

entage of both the ground and analog states are reasonably simple even if the wave functions themselves are not. This indication is similar to the more definite results recently obtained³ for certain states in the same mass region with $T_f = T_i$.

It is also of interest to investigate why no other states with $T = T_z + 2$ are produced with observable strength. The case of $^{42}\text{Ca}(p, t)^{40}\text{Ca}$ will be illustrative. The first excited $T = 2$ state in ^{40}Ca would be the 2^+ analog to the 1.46-MeV state in ^{40}Ar . If its wave function were comprised only of the term $[(1d_{3/2})_{21}^{-2}(1f_{7/2})_{01}^2]_{22}$, then DWBA calculations similar to those summarized in Table IV indicate that its intensity would be comparable to the 0^+ , $T = 2$ state. However, unlike the latter state, its configuration should not be dominated by a single term, and most other contributing terms—such as $[(1d_{3/2})_{01}^{-2}(1f_{7/2})_{21}^2]_{22}$ —have no spectroscopic strength for production from the simple target wave function. Evidently, this results in a significant reduction of the intensity with which the state is produced in the (p, t) reaction. Similar arguments apply to other $T = 2$ states in ^{40}Ca as well as to excited analog states in all the nuclei investigated.

Using the IMME [Eq. (1)] and measured energies from Table II, masses can be predicted for a number of neutron-deficient nuclei which are as yet unobserved. The results are given in Table V together with the predictions of Kelson and Garvey.⁴ Both sets of predictions agree throughout.

The method followed in this experiment has been used previously by us to identify analog states with $T \leq 2$ (where $T > |T_z|$). It has been restricted to these low values of T by the fact that the ratio in Eq. (2) is inversely proportional to $(2T_f - 1)$, and for analog states with higher values of T it was anticipated that the (p, t) cross section could be prohibitively small. The observation and firm identification of $T = 3$ states in mass 38 indicate that higher-isospin states can in fact be adequately studied. Consequently, it appears that such investigations as these can be extended to heavier nuclei, particularly those in the $(1f_{7/2})$ shell.

ACKNOWLEDGMENT

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Inelastic Scattering of 42-MeV Alpha Particles by $^{25}\text{Mg}^\dagger$

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Cross sections have been measured for elastic and inelastic scattering of 42-MeV α particles by ^{25}Mg in the range $\theta(\text{lab}) = 10^\circ - 60^\circ$. Six inelastic α groups were identified at the following measured $-Q$ values: 1.59, 1.94, 2.55, 2.75 (doublet), 3.400 (doublet), and 4.05 MeV. The cross sections are analyzed in terms of the distorted-wave Born-approximation version of the extended optical model and the smoothed Fraunhofer inelastic diffraction model. A general discussion is given of the transition strengths of odd-mass nuclei within the context of the strong-coupling rotational model or modifications thereof, and application is made to the present results. The main conclusions are: Inelastic scattering within the ground-state band supports the rotational model in its simplest form. Comparison of the elastic cross sections from ^{24}Mg , ^{25}Mg , and ^{26}Mg indicates the presence of a quadrupole contribution to the elastic cross section from ^{25}Mg , consistent with the strong-coupling prediction. The $\lambda = 2$ single-nucleon contribution to transitions into the lowest $K = \frac{1}{2}^+$ band appears to be very small. The excitation corresponding to the known level at 4.057 MeV suggests that this level arises from coupling a γ vibration to the ground-state band. The second $K = \frac{1}{2}^+$ band may have the same origin, although other evidence indicates that there is at least a sizable admixture of a single-nucleon orbital to this band.

I. INTRODUCTION

THE present paper is the second in a series concerned with the elastic and inelastic scattering of 42-MeV α particles from isotopes which lie in the

middle of the s - d shell. The motivation for the experiments, the general experimental procedures, and methods of theoretical analysis have already been presented in the paper¹ discussing scattering from ^{24}Mg ; only those experimental and theoretical points not already discussed that are pertinent to the nucleus ^{25}Mg will be discussed in Secs. II and III. The angular distributions, their analysis, and interpretations are presented in Sec. IV.

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¹I. M. Naqib and J. S. Blair, *Phys. Rev.* **165**, 1250 (1968).

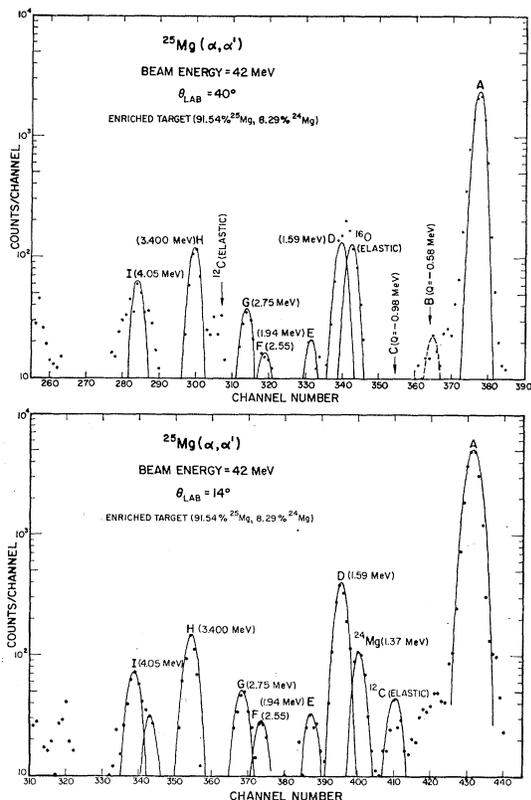


FIG. 1. Two typical pulse-height spectra of α particles scattered by an enriched ^{25}Mg target. Peaks are labelled by the corresponding excitation energies as determined in this experiment. The energy for peak *H* was taken from Ref. 5 and used for calibration of spectra.

II. EXPERIMENTAL PROCEDURES AND PULSE-HEIGHT SPECTRA

The target used in the present experiment was an isotopically enriched ^{25}Mg foil (91.54% ^{25}Mg , 8.29% ^{24}Mg) with a thickness of 3.3 mg/cm². This was thicker than the targets of ^{24}Mg , ^{26}Mg , and ^{27}Al also studied (all less than 1.7 mg/cm²), and thus it was necessary to change the target orientation after each 10-deg change in counter angle to minimize the broadening of peaks due to target thickness.^{3,4}

Two typical pulse-height spectra are shown in Fig. 1. The energy resolution, 150–160 keV (full width at half-maximum), is a shade inferior to that obtained with the targets ^{24}Mg , ^{26}Mg and ^{27}Al .

In addition to the elastic group *A*, the inelastic groups *D*, *E*, *F*, *G*, *H*, and *I* were identified; the corresponding measured values for *Q* are given in Table I. For reference, a diagram showing the energies⁵ and

spin-parity assignments^{6,7} of known low-lying levels of ^{25}Mg through the level at 4.057 MeV is presented in Fig. 2. This diagram shows, in addition, a classification of these levels into rotational bands.⁶

Comparison of Table I and Fig. 2 indicates that there is a one-to-one correspondence between the three peaks *D*, *E*, and *F*, and known levels. Concerning the other three groups, we note:

(i) Group *G* (2.75 MeV) is probably receiving contributions from both the level at 2.738 MeV and the level at 2.801 MeV.

(ii) In principle, both members of the doublet near 3.40 MeV can contribute to peak *H*. We shall see, however, that analysis of the observed angular distribution suggests only weak excitation of the $\frac{3}{2}^-$ member of the doublet. For this reason the peak has been assigned the energy 3.400 MeV; this value has been used for energy calibration of spectra and the subsequent assignment of experimental *Q* values to the other peaks.

(iii) Group *I* (4.05 MeV) receives its main contribution from the level at 4.057 MeV. A shoulder due to the level at 3.905 MeV has been taken into account in the data analysis. Contamination from the negative-parity level at 3.970 MeV appears to be small, but cannot be entirely excluded.

(iv) We did not discern above background any excitation to the known levels at 0.585 and 0.975 MeV (referred to as groups *B* and *C*, respectively).

III. TRANSITION STRENGTHS FOR ODD-MASS NUCLEI

A. General

The observed inelastic cross sections will be analyzed in terms of the distorted-wave Born-approximation (DWBA) version of the optical model and the smooth-edge Fraunhofer model. In the DWBA model, the inelastic cross section for single excitation of collective

TABLE I. Observed inelastic groups and corresponding measured *Q* values.

α Group	$-Q$ (MeV)
<i>D</i>	1.59 ± 0.03
<i>E</i>	1.94 ± 0.03
<i>F</i>	2.55 ± 0.04
<i>G</i>	2.75 ± 0.06
<i>H</i>	3.400 ± 0.005^a
<i>I</i>	4.05 ± 0.05

^a Given by Ref. 5 and used here for absolute reaction energy calibration.

² Obtained from Isotopes Sales Department, Oak Ridge National Laboratory, Oak Ridge, Tenn.

³ B. L. Cohen, Rev. Sci. Instr. **30**, 415 (1959).

⁴ I. Naqib and D. K. McDaniels, Rev. Sci. Instr. **31**, 1358 (1960).

⁵ P. M. Endt and C. van der Leun, Nucl. Phys. **A105**, 1 (1967).

⁶ J. F. Sharpey-Schafer, R. W. Ollerhead, A. J. Ferguson, and A. E. Litherland, Can. J. Phys. **46**, 2039 (1968).

⁷ G. J. McCallum and B. D. Sowerby, Phys. Letters **25B**, 109 (1967).

surface modes may be written in the form

$$(d\sigma/d\Omega)(I \rightarrow I') = \sum_{\lambda} S_{\lambda}(I \rightarrow I') (d\bar{\sigma}/d\Omega)(\lambda, Q). \quad (3.1)$$

Here $(d\bar{\sigma}/d\Omega)(\lambda, Q)$ is a reduced cross section for angular-momentum transfer λ , while S_{λ} is the corresponding collective transition strength defined by

$$S_{\lambda}(I \rightarrow I') = (2I+1)^{-1} \sum_{\mu, M, M'} |\langle I', M' | \xi_{\lambda, \mu} | I, M \rangle|^2, \quad (3.2)$$

where $\xi_{\lambda, \mu}$ is a collective coordinate describing the displacement of the nuclear surface. For the Fraunhofer model, the inelastic cross section may be written analogously

$$(d\bar{\sigma}/d\Omega)(I \rightarrow I') = \sum_{\lambda} S_{\lambda}(I \rightarrow I') (d\ddot{\sigma}/d\Omega)(\lambda), \quad (3.3)$$

where $(d\ddot{\sigma}/d\Omega)(\lambda)$ is the Fraunhofer reduced cross section [Eq. (4.9) of Ref. 1].

A main objective of inelastic scattering experiments is the determination of transition strengths. For excitation of even-mass nuclei, it has become traditional to express these strengths in terms of the deformation distances δ_{λ} or dimensionless deformation parameters β_{λ} of a permanently deformed axially symmetric nucleus

$$S_{\lambda}(0 \rightarrow \lambda) = \delta_{\lambda}^2 = (\beta_{\lambda} R)^2, \quad (3.4)$$

where R is either the radius of the optical potential or the strong absorption radius of the Fraunhofer model. The parametrization is used whether or not the rotational model is valid for the transition in question. For excitation of odd-mass nuclei, the rotational model parametrization of the strengths is, in general, not appropriate since the rotational-model expression for the strength contains several angular-momentum quantum numbers as well as the deformation distance. It is more appropriate here to give directly the values of S_{λ} or $\sqrt{S_{\lambda}}$, and to present values for δ_{λ} only in those cases where there is some indication that the excitation lies within a rotational band.

B. Strong-Coupling Rotational Model

The axially symmetric rotational model implies that the nuclear wave functions may be written as

$$|IMK\rangle = [(2I+1)/16\pi^2]^{1/2} [D_{M, K}^{I*}(\alpha\beta\gamma)\phi_K(q') + (-)^{I-K} D_{M, -K}^{I*}(\alpha\beta\gamma)R\phi_K(q')]. \quad (3.5)$$

Here, $D_{M, K}^{I*}(\alpha\beta\gamma)$ is an element of the rotation matrix, following the conventions of Rose,⁸ where the Euler angles α , β , and γ give the orientation of the body-fixed axes. $\phi_K(q')$ is an internal wave function whose coordinates q' are referred to the body-fixed frame and whose projection of angular momentum on

the z' axis in the body-fixed frame is K . R is the operator for rotation by angle π about the y' axis such that, when it acts on eigenfunctions of total angular momentum $\phi_{J, K}(q')$, it gives

$$R\phi_{J, K}(q') = (-)^{J-K} \phi_{J, -K}(q'). \quad (3.6)$$

For an in-band transition, one in which there is no change in the internal wave functions, the transition strength is easily calculated to be

$$S_{\lambda}(I \rightarrow I') = (I\lambda K 0 | I'K)^2 \delta_{\lambda}^2, \quad (3.7)$$

where δ_{λ} is the deformation distance corresponding to multipolarity λ and to the particular band in question. In this extreme version of the rotational model, the transition strength between two states lying in differing bands is zero. Thus, this model places very stringent conditions on the strengths, conditions which are easily subject to experimental test. The ratios of inband strengths depend solely on the angular-momentum quantum numbers, and the out-of-band strengths should be negligible in comparison to the inband strengths.

C. Quadrupole Contribution to Elastic Scattering

In any situation where the ground state of the target nucleus may have a quadrupole moment, one anticipates that there will be a quadrupole contribution to the observed elastic scattering. When the strong-coupling rotational model is applicable, rather simple predictions can be made for this contribution as well as those of higher multipolarities:

(1) With the further assumption that the scattering amplitudes may be considered only through terms linear in the collective surface coordinates (an approximation made in both DWBA calculations and the usual linear form of Fraunhofer model) the cross section for elastic scattering becomes⁹

$$(d\sigma/d\Omega)(I \rightarrow I) = (d\sigma/d\Omega)(0, 0) + \sum_{\lambda \neq 0, \text{ even}} (I\lambda K 0 | IK)^2 \delta_{\lambda}^2 (d\bar{\sigma}/d\Omega)(\lambda, 0), \quad (3.8)$$

where $(d\sigma/d\Omega)(0, 0)$ is the cross section for elastic scattering from the undeformed nucleus, and where $(d\bar{\sigma}/d\Omega)(\lambda, 0)$ represents the reduced inelastic cross section for either the DWBA or Fraunhofer models with $Q=0$.

(2) The above expression can be justified for less restrictive situations; thus Satchler¹⁰ has shown that it also holds when all contributions to the cross section through second order in the collective coordinates are retained, provided that the adiabatic approximation is invoked. Now, though, the meaning of $(d\sigma/d\Omega)(0, 0)$ is altered; it is here the adiabatic cross section for elastic scattering from an even-even nucleus with in-

⁸ M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

⁹ J. S. Blair, *Phys. Rev.* **115**, 928 (1959).

¹⁰ G. R. Satchler, *Nucl. Phys.* **45**, 197 (1963).

trinsic deformations δ_λ , correct through second order in such deformations.

(3) When full advantage is taken of the adiabatic approximation, however, Eq. (3.8) and its inelastic analog [Eq. (3.1) and (3.7)] can be replaced by a generalization valid to all orders in the deformation parameters. Specifically, the cross section to any member of the ground-state rotational band is

$$(d\sigma/d\Omega)(I \rightarrow I') = \delta_{I,I'}(d\sigma/d\Omega)(0, 0) + \sum_{\lambda \neq 0, \text{ even}} (I\lambda K 0 | I'K)^2 (d\sigma/d\Omega)(\lambda, 0), \quad (3.9)$$

a result implicit in the earliest applications¹¹ of the adiabatic approximation. In this expression, $(d\sigma/d\Omega)(0, 0)$ and $(d\sigma/d\Omega)(\lambda, 0)$ are the elastic and in-band inelastic cross sections, correct through all orders in the deformation parameters, for scattering from a spin-zero nucleus whose intrinsic deformations and other optical parameters are the same as those of the odd-mass nucleus in its ground-state band.

Only three assumptions are necessary for the derivation of Eq. (3.9):

(i) The adiabatic approximation is applied to the nuclear coordinates, from which it follows that the scattering amplitudes $f_{I',M';I,M}(\theta)$ may be written

$$f_{I',M';I,M}(\theta, \phi) = \langle I', M' | f(\xi; \theta, \phi) | I, M \rangle, \quad (3.10)$$

where $f(\xi; \theta, \phi)$ is the elastic scattering amplitude for scattering from a nucleus with fixed coordinates ξ .

(ii) The only nuclear coordinates entering $f(\xi; \theta, \phi)$ are the collective angular coordinates, α and β , specifying the orientation of an axially symmetric nucleus with fixed deformations δ_λ . This assumption allows us to make a spherical harmonic expansion of $f(\xi; \theta, \phi)$,

$$f(\alpha, \beta; \theta, \phi) = \sum_{\lambda, \mu} (4\pi)^{1/2} f_{\lambda, \mu}(\theta, \phi) Y_{\lambda, \mu}^*(\alpha, \beta). \quad (3.11)$$

The deformations δ_λ are simply constant parameters entering these expressions.

(iii) The nuclear wave functions are those of the strong-coupling rotational model Eq. (3.5). Evaluation of the nuclear matrix elements [Eq. (3.10)] then leads straightaway to the basic result Eq. (3.9), since

$$(d\sigma/d\Omega)(\lambda, 0) = \sum_{\mu} |f_{\lambda, \mu}(\theta, \phi)|^2. \quad (3.12)$$

Not the least of the virtues of Eq. (3.9) is that it can be applied directly to experimental cross sections of neighboring odd- and even-mass nuclei even when the deformations are large, since the cross sections presumably contain the contributions to all orders in the deformation parameter. The cross sections $(d\sigma/d\Omega)(\lambda, 0)$ for $\lambda \geq 4$ may well receive their largest contributions from multiple excitation processes. The Achilles heel

of this equation is the adiabatic approximation itself, since coupled-channel¹² and DWBA¹³ calculations do indicate many instances where the nonadiabatic corrections are important.

D. Out-of-Band Transitions—Single-Nucleon Excitation

Transitions between bands whose internal wave functions differ in one nucleon orbital can proceed through an effective potential $V(\mathbf{r}, \mathbf{q})$ acting between the projectile, with coordinate \mathbf{r} , and the last target nucleon, with coordinates \mathbf{q} . This potential has the multipole decomposition

$$V(\mathbf{r}, \mathbf{q}) = \sum_{\lambda, \mu} (2\lambda+1)^{1/2} g_\lambda(r, q) Y_{\lambda, \mu}(\hat{q}) Y_{\lambda, \mu}^*(\hat{r}). \quad (3.13)$$

The resulting DWBA expression for the inelastic scattering cross section is, in general, rather unwieldy, since, because of the symmetry properties of the nuclear wave functions, the scattering amplitudes contain terms in which the transfer of intrinsic projected angular momentum is $(K'+K)$ as well as $(K'-K)$.

In the cases where only the $K'-K$ terms are present, however, the cross section takes the simple form

$$(d\sigma/d\Omega)(I, K \rightarrow I', K') = \sum_{\lambda} (I\lambda K, K'-K | I'K')^2 \times (d\sigma/d\Omega)(\lambda; K \rightarrow K'), \quad (3.14)$$

so that all dependence on total angular momenta is isolated in the multiplicative Clebsch-Gordan factors; here, $(d\sigma/d\Omega)(\lambda; K \rightarrow K')$ is the "intrinsic single-nucleon cross section" for multipolarity λ and is given by

$$(d\sigma/d\Omega)(\lambda; K \rightarrow K') = [m/2\pi\hbar^2]^2 (k'/k) \times \sum_{\mu} | \int d\mathbf{r} \chi^{(-)*}(\mathbf{k}', \mathbf{r}) \times \langle \phi_{K'} | g_\lambda(r, q) Y_{\lambda, K'-K}(\hat{q}) | \phi_K \rangle Y_{\lambda, \mu}^*(\hat{r}) \chi^{(+)}(\mathbf{k}, \mathbf{r}) |^2, \quad (3.15)$$

where m is the reduced mass, k and k' are initial and final momenta, and $\chi^{(+)}$ and $\chi^{(-)*}$ are the usual distorted wave functions.¹³ For the out-of-band transitions of concern in this paper ($K = \frac{5}{2}$ to $K' = \frac{1}{2}$), the terms involving intrinsic projected angular-momentum transfer of $(K'+K)$ units are absent from the important $\lambda=2$ amplitudes. Consequently, in our applications no serious error is incurred through using Eq. (3.14) instead of the general expression.

Since the projectile interacts with only one nucleon and induces a change in its orbital wave functions, we expect single-nucleon excitation to be inhibited in comparison to collective excitation. Further, to the extent

¹² T. Tamura, Rev. Mod. Phys. **37**, 679 (1965).

¹¹ S. I. Drozdov, Zh. Eksperim. i Teor. Fiz. **30**, 786 (1956) [English transl.: Soviet Phys.—JETP **3**, 759 (1956)].

¹³ R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).

that the core wave functions of the two bands are different, there will be additional inhibition of single-nucleon excitation.

E. Out-of-Band Transitions—Band Mixing

Because there is inhibition of single-nucleon excitation, we now inquire whether small amounts of collective excitation might be present in presumed out-of-band transitions, thereby masking the single-nucleon contribution. Let us consider an out-of-band transition where both the initial and final states may contain admixtures of the same intrinsic wave functions. Specifically, we assume that the initial nuclear wave function is

$$|IM\rangle = a(I, K) |IMK\rangle + a(I, K') |IMK'\rangle \quad (3.16)$$

and the final wave function is

$$|I'M'\rangle = a'(I', K) |I'M'K\rangle + a'(I', K') |I'M'K'\rangle. \quad (3.17)$$

The resulting collective transition strength is then

$$S_\lambda(I \rightarrow I') = |\delta_\lambda(K) \langle I\lambda K0 | I'K \rangle a(I, K) a'^*(I', K) + \delta_\lambda(K') \langle I\lambda K'0 | I'K' \rangle a(I, K') a'^*(I', K')|^2, \quad (3.18)$$

where $\delta_\lambda(K)$ and $\delta_\lambda(K')$ are the deformation parameters for the two bands.

For the applications in this paper, certain simplifications in this formula will occur, since we will be concerned only with out-of-band transitions from a ground state with $I = \frac{5}{2}$ containing primarily the $K = \frac{5}{2}$ band to states containing primarily a $K' = \frac{1}{2}$ band. When $I' = \frac{1}{2}$ or $\frac{3}{2}$, such final states will be pure, and the transition proceeds only through the admixture in the initial state. When $I' = \frac{5}{2}$, orthogonality with the ground state requires that $[a(\frac{5}{2}, \frac{5}{2}) a'^*(\frac{5}{2}, \frac{5}{2}) + a(\frac{5}{2}, \frac{1}{2}) a'^*(\frac{5}{2}, \frac{1}{2})]$ vanishes. If the deformation parameters of both bands are the same, then

$$\begin{aligned} S_2(\frac{5}{2} \rightarrow \frac{5}{2}) &= \delta_2^2 |a(\frac{5}{2}, \frac{1}{2}) a'^*(\frac{5}{2}, \frac{1}{2})|^2 \\ &\quad \times \{ [15/42]^{1/2} + [8/35]^{1/2} \}^2 \\ &= \delta_2^2 |a(\frac{5}{2}, \frac{1}{2})|^2 |a(\frac{5}{2}, \frac{5}{2})|^2 \quad (1.16). \end{aligned} \quad (3.19)$$

In concluding this section, we note: (a) The amplitudes for single-nucleon and collective excitation will interfere, and thus we should be prepared to find observed strengths which deviate considerably from the predictions of Eq. (3.18), even when there is appreciable band mixing. (b) The usual source of band mixing is rotation particle coupling, but this is not expected to lead to much mixing between bands whose projected intrinsic angular momenta differ by two units. (c) If band mixing is present, one sees from Eq. (3.18) that there will also be some modification of our earlier predictions for inband transitions.

F. Out-of-Band Transitions—Core Excitations

Transitions induced by a permanently deformed axially symmetric optical potential are not the only variety of collective excitation. For example, natural-parity transitions to the $K = 2^+$ band in ^{24}Mg are commonly regarded as collective in character since their strengths,^{14,15} while less than those of quadrupole in-band transitions, are larger than single-particle estimates. For an even-even nucleus, such collective transition strengths have, in the rotational model, the form (for $\kappa' \neq 0$)

$$S_\lambda(0, 0 \rightarrow \lambda, \kappa') = 2 |\langle \phi_{\kappa'}(c) | \xi_{\lambda\kappa'} | \phi_0(c) \rangle|^2, \quad (3.20)$$

where $\xi_{\lambda\kappa'}$ is the surface-displacement coordinate referred to the body-fixed frame, while $\phi_{\kappa'}(c)$ and $\phi_0(c)$ are the intrinsic wave functions of the core; the factor 2 arises because the ground-state wave function, in contrast to Eq. (3.5), comprises but a single term.

In the neighboring odd-mass nucleus, bands can be formed whose intrinsic functions are products of core excitation and single-nucleon wave functions¹⁶; transitions will be relatively strong to bands whose intrinsic functions are

$$\phi_{K'-K \pm \kappa'} = \phi_{\pm\kappa'}(c) \phi_K(\mathbf{q}'). \quad (3.21)$$

The corresponding collective transition strength is then

$$S_\lambda(I, K \rightarrow I', K') = \frac{1}{2} \langle I\lambda K, K' - K | I'K' \rangle^2 S_\lambda(0, 0 \rightarrow \lambda, \kappa'). \quad (3.22)$$

If there is small interaction between the collective and single-particle motion, the differences in energy between the core-excited states in the odd- and even-mass nuclei will involve only differences of various rotational energies, which are often easily estimated.

The preceding discussion has been couched in terms appropriate to an extended optical potential containing collective coordinates. The same relative intensities result, however, when a microscopic description is employed in which the core excitation is induced by a scalar effective potential acting between the projectile and target nucleons.

IV. ANGULAR DISTRIBUTIONS AND DISCUSSION

A. In-Band Transitions

The measured elastic cross section A and the cross sections for the two strongest inelastic transitions D (1.61 MeV) and H (3.400 MeV) are shown in Fig. 3. A detailed discussion of the elastic scattering, particularly its behavior at the larger angles, will be de-

¹⁴ D. L. Hendrie, B. G. Harvey, J. Mahoney, and J. R. Meriwether, *Bull. Am. Phys. Soc.* **12**, 555 (1967).

¹⁵ S. J. Skorka, J. Hertel, and T. W. Retz-Schmidt, *Nucl. Data A2*, 347 (1967).

¹⁶ O. Nathan and S. G. Nilsson, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Co., Amsterdam, 1965), p. 601.

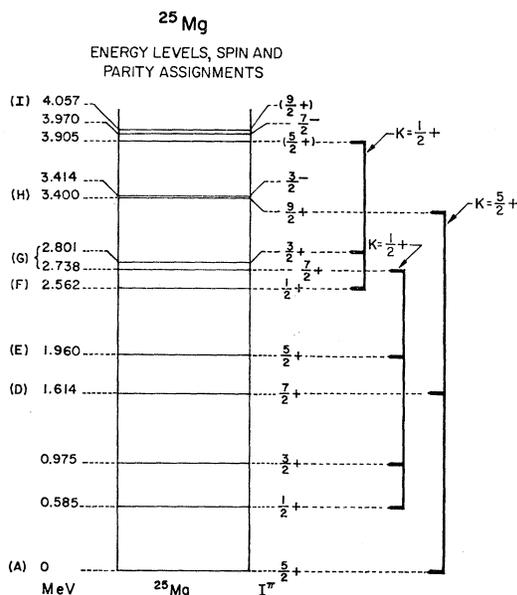


FIG. 2. Energy level diagram of ^{25}Mg . The energies are taken from Ref. 5, the spin-parity assignments from Refs. 6 and 7; the rotational band classification is that given in Ref. 6.

ferred until Sec. IV B. For present purposes, it suffices to say that the location of the diffraction oscillations and the magnitudes of the more forward maxima are duplicated by the same "smooth cutoff" calculation¹ used for ^{24}Mg except that the Fraunhofer radius R_0 should be increased from 6.06 to 6.15 F; the previous ratio of the diffuseness distance d to R_0 need not be changed. A spherical optical-model calculation using the parameters of Ref. 1 provides a similarly satisfactory account of the elastic scattering, except that the midpoint radius should be increased, by the same amount as was the Fraunhofer radius, from 4.76 to 4.85 F. These parameters are then used for all subsequent Fraunhofer and DWBA inelastic calculations.

Our best Fraunhofer and DWBA fits to the inelastic angular distribution D , assuming only $\lambda=2$ excitation, are given in Fig. 4. The corresponding quadrupole transition strengths are 0.93 and 0.96 F^2 , respectively. The quality of the fits are comparable to those found for excitation of the first 2^+ level in ^{24}Mg at 1.37 MeV.

Perhaps the most striking aspect of Fig. 3 is the similarity of the two inelastic angular distributions. The ratios of the cross sections $(d\sigma/d\Omega)(H)/(d\sigma/d\Omega)(D)$ are nearly constant at the maxima. Specifically, these have the values 0.33, 0.31, 0.33, and 0.31 at 13° , 26° , 39° , and 55° , respectively. When account is taken of the Q dependence of the DWBA cross sections, we find that the average ratio of the $\lambda=2$ DWBA transition strengths is 0.36.

It has long been popular to regard the 1.611- and 3400-MeV levels as members of the ground-state

band.¹⁷ This interpretation is supported not only by the large inelastic cross sections at these levels but also by the above ratio of quadrupole strengths. According to Eq. (3.7), this ratio should be $(7/20)=0.35$. The deformation parameter in the ground-state band, inferred from the DWBA analysis, is 1.42 F, which is somewhat smaller than the value found¹ for the presumed ground-state band in ^{24}Mg , 1.68 F.

The strong excitation of these in-band transitions and the similarity of the observed relative intensity to the prediction of the strong-coupling model have been noted in previous studies¹⁸⁻²² of inelastic scattering. The observed ratios of cross sections for proton^{20,21} and deuteron¹⁸ scattering, 0.42 and 0.41, are slightly higher than our own. However, only the (α, α') results display

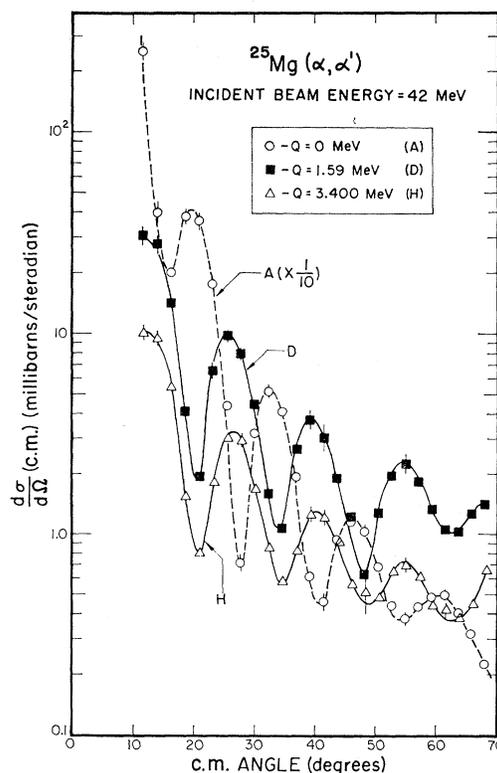


FIG. 3. The measured angular distributions for the elastically scattered α particles and for the pronounced inelastic groups D and H that correspond to in-band excitation of ^{25}Mg .

¹⁷ A. E. Litherland, H. McManus, E. B. Paul, D. A. Bromley, and H. E. Gove, Can. J. Phys. **36**, 378 (1958).

¹⁸ A. G. Blair and E. W. Hamburger, Phys. Rev. **122**, 566 (1961).

¹⁹ G. Schrank, E. K. Warburton, and W. W. Daehnick, Phys. Rev. **127**, 2159 (1962).

²⁰ G. M. Crawley and G. T. Garvey, Phys. Letters **19**, 228 (1965).

²¹ G. M. Crawley and G. T. Garvey, Phys. Rev. **167**, 1070 (1968).

²² G. Bruge, J. C. Faivre, G. Vallois, A. Bussiere, and P. Roussel, J. Phys. **1**, 44 (1966).

such a detailed similarity between the two angular distributions D and H and between the distributions and the characteristic quadrupole patterns of neighboring even-mass nuclei. It should be remarked that the relative intensity emerging from the inelastic scattering studies disagrees with the value deduced from measurements of EM lifetimes,⁶ $0.025 + 0.060$ and $0.025 - 0.021$. The uncertainties in the EM determination are considerable, but even the largest value consistent with the stated errors is a factor of 4 less than the inelastic scattering value.

The analyses so far made in this section are incomplete in that no account has been taken of possible $\lambda=4$ contributions. The relevant Clebsch-Gordan factors are, in themselves, almost sufficient to guarantee that the $\lambda=4$ contribution to angular distribution D is negligible. However, a measurable $\lambda=4$ contribution to H is expected. According to the adiabatic prediction [Eq. (3.9)], we have

$$\begin{aligned} (d\sigma/d\Omega) \left(\frac{5}{2} \rightarrow \frac{9}{2}\right) &= 0.167(d\sigma/d\Omega)(2, 0) \\ &+ 0.379(d\sigma/d\Omega)(4, 0). \end{aligned} \quad (4.1)$$

If we now assume that the ratio of $(d\sigma/d\Omega)(4, 0)$ to $(d\sigma/d\Omega)(2, 0)$ for 42-MeV α particles bombarding ^{24}Mg is the same¹⁴ as at corresponding points in the diffraction patterns for 50-MeV α particles, we find, (a) that the predicted cross sections at the first four maxima of H would be increased by 1, 11, 18, and

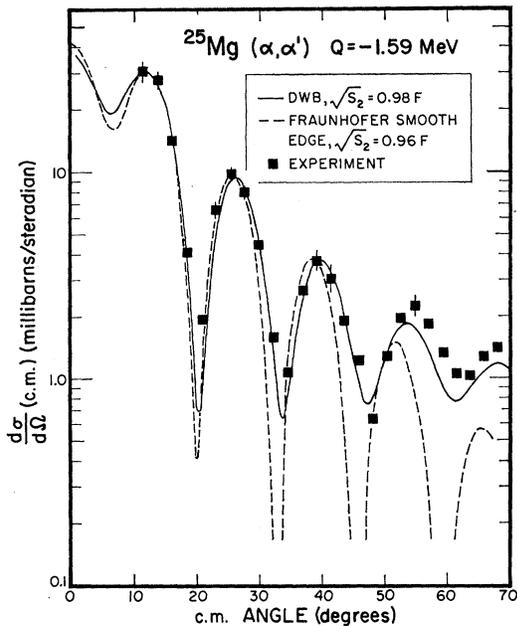


FIG. 4. The measured angular distribution D (1.59 MeV) compared with DWBA and Fraunhofer predictions; only $\lambda=2$ excitation is assumed. The theoretical curves were normalised to give their best fit to the measured cross sections at the forward maxima.

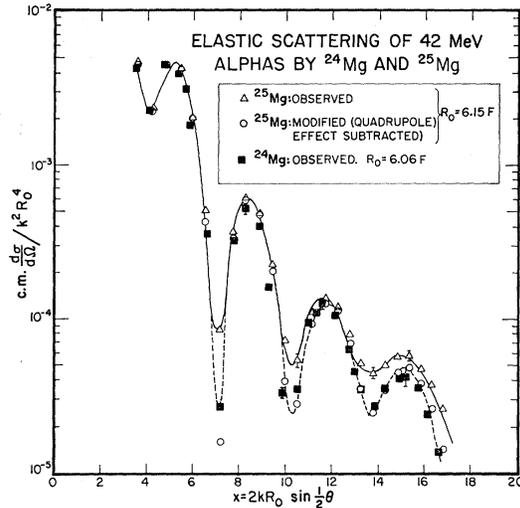


FIG. 5. Comparison of the universal plots for elastic scattering by ^{24}Mg and by ^{25}Mg . The two ^{25}Mg curves correspond to whether the quadrupole effect was subtracted from the measured cross section or not.

17%, respectively, and (b) that the minima would be filled somewhat in comparison to the pure $\lambda=2$ prediction. We find no positive evidence for such $\lambda=4$ contributions: The contributions at the maxima should have revealed themselves through some increase in the ratio of D to H at successive maxima. The experimental errors and the unknown contribution of the $\frac{3}{2}^-$ member of the doublet prevent us from basing any conclusions on the appearance of the minima.

B. Elastic Scattering

We now inquire whether there is an observable quadrupole contribution to the elastic scattering. Since we cannot directly measure $(d\sigma/d\Omega)(0, 0)$ (the elastic scattering cross section from a hypothetical spin-zero nucleus whose deformation and other properties are those of the ^{25}Mg ground-state band), we compare the observed elastic scattering from ^{25}Mg to that from its neighbors ^{24}Mg and ^{26}Mg , hoping that this replacement is not accompanied by any critical change in nuclear properties. We anticipate some differences in $(d\sigma/d\Omega)(0, 0)$ since the measured deformation parameters^{1,23} for ^{24}Mg , ^{25}Mg and ^{26}Mg are 1.68, 1.42, and 1.40 F, respectively.

To eliminate some purely geometrical and kinematical effects, universal plots are made of the cross sections, i.e., the ordinate is the cross section divided by $(k^2 R_0^4)$ and the abscissa is $[2kR_0 \sin \frac{1}{2}\theta]$. Figure 5 contains the comparison for ^{25}Mg and ^{24}Mg , while Fig. 6 contains the comparison for ^{25}Mg and ^{26}Mg . For the latter nucleus it has been found²³ that $R_0 = 6.11$ F. In

²³ I. M. Naqib and J. S. Blair (to be published).

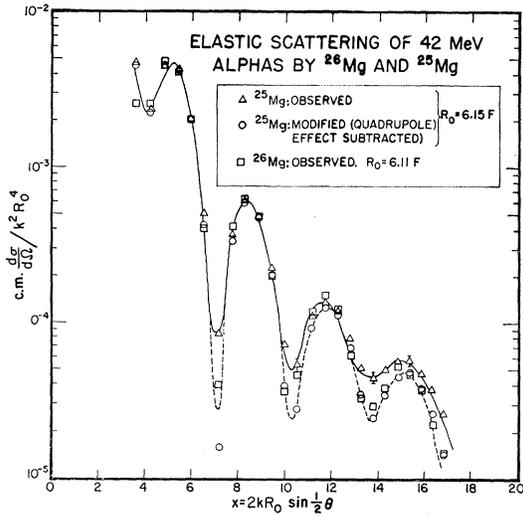


FIG. 6. A comparison, similar to that in Fig. 5, for elastic scattering by ^{25}Mg and ^{26}Mg .

addition, these figures contain the ^{25}Mg modified elastic scattering cross section, where the quadrupole contribution predicted by Eq. (3.9) is subtracted from the observed elastic scattering cross section for ^{25}Mg . In our numerical calculations, this quadrupole contribution is taken to be three-quarters of the observed inelastic cross section D . The $\lambda=4$ contribution to ^{25}Mg elastic cross section is predicted to be completely negligible over the range of angles here considered.

While the modified cross section lies closer to the ^{24}Mg and ^{26}Mg data in the minima near $x=7$ and $x=10$, we think that the most significant comparisons occur in the region beyond $x=13$, particularly in the broad minimum near $x=14$, and that these comparisons provide confirmation for a quadrupole contribution whose magnitude is of the order of that predicted in the strong-coupling model. Further, the fact that the modification of the ^{25}Mg elastic scattering curve agrees well with both the ^{24}Mg and ^{26}Mg curves supports our initial "hopeful" assumption that there are no critical differences in $(d\sigma/d\Omega)(0,0)$ for the three isotopes. The one disquieting circumstance is the ^{25}Mg - ^{26}Mg comparison at the maximum near $x=12$, where even the ^{25}Mg observed universal cross section is less than the ^{26}Mg cross section by some 10%. We think, however, that this discrepancy is not as important as the comparison near $x=14$, where the observed ^{25}Mg cross section exceeds that for ^{26}Mg by 50%.

C. Transitions to the Lowest $K=\frac{1}{2}^+$ Band

A substantial body of evidence^{6,17} has indicated that levels B (0.585 MeV), C (0.975 MeV), and E (1.960 MeV), as well as the level at 2.738 MeV contributing to group G , are members of a rotational band built on a single-nucleon orbital with quantum number $K=\frac{1}{2}^+$.

Concerning excitation of these levels in the present

experiment, we make the following general observations:

(1) No measurements of the cross sections to levels B and C were possible at angles less than 36° (c.m.) due to oxygen and carbon contaminations; at higher angles the excitations were so weak that only upper limits on the cross sections could be established. For values of θ (c.m.) between 36° and 46° , the upper limits are less than 0.16 mb/sr; at larger angles, the upper limits are less than 0.1 mb/sr. In other words, at the last two maxima of D , located near 39° and 54° , respectively, the upper limits on the cross sections to either B or C are less than the cross section for D by more than a factor of 20.

(2) The excitation of E , though weak, is measurable. This cross section, as well as those for excitation of groups F through I , are shown in Fig. 7. Although the cross section for E is not well fitted by the $\lambda=2$ predictions of either the DWBA or Fraunhofer models, it is not a gross distortion to label it as a $\lambda=2$ excitation. The magnitude of the cross section is roughly a factor of 10 less than that for level D and thus exceeds the upper limits on those to B or C by at least a factor of two.

(3) Since the two contributors to G are not resolved, we content ourselves with but one observation about

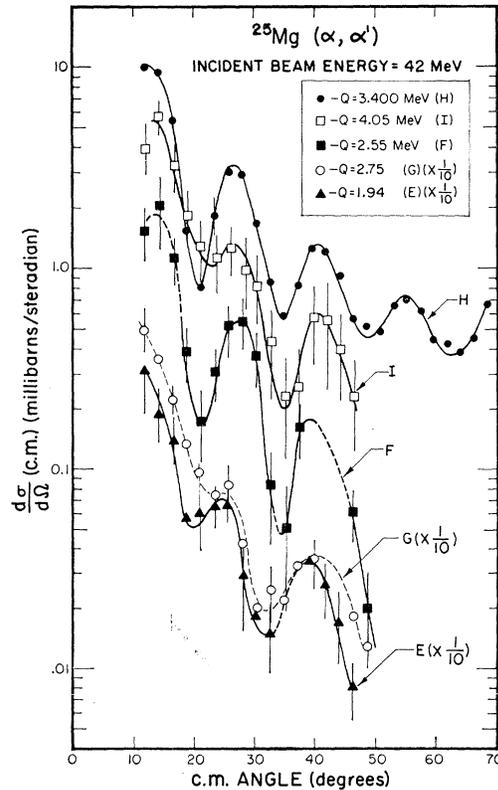


FIG. 7. Measured angular distributions for inelastic α groups E - I , which correspond to the observed out-of-band transitions.

the excitation of the 2.738-MeV level, namely, that it cannot appreciably exceed that of level E .

The marked inhibition of transitions to levels B and C is consistent with the rotational model, provided that the cross section for single-nucleon excitation is very small. However, the observed excitation of E requires that there be some modification of the simplest form of the rotational model. For a single-nucleon transition, the relative strengths for excitation of the $I = \frac{1}{2}, \frac{3}{2},$ and $\frac{5}{2}$ levels are $(14/42), (16/42),$ and $(9/42)$, respectively, which are quite contrary to observation.

Band mixing between the ground-state and $K = \frac{1}{2}^+$ bands may provide a solution of this dilemma. According to the discussion of Sec. III E, the collective transition strengths to the three levels would be $0.20, 0.057,$ and $1.16 |a(\frac{5}{2}, \frac{5}{2})|^2$, respectively, times $\delta_2^2 |a(\frac{5}{2}, \frac{1}{2})|^2$. Since the amount of mixing need not be large, the quantity $|a(\frac{5}{2}, \frac{5}{2})|^2$ will be close to unity, and, consequently, the predicted relative strengths are consistent with inhibition of transitions to the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ levels. Matching the $\lambda = 2$ DWBA angular distribution to the observed cross section for excitation of level E results in the value $0.078 F^2$ for the corresponding transition strength. Thus a 4% admixture of the $K = \frac{1}{2}^+$ band in the ground-state probability suffices to explain this magnitude. Such an admixture will not substantially modify our description of in-band transitions.

Although the above argument invoking band mixing is attractive, we are reluctant to insist on its validity. According to this model, the shape of the angular distribution E should be the same as those of the strong rotational transitions D and H . In fact, though, there is considerable deviation from the rotational pattern, particularly near the first minimum of E . Since the total angular momenta of the initial and final states are the same, it is possible for transition E to proceed via monopole excitations. Accordingly, we have also examined this possibility. The DWBA and Fraunhofer fits with $\lambda = 0$ to angular distribution E are distinctly inferior to those with $\lambda = 2$. For $\lambda = 0$ deep minima occur at 9.5° and 22.5° , while maxima occur at 15° and 27.5° . On the other hand, experience has shown¹ that known monopole transitions are generally not well described by such calculations. Direct comparison of angular distribution E and that²³ leading to the 0^+ level at 3.57 MeV in ^{26}Mg shows considerable overlap out to 30° , and suggests that the bulk of transition E may well proceed with no transfer of angular momenta, in spite of the failure of the theoretical $\lambda = 0$ calculations.

D. Other Out-of-Band Transitions

Except for the in-band transitions D and H the largest inelastic cross section appears to be that of group I which we think, as mentioned earlier, to be almost entirely attributable to the excitation of the level at

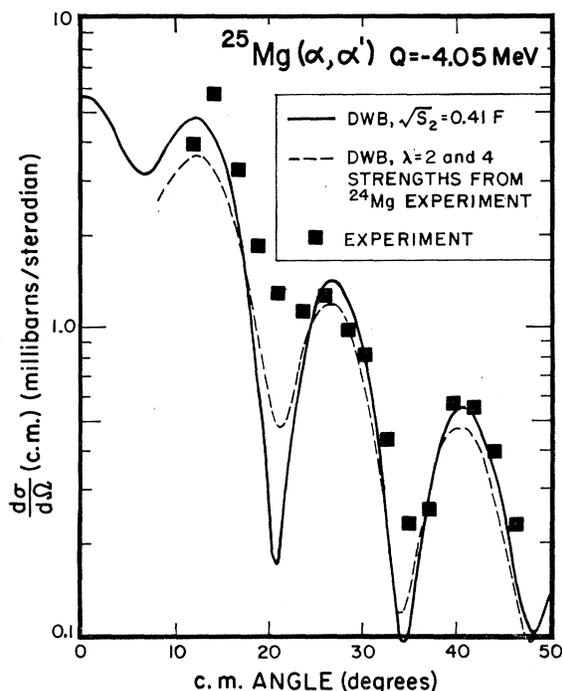


Fig. 8. Comparison of the measured angular distribution I (4.05 MeV) with DWBA predicted cross sections. The dashed theoretical curve represents a combination of $\lambda = 2$ and $\lambda = 4$ contributions with strengths determined by ^{24}Mg experiments (see text). The solid curve is the best fit for $\lambda = 2$ only.

4.057 MeV. Röpke *et al.*²⁴ have made the assignment $I^\pi = K^\pi = \frac{9}{2}^+$ to the analog of this level in ^{25}Al at 4.038 MeV. Furthermore, they have located another $I^\pi = \frac{9}{2}^+$ level in ^{25}Al , which is much more plausibly the missing member of the first $K = \frac{1}{2}^+$ band, this being the analog of a level in ^{25}Mg at 4.708 MeV. The moderately large cross section and the angular-momentum assignments make it natural to follow the suggestion of Litherland *et al.*¹⁷ and McPherson,²⁵ that the 4.057-MeV level is a core excited state. Specifically, we here consider the intrinsic state with $K = \frac{9}{2}^+$ to involve a $K = 2^+$ excitation similar to that whose band head in ^{24}Mg lies at 4.23 MeV, coupled to the orbital of the ground-state band.

The observed inelastic cross section at the 4.057-MeV level is consistent with this supposition. The angular distribution shown in Fig. 7 is similar to the $\lambda = 2$ patterns of stronger transitions, although there is considerable filling in of the minimum near 20° . In the excited-core model, the predicted cross section is

$$(d\sigma/d\Omega) \left(\frac{5}{2}, \frac{5}{2} \rightarrow \frac{9}{2}, \frac{9}{2} \right) = \frac{1}{2} (d\sigma/d\Omega) (0, 0 \rightarrow 2, 2) + (5/66) (d\sigma/d\Omega) (0, 0 \rightarrow 4, 2).$$

The resulting curve is compared with the experimental

²⁴ H. Röpke, N. Anyas-Weiss, and A. E. Litherland, *Phys. Letters* **27B**, 368 (1968).

²⁵ D. McPherson, Progress Report of the Atomic Energy of Canada Limited, No. AECL-1831, 1963 (unpublished).

cross section in Fig. 8, where for $(d\sigma/d\Omega)(0, 0 \rightarrow \lambda, 2)$ we insert the predicted DWBA cross section using the measured ^{24}Mg strengths^{1,14} $S_2(0, 0 \rightarrow 2, 2) = 0.24 \text{ F}^2$ and $S_4(0, 0 \rightarrow 4, 2) = 0.48 \text{ F}^2$. The $\lambda=4$ contribution does not fill the first minimum to the extent observed, but it does increase the second and third maxima by 15 and 22%, respectively, over those predicted for a quadrupole contribution only. We note that the predicted cross section is somewhat less than that observed, although it lies within the experimental errors at the maxima. Also shown is the best DWBA fit obtained when only the quadrupole contribution is retained. The quadrupole strength S_2 is then 0.168 F^2 . We feel that a more reliable value is obtained when a $\lambda=4$ contribution to the cross section equal to that found above is included. Accordingly, our preferred value for the quadrupole strength is $0.144 \pm 0.036 \text{ F}^2$.

With the assumption that the predominant angular momentum in the intrinsic core-excitation wave function is 2, the energy of the $I^\pi = K^\pi = \frac{9}{2}^+$ state, relative to the ground state, would be $9A_{1/2} - 5A_{5/2} + \epsilon_{\text{core}}$ (where A is the moment of inertia parameter $\hbar^2/2\mathcal{J}$). Similarly, with the same core excitation, the lowest member of the $K^\pi = 2^+$ band in a neighboring even-mass nucleus would be $4A_2 + \epsilon_{\text{core}}$. If the moment of inertia parameters $A_{9/2}$ and A_2 are assumed equal and the core excitation is taken to be the first $K = 2^+$ state of ^{24}Mg , then the head of the $K = \frac{9}{2}^+$ band is estimated to lie at 3.71 MeV. We recognize in making these estimates, however, that the observed¹ large single excitation of the 4^+ member of the $K = 2^+$ band in ^{24}Mg casts considerable doubt on the initial assumption of this paragraph. We also expect the parameters associated with core excitation in ^{25}Mg to be somewhat different from those of ^{24}Mg . Values intermediate between those of ^{24}Mg and ^{26}Mg are probably more realistic.

A straightforward calculation shows similarly that the head of the $K = \frac{1}{2}^+$ concomitant band should lie ($4A_{9/2}$) below the head of the $K = \frac{9}{2}^+$ band. Again assuming that $A_{9/2} \approx A_2$ and that the 4.057-MeV level is the head of the $K = \frac{9}{2}^+$ band, we estimate the head of the $K = \frac{1}{2}^+$ band to lie at 3.56 MeV.

Presuming that we have correctly located the band head of the $K = \frac{9}{2}^+$ core excitation band, we must now ask where are the members of the concomitant $K = \frac{1}{2}^+$ band? Since we expect this band to commence at an energy lower than that of the $K = \frac{9}{2}^+$ band, the only reasonable candidates appear to be²⁵ the members of the second $K = \frac{1}{2}^+$ band which starts at 2.562 MeV. This energy is about 1.0 MeV below the estimated value of 3.56 MeV, but the discrepancy may well be the result of the approximate nature of our energy calculations. The observed cross sections of the present experiment are consistent with this suggestion: Only quadrupole interactions can contribute to excitation of the 2.652-MeV level, and the corresponding transition strength extracted from the DWBA analysis, $0.058 \pm 0.014 \text{ F}^2$, is roughly one-third of that to the 4.057-MeV

level, in accordance with the intensity relations of Sec. III F. Further, the angular distribution for this transition, though associated with large statistical errors, does conform well to the classic $\lambda=2$ pattern for collective excitations. The observed excitation of other members of this band, or lack thereof, does not contradict this model. Although the transition to the $I^\pi = \frac{3}{2}^+$, $K^\pi = \frac{1}{2}^+$ state at 2.801 MeV cannot be resolved, the observed cross section for group G does not greatly exceed the cross section for group F except at some forward angles. Consequently, the transition strengths to the 2.801-MeV level cannot be much greater than those predicted by the intensity relations [Eq. (3.22)].

There are, however, certain serious objections to our suggestion that the second $K^\pi = \frac{1}{2}^+$ band is based on an excited-core configuration:

(a) Deuteron stripping studies²⁶⁻²⁹ indicate substantial spectroscopic factors to these levels, while the core-excitation model predicts that such factors are zero.

(b) The decoupling parameter for a $K = \frac{1}{2}^+$ core-excitation band is predicted to be zero.¹⁶ The observed energy intervals, however, do not follow regular rotational spacing, but rather yield a substantial decoupling parameter $a = -0.47$ as well as a rather small rotational energy parameter, $A_{1/2} = 0.15 \text{ MeV}$.

The traditional description of the second $K^\pi = \frac{1}{2}^+$ band, that it is based on the Nilsson single-nucleon orbital No. 11, is not in contradiction with our experiment and has less difficulty with the points mentioned above. The ratio of the intrinsic single-nucleon cross section $(d\sigma/d\Omega)(2; \frac{5}{2} \rightarrow \frac{1}{2})$ to $(d\sigma/d\Omega)(\lambda, Q=0)$, the intrinsic cross section of the ground-state band, would then be 0.09, and, though this is larger than the upper limit to the corresponding ratio for the first $K = \frac{1}{2}^+$ band, this small value is consistent with the notion that we are dealing with a single-nucleon transition. The predicted relative intensities (for the $\lambda=2$ contributions) to different members of the band are the same as those of the excited-core model and thus are not contradicted by our measurements. The single-nucleon description can accommodate the decoupling parameter observed for this band.¹⁷ The most recent stripping analyses indicate,^{28,29} however, that the observed $\lambda=0$ spectroscopic factor is a factor of 2.5-5 less than what is predicted with the single-nucleon description, although the observed $\lambda=2$ factors are consistent with the description. It is also worth noting that another level, at 5.465 MeV, is populated²⁶ through $\lambda=0$ stripping and has a spectroscopic factor which is considerably larger than that observed for the 2.565-MeV level. Further, the question concerning the whereabouts of the $K^\pi = \frac{1}{2}^+$ core-excitation band reappears if one adopts the single-nucleon model for the second

²⁶ R. Middleton and S. Hinds, Nucl. Phys. **34**, 404 (1962).

²⁷ B. Cujec, Phys. Rev. **135**, B1305 (1964).

²⁸ H. Fuchs, D. Grabisch, P. Kraaz, and G. Röscher, Nucl. Phys. **A110**, 65 (1968).

²⁹ E. Rost, Phys. Rev. **154**, 994 (1967).

TABLE II. Values obtained for the quadrupole transition strengths S_2 as well as $\sqrt{S_2}$.

$-Q^a$ (MeV)	Spin parity ^b of the excited state	$\sqrt{S_2}$	S_2	$\sqrt{S_2}$	S_2
		F	F ²	F	F ²
		Fraunhofer		DWBA	
0.585 ^c	$\frac{3}{2}^+$	<0.2	<0.04	<0.2	<0.04
0.975 ^c	$\frac{3}{2}^+$	<0.2	<0.04	<0.2	<0.04
1.614 ^d	$\frac{7}{2}^+$	0.96±0.03	0.93±0.06	0.98±0.03	0.96±0.06
1.960 ^e	$\frac{3}{2}^+$	(0.28±0.03)	(0.078±0.017)	(0.28±0.03)	(0.078±0.017)
2.562 ^f	$\frac{1}{2}^+$	0.23±0.03	0.053±0.014	0.24±0.03	0.058±0.014
2.738 ^g	$\frac{7}{2}^+$	0.39–0.27	0.15–0.07	0.42–0.28	0.18–0.08
2.801 ^h	$\frac{3}{2}^+$				
3.400 ^{d,h}	$\frac{9}{2}^+$	0.56±0.02	0.31±0.02	0.59±0.02	0.35±0.02
4.057 ⁱ	$(\frac{9}{2})^+$	0.34±0.03	0.117±0.03	0.38±0.05	0.144±0.036

^a The excitation energies here tabulated are those of Endt and van der Leun (Ref. 5) rather than the values measured in the present experiment (Table I).

^b We quote the assignments of Sharpey-Schafer *et al.* (Ref. 6).

^c Only upper limits on the cross sections at those levels could be established.

^d Good fits to both Fraunhofer and DWBA $\lambda=2$ single-excitation angular distributions.

^e Inferior $\lambda=2$ fits; there may well be a large $\lambda=0$ contribution.

^f Reasonable $\lambda=2$ fits.

^g Probably both states contribute appreciably to the cross section. Inferior $\lambda=2$ fits.

^h We have assumed that only the $\frac{9}{2}^+$ member of the doublet at 3.40 MeV has an appreciable cross section.

ⁱ We cannot exclude the possibility of some contamination from the neighboring levels at 3.905 and 3.970 MeV. The quoted quadrupole strengths allow for an anticipated $\lambda=4$ contribution (see text); analysis with $\lambda=4$ contribution set equal to zero gives a quadrupole strength well within the stated errors.

$K\pi=\frac{1}{2}^+$ band. It is possible that all difficulties would disappear were the intrinsic state a linear combination of both the core-excitation and single-nucleon configurations, but we suspect that with such a description some of the simplicities of both extreme models would then be lost.

E. Summary of Transition Strengths

The values deduced from the quadrupole transition strengths and their square roots are listed in Table II. The DWBA strengths are then used to estimate G_2 , the reduced electromagnetic (EM) quadrupole transition probabilities from the ground state in units of the Weisskopf single-particle strength. More properly, these should be considered as reduced isospin zero-transition probabilities.³⁰ Estimate $G_2^{(a)}$ is the traditional one, already used in Ref. 1, in which the spherical charge density is assumed to be uniform within a radius, $R_{EM}=1.2A^{1/3}$ F. This gives

$$G_2^{(a)} = (5/4\pi) (Z^2/R_{EM}^2) S_2(I \rightarrow I'). \quad (4.2)$$

Estimate G_2 is calculated using a Fermi distribution for the charge density³¹ with parameters chosen to fit elastic electron scattering data, and has been recommended³⁰ as a realistic improvement over the traditional estimate. In the present case G_2 equals $1.29G_2^{(a)}$. These quantities are given in Table III together with measured EM transition probabilities.⁶ On the whole, the latter are consistent with the values for G_2 , except for

³⁰ A. Bernstein, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum Press, New York, to be published), Vol. III.

³¹ L. W. Owen and G. R. Satchler, Nucl. Phys. **51**, 155 (1964).

the transition to the level at 3.400 MeV already commented upon in Sec. IV A. We should not expect more than order-of-magnitude agreement between these quantities for noncollective transitions.

Good energy resolution was achieved by Blair and Hamburger¹⁸ in their (d, d') experiment using 15-MeV deuterons and a magnetic spectrograph for particle detection. Although an angular distribution was obtained for only the strongest transition, cross sections to all the levels through 4.057 MeV were measured at $\theta_{lab}=29.7^\circ$, where the cross section to the 1.614-MeV state was a maximum. In the last column of Table III we list the relative cross sections at this angle, normalized so that the value for the 1.614-MeV level equals what we have deduced for G_2 . It is interesting to find that such relative cross sections correlate very well with our measured strengths, although we appreciate that this is a rather optimistic usage of incomplete and unanalyzed data. We note particularly (i) the cross section to the state at 3.970 MeV is less than one-tenth of that for the level at 4.057 MeV, thus giving support to our neglect of contamination from the 3.970-MeV state. (ii) The cross section to the $I^\pi=\frac{3}{2}^+$, $K\pi=\frac{1}{2}^+$ level at 2.801 MeV is three times stronger than that to the neighboring member of the first $K=\frac{1}{2}^+$ band at 2.738 MeV, transitions which were unresolved in our experiment.

When normalized in a similar fashion, the relative cross sections for inelastic scattering of 17-MeV protons^{20,21} tend to be somewhat larger than was the case for deuteron or α -particle scattering. However, since the proton angular distributions for even the strongest transitions do not display patterns clearly characteriz-

TABLE III. Values of G_2 , the collective enhancement ratio.

$-Q$ (MeV)	G_2^a	G_2	G_2 (EM)	Relative (d, d) cross section ^a
0.585	<0.19	<0.25	0.053±0.007	0.22
0.975	<0.19	<0.25	0.16±0.04	0.20
1.614	4.5±0.3	5.8±0.4	6.67±1.6	5.8
1.960	(0.36±0.08)	(0.46±0.10)	0.13±0.04	0.36
2.562	0.27±0.07	0.35±0.09	...	0.34
2.738			<0.08	0.17
	0.84 to 0.37	1.08 to 0.48		
2.801			...	0.51
3.400	1.63±0.10	2.10±0.13	0.17 _{-0.18} ^{+0.27}	2.34
3.905	0.39
3.970	<0.09
4.057	0.67±0.17	0.86±0.22	0.62±0.17	0.96

^a Normalized so that the cross section for the excitation of the 1.611-MeV level equals 5.8, the corresponding G_2 value in the third column.

ing quadrupole excitation over a range of isotopes and further are not well fitted by DWBA calculations, it is likely that the excitation mechanism for protons is often more complicated than that for α particles at our energy.

The inelastic scattering of 44-MeV α particles²² has been analyzed in terms of the smooth-cutoff model of Blair, Sharp, and Wilets,³² and the values extracted for $\sqrt{S_2}$ have been presented in graphical form. These values seem to be somewhat smaller than our own.

V. CONCLUSIONS

The relative strengths of the ground-state in-band transitions show remarkable agreement with the predictions of the strong-coupling model for $\lambda=2$ excitation only. The deformation distance, which characterizes the intrinsic structure of this band, as extracted from the DWBA in-band transition strengths, is 1.42 F. This is to be compared with the value of 1.68 F obtained¹ for the ground-state band in ²⁴Mg.

Comparison of the universal plots of elastic scattering from ²⁵Mg with those of the even-even neighboring nuclei ²⁴Mg and ²⁶Mg exhibit marked differences in cross section between ²⁵Mg and its even-mass neighbors, especially at the third and fourth diffraction minima. When, however, the quadrupole contribution to ²⁵Mg elastic scattering, as predicted by the strong-coupling model, is subtracted out, agreement among the elastic scattering angular distributions was restored.

Our analysis of the cross section to the 4.057-MeV level, the largest observed of the out-of-band transitions, strongly supports the assignment of $I^\pi = K^\pi = \frac{9}{2}^+$

³² J. S. Blair, D. Sharp, and L. Wilets, Phys. Rev. **125**, 1625 (1962).

for this state. The level is interpreted as a $K=2^+$ core excitation coupled to the ground state $K^\pi = \frac{5}{2}^+$ orbital.

The results on the out-of-band transitions to the members of the second $K = \frac{1}{2}^+$ band (which commences at 2.565 MeV) are not inconsistent with the interpretation of this band as the second $K^\pi = \frac{1}{2}^+$ band predicted by the core-excitation model. However the popular (alternative) interpretation, that it is based on Nilsson's single-particle orbital 11, is supported by the observed energies of the band members and by the results of stripping reactions which populate these levels, and furthermore is not contradicted by our scattering experiment. It is possible that the correct interpretation is more complex than the picture offered by either of the extreme models.

The observed strong inhibition of the transitions to the lowest two members of the first $K^\pi = \frac{1}{2}^+$ band (which commences at 0.58 MeV) provides further evidence of the validity of the strong-coupling model in its application to ²⁵Mg. The observed enhancement of the transition to the $\frac{5}{2}^+$ member of this band can be accounted for by assuming a small ground-state admixture in the configuration of this state. We think it is equally possible, however, that the enhancement is caused mainly by a monopole transition similar to that observed in the neighboring even-mass nucleus ²⁶Mg.

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