a thin-walled glass vial obtained from the New England Nuclear Corporation. The source consisted of 1 cm^3 of carrier-free ^{133}Xe gas at a pressure of 40 Torr at standard temperature and had an activity of approximately 5 mCi at the start

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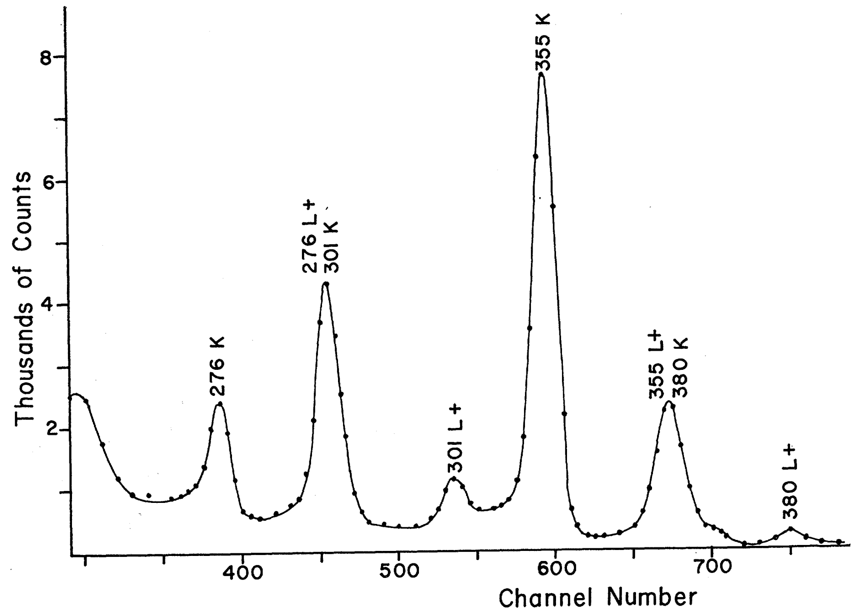

FIG. 3. Conversion electrons from the higher transitions of ^{133}Cs, observed in a 200-mm^2×2-mm Si(Li) detector at 77°K.

of the measurements. The attenuation of the 0.044-in. glass walls and other material between the source and the detector was experimentally measured. No correction for self-absorption in the low-pressure Xe gas was made. The radiations were detected in a $1\frac{1}{2}$-in.-diam by $\frac{1}{4}$-in.-thick NaI(Tl) crystal with a 0.005-in.-thick Be window. The general procedure and the correction for the x-ray escape peak was similar to that of the Erman and Sujkowski[23] with the exception of the gaseous source and the geometry used in the present work. An effort was made to achieve a so-called "good geometry," i.e., parallel beam geometry, by placing the source inside of an iron shield, 2 m from the detector. The spectrum is shown in Fig. 2.

$\alpha_K(81)$ was determined from the equation

$$\alpha_K = N_x R(\gamma x)(1-e_\gamma)/N_\gamma \omega_K(1-e_x),$$

where N_x and N_γ are the areas under the full energy peaks, $R(\gamma x)$ is the factor which corrects the ratio N_x/N_γ for the x-ray and γ-ray attenuations in the glass, air, and Be window, e_γ and e_x are the γ- and x-ray excape fractions, and ω_K is the K-shell fluorescence yield. The value of ω_K used was the average of

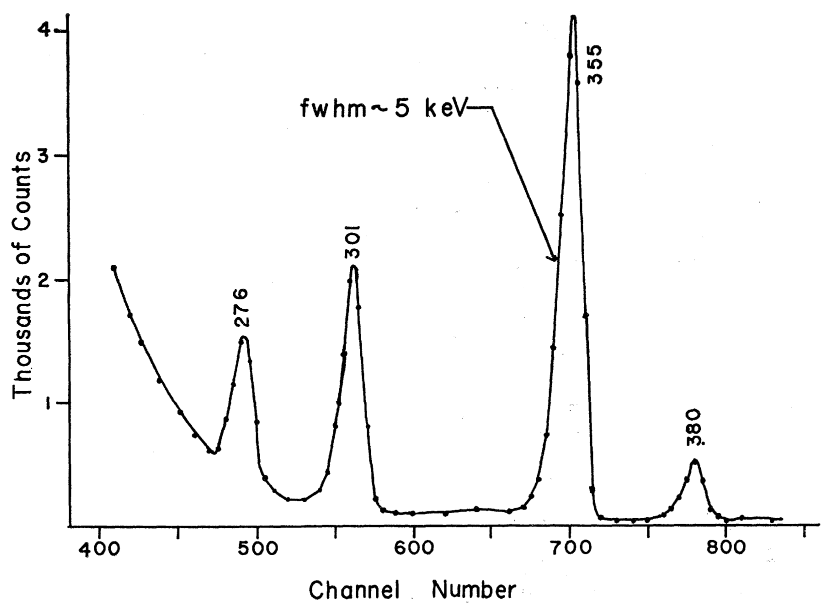

FIG. 4. γ rays from the higher transitions in ^{133}Cs, observed in a 200-mm^2 ×2-mm Si(Li) detector covered with a 350-mg/cm^2 aluminum electron absorber.

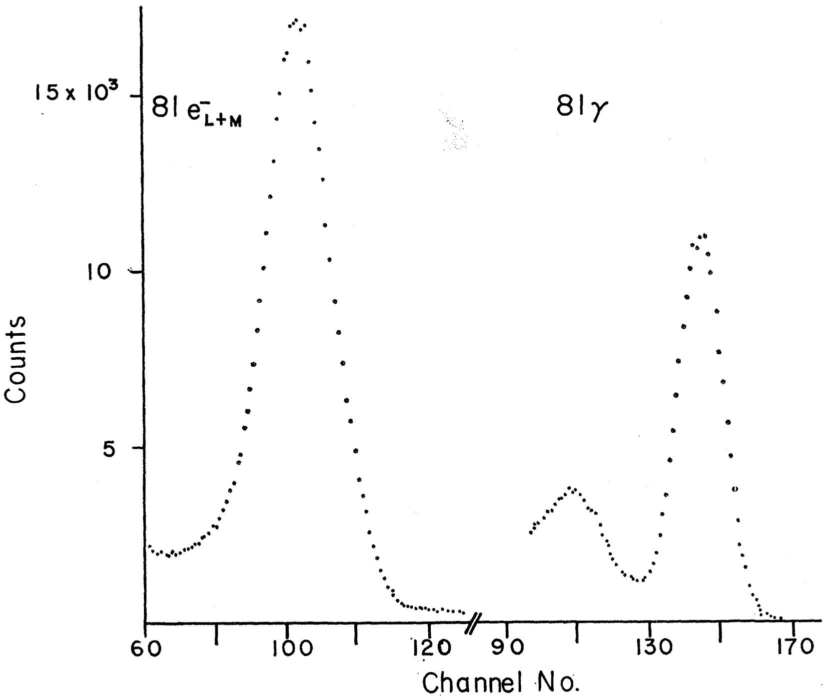

FIG. 5. *L*-, *M*-, and *N*-shell conversion electrons and γ rays from the 81-keV transition [observed in a 200-mm^2 Si(Li) detector at 77°K], in the calibrated geometry.

the results, for $Z=55$, given by Fink *et al.*,[29] e_γ was taken from the γ escape peak and e_x was the same as that used in Ref. 23. $\alpha_K(81)$ is found to be 1.46 ± 0.05.

The higher-shell conversion coefficient $\alpha_{L+M+N}(81)$ was measured directly using a single 100-mm^2-area by 2-mm-deep Si(Li) detector, cooled to approximately 77°K, to detect both the γ rays and conversion electrons. The observed spectra of conversion electrons and of γ rays from the higher transitions of ^{133}Cs are shown in Figs. 3 and 4, respectively. The detector and geometry were carefully calibrated using the well-known internal-conversion-coefficient standards of the 88-, 145-, 166-, 279-, 392-, 570-, 662-, and 835-, and 1064-keV transitions in the decays of ^{109}Cd, ^{141}Ce, ^{139}Ce, ^{203}Hg, ^{113}Sn, ^{207}Bi, ^{137}Cs, ^{54}Mn, and ^{207}Bi, respectively. The γ rays were observed by moving an externally controlled, 350-mg/cm^2 aluminum plate between the source and the detector. For the measurement of $\alpha_{L+M+N}(81)$, only electrons and γ rays in coincidence with the 356 γ ray were counted in order to eliminate the interference from electrons of the 80-keV transition. The 356-keV γ rays were detected with a 3-in.×3-in. NaI(Tl) scintillation detector. Standard coincidence techniques were used in which the coincidence circuit output pulses are used to gate a multichannel analyzer. There was some interference from Compton scattered γ rays in the tail of the electron peak. The correction for this effect was made by placing the source and Si(Li) detector in a magnetic spectrometer with specially designed baffles which transmit electrons over an energy interval of approximately 100 keV. The shape of the line thus observed was used to correct the tail of the line observed in the calibrated geometry. Figure 5 shows the corrected electron line and the γ line, both observed in the Si(Li) detector. The 356γ-$81e_{LMN}^-$ directional correlation was investigated and found to be weak, i.e., $A_2(356\gamma\text{-}81e_{LMN}^-)\simeq 0.01$. In the $\alpha_{L+M+N}(81)$ experiment the Si(Li) detector was located 3 cm from the source and the 3-in.×3-in. NaI(Tl) detector was located 2 cm from the source. The large solid angle subtended by the detectors and the weak correlation result in a correction of approximately 0.5% to the conversion coefficient for correlation effects. The result of this measurement is $\alpha_{L+M+N}=0.314\pm0.020$. This result was subdivided into its subshell contributions using the relative intensities of Siegbahn *et al.*[22] The resulting subshell coefficients are given in Table III. The L_I and M_I conversion coefficients are also sensitive to λ since their main contributions come from electron wave functions whose initial and final states are also characterized by the quantum number $\kappa=-1$, where $-\kappa$ is the eigenvalue of the operator $(\boldsymbol{\sigma}\cdot\mathbf{l}+1)$. From the values $\alpha_{L_\text{I}}=0.195\pm0.020$ and $\alpha_{M_\text{I}}=0.044\pm0.009$, the lower of the two implied probable ranges of λ are found to be -11 to $+2$ and -24 to $+2.5$, respectively. Although these are not in conflict with the range of λ of $+0.5$ to $+5.1$ implied by $\alpha_K(81)=1.46\pm0.05$ (see Fig. 6), the larger uncertainties render them less useful in determining λ,

[29] R. W. Fink, R. C. Jopson, H. Mark, and C. D. Swift, Rev. Mod. Phys. **38**, 513 (1966).

TABLE III. Experimental and theoretical ICC values for the 81-keV transition.

Shell	Expt	Theor ($M1$)	Theor ($E2$)	Theor ($\delta=-0.152$)
K	1.46 ± 0.050	1.499(0)	2.360(0)	1.5185
L_{I}	0.195 ± 0.020	0.1814(0)	0.2036(0)	0.1819
L_{II}	0.029 ± 0.004	0.1350(−1)	0.5549(0)	0.0268
L_{III}	0.022 ± 0.004	0.3072(−2)	0.6629(0)	0.0180
M_{I}	0.044 ± 0.009	0.3582(−1)	0.367(−1)	0.0358

The region of overlap for these three independently determined ranges of λ is $0.5\leq\lambda\leq2.0$. The higher value of $\lambda(81)$ implied by these measurements is 257 ± 3. It should be noted that there is always a fairly large value of λ possible ($\lambda\sim100$–200) owing to the presence of the λ^2 term in the theoretical expression for the conversion coefficient and the fact that b_2 is generally negative for the K, L_{I} and M_{I} shells.

The retardation factor for the 81-keV transition was reevaluated using the Weisskopf half-life $[T_{1/2}(M1)=0.423\times10^{-11}]$ used by Thun *et al.*,[18] the α_K, α_{L+M+N}, and δ of the present work, and the total half-life 6.31×10^{-9} sec reported by Bodenstedt *et al.*[12] The result is $R(M1, 81)\simeq420$.

III. 161-keV TRANSITION

The 161-keV transition is generally thought to be a predominantly $M1$ transition between the $\frac{5}{2}+$ 161-keV level and the $\frac{7}{2}+$ ground state. Thun *et al.*[18] give the $M1$ retardation factor $R(M1)\simeq320$. The K/L_{III} and $L_{\text{I}}/L_{\text{II}}$ measurements of Hennecke and his co-workers[7] are consistent with the 276γ-161γ directional correla-data only if $\lambda=45\pm10$ or $\lambda=180\pm20$. The theoretical calculations of the penetration factors of Church and

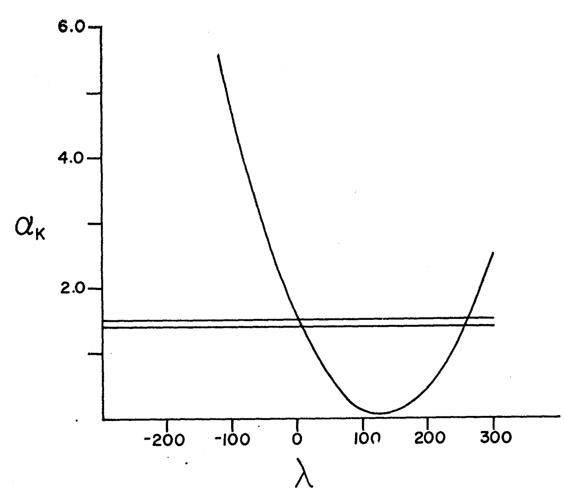

FIG. 6. Theoretical plot of $\alpha_K(81)$ versus λ and intersection of the experimental data.

Weneser[30] and Reiner[31] were used in evaluating λ in Ref. 7.

We have investigated the penetration effects in this transition by remeasuring the 276γ-161γ directional correlation using both liquid and solid sources, measuring the 276γ-161e_K^- and 276e_K^--161γ directional correlations and measuring both $K/(L+M)$ and α_K.

The results of previous 276γ-161γ directional correlations are presented in Table I. All of the A_2 values are in good agreement; however, the A_4 value of Yin and Wiedenbeck[13] is disturbing for two reasons: First, all of the values of A_2 imply two ranges of δ, namely, $\delta(161)\sim0.6$ and $\delta(161)\sim2.5$. The A_4 of Ref. 13 favors the larger value in which case no penetration is necessary in order to explain the conversion data; and, second, the large fluctuation in A_4 and the very small A_4 observed by Meder[16] raises the question of the reliability of the A_4 measurements in this cascade. The

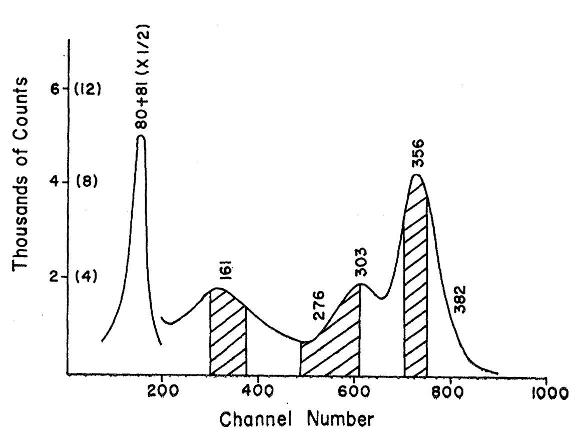

FIG. 7. γ-ray spectrum of ^{133}Cs observed in a 2-in.×2-in. NaI(Tl) crystal with the energy regions accepted in the γ-γ directional corresslations in which this size detector was used.

spin assignment of $\frac{5}{2}$ was made on the grounds that $A_4=0$ for an intermediate state spin of $\frac{3}{2}$. If the spin were $\frac{3}{2}$ then the 276 transition would probably be predominantly $M1$; however, the measured value of $[K/(L+M)]_{276}=4.75\pm0.15$ agrees more closely with the theoretical $E2$ value of 4.84 than with the $M1$ value of 6.48. This supports the prior spin assignment of $\frac{5}{2}$ for the 161-keV level.

The investigation of possible extra nuclear effects in the ($e_K^-\gamma$) and (γe_K^-) correlations was the main reason for our ($\gamma\gamma$) correlation measurements of this cascade.

The 276γ-161γ correlation was measured in the liquid source described earlier and in a solid source of $BaCl_2$ deposited on a 0.25-mil aluminized Mylar strip approximately 1 mm in width and 4 mm long. Both γ rays were detected in 2-in.×2-in. NaI(Tl) detectors,

[30] E. L. Church and J. Weneser, Phys. Rev. **104**, 1382 (1956).
[31] A. S. Reiner, Nucl. Phys. **5**, 554 (1958).

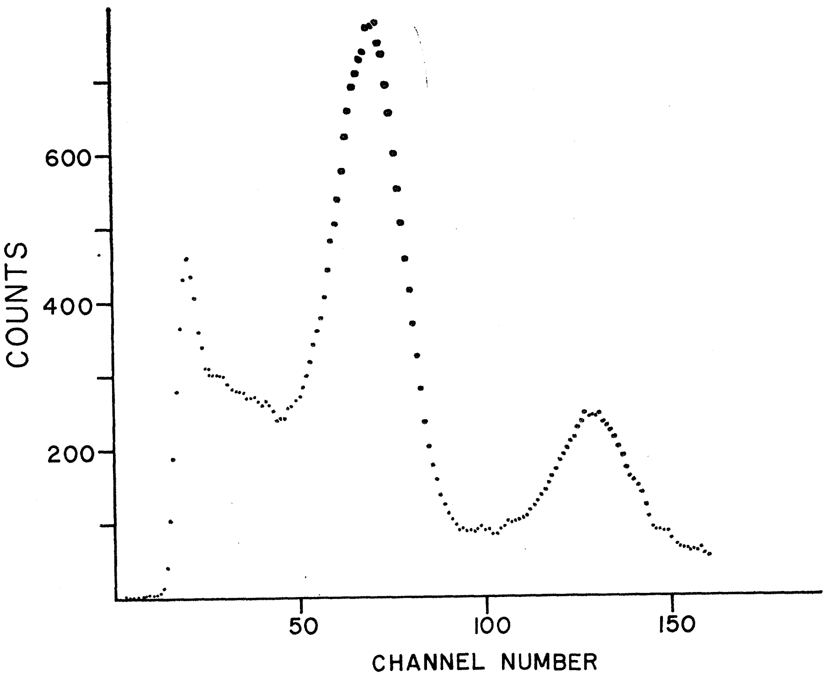

FIG. 8. Coincidence spectrum of conversion electrons from the 161-keV transition observed during the 276γ-$161e_K^-$ directional correlation. The K and $L+M+\cdots$ peaks are shown.

shielded by 1 mm of cadmium and placed 7 cm from the source axis, in both experiments. The γ spectrum and accepted regions for both γ rays are shown in Fig. 7. Standard coincidence techniques were used with a resolving time of approximately 100 nsec. The results were in very close agreement which indicates that the 276γ-$161e_K^-$ correlation is not measurably influenced by extra-nuclear effects. The values $A_2=-0.420\pm0.009$ and $A_4=0.028\pm0.010$ are in general agreement with prior results and imply $\delta(161)=+0.58\pm0.02$, where, as before, the quoted errors are purely statistical. This value of $\delta(161)$ is consistent with the lower values of $\delta=0.6\pm0.1$ from the $(e_K^-\gamma)$ correlation described below and with the value of $\delta=0.59\pm0.08$ and $\delta=0.69\pm0.05$ found by Meder from his measurements of the $276e_K^-$-161γ and 276γ-161γ directional correlations, respectively.[16]

The 276γ-$161e_K^-$ correlation was measured using the solid source described above. The 276-keV γ rays were detected in a 2-in.×2-in. NaI(Tl) detector while the conversion electrons from the 161-keV transition were detected in a 200-mm^2×1-mm-thick Si(Li) detector maintained at approximately 77°K. The coincidence pulses were used to gate a multichannel analyzer in order to record the coincidence-gated conversion-electron spectrum at each of the positions. A typical coincidence spectrum for the 135° position is shown in Fig. 8. The coincidence spectra collected at the several positions are used to correct for the continuum, under the peaks, which can be caused by Compton scattering of γ rays between the detectors. The resulting correlation coefficients were found to be $A_2=-0.256\pm0.020$ and $A_4=-0.090\pm0.025$. In addition, the $276e_K^-$-161γ directional correlation was measured with the result $A_2=-0.7\pm0.1$. The large error in this correlation is due to the Compton edge of the 382-keV γ ray which can scatter from the Si(Li) detector into the NaI(Tl) detector leaving the proper energy in each so that it is detected under the $276e_K^-$ electron peak. Correlations, with the Si(Li) detector covered by 350 mg/cm^2 of aluminum, were measured in order to investigate the effects of γ-ray scattering between the detectors during the $(e_K^-\gamma)$ and (γe_K^-) correlations. The effect was found to be significant only in the $276e_K^-$-161γ case. $\lambda(161)$ was obtained by intersecting the experimental $A_2(\gamma e_K^-)$ with a theoretical plot as shown in Fig. 9. The result is $\lambda(161)=40\pm10$.

There is large disagreement among the values of $\alpha_K(161)$ which appear in the literature. Thun *et al.*[18] give $\alpha_K(161)=0.39\pm0.13$, Bosch *et al.*[28] give $\alpha_K(161)=0.18\pm0.03$, and Notea and Gurfinkel[32] give $\alpha_K(161)=0.29\pm0.08$. We have determined $\alpha_K(161)$ both directly and by remeasuring its relative conversion-electron intensity and using the relative γ-ray intensities of Ref. 32.

A magnetic spectrometer, with special high transmission baffles and with a cooled Si(Li) detector at the focus, was used for the $\alpha_K(161)$ measurements. In the first measurement a 4-cm^3 Ge(Li) detector was positioned near the source end of the spectrometer and the $e_K^-(161)$ and 161γ lines were counted, in the Si(Li) and Ge(Li) detectors, respectively. The geometry was

[32] A. Notea and Y. Gurfinkel, Nucl. Phys. **A107**, 193 (1968).

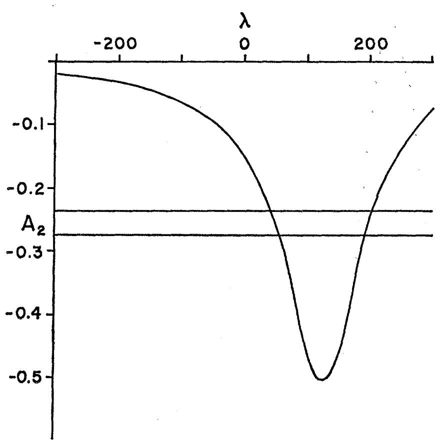

FIG. 9. Theoretical plot of $A_2(276\gamma\text{-}161_K^-)$ versus λ and intersection of the experimental data.

then calibrated by performing a similar experiment using the 166-keV transition in ^{139}Ce for which α_K is a well-known calibration standard ($\alpha_K=0.2142\pm0.0015$).[33] The actual electron energy difference between the ^{139}Ce electron and the ^{133}Cs electron is only about 0.05 keV rather than 4 keV due to the difference between Ce and Cs K-shell binding energies. The result of this calibrated measurement is $\alpha_K(161)=0.21\pm0.02$. As a check the intensity of the $e_K^-(161)$ line relative to the $e_K^-(356)$ line was remeasured with this system as 0.115 ± 0.005. The relative γ-ray intensities of Ref. 32 $N(161)/N(356)=0.0121\pm0.0005$ were used. $\alpha_K(356)$ was directly measured as 0.020 ± 0.001 using the calibrated Si(Li) spectrometer described in Sec. II.[34] $\alpha_K(161)$ was then determined as follows:

$$\alpha_K(161)=\frac{N(161e_K^-)N(356\gamma)}{N(356e_K^-)N(161\gamma)}\,\alpha_K(356).$$

Using this method, we find $\alpha_K(161)=0.202\pm0.027$. The direct measurement has a smaller error, hence it was used to determine λ by comparing α_K to the calculations of Pauli.[3] The result is $\lambda(161)=30_{-11}^{+12}$ (or 220 ± 10).

The $K/(L+M)$ (161) conversion ratio was measured using the magnetic spectrometer with Si(Li) detector installed as described above. The result is $K/(L+M)=3.75\pm0.25$ which, when compared to the theoretical values, implies $\lambda(161)=53\pm10$ (or 185 ± 15).

The average of the values of $\lambda(161)$ from all three experiments with the root-mean-square error is found to be $\lambda(161)=41\pm6$. The average of the higher possible range of λ implied by the data is 200 ± 7.

The retardation factor for this transition was re-evaluated using the Weisskopf half-life $[T_{1/2}(M1)=0.539\times10^{-11}$ sec$]$ used in Ref. 18, the branching ratio of 0.12 calculated using the electron intensities of Refs. 7 and 18, the mixing ratio $\delta(161)$, conversion coefficient $\alpha_K(161)$ of the present work, and the total half-life of $85\pm15\times10^{-12}$ sec of the 161-keV level measured by Agarwal *et al.*[15] The result is $R(M1,\,161)\simeq230$.

IV. 303-keV TRANSITION

The 303-keV transition is commonly accepted as a predominantly $M1$ transition with a mixing ratio determined from existing γ-γ directional correlations[7] $\delta(303)=0.006\pm0.022$. The retardation factor for this transition is given in Ref. 7 as $R(M1)\simeq97$.

The $303e_K^-$-81γ directional correlation was measured with the result: $A_2=+0.003\pm0.008$ which allows any value of $\lambda(303)$ from -60 to $+90$. The K-shell conversion coefficient is far more helpful in delimiting $\lambda(303)$. Values of $\alpha_K=0.038\pm0.003$,[28] 0.037 ± 0.005,[18] and 0.036 ± 0.004[35] appear in the literature. The values are in good agreement; however, a remeasurement was undertaken in an attempt to increase the accuracy of this value. This case turns out to be ideal for the single-detector method described earlier and an accuracy of better than 5% was achieved. We find $\alpha_K(303)=0.039\pm0.002$. This value of α_K implies $\lambda(303)=-2.0_{-3.1}^{+3.3}$ when compared to the theoretical values of Pauli.[3] The higher range of $\lambda(303)$ is found to be 237 ± 3.[36]

V. SUMMARY AND CONCLUSIONS

Internal-conversion-coefficient measurements and a remeasurement of the 356γ-81γ directional correlation in ^{133}Cs were used to determine the multipole mixing ratio and penetration parameter $\delta(81)=-0.152\pm0.006$ and $\lambda(81)=2.8\pm2.3$, respectively. The measurements of the K-shell conversion coefficient, the $K/(L+M)$ conversion ratio of the 161-keV transition and the 276γ-161γ, 276γ-$161e_K^-$, and $276e_K^-$-161γ directional correlations were used to determine δ and λ for the 161-keV transition as $\delta(161)=0.58\pm0.02$ and $\lambda(161)=41\pm6$. This rather large λ is in good agreement with the value of 45 ± 10 reported by Hennecke[7] on the grounds of K/L_{III} and $L_{\text{I}}/L_{\text{III}}$ measurements and the value of 41_{-9}^{+7} found by Meder[16] using the results of directional correlation and $K/(L+M)$ measurements. A remeasurement of $\alpha_K(303)$ was undertaken in order to reduce the uncertainties appearing in the literature. The result was $\alpha_K(303)=0.039\pm0.002$ from which a value of $\lambda(303)=-2.0_{-3.1}^{+3.3}$ was determined. Greater accuracy than that which appears in the literature was desirable because almost nothing about $\lambda(303)$

[33] *Internal Conversion Processes*, edited by J. H. Hamilton (Academic Press Inc., New York, 1966), p. 643.
[34] F. T. Avignone and H. H. Knox, Rev. Sci. Instr. **40**, 1046 (1969).
[35] K. C. Mann and R. P. Chaturvedi, Can. J. Phys. **41**, 932 (1963).
[36] We recognize that there will always be a possible high range of λ and we choose to consider only the lower ranges in the remaining discussion.

can be concluded from the correlation coefficient $A_2(303e_K^- \text{-} 81\gamma) = +0.003 \pm 0.008$.

In addition, the conversion coefficients of the $L_{\rm I}$, $L_{\rm II}$, $L_{\rm III}$, and the $M_{\rm I}$ shell of the 81-keV transition were determined by directly measuring $\alpha_{L+M+N}(81)$ in a coincidence experiment and comparing the result to the measured relative subshell intensities of Siegbahn *et al.*[22] These coefficients are in good agreement with the calculated subshell coefficients of Pauli.[3]

It is interesting to recall that one of the possible causes of enhancement of the penetration parameter λ is a strong retardation of the $M1$ γ-ray transition. We have reevaluated the retardation factors for the 81- and 161-keV transitions using the present values: $\delta(81) \simeq -0.152$, $\alpha_K \simeq 1.46$, $\delta(161) \simeq 0.58$, and $\alpha_K(161) \simeq 0.21$. The results are $R(M1, 81) \simeq 420$ and $R(M1, 161) \simeq 230$. The retardation factor $R(M1, 303) \simeq 97$ is given in Ref. 7. It is clear that in this case, some process other than retardation of the $M1$ γ transition plays a dominant role in the penetration effect, causing $\lambda(161)$ to be significantly larger than $\lambda(81)$. The authors suggest that this case could be useful in testing the accuracy of nuclear models in the nondeformed nuclide region and that a theoretical calculation of all three quantities $\lambda(81)$, $\lambda(161)$, and $\lambda(303)$ might be very helpful in furthering the understanding of penetration effects and the mechanism of their enhancement.

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Beta-Gamma Angular Correlations and Shape-Factor Measurements in ^{186}Re and ^{188}Re

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The A_2 coefficients in the directional correlation between the nonunique first forbidden 0.935-MeV β group and the 0.137-MeV γ ray in the decay of ^{186}Re, and in the directional correlation between the nonunique first forbidden 1.980-MeV β group and the 0.155-MeV γ ray in the decay of ^{188}Re, have been measured as a function of energy. An investigation of perturbation caused by a magnetic hfs-type interaction yielded a negative result. Both of these β spectra were found to have the "allowed" shape over the region examined.

I. INTRODUCTION

The β-γ angular correlation coefficients for the 0.935-MeV β, 0.137-MeV γ cascade in ^{186}Re and the 1.980-MeV β, 0.155-MeV γ cascade in ^{188}Re have been previously measured by a number of authors.

In particular, the A_2 coefficients in ^{186}Re have been measured with good statistical accuracy by Novey *et al.*[1] The 1.980-MeV β, 0.155-MeV γ directional correlation coefficients in ^{188}Re have been determined by Wyley *et al.*,[2] but their results are not very useful for theoretical calculations because of the relatively large statistical errors.

The question of attenuation of the β-γ angular correlation pattern, because of the half-life ($\sim$0.5 nsec) of the intermediate state of ^{186}Os has been raised by Novey.[1] If the value of A_2 is significantly attenuated in the ^{186}Re correlation, it will also be attenuated in the ^{188}Re case. There has been no further mention of this in the literature, and it is still an open question.

Different results have been reported by a number of authors for the β-spectrum shape factors for these decays. Porter *et al.*[3] reported an energy-dependent shape factor for the 0.935-MeV ^{186}Re β spectrum. Koerts[4] and also Bashandy[5] reported an allowed shape but neither author indicated the accuracy with which the energy dependence of the shape factor may be deduced. Johns *et al.*[6] also reported an allowed shape for this

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