Fast Neutrons Incident on Vanadium†

A. B. SMITH, J. F. WHALEN, AND K. TAKEUCHI
Argonne National Laboratory, Argonne, Illinois 60439

(Received 18 August 1969)

Total neutron cross sections and elastic and inelastic neutron-scattering cross sections of vanadium are experimentally studied. Total neutron cross sections are determined with good resolution ($\gtrsim$1 keV) from 0.1 to 1.45 MeV. Differential elastic and inelastic scattering angular distributions are measured at incident-neutron energy intervals of $\lesssim$10 keV and with resolutions of $\sim$20 keV from 0.3 to 1.5 MeV. The inelastic excitation of states at 330±10 and 926±10 keV is observed. The experimental results are interpreted in terms of the optical model and statistical concepts inclusive of resonance-width fluctuations. The observed intermediate structure is discussed in the context of strongly overlapping resonances, distributions in resonance widths and spacing, and an intermediate optical model.

1. INTRODUCTION

THE objective of the work reported here was to improve the understanding of the total neutron cross section and of the elastic and inelastic scattering cross sections of vanadium at incident energies up to 1.5 MeV. Natural vanadium is nearly 99.76% monoisotopic, lies near the peak of the s-wave strength function, is magic in neutron number, and has relatively low-lying states which are appreciably excited by inelastic neutron-scattering processes. Collectively, these properties are unusual, and many of them have been associated with the prevalence of intermediate structure in neutron processes.[1] The relatively large inelastic excitations make possible the assay of such intermediate structure in several reaction channels. In the present work, considerable attention is given to the experimental accuracies and energy resolutions in an effort to well define the energy-dependent structure.

In the subsequent discussion, Sec. 2 outlines the techniques employed in the experimental portions of the study. In Sec. 3, the experimental results are presented and comparisons made with previously reported values.[2] Section 4 is devoted to the physical interpretation of the measured results. A broad energy average of the measured values is described in terms of a conventional optical potential[3] and statistical concepts[4] inclusive of resonance-width fluctuations.[5] The observed intermediate structure is discussed in the context of (a) strongly overlapping compound-nucleus resonances,[6] (b) fluctuations in widths of isolated or partially overlapping resonances,[7,8] and (c) an intermediate optical potential assuming reaction processes proceeding through "doorway" states.[9,10]

2. EXPERIMENTAL METHOD

Fast-neutron time-of-flight techniques were employed in all of the scattering measurements and a portion of the total cross-section determinations. The experimental neutron-scattering apparatus, inclusive of pulsed and bunched Van de Graaff accelerator, multiangle detection system, on-line computer usage, and associated data-processing procedures, has been described in detail and will not be further defined herein.[11,12] All measurements employed the reaction $Li^7(p, n)Be^7$ as a neutron source.[13] The scattering samples were right cylinders (2.0 cm diam and 2.0 cm high) fabricated of natural vanadium metal with a chemical purity of >98%. All scattering measurements were made relative to the known differential elastic scattering cross sections of carbon, and corrected for multiple scattering, incident-beam attenuation, and angular-resolution effects.[11,14] Total neutron cross sections were determined from neutron transmissions measured using a pseudo-white pulsed-source technique or the monoenergetic source method.[12,15] The latter method was employed exclusively below incident-neutron energies of $\sim$500 keV. Transmission samples of natural vanadium varied in geometry but generally were such as to provide transmissions of greater than 50%.

† Work supported by the U.S. Atomic Energy Commission.

[1] H. Feshbach, in *Proceedings of the Conference on Nuclear Structure Study with Neutrons* (North-Holland Publishing Co., Amsterdam, 1966); see also H. Feshbach *et al.*, Ann. Phys. (N.Y.) **41**, 230 (1967).

[2] A detailed report of this work including a complete numerical tabulation of experimental results can be found in the Argonne National Laboratory Report No. ANL-7564 (unpublished). The numerical results have also been forwarded to the National Neutron Cross Section Center, Brookhaven National Laboratory.

[3] P. Hodgson, in *Optical Model of Elastic Scattering* (Oxford University Press, London, 1963).

[4] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).

[5] P. Moldauer, Rev. Mod. Phys. **36**, 1079 (1964).

[6] T. Ericson, Ann. Phys. (N.Y.) **23**, 390 (1963).

[7] A. Agodi and G. Pappalardo, Nucl. Phys. **47**, 129 (1963).

[8] A. Carlson and H. Barschall, Phys. Rev. **158**, 1142 (1967).

[9] K. Takeuchi, thesis, University of Michigan, Ann Arbor, Mich., 1968 (unpublished).

[10] R. Lipperheide, Z. Physik **202**, 58 (1967).

[11] A. Smith *et al.*, Nucl. Instr. Methods **50**, 277 (1967).

[12] J. Whalen, Nucl. Instr. Methods **39**, 185 (1966).

[13] J. Gibbons and H. Newson, in *Fast Neutron Physics*, edited by J. Marion and J. Fowler (Interscience Publishers, Inc., New York, 1960), Vol. 1, p. 133.

[14] A. Langsdorf *et al.*, Argonne National Laboratory Report No. ANL-5567 (unpublished); see also A. Langsdorf *et al.*, Phys. Rev. **107**, 1077 (1957).

[15] E. Barnard *et al.*, Nucl. Phys. **A118**, 321 (1968).

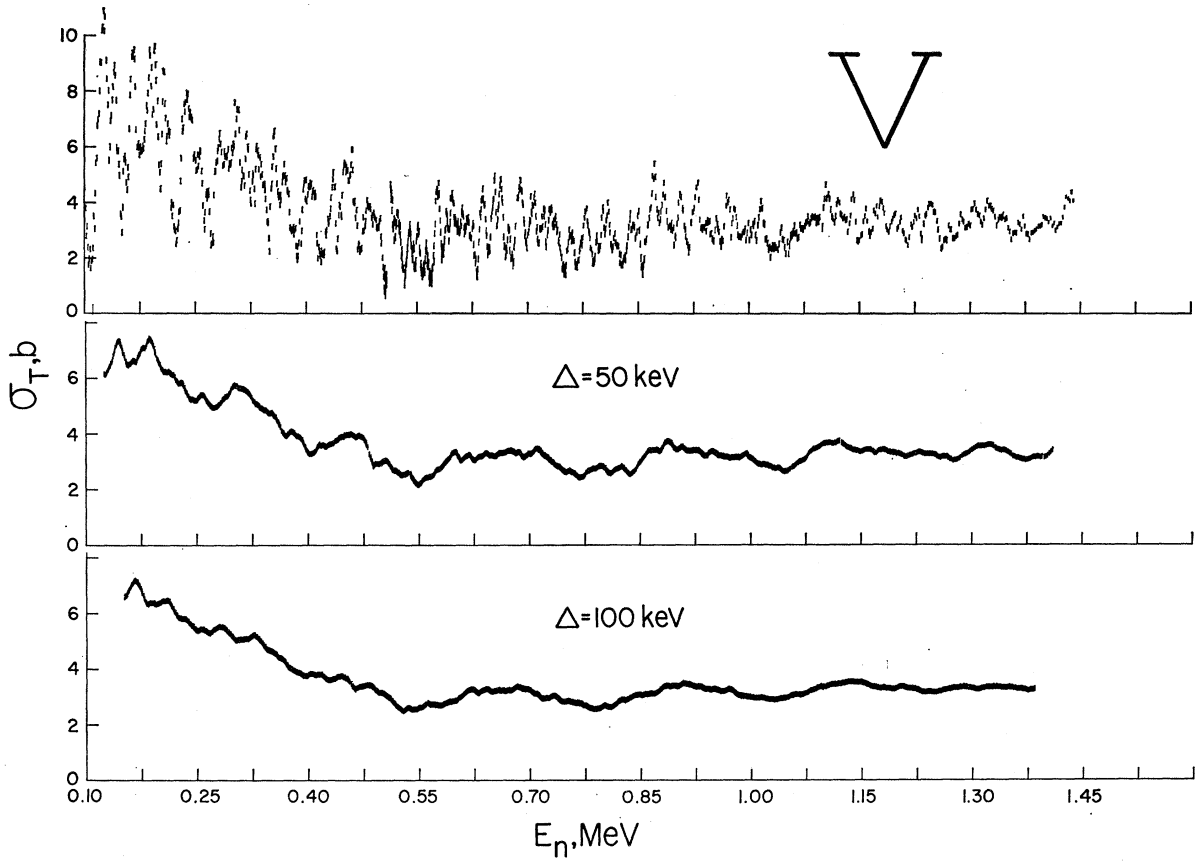

FIG. 1. Total neutron cross sections of vanadium. (a) Top, measured data. Vertical bars indicate statistical error (b) Center, data averaged over a 50-keV interval. (c) Bottom, data averaged over a 100-keV interval.

3. EXPERIMENTAL RESULTS

A. Total Neutron Cross Sections

The total cross section measured over the interval 0.1–1.45 MeV is shown in the upper portion of Fig. 1. At incident-neutron energies of $\lesssim$0.5 MeV, the experimental energy resolution was 1.5–2.5 keV. At higher energies, the velocity resolution was estimated to be 0.1–0.12 nsec/m. Below 0.5 MeV, the incident-energy calibration was believed accurate to $\pm$3 keV. The uncertainty of the calibration increased above 0.5 MeV to a maximum of $\lesssim$8 keV at incident energies of $\sim$1.5 MeV. Statistical uncertainties in the measured cross sections were $\sim$1–3% in the majority of the measurements. "In-scattering" corrections were estimated, found small, and neglected.

In general, the present results are consistent with those obtained by other workers.[16] Values reported by Cabe *et al.* in the interval 0.4–1.2 MeV follow the energy average of the present higher-resolution values.[17] The present work is consistent with the detailed resonance results reported by Rohr and Friedland and by Firk *et al.* for incident neutron energies of $\lesssim$200 keV.[18,19]

B. Elastic Cross Sections

The differential elastic neutron-scattering cross sections were determined at incident-neutron energy intervals of $\lesssim$10 keV with an incident-neutron energy resolution of 20$\pm$5 keV. The angular range of the measurements at each incident energy was from $\approx$25° to $\approx$155°, and usually covered in eight angular measurements. Typical values of the laboratory scattering angles were 27°, 38°, 53°, 69°, 84°, 114°, 129°, and 154°.

Angular resolutions varied but were in the order of a few degrees, and the scattered-neutron velocity resolution of 1.5–2.0 nsec/m was sufficient to resolve the elastic scattered neutrons from inelastic scattered components.

[16] D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratory Report No. BNL-325, 1958, 2nd ed. (unpublished); see also M. D. Goldberg *et al.*, Second Supplement, 2nd ed., 1966, Vol. 11A (unpublished).

[17] J. Cabe *et al.*, Nucl. Phys. **A102,** 92 (1967).
[18] G. Rohr and E. Friedland, Nucl. Phys. **A104,** 1 (1967).
[19] F. Firk *et al.*, Proc. Phys. Soc. (London) **82,** 477 (1963).

The experimental results are expressed in the form

$$\frac{d\sigma}{d\Omega} = \sum_{i=0}^{4} B_i P_i, \qquad (1)$$

where P_i are Legendre polynomials expressed in the laboratory system and B_i are parameters derived by least-squares fitting Eq. (1) to experimentally measured differential distributions. The expansion of Eq. (1) is terminated at $i=4$, as the magnitudes of higher-order terms do not generally exceed their corresponding uncertainties. Expressed in the above forms, the results are descriptive of the measured data points and parameter values obtained from measurements at slightly different incident energies or scattering angles and are easily comparable. However, extrapolation of the results [expressed in the form of Eq. (1)] beyond the measured angular interval of $\approx 25°$–$\approx 155°$ is not necessarily valid.

As expected from the complex resonance nature of the total neutron cross section, the elastic scattering cross sections varied rapidly with incident energy. Small experimental variations in incident-neutron energy and/or energy spread could, and did, result in quite different measured cross-section values. In order to provide consistent experimental results, the measured differential elastic cross sections were progressively averaged over a squared energy increment with a width of several experimental resolutions. Results obtained with a 50-keV averaging increment are shown in Fig. 2. Assay of the experimental errors was a complex and subjective matter. Generally, it was thought that the elastic scattering cross sections were determined to an accuracy of $\sim 8\%$, inclusive of uncertainties in the cross section of the carbon standard. The illustrated uncertainties in the angular distribution coefficients are indicative of standard deviations derived from the least-squares-fitting procedures.

The elastic scattering results were generally consistent with the total cross section (see Sec. 3 A) as indicated in Fig. 3, and with the reported total scattering results of Ref. 14. The agreement between the present results and those obtained using a soft-fission spectrum[20] was less satisfactory.

C. Inelastic Neutron Scattering

The inelastic excitation of states in vanadium at 330 ± 10 and 926 ± 10 keV was observed. The excitation energies were determined by calibration of time-of-flight scales using the 845-keV state in ^{56}Fe. The observed states were attributed to the reported levels at 320 and 930 keV, and the latter published values were accepted as the more accurate energies.[21] Additional

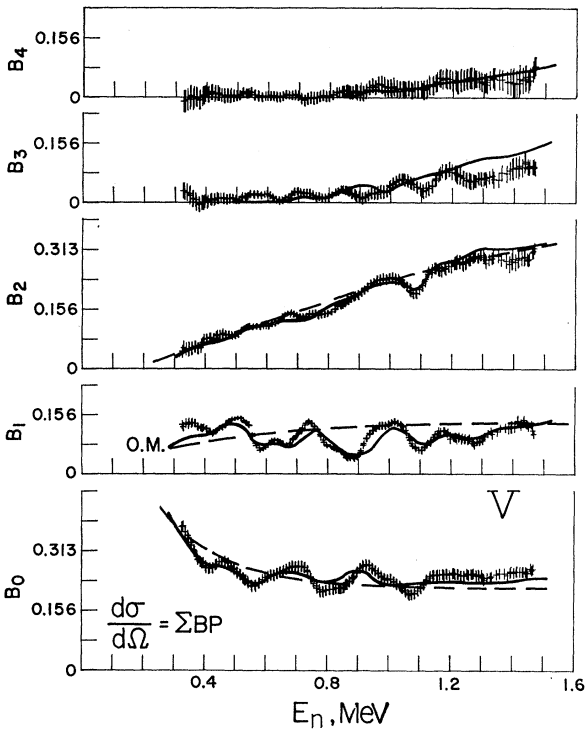

FIG. 2. Differential elastic scattering cross sections of vanadium given in the format of Eq. (1); B_i are in b; a 50-keV average of the measurements is indicated by the data points. The solid and dashed curves are the result of calculation as described in Sec. 4 A of the text.

states in vanadium have been reported at 480, 645, and 1160 keV.[21–23] It was judged that these states would have been observed in the present work if the respective excitation cross sections were $\gtrsim 10$, $\gtrsim 2$, and $\gtrsim 4$ mb/sr. Within these uncertainties, there was no evidence for the existence of such states. This conclusion was consistent with the results of recent inelastic neutron-scattering measurements at higher energies,[24] and with shell-model calculations.[25]

The differential cross sections for the excitation of the 320- and the 930-keV states were measured from several hundred keV of the respective thresholds to a maximum incident energy of 1.5 MeV. In general, the measurements were made at the same scattering angles as employed in the elastic studies. The observed angular distributions did not deviate appreciably from isotropy. The differential cross sections for the excitation of the 320-keV state were fitted, by the method of least squares, with an expansion in Legendre polynomials through the P_2 term. The coefficients of the expansions showed energy-dependent fluctuations, and these were qualitatively correlated between P_0 and P_2 coefficients.

[20] A. G. Guseinov, Atomnaya Energiya Vol. **18**, 409 (1965).
[21] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington, D.C., 1963), detailed references contained therein; see also M. O'Brien, Nucl. Phys. **A104**, 609 (1967); M. Mazari *et al.*, Phys. Rev. **112**, 1691 (1958); M. Lopez *et al.*, Nucl. Phys. **A94**, 673 (1967).
[22] H. J. Hausman *et al.*, Phys. Rev. **88**, 1296 (1952).
[23] S. Ofer and R. Wiener, Phys. Rev. **107**, 1639 (1957).
[24] J. Towle (private communication); see also *Proceedings of the Conference on Study of Nuclear Structure with Neutrons* (North-Holland Publishing Co., Amsterdam, 1966), paper No. 37.
[25] N. Auerbach, Phys. Letters **34B**, 260 (1967).

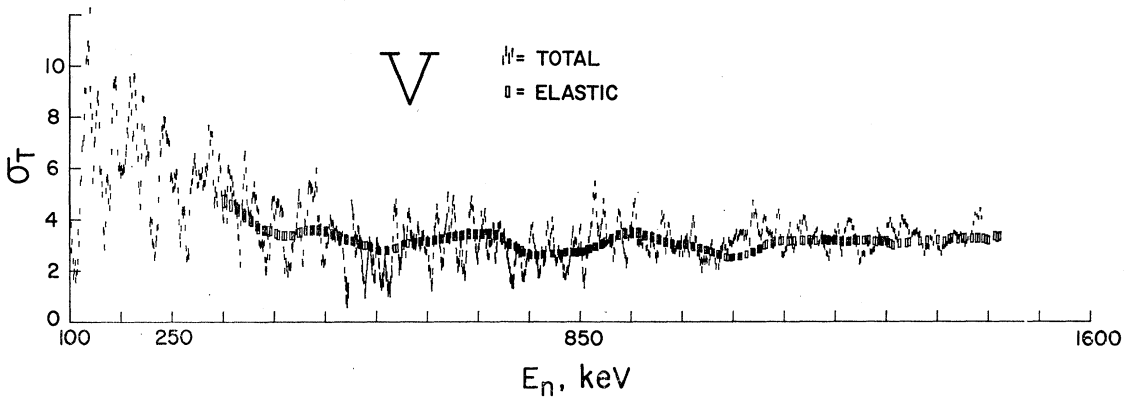

FIG. 3. Comparison of experimental total neutron cross sections (vertical bars) and total elastic scattering cross sections (boxes) of vanadium.

However, both P_1 and P_2 coefficients were small, and the excitation cross sections obtained by angle-integrating the Legendre expansions differed only slightly (1→5%) from those derived from a simple average of the differential measurements multiplied by 4π. The latter approximating procedure was followed in obtaining the inelastic excitation cross sections shown in Fig. 4. As was true of the elastic cross sections, the inelastic cross sections indicated partially resolved resonance structure. The cross-section uncertainties shown in Fig. 4 are subjective estimates of standard deviations and include uncertainties in the carbon reference standard.

The present results can be compared with the few previously reported values. Towle reports cross sections for the excitation of the 320- and 930-keV states at incident energies of ≈1.5 MeV. These are similar to the results of the present work.[22] Results of $(n, n'\gamma)$ measurements by Mathur *et al.* at 0.8, 1.0, and 1.5 MeV are consistent with the values obtained in the present work when cognizance is taken of the anisotropy of the emitted quanta and the effects of branching ratios.[26] Barrows reports $(n, n'\gamma)$ results at incident energies of 1.83 MeV and above.[27] When extrapolated downward to the 1.5-MeV maximum energy of the present work, Barrow's results are ≈20% higher than those reported here. This may in part be due to the use of the rapidly varying Fe$(n, n'\gamma)$ ($Q=-0.85$ MeV) cross section as a reference standard in Barrows's work.

4. PHYSICAL INTERPRETATIONS

A. Optical-Model and Statistical Calculations

The optical-model and statistical methods were used in the phenomenological interpretation and extrapolation of the experimental results.[3,4,28,29] The potential employed was based upon the surface-absorption model of Moldauer.[30] This model has been extended by Engelbrecht and Fiedeldey to an equivalent nonlocal potential of the form[31]

$$V_c(r) = -(V+iU)f_1(r) - if_2(r)W + V_{so}(h/\mu_\pi c)^2\boldsymbol{\sigma}\cdot\mathbf{l}r^{-1}(df_1/dr),$$

where

$$\begin{aligned} &f_1(r) = [1+\exp(r-R_1)/a_1]^{-1}, \\ &f_2(r) = \exp\{-[(r-R_2)/a_2]^2\}, \\ &R_1 = r_1 + r_0A^{1/3}, \quad R_2 = R_1 + r_2, \\ &r_0 = 1.16\text{ F}, \quad r_1 = 0.6\text{ F}, \quad r_2 = 0.5\text{ F}, \\ &a_1 = 0.62\text{ F}, \quad a_2 = 0.5\text{ F}, \\ &V = V_0 - 0.25E, \\ &W = W_0 - 0.2E, \\ &V_{so} = 7.0, \\ &U = 0.125E - 0.004E^2 \text{ (with all energies in MeV).} \end{aligned} \tag{2}$$

Letting $E\to 0$, $U\to 0$, and setting $V_0=46$ and $W_0=14$ MeV, Eq. (2) becomes the potential of Moldauer. This potential and the Hauser-Feshbach formalism give a good description of the elastic scattering observed in this work [indicated by the dashed curves of Fig. (2)] and lead to calculated $l=0$ strength functions similar to those reported from experiment.[18,32]

[26] Mathur *et al.* (private communication).
[27] D. Barrows, thesis, University of Kentucky, 1965 (unpublished).
[28] L. Wolfenstein, Phys. Rev. **82**, 690 (1951).
[29] F. Perey, in *Proceedings of the Conference on the Study of Nuclear Structures with Neutrons* (North-Holland Publishing Co., Amsterdam, 1966), p. 418.
[30] P. A. Moldauer, Nucl. Phys. **47**, 65 (1963).
[31] C. Engelbrecht and H. Fiedeldey (private communication).
[32] J. Harvey as quoted by K. Seth, Nucl. Data **2**, No. 3 (1966).

A detailed description of the structure observed in the elastic scattering was sought by varying V_0 and W_0 ($U=0$ and other parameters maintained constant) to obtain the best fit to the individually observed elastic scattering angular distributions.[33] The calculated results were inclusive of compound elastic scattering contributions obtained from the Hauser-Feshbach formula and the reported excited structure of vanadium[21] (see Fig. 5). The angular distributions calculated using this fitting procedure were descriptive of individual measurement, as illustrated in Fig. 6, and followed the intermediate structure observed in the elastic scattering, as indicated by the solid curves of Fig. 2. V_0, W_0, and associated transmission coefficients obtained from the fitting reflected the intermediate structure, which fluctuated with energy.

The derivation of the above fluctuating potential parameters by detailed fitting was not above question. However, V_0 and W_0 obtained from the fitting procedure approached asymptotic values of 43 and 8 MeV, respectively, above energies of about 1.1 MeV. Elastic scattering cross sections calculated from these "asymptotic parameters" were in reasonable agreement with experiment above $\approx$0.8 MeV, as indicated by the solid curves in Fig. 7, but the agreement degenerated at lower energies, and the calculated $l=0$ strength function was about half that reported from microscopic measurement.

The fluctuating potential parameters obtained from the above fitting procedures and the Hauser-Feshbach formula were used to calculate the inelastic excitation cross sections (Fig. 4). The calculated values qualitatively display structure similar to that experimentally observed, though the calculation was based upon the potential obtained entirely from the fit to the elastic

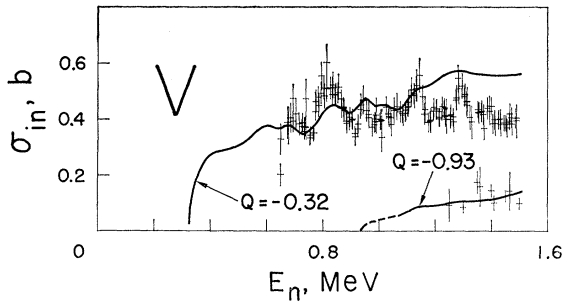

FIG. 4. Measured cross sections for the excitation of states in vandadium at 0.32 and 0.93 MeV. The solid curve was obtained by calculation as described in Sec. 4 A of the text. Estimated experimental uncertainties are indicated by vertical portion of the data crosses.

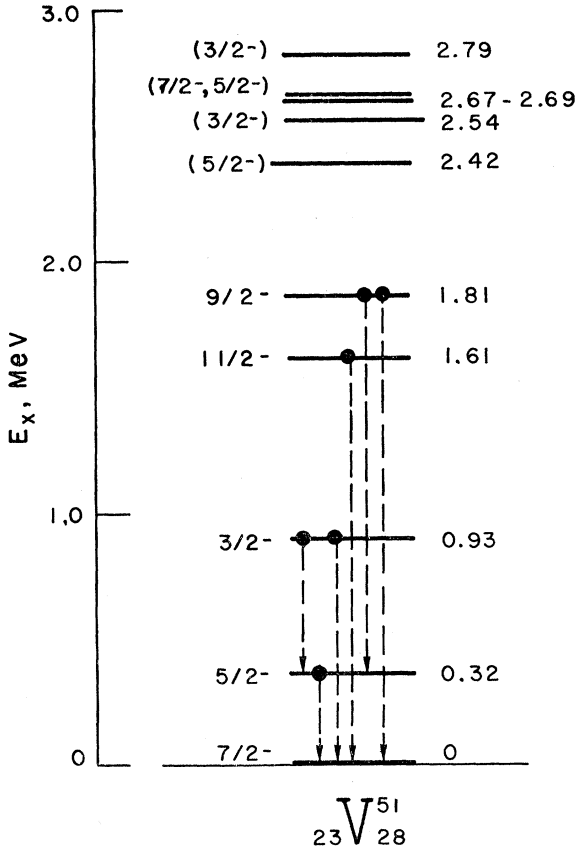

FIG. 5. Excited structure of vandium taken from Ref. 21 and used in calculations of Sec. 4 A. The structure was consistent with the present experimental results.

data. Inelastic excitation cross sections calculated from the "asymptotic potential" ($V_0=43$, $W_0=8$) were in fair agreement with experiment at energies above $\approx$1.0 MeV, as shown by the solid curves of Fig. 8, but were less descriptive of experiment at lower energies. The low-energy discrepancies tended to be potential-independent and were evident in other reported calculations employing different potentials.[24,34] Results of calculations made with changes of one unit of spin for both the 320- and 930-keV states compared poorly with the experimental results. This, together with the previously reported evidence,[24] made it difficult to accept spin and parity assignments differing from those shown in Fig. 5.

It is known that compound-nucleus resonance widths fluctuate and interfere in a statistically describable manner and that transmission coefficients are no longer described by $T=2\pi\langle\Gamma\rangle D^{-1}$ when $T\rightarrow 1$. The effects of such resonance-width fluctuation and interference have been extensively discussed by Moldauer.[5,35] He derives

[33] All optical-model calculations employed one or both of the following computing programs: E. Auerbach, ABACUS-II (Program Operation and Input Description), Brookhaven National Laboratory Report No. BNL-6562 (unpublished); ABACUS II (revised version) 1962 (unpublished); P. Moldauer *et al.*, NEARREX (A Computer Code for Nuclear Reaction Calculation), Argonne National Laboratory Report No. ANL-6978, 1964 (unpublished).

[34] E. M. Pennington and J. C. Gajniak, Argonne National Laboratory Report No. ANL-7387, 1968 (unpublished).

[35] P. Moldauer, Phys. Rev. **123**, 968 (1961).

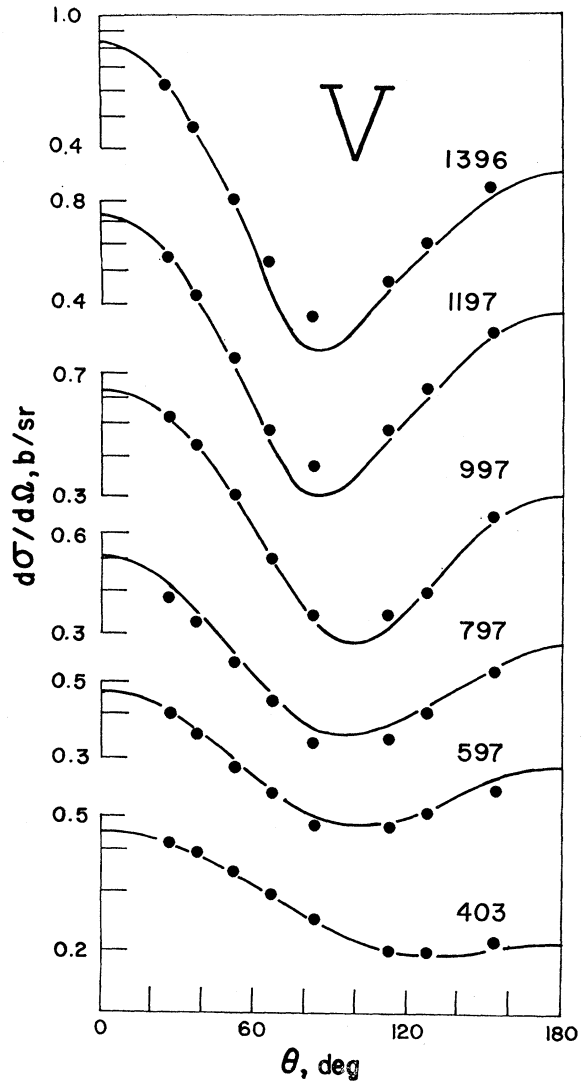

FIG. 6. Comparison of measured (points) and calculated (lines) differential elastic distributions at selected incident-neutron energies (keV). See Sec. 4 A of text for discussion of the calculations.

a "corrected" transmission coefficient θ_α of the form

$$\theta_\alpha = 2Q_\alpha^{-1}[1-(1-Q_\alpha T_\alpha)^{1/2}]. \quad (3)$$

For well-isolated resonances $\Gamma \ll D$, T is small and $\theta \to T$. Using the "asymptotic potential" and the corrected transmission coefficients of Eq. (3), both elastic and inelastic cross sections were calculated for the limiting cases $Q=1$ (isolated resonances) and $Q=0$ (strongly overlapping resonances). The results are indicated in Figs. 7 and 8. Results obtained with intermediate choices of the Q parameter lay between the limits set by the above extreme cases. The correction factors and choice of the Q parameter did not lead to calculated elastic scattering differing appreciably from that obtained with the uncorrected Hauser-Feshbach formalism. The effect of the correction factors on the calculated inelastic cross sections was appreciable, the corrected values lying 20–30% below the Hauser-Feshbach results and in poorer agreement with experiment. These qualitative effects of the correction factors were not appreciably dependent upon the choice of the optical potential. Thus, comparison with the present experiments gave little support to the use of correction terms, though comparisons by other workers at higher energies indicated the desirability of the fluctuation correction.[24]

It is interesting to examine the applicability of the "asymptotic potential" over a wide energy range since the parameters of Eq. (2) are energy-dependent and the volume absorption becomes significant at higher energies. The calculated total neutron cross section compares well with experiment to 10.0 MeV.[16] The calculated elastic angular distributions are in agreement with experimental results reported at incident energies of 2.35, 3.0, and 7.05 MeV (see Fig. 9, for example).[24,36,37] At 14.7 MeV, the agreement with experi-

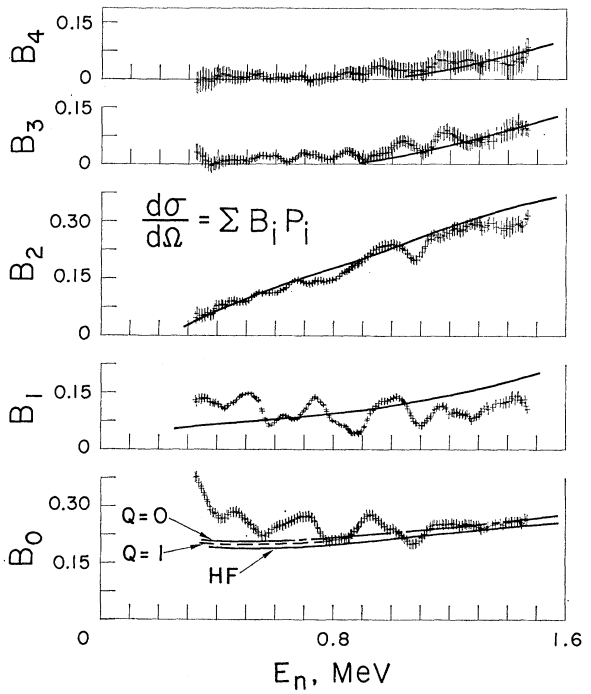

FIG. 7. Measured (crosses) and calculated (curves) elastic distribution of vanadium. The "asymptotic potential" form of Eq. (2) was used. The Hauser-Feshbach formalism was employed to estimate compound elastic contributions with the result indicated by solid curves. Curves denoted as $Q=0$ and $Q=1$ were obtained by applying correction factors as described in Sec. 4 A of text. All calculated results lead to essentially identical $B_1 \to B_4$ coefficients.

[36] R. Becker *et al.*, Nucl. Phys. **89**, 154 (1966).

[37] B. Holmqvist and T. Wiedling, in *Proceedings of the Second Conference on Neutron Cross Section and Technology, Washington, D.C., 1968*, edited by D. T. Goldman, (U.S. Government Printing Office, Washington, D.C., to be published), Vol. 1, p. 845.

ment was less satisfactory,[38] although it is not clear whether the discrepancies were due to defficiencies in the calculation or experiment or both. Inelastic scattering calculated with the "asymptotic potential" and the Hauser-Feshbach formalism was in relatively good agreement with values measured at 2.35 MeV.[24]

B. Correlations and Fluctuations

Correlations and fluctuations in the measured cross sections were examined with the intent of ascertaining the statistical properties of the observables and their physical significance. The interpretation took several forms: (a) a search for a quantitative measure of intermediate resonance structure, (b) an analysis based upon the premise of strongly overlapping resonances (Ericson fluctuations),[6,39,40] and (c) a derivation of average level density from fluctuations in widths and spacings of compound-nucleus resonances.[7,8]

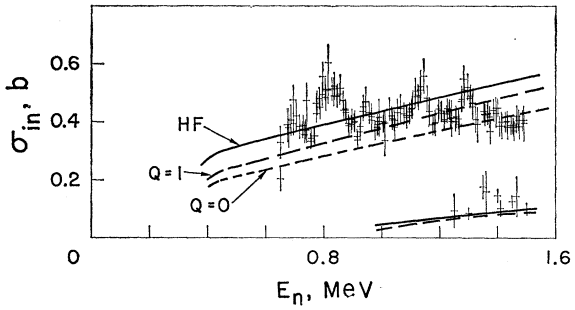

FIG. 8. Calculated (curves) and measured (crosses) cross sections for the excitation of the 0.32- and 0.93-MeV states in vanadium. The interpretation of the calculated curves is discussed in Sec. 4 A of the text.

1. *Existence of Intermediate Resonance Structure*

Intermediate structure can be attributed to measured cross sections by qualitative inspection. Such procedures can be deceptive.[41] A quantitative approach makes use of the function[8,42]

$$C(R)=N^{-1}\sum_{i=1}^{N}[\sigma(E_i)-\bar{\sigma}(E_i)]^2, \qquad (4)$$

where

$$\bar{\sigma}(E_i)=R^{-1}\int_{E_i-R/2}^{E_i+R/2}\sigma(E)\,dE.$$

$C(R)$ increases with R, asymptotically approaching a constant value for R much larger than the average structure width of the fluctuating cross section. In the presence of two very different resonance widths (Γ_1 and Γ_2, where $\Gamma_1 \ll \Gamma_2$) $C(R)$ increases with R, approaching an initial plateau value for $\Gamma_1 < R < \Gamma_2$, and then rises to a second and final plateau for $R > \Gamma_2$. We note that Γ_1 can be associated with the compound-nucleus width (Γ_{CN}) and Γ_2 with a much larger intermediate resonance width (Γ_{IS}). The experimental resolution employed in the present total cross-section measurements was $\gtrsim\Gamma_{CN}$, sufficient to clearly indicate inter-

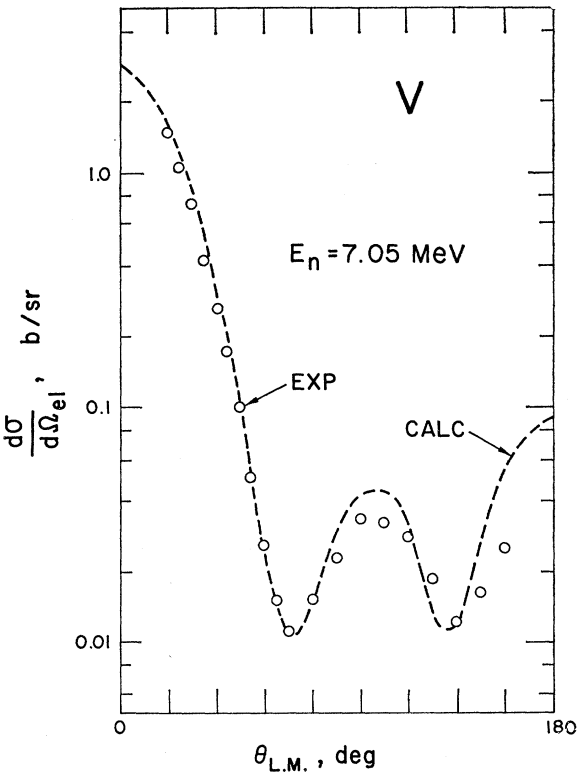

FIG. 9. Elastic scattering from vanadium at 7.05 MeV. Points are measured values reported by Holmqvist and Weidling (Ref. 37). The curve was calculated using "asymptotic potential" discussed in Sec. 4 A of the text.

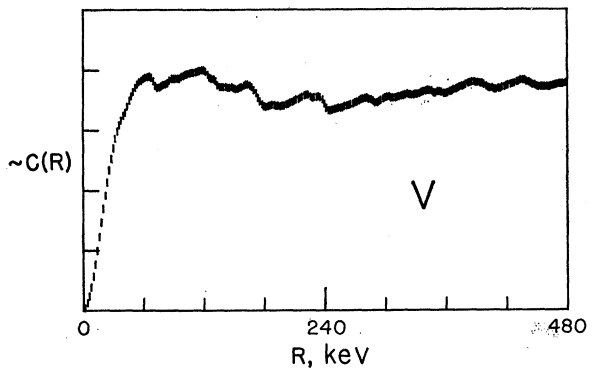

FIG. 10. Function $C(R)$, Eq. (4) of the text, evaluated from the measured total neutron cross sections of vanadium for $0<R<$ 480 keV.

[38] Western *et al.*, General Dynamics, Fort Worth, Report No. AFWL-TR-65-216, 1965, Vol. II (unpublished).
[39] T. Ericson and T. Mayer-Kuckuk, Ann. Rev. Nucl. Sci. **16**, 183 (1966).
[40] T. Ericson, Phys. Letters **4**, 258 (1963).
[41] P. P. Singh *et al.*, Phys. Letters **23**, 255 (1966).
[42] G. Pappalardo, Phys. Letters **13**, 320 (1964).

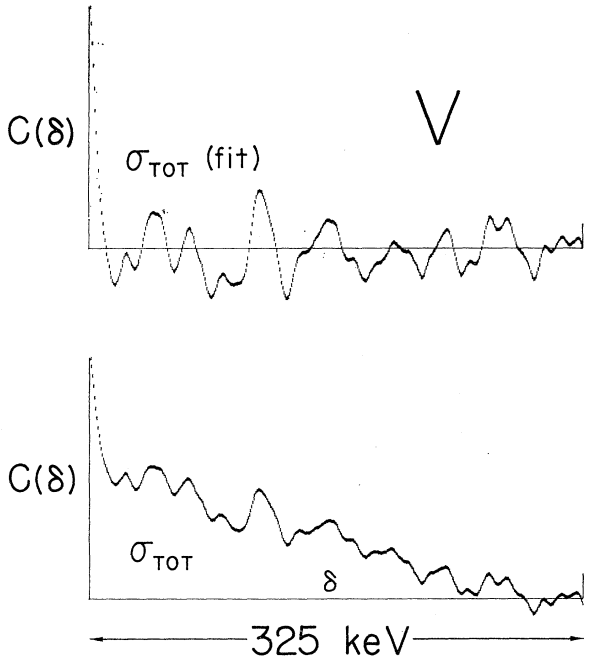

FIG. 11. Lower: $C(\delta)$ [Eq. (6)] evaluated from the total neutron cross sections on vanadium. The effects of direct reactions and the finite sample range are evident. Upper: $C(\delta)$ evaluated from the total neutron cross sections corrected for direct reactions.

mediate structure if present to an appreciable extent. The function $C(R)$, calculated from the total cross-section values determined in the present experiment, is shown in Fig. 10. The function rises to a single plateau of an oscillatory nature characteristic of finite-range deviations (FRD) inherent to the limited experimental sample.[43,44] This behavior of $C(R)$ is consistent with a single average width of a few keV. The intermediate structure qualitatively apparent in the total neutron cross sections (see Fig. 1) must, if valid, be of such a magnitude as to be masked by FRD and statistical fluctuations, and to lead to an inconclusive interpretation of the $C(R)$ distribution. $C(R)$, derived from the measured elastic scattering cross sections, oscillated with such an amplitude as to preclude any reasonable assay of intermediate structure.

2. *Limit of Strongly Overlapping Resonances* $\Gamma \gg D$

Fluctuations have been observed in cross sections at energies where the compound-nucleus width is expected to greatly exceed the spacing ($\Gamma_{CN} \gg D_{CN}$).[8,16,45–47] Ericson has studied these fluctuations theoretically and related observable quantities to average compound-nucleus properties.[6,39,40] As used in simplified form here, his interpretation is based upon a number of assumptions, primary of which are (a) the reactions are statistical, (b) $\Gamma \gg D$, (c) $\Gamma^J \rightarrow \Gamma = \text{const}$, (d) experimental resolutions $< \Gamma$, and (e) there exist no single- or several-particle resonance effects. Qualitatively, $\Gamma/D = n/2\pi$, where n is the number of effective exit channels.[48] In the present experiments, it was likely that n was not $\gg 2\pi$; rather, $l=0$ strength functions[18] indicated that Γ/D was more nearly unity. $\Gamma^J \rightarrow \Gamma = \text{const}$ implies a number of available exit channels and a large nuclear moment of inertia, not particularly characteristic of the present situation. The effect of several particle resonances may be appreciable over much of the measured energy interval. Further, the experimental resolution employed in the present total cross-section measurements was $\lesssim \Gamma$ only in limited and lower energy regions. Thus, this simplified interpretation of the present work in the framework of Ericson fluctuations is open to considerable question. The approach is pursued here only in the spirit of a qualitative experimental assay. Ericson defines the total cross-section correlation function[6]

$$F(\epsilon) = \langle [\sigma(E+\epsilon) - \langle \sigma(E) \rangle][\sigma(E) - \langle \sigma(E) \rangle] \rangle,$$

where

$$F(\epsilon) = \frac{\Gamma^2}{\epsilon^2+\Gamma^2} \kappa \frac{D_0}{\pi\Gamma} \frac{(\pi\lambda^2)^2}{(2i+1)(2I+1)} \sum_l (2l+1) T_l^2 \quad (5)$$

and κ is a measure of the distribution of partial widths around the mean ($\sim$1.5 Porter-Thomas distribution[49]), $D^J = D_0/(2J+1)$, T_l = optical-model transmission coefficient, and i and I are the spins of the incident and target particle, respectively. In the ratio form, Eq. (5) is given by

$$C(\delta) = \langle \sigma(E+\delta)\sigma(E) \rangle / \langle \sigma(E+\delta) \rangle \langle \sigma(E) \rangle - 1 = N^{-1}(1-Y^2)\Gamma^2/(\delta^2+\Gamma^2), \quad (6)$$

where N is the number of contributing independent channels and Y is the proportion of direct reactions. Equation (6) is independent of D_0 and κ.[6,44] Further,

$$C(0) = N^{-1}(1-Y^2)$$

and

$$F(0) = \kappa(D_0/\pi\Gamma)[(\pi\lambda^2)^2/(2i+1)(2I+1)] \times \sum_l (2l+1) T_l^2. \quad (7)$$

In the following, Eqs. (6) and (7) are employed to obtain N and Γ from the measured total cross sections and, with the T_l values calculated using the "asymptotic potential" of Sec. 4 A, the value of D_0. Corrections are made for the effect of the finite experimental

[43] I. Hall, Phys. Letters **10**, 199 (1964).
[44] I. Hall, Oxford University Nuclear Physics Laboratory Report No. 37-64, 1964 (unpublished).
[45] H. Vonach and J. Huizenga, Phys. Rev. **138**, B1372 (1965).
[46] G. Calvi *et al.*, Nucl. Phys. **48**, 408 (1963).
[47] D. W. Glasgow and D. G. Foster, Hanford Laboratory Report No. HW-SA-2875, 1963 (unpublished).
[48] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[49] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).

TABLE I. Results of correlation analysis of measured total cross sections.

Energy interval (MeV)	0.10–1.44 (entire range)	0.10–0.42	0.21–0.53	0.32–0.64	0.43–0.75	0.53–0.86	0.64–0.97	0.75–1.07	0.85–1.18	0.97–1.29
$10^2C(0)$	3.78	10.1	4.73	3.53	3.00	2.34	1.80	1.52	1.12	0.81
$10b^2F(0)$	11.03	30.3	13.84	9.57	8.48	6.47	5.46	4.39	3.46	2.35
Γ (keV)	4.0	5.1	4.4	3.7	3.8	3.8	3.9	4.0	3.2	3.0
D_0 (keV)	410.	540.	304.	223.	225.	189.	171.	162.	115.	83.
$D=D_0/(2J+1)$ [a]	49.	65.	36.	26.	27.	23.	20.	19.	13.	10.
$10^{-4}(\Gamma/D)_0$ [a,b]	0.96	1.5	2.0	2.0	1.8	2.0	2.2	2.2	2.4	2.8

[a] Assuming $2J+1\sim 8$.

[b] All values corrected to 1 eV.

resolution.[50,51] These corrections are not negligible, particularly at higher energies where the assumptions of the theory become more applicable to the experiment.

$C(\delta)$, calculated from the measured total cross sections, is shown in the lower portion of Fig. 11. The function decreases rapidly from a maximum at $\delta=0$ to about half magnitude, then slowly decreases in an oscillatory manner with increasing δ. The slow decrease at large δ is characteristic of a direct reaction (DR) modulation, and the oscillations are attributed to the finite sample size.[43,44] The DR modulation (shape scattering) could be removed from the measured data using results of a suitable optical-model calculation, but a model calculation of sufficient accuracy is difficult to achieve. As a phenomenological alternative, the DR component was determined by means of a least-squares fit of a cubic energy dependence to the measured total cross sections. The order of the fit was such as to well describe the coarse energy dependence without unduly distorting fine or intermediate cross-section structure. The smoothly varying cross section determined from this fitting procedure was subtracted from the measured cross section to obtain the "fluctuating cross section" for subsequent correlation analysis. $F(\epsilon)$ and $C(\delta)$ were evaluated from this fluctuating total cross section over the entire measured energy interval and a number of subintervals thereof. Typical of the results is $C(\delta)$ calculated for the entire interval (upper portion of Fig. 11). FRD oscillations remain evident and are of an rms magnitude consistent with the estimates of Hall.[43,44] Neither in this example, nor in any other $C(\delta)$ calculation, was a significant deviation from a simple Lorentzian form noted, as would be expected from appreciable intermediate resonance structure with a width $\Gamma_{IS}\gg\Gamma_{CN}$. Values for $C(0)$, $F(0)$, Γ, and D_0, derived from the correlation analysis of the total cross sections and carried out over the entire measured interval and subintervals thereof, are summarized in Table I. Uncertainties in the calculated $C(0)$ and $F(0)$ values are large; FRD alone lead to standard deviations of 30% or more. The Γ values are remarkably consistent, and D_0 shows the expected energy dependence. However, D_0 tends to be an order of magnitude larger, and Γ a factor of 2 to 3 larger, than indicated by the level-density formulas that are the results of detailed low-energy resonance studies and the analysis of average cross sections at low energies.[18,52,53] These discrepancies tend to be compensated for when the $l=0$ strength function is estimated, resulting in a value $\approx\frac{1}{5}$ of that reported from detailed resonance studies and $\approx\frac{1}{2}$–$\approx\frac{1}{3}$ of that obtained from average low-energy cross sections. In view of the doubtful validity of the theory in the present experimental context, the results are probably remarkable for their agreement with values obtained by other and more proper methods, and may indicate that such analysis of total neutron cross sections are not sensitive to these theoretical concepts and underlying premises.

As is evident from Eq. (6), correlations are damped as the number of exit channels increases, and thus should become more pronounced as the number of reaction channels is restricted. In an effort to obtain more definitive results, $C(\delta)$ was evaluated for the B_l coefficients of the elastic channel and for the excitation cross section of the inelastic channel ($Q=-0.32$ MeV). Unfortunately, all of these correlation functions showed strong FRD oscillations because of the limited extent of the available sample. These oscillations and the experimental resolutions appreciably exceeding Γ_{CN} precluded any significant interpretation of the $C(\delta)$ derived from either the elastic or inelastic scattering measurements.

The cross correlation function

$$C_{ab}=\kappa G_{ab}, \tag{8}$$

where

$$G_{ab}=\frac{\langle\sigma_a\sigma_b\rangle}{\langle\sigma_a\rangle\langle\sigma_b\rangle}-1,$$

$$\kappa=(C_a(0)C_b(0))^{-1/2},$$

[50] C. Brink, in *Proceedings of the Conference on Nuclear Structure Study with Neutrons* (North-Holland Publishing Co., Amsterdam, 1966), p. 278.

[51] K. Tsukada and T. Lee, Phys. Letters **11**, 141 (1964).

[52] K. K. Seth *et al.*, Phys. Letters **13**, 70 (1964).

[53] A. Gilbert and A. G. W. Cameron, Can. J. Phys. **43**, 1446 (1965).

is of interest.[6,44] C_{ab} was determined for the B_l coefficients of the elastic cross section, the elastic and total cross sections, and the elastic and inelastic ($Q=-0.32$ MeV) cross sections. As expected, B_0 ($\propto$ elastic cross section) was strongly correlated with the total cross section (see Fig. 3). Between B_l values, only the B_0-B_1 correlation was significantly larger than FRD errors alone. This is in contrast to the concepts of Ericson[6] which indicate small correlations between even and odd B_l coefficients. There was no significant numerical correlation between elastic and inelastic cross sections, though experimental uncertainties and the limited range of the samples might have masked the small effect that seems qualitatively present from a visual inspection of the respective cross sections.

3. *Cross-Section Fluctuations and Distributions of Widths and Spacings*

In this section, structure in the measured total cross sections of vanadium is considered in the context of fluctuations in compound-nucleus resonance spacings and widths after the manner of Carlson and Barschall[8] and of Agodi and Pappalardo.[7] It is assumed that the experimental resolution appreciably exceeds the average compound-nucleus width, and experimental averages are constructed to insure the validity of the premise. Within these averages, the interference between potential and compound scattering is neglected and the former's smooth energy dependence is removed by the cubic fitting procedure described above. It is further assumed that the partial widths are small compared to their spacings.

With the above premises, Carlson and Barschall derived the total cross-section variance

$$S^2(\text{variance})=(\pi\lambda^2)^2\sum_{J\pi}(g^2/\langle N_{J\pi}\rangle)\times[k_w\sum_{ls}(T_{ls}{}^J)^2+k_n(\sum_{ls}T_{ls}{}^J)^2], \quad (9)$$

where $g=(2J+1)/2(21+1)$, $\langle N_{J\pi}\rangle$ is the average number levels of a given $J\pi$ in the energy interval Δ, and $T_l{}^J$ are transmission coefficients.[4] The derivation makes use of the definitions

$$T_{ls}{}^J=2\pi\langle\Gamma_{ls}{}^J\rangle D_{J\pi},$$

$k_w=$ variance $(\Gamma_{ls}{}^J)/\langle\Gamma_{ls}{}^J\rangle=2$ for a Porter-Thomas distribution of widths (Ref. 49),

$k_n=$ variance $(N_{J\pi})/\langle N_{J\pi}\rangle=0.27$ for a Wigner distribution of spacings (Ref. 54).

Further, it is assumed that the distributions used in calculating the variances are independent of quantum number and that the widths associated with differing quantum numbers and spacings for the same quantum number are uncorrelated. Some of the above assumptions are not without question.[5,55] Separating the energy and spin dependence of the level density in the manner of Gilbert and Cameron,[53] $\langle N_{J\pi}\rangle/\Delta=G(E)H(J\pi)$ and Eq. (9) reduces to

$$S^2\Delta=(\pi\lambda^2)^2[1/G(E)]\sum_{J\pi}[g^2/H(J\pi)]\times k_w\sum_{ls}(T_l{}^J)^2+k_n(\sum_{ls}T_l{}^J)^2]. \quad (10)$$

Using $T_{ls}{}^J$ obtained from the "asymptotic potential" derived in Sec. 4 A, $H(J\pi)$ from Ref. 53, and S^2 determined from averages of measured values over the interval Δ, $G(E)$ can be obtained. Results for several energy intervals of the measured total cross sections and a number of averaging increments Δ are given in Table II, and the level density derived from the present experiments $[\rho_{\text{expt}}=G(E)H(J\pi)]$ is compared with that given by the formula of Gilbert and Cameron (ρ_{theor}).[53] For averaging increments $\Delta\gg\Gamma$ (i.e., $\Delta\gtrsim 25$ keV), the ρ_{expt} values obtained from the experiments are within a factor of 2 of those given by the density formula, and are similar to values deduced from resonance studies at low energies.[18] The results were not particularly sensitive to the distributions of widths and spacings used in evaluating k_w and k_n. Assuming exponential distributions ($k_w=k_n=1$), or even neglecting width effects ($k_w=0$, $k_n=1$) as in Ref. 7, the results were similar to those obtained with Porter-Thomas and Wigner distributions. Within the experimental uncertainties of 30–50%, the observed structure of the total cross section is consistent with the interpretation based upon recognized distributions of compound-nucleus resonance widths and spacings.

The expression $2\pi\langle\Gamma/D\rangle=T$, used in deriving Eq. (9), is known to be invalid when T approaches unity, as it tends to in the present experiments. Considering resonance interference and S-matrix unitarity, Moldauer has derived the relation $2\pi\langle\Gamma/D\rangle=\ln[1/(1-T)]$, valid for large T.[56] The use of this relation in evaluating Eq. (10) increased the level densities derived from experiment, by about a factor of 3, to values appreciably larger than indicated by either the level-density formula or detailed resonance measurements. Such differences may in part be due to the inappropriateness of Eq. (10) in the present experimental context. The equation is valid for nonoverlapping resonances, and the Bethe randomness hypothesis[7,57] is invoked to extend the validity to situations where partial widths alone are small compared to spacings. It is not clear that such an extrapolation is valid in the present experimental application.

C. Intermediate Optical Model

The intermediate structure quantitatively evident in the energy-averaged total, elastic, and inelastic cross

[54] M. A. Preston, in *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Mass., 1962).

[55] F. Dyson and M. Mehta, J. Math. Phys. **4**, 701 (1963).

[56] P. A. Moldauer, Argonne National Laboratory Report No. ANL-7467, 1968 (unpublished).

[57] H. A. Bethe, Rev. Mod. Phys. **9**, 69 (1937).

TABLE II. Comparison of experimentally and theoretically derived level densities.

Averaging increment (keV)		Incident energy interval (MeV) 0.10–0.545 av=0.327	0.545–0.990 av=0.767	0.990–1.435 av=1.212
$\Delta\sim 1$–3[a]	$S^2\Delta$	$7.49\times10^{-3}b^2$ MeV	1.8×10^{-3}	0.56×10^{-3}
	ρ_{expt}	3.37×10^{3} MeV^{-1}	2.77×10^{3}	8.16×10^{3}
	ρ_{theor}[b]	1.27×10^{3} MeV^{-1}	1.73×10^{3}	2.34×10^{3}
	ρ_{expt}/ρ_{theor}	2.6(2.5,[c] 1.3[d])	1.59(1.8,[c] 1.2[d])	3.47(4.5,[c] 3.2[d])
$\Delta=10$	$S^2\Delta$	1.65×10^{-2}	0.38×10^{-2}	0.13×10^{-2}
	ρ_{expt}	1.53×10^{3}	1.34×10^{3}	3.50×10^{3}
	ρ_{expt}/ρ_{theor}	1.20(1.1,[c] 0.6[d])	0.78(0.9,[c] 0.59[d])	0.78(1.93,[c] 1.36[d])
$\Delta=25$	$S^2\Delta$	0.136×10^{-1}	0.477×10^{-2}	0.216×10^{-2}
	ρ_{expt}	1.85×10^{3}	1.09×10^{3}	2.14×10^{3}
	ρ_{expt}/ρ_{theor}	1.46(1.4,[c] 0.7[d])	0.63(0.7,[c] 0.5[d])	0.91(1.2,[c] 0.8[d])
$\Delta=50$	$S^2\Delta$	0.109×10^{-1}	0.495×10^{-2}	0.265×10^{-2}
	ρ_{expt}	2.37×10^{3}	1.05×10^{3}	1.76×10^{3}
	ρ_{expt}/ρ_{theor}	1.82(1.7,[c] 0.9[d])	0.61(0.7,[c] 0.6[d])	0.74(1.0,[c] 0.7[d])
$\Delta=75$	$S^2\Delta$	0.106×10^{-1}	0.546×10^{-2}	0.234×10^{-2}
	ρ_{expt}	2.30×10^{3}	0.95×10^{3}	1.97×10^{3}
	ρ_{expt}/ρ_{theor}	1.87(1.7,[c] 0.9[d])	0.549(0.6,[c] 0.4[d])	0.842(1.1,[c] 0.8[d])

[a] Raw experimental data, resolution as per Sec. 3 A.
[b] Calculated from form of Gilbert and Cameron (Ref. 53).
[c] Ratio assuming exponential distribution of widths and spacings ($k_w=1$, $k_n=1$).
[d] Ratio assuming only exponential spacing distribution ($k_w=0$, $k_n=1$).

sections of vanadium can be characterized by a width Γ_{IS} and a spacing D_{IS}, and is large compared to the structure of the compound nucleus and small relative to that of the single-particle or diffraction "giant resonances" (i.e., $\Gamma_{CN}<\Gamma_{IS}<\Gamma_{SP}$, $D_{CN}<D_{IS}<D_{SP}$). Structure of the type observed here has been qualitatively interpreted in terms of "doorway-state" processes.[1,58,59] The "doorway state" has been quantitatively studied in the context of CN fine structure by Lejeune and Mahanx[60] and by Takeuchi.[9] An "intermediate optical model" has been suggested by Lipperheide,[10,61] Feshbach *et al.*,[1] and Takeuchi.[9] This concept has the advantages of simplicity of application and avoiding excessive demands on experimental energy resolution. The "intermediate optical model" proposed by Takeuchi is used in the following comparisons with experimental results.[9,62] It assumes well-isolated doorway states and no direct reactions, and is restricted to two open s-wave channels (elastic and inelastic). With these conditions, Takeuchi constructs an optical potential differing from the conventional form by energy-dependent factors $F_i(E)$ as follows[9,62]:

$$V^{opt}=-\bar{V}-i\bar{W}+\bar{V}_{so}\mathbf{l}\cdot\mathbf{s}+F_1+iF_2, \qquad (11)$$

where

$$F_1=\sum_s S_s\frac{E-E_s}{(E-E_s)^2+[\Gamma_s^{\downarrow}/2+\Gamma_s^{\uparrow}/4]^2},$$

$$F_2=\sum_s S_s\frac{-\Gamma_s^{\downarrow}/2}{(E-E_s)^2+[\Gamma_s^{\downarrow}/2+\Gamma_s^{\uparrow}/4]^2},$$

and where E_s is the energy of the sth doorway state, $\Gamma_s^{\downarrow}$ is the decay width of the sth doorway to the CN, $\Gamma_s^{\uparrow}$ is decay width to the continuum, and S_s is the strength of interaction with the s doorway.

The imaginary component $\bar{W}$ of Eq. (11) is due to reactions not proceeding explicitly through the identified doorway states.

The potentials $\bar{V}$, $\bar{W}$, and V_{so} are assumed to follow the spacial distributions of the phenomenological potential given in Eq. (2). The doorways are, of course, classified as to $J\pi$, each with associated strengths, resonant energies E_s, and widths $\Gamma_{J\pi}{}^{\downarrow}$. Averaged over energies that are large compared to those characteristic of the doorways, $F_1\rightarrow 0$, $F_2\rightarrow$ const, and Eq. (11) reduces to the "conventional" optical potential.[9] Other aspects of the intermediate potential such as nonlocality and energy dependence have been extensively discussed.[9,10,62]

General application of Eq. (11) to the interpretation of experiment leads to the adjustment of a large number of parameters descriptive of each of a number of contributing doorway states. This would be true, for example, of charged-particle reactions at relatively high

[58] L. S. Rodberg, in *Intermediate Structure in Nuclear Reactions* (University of Kentucky Press, Lexington, Ky., 1967).
[59] R. H. Lemmer, in *Intermediate Structure in Nuclear Reactions* (University of Kentucky Press, Lexington, Ky., 1967).
[60] A. Lejeune and C. Mahanx, Z. Physik **207**, 35 (1967).
[61] R. Lipperheide, Nucl Phys. **89**, 97 (1966).
[62] K. Takeuchi (to be published); see also Argonne National Laboratory Report No. ANL-7564, 1969 (unpublished) for theoretical outline.

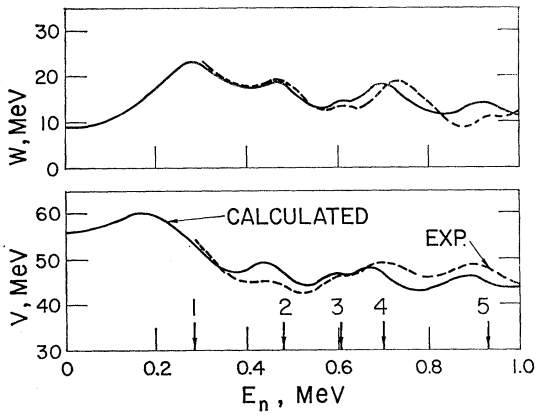

FIG. 12. Real (V) and imaginary (W) portions of the intermediate optical potential CAL, compared with the phenomenological optical potential derived from a fit to the measured elastic scattering distributions EXP. Positions of doorway states are indicated by arrows referring to values of Table III.

energies involving a number of angular momenta.[63] The number of parameters is sharply reduced in the consideration of the present experiments at incident energies <1.0 MeV. Also, $l=0$ processes are predominant with possible doorway configurations of 3^- and 4^-. Only the former can contribute to the inelastic channel ($Q=-0.32$ MeV). The present interpretation is further simplified by assuming the reactions are spin-independent.

The procedure followed was to select by inspection average $\bar{V}$ and $\bar{W}$ values and a limited number of doorways characterized by E_s, $\Gamma_s{}^{\downarrow}$, $\Gamma_s{}^{\uparrow}$, and a strength S_s, and calculate from Eq. (11) a $V^{\rm opt}$ for comparison with the real and imaginary portions of the potential derived from a detailed fit to the measured elastic distributions (see discussion of Sec. 4 A). Commensurate with the assumption of isolated doorways and the known compound-nucleus structure, the number of doorway states considered was restricted to $\lesssim 5$/MeV. Suitable adjustment of doorway parameters was made where indicated, and the above procedure repeated until a reasonable agreement was achieved between $V^{\rm opt}$ and the experimentally derived potential. The "intermediate" potential was then used to calculate elastic and inelastic distributions for direct comparison with the measured values. Typical of the results or the comparisons of real and imaginary portions of $V^{\rm opt}$ and the experimentally derived potential are shown in Fig. 12, and the elastic and inelastic results as measured and as calculated from $V^{\rm opt}$ are shown in Fig. 13. The doorway parameters derived from these fitting procedures, assuming $\Gamma_{s1}{}^{\uparrow}=\Gamma_{s2}{}^{\uparrow}=\Gamma_s{}^{\uparrow}/2$, are given in Table III. In order to avoid an upper-energy "end effect", three additional doorways were postulated, distributed uniformly between 1.1 and 1.6 MeV. Their effect on the calculated results below ~ 0.8 MeV was small. No additional doorway states were assumed below the minimum experimental energy of ~ 0.3 MeV.

$V^{\rm opt}$ qualitatively describes the features of the experimentally derived potential (see Fig. 12), with the more serious differences at the higher incident energies where $l\neq 0$ contributions are of increasing importance. $V^{\rm opt}$ also provides a similar qualitative description of the measured elastic angular distributions (see Fig. 13). The structure of the B_1 coefficient is particularly well portrayed by calculations based upon $V^{\rm opt}$. The correlation between measured and calculated inelastic scattering is less satisfactory, but the former did not appreciably extend over much of the valid energy range of $V^{\rm opt}$. From the doorway parameters and T_0 calculated at the peaks of the respective states using the "asymptotic potential" of Sec. 4 A, the ratios $\Gamma^{\uparrow}/\Gamma^{\downarrow}$ and the total doorway width $\Gamma=\Gamma^{\uparrow}+\Gamma^{\downarrow}$ can be evaluated[1,9,62] with the respective results given in Table III. Though speculative, the quantities are of magnitudes commensurate with the estimates of theory.[1,9] Averaging F_1 and F_2 of Eq. (11) over energy intervals much greater than $\Gamma_{\rm IS}$ or $D_{\rm IS}$ leads to a $V^{\rm opt}$ that reasonably describes the measured results averaged

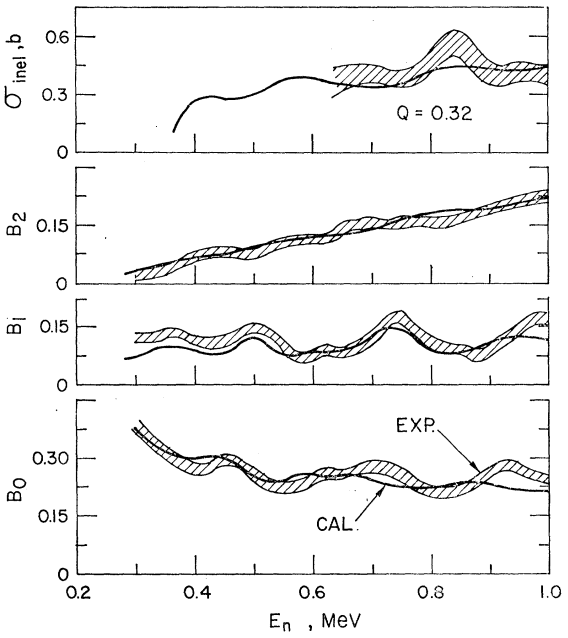

FIG. 13. Comparison of experimental and calculated elastic and inelastic scattering results. Values obtained from the intermediate optical model are indicated by solid curves. Ranges of significance of experimental results are shown as shaded areas The lower three sections of the figure pertain to differential elastic distributions expressed in the form of Eq. (1). The upper section indicates the cross sections for the inelastic excitation of the 320-keV state.

[63] R. H. Davis, in Symposium on Recent Progress in Nuclear Physics with Tandems, Heidelberg, Germany, 1966, Session 5-A (unpublished).

over a 200-keV energy interval. However, the s-wave strength function calculated from V^{opt} for $E \rightarrow 0$ is approximately a factor of 7 smaller than indicated by direct observation[18] or systematics.[54] This may reflect the presence of doorways near zero or negative energies not identified by the present experiments.

The above interpretation is qualitative and, certainly, nonunique. It is reasonably free of gross assumptions. The result is sensitive to the choice of resonance parameters but far less than at higher incident energies where, for example, $l>0$ contributions would be significant. The interpretation is a logical development of the general optical potential. It does provide qualitative knowledge of the properties of doorway resonances without detailed understanding of CN resonance structure, in contrast to the direct correlation between explicit CN and doorway resonance parameters in the manner of Monaham and Elwyn.[64]

5. CONCLUDING REMARKS

The observed total and elastic and inelastic scattering cross sections were characterized by both a fine and intermediate energy-dependent structure. Though of good resolution, the results did not provide a definition commensurate with compound-nucleus resonance analysis, and the interpretation was confined to the assay of the intermediate structure and the average energy dependence of the observed quantities.

A phenomenological optical potential was shown descriptive of total and elastic scattering cross sections of vanadium over a wide energy range. Results of calculations based upon this potential and statistical theory were qualitatively descriptive of the observed inelastic scattering, but quantitatively deviated from measurement, particularly near the reaction thresholds. Consideration of alternative spin-parity assignments and/or the effects of resonance-wdith fluctuation and interference failed to enhance the agreement between calculation and experiment.

An attempt to employ correlation analysis in a quantitative assay of the observed intermediate structure was inconclusive because of the effects of the finite sample range, the presence of direct reactions, and the experimental uncertainties.

The suitability of Ericson's concepts in the context of the present experiments is questionable. However, an interpretation based upon them led to reasonable strength functions, though the deduced Γ and D were inconsistent with both previously reported experimental evidence and the basic premise ($\Gamma \gg D$).

TABLE III. Doorway potential parameters used with Eq. (11).[a]

No. of state	E_s (MeV)	$\Gamma_s^{\uparrow}/\Gamma_s^{\downarrow}$	Γ_s (tot) (MeV)	T_0 Peak	Strength (MeV)
1	0.28	0.3	0.31	0.74	1.80
2	0.48	2.02	0.135	0.88	0.70
3	0.60	3.52	0.11	0.54	0.22
4	0.70	1.99	0.17	0.89	0.99
5	0.93	2.33	0.27	0.78	0.97

[a] $\bar{V}$ and $\bar{W}$ of Eq. (11) are, respectively, 48.0 and 6.5 MeV.

An experimental interpretation of the observed structure in the total cross section, based upon known compound-nucleus distributions of resonance widths and spacings that assumed a nonoverlapping of partial resonance widths, resulted in compound-nucleus level densities similar to those predicted by systematics and by extrapolation from detailed low-energy resonance studies. The result was not particularly sensitive to the form of the distributions of either Γ or D even to the omission of the former.

The results of calculation employing an intermediate optical potential based upon the premise of a few isolated doorway states were in qualitative agreement with the experimental elastic distributions and the phenomenological potential derived from them. This intermediate model employed relatively few doorway parameters, and comparison with experiment led to parameter values of the magnitude expected from theoretical estimates. A wider application of the method may assist in determining some of the statistical properties of doorway states.

In particular, the last two of the above interpretations of the intermediate structure appear physically applicable, and result in qualitative agreement with experiment. The approaches are not unique, and other interpretations have led to similar qualitative successes. The various interpretations are not necessarily mutually exclusive and may each apreciably contribute to the physical reality.

ACKNOWLEDGMENTS

The authors acknowledge the assistance of a number of members of Argonne National Laboratory staff. They are particularly indebted to Dr. P. A. Moldauer for theoretical assistance and guidance.

[64] J. Monahan and A. Elwyn, Phys. Rev. Letters **20**, 1119 (1968).