Parametrization of Energy-Averaged and Fine-Energy-Resolution Cross Sections for the $^{40}Ca(\alpha, \alpha_0)^{40}Ca$ Reaction 5.5 to 17.5 MeV*

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The description of energy-averaged angular distributions for the $^{40}Ca(\alpha, \alpha_0)^{40}Ca$ reaction from 5.5 to 17.5 MeV as the incoherent sum of an l-dependent-absorption optical-model cross section and a Hauser-Feshbach compound-elastic-scattering cross section is discussed. The parameters for both cross sections are determined throughout the energy range. Statistical-model calculations of the Hauser-Feshbach parameters ρ and σ are in good agreement with those extracted from the data. The compound-elastic-scattering cross sections obtained by analyzing the energy-averaged angular distributions are compared with those obtained in an Ericson fluctuation analysis of the fine-resolution excitation curves.

I. INTRODUCTION

THE combined $^{40}Ca(\alpha, \alpha_0)^{40}Ca$ data obtained by John *et al.*[1] and by Robinson *et al.*[2] are analyzed in this paper both as energy-averaged angular distributions and as fine-energy-resolution excitation functions. These measurements were made between beam energies of 5 and 18 MeV. At these energies α-particle scattering by ^{40}Ca is characterized by rapid variations of the differential cross section with energy. Previous attempts to parametrize the strongly energy-dependent angular distributions in terms of reaction models have proceeded along three alternate paths.

The first is an extended absorption model in which resonance terms are added to the Ackhiezer-Pomeranchuk-Blair-McIntyre model[3] or the optical model.[4,5] However, it is assumed in this type of analysis that the anomalies in the cross section are well enough separated to identify the resonating partial wave or waves, a condition which is not generally valid in the present case.

The second method followed by Robinson *et al.*[2] consists of fitting each angular distribution by allowing several optical-model parameters to vary.[6] This procedure determines an average optical potential, the deviations from which describe the energy dependence of the cross section. Fits to the data were obtained for energies between 12 and 18 MeV which are fair by present standards.

In the third method, John *et al.*[1] showed the importance of compound elastic scattering for bombarding energies below 12 MeV. The experimental cross sections were energy-averaged to remove the fine structure. The results were compared with computed cross sections which were incoherent sums of a direct cross section given by the standard optical model and a compound elastic cross section given by a Hauser-Feshbach term. Good fits were obtained in the bombarding energy range 5.5–10 MeV, but above this energy the fits deteriorated rapidly.

In the present analysis, the imaginary part of the optical-model potential is l-dependent, following the technique briefly introduced by Bisson and Davis.[7] They obtained excellent fits to the $^{40}Ca(\alpha, \alpha_0)^{40}Ca$ scattering data over the entire energy range of 5.5–17.5 MeV with the l-dependent modification of the optical model for the direct scattering along with a Hauser-Feshbach term for the compound elastic scattering. An expanded discussion of the analysis is given here. Further, an Ericson-type fluctuation analysis[8] has been applied to the fine-energy-resolution excitation functions. Compound-elastic-scattering cross sections are thus obtained in two ways, and the results are compared. Extracted values of the Hauser-Feshbach term parameters σ and ρ are compared with the predictions of the statistical model.

II. AVAILABLE DATA AND DATA HANDLING

The data on the elastic scattering of α particles by ^{40}Ca were measured by Robinson *et al.*[2] and by John *et al.*[1] with the Florida State University Tandem Van de Graaff Accelerator. In the energy range from 5.0 to 12.5 MeV, John *et al.*[1] obtained 16-point an-

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[1] J. John, C. P. Robinson, J. P. Aldridge, and R. H. Davis, Phys. Rev. **177**, 1755 (1969).

[2] C. P. Robinson, J. P. Aldridge, J. John, and R. H. Davis, Phys. Rev. **171**, 1241 (1968).

[3] E. B. Carter, G. E. Mitchell, and R. H. Davis, Phys. Rev. **133**, B1421 (1964).

[4] T. Tamura and T. Teresana, Phys. Letters **8**, 41 (1964).

[5] H. J. Kim, Phys. Letters **19**, 296 (1965).

[6] J. P. Aldridge, G. E. Crawford, and R. H. Davis, Phys. Rev. **167**, 1053 (1968).

[7] A. E. Bisson and R. H. Davis, Phys. Rev. Letters **22**, 542 (1969).

[8] T. Ericson and T. Mayer-Kuckuk, Ann. Rev. Nucl. Sci. **16**, 183 (1966).

gular distributions every 10 keV and 64-point angular distributions every 100 keV. In the energy range from 12.0 to 17.9 MeV, Robinson *et al.*[2] obtained 16-point angular distributions every 25 keV and 64-point angular distributions every 100 keV.

In this new analysis of all the data, an averaging procedure similar to that of John *et al.*[1] is followed. The experimental cross section is weighted by a Lorentzian function $\rho(E-E_0)$, defined by

$$\rho(E-E_0)=I^2/[(E-E_0)^2+I^2], \tag{1}$$

where $2I$ represents the full width at half-maximum, and E_0 is the energy at which the average value is computed. Between 5.0 and 12.5 MeV, the 16-angle 10-keV data are averaged with $2I=0.5$ MeV. Because of the pronounced structure in the high-energy angular distributions, the 64-angle data at 100-keV intervals have been averaged in the energy range from 12.0 to 17.9 MeV with $2I=1.0$ MeV. The averaging interval $2\Delta E$ was limited to 2.0 MeV over the full energy range.

The average cross section

$$\langle\sigma(E_0)\rangle=\sum_{E_0-\Delta E}^{E_0+\Delta E}\sigma(E)\rho(E-E_0)\Big/\sum_{E_0-\Delta E}^{E_0+\Delta E}\rho(E-E_0) \tag{2}$$

was calculated for all angles every 0.5 MeV throughout the bombarding energy range 5.5–17.5 MeV.

III. ANALYSIS OF ENERGY-AVERAGED ANGULAR DISTRIBUTIONS

A. Shape and Compound Elastic Scattering

Energy-averaged differential cross sections $\langle\sigma\rangle$ have been calculated as an incoherent sum

$$\langle\sigma\rangle=\sigma_d+\langle\sigma\rangle_c \tag{3}$$

of a direct-reaction cross section σ_d obtained by the optical model and a compound elastic cross section $\langle\sigma\rangle_c$. The modified optical model used to calculate the direct-reaction cross section σ_d is described in Sec. III B. The compound elastic cross section $\langle\sigma\rangle_c$ for the ^{40}Ca(α, α_0) reaction has been calculated using a Hauser-Feschbach formula

$$\langle\sigma\rangle_c=(W_{cc}/4k^2)\sum_l(2l+1)^2 \times(T_c{}^lT_c{}^l/\sum_{c''}T_{c''}{}^l)P_l{}^2(\cos\theta), \tag{4}$$

where k is the wave number of the incoming α particle and the quantities $T_c{}^l$ are taken as optical-model transmission coefficients for the elastic channel c. The width correction factor W_{cc} accounts for the fact that the decay of the compound nucleus into the elastic channel c is enhanced by a factor W_{cc} relative to the inelastic channels.

A difficulty in evaluating Eq. (4) is the occurrence of the term $\sum_{c''}T_{c''}{}^l$ because it involves all open exit channels in which the compound nucleus can decay. This term can be calculated, however, in terms of statistical-model quantities,[9] and the following formula has recently been given by Eberhard *et al.*[10]:

$$\sum_{c''}T_{c''}{}^J=2\pi(\Gamma_0/D_0)(2J+1) \times\exp[-J(J+1)/2\sigma_{\rm res}{}^2(1+\omega_{\rm res})]. \tag{5}$$

Here, J is the spin of the compound-nucleus levels populated in the ^{40}Ca+α reaction and the relation $J=l$ holds in this case since the α particle and ^{40}Ca are spin-zero particles. The quantities Γ_0 and D_0 are the mean width and spacing of compound levels with spin $J=0$, and $\sigma_{\rm res}{}^2$ is an averaged value of the spin cutoff parameters for the various residual nuclei reached by the decay of the compound nucleus. The correction term $\omega_{\rm res}\approx(5/3)A_\nu/A_{\rm res}$ depends on the mass A of particles ν by which the compound nucleus decays, and on the mass $A_{\rm res}$ of the corresponding residual nucleus. Combining Eqs. (4) and (5) along with the abbreviations

$$\sigma^2=\sigma_{\rm res}{}^2(1+\omega_{\rm res}) \tag{6}$$

and

$$\rho=2\pi(\Gamma_0/D_0)W_{cc}{}^{-1}, \tag{7}$$

the compound elastic cross section $\langle\sigma\rangle_c$ becomes

$$\langle\sigma\rangle_c=(4k^2\rho)^{-1}\sum_l(2l+1)\frac{(T_c{}^l)^2P_l{}^2(\cos\theta)}{\exp[-l(l+1)/2\sigma^2]}. \tag{8}$$

For the calculations presented in this paper, the quantities σ and ρ were treated as free parameters and were adjusted simultaneously with the optical-model parameters.

In Eq. (3), the interference effect between the optical-model amplitude and the compound elastic one has been neglected. The use of Eq. (3) is therefore only justified if an energy-averaging interval can be found which is sufficiently larger than the mean width of the fine structure in the excitation functions but within which the direct-reaction cross section changes only smoothly. The experimental data used here for a comparison with Eq. (3) have been averaged using the Lorentzian weighting procedure of Sec. II over approximately 1 MeV, which is much larger than the mean width of the fine structure [about 6 keV at 6 MeV and about 30 keV at 17 MeV bombarding energy (Fig. 7)].

B. *l*-Dependent Absorption

While a shape-plus-compound-elastic-scattering description proved satisfactory below 10.5-MeV bombarding energy, it progressively deteriorated above that energy. Computed back-angle cross sections were generally too low. This led Bisson and Davis to in-

[9] W. von Witsch, P. von Brentano, T. Mayer-Kuckuk, and A. Richter, Nucl. Phys. **80**, 394 (1966).

[10] K. A. Eberhard, P. von Brentano, M. Böhning, and R. O. Stephen, Nucl. Phys. **A125**, 673 (1969).

troduce an l dependence in the imaginary part of the optical model.[7] No modification of the real part proved necessary in order to obtain good fits throughout the energy range 5.5–17.5 MeV.

The empirical requirement for the l-dependent imaginary part is that the imaginary strength be reduced as l increases. A Woods-Saxon form with the argument l was chosen. This and similar forms can be qualitatively justified as a result of momentum- and energy-conservation requirements.[11] It has subsequently been successfully used in the analysis of ^{16}O scattering.[12–14]

The l-dependent modified optical-model potential used is given by

$$V(r)=-Uf_M(r)-F(l)\times 4ia_IW_P(d/dr)[f_I(r)]+V_C, \tag{9}$$

where

$$V_C=(Z_pZ_te^2/2R_C)[3-(r/R_C)^2], \qquad r\le R_C$$
$$=Z_pZ_te^2/r, \qquad r>R_C$$

and

$$F(l)=\{1+\exp[(l-l_c)/\Delta l]\}^{-1},$$
$$f(r)=\{1+\exp[(r-R)/a]\}^{-1}.$$

The angular momentum cutoff parameter l_c and the diffuseness Δl in the cutoff are average values representing all open exit channels except the elastic one. The magnitude and energy dependence of l_c is given by Chatwin *et al.*[12] as $l_c=\bar{R}\times[E(\text{c.m.})+\bar{Q}]^{1/2}$ where $\bar{R}$ is an average interaction radius and $\bar{Q}$ an average $\bar{Q}$ value. For simplicity we have assumed $\bar{Q}=0$ and have used $\bar{R}=5.2$ F which corresponds, e.g., to a radius parameter of $r_0=1.06$ F for the $\alpha+^{40}Ca$ channel. This relatively small $\bar{R}$ value may partly compensate the effect of using $\bar{Q}=0$. The large value of $\Delta l=4$, along with the unusually deep imaginary potential depths at higher energies, however, seems to indicate the need for a more realistic treatment of l_c.

C. Fits to Energy-Averaged Data

Energy-averaged angular distributions every 0.5 MeV were fitted with the l-dependent modified optical model along with the addition of a compound-elastic-scattering term.

The quality of the fits was determined by the usual χ^2 criterion

$$\chi^2=N^{-1}\sum_{i=1}^{N}\left(\frac{\sigma(\theta_i)_{\text{expt}}-\sigma(\theta_i)_{\text{calc}}}{\Delta\sigma(\theta_i)_{\text{expt}}}\right), \tag{10}$$

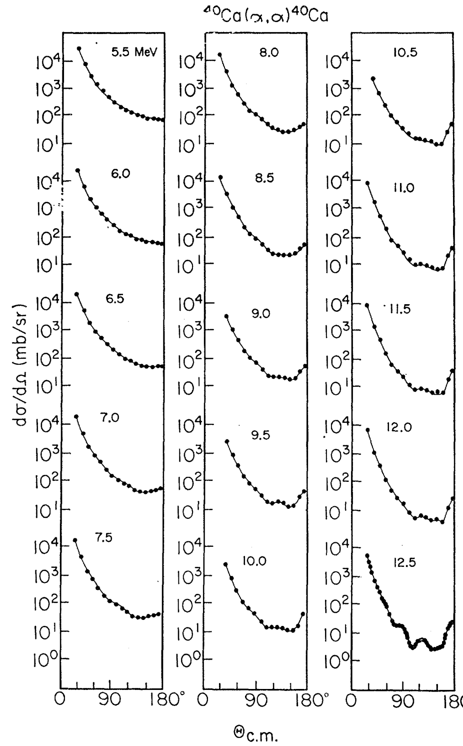

FIG. 1. Energy-averaged angular distributions for the $^{40}Ca(\alpha,\alpha_0)^{40}Ca$ reaction from 5.5 to 12.5 MeV. The solid lines are calculated from the extended optical model described in the text.

where N is the number of data points, $\sigma(\theta_{i\text{expt}})$ is the measured cross section, $\hbar(\theta_i)_{\text{calc}}$ is the calculated one, and $\Delta\sigma(\theta_i)_{\text{expt}}$ is the error in the experimental cross section.

The optical-model parameters describing the geometry were taken from Robinson *et al.*[2] and John *et al.*,[1] namely, $R_M=5.2$ F, $R_I=5.0$ F, $R_C=5.2$ F, and $a_I=0.30$ F. With the single exception of finding that $a_M=0.53$ F gave a lower χ^2 value, these were the parameters used throughout the energy range.

Different values of Δl were tried with satisfactory fits obtained for Δl ranging from 3.5 to 4.5. However, $\Delta l=4$ gave slightly lower values of χ^2 and was used throughout the energy range.

With the parameters describing the geometry fixed, the parameters U, W_P, σ, and ρ were varied. A coarse grid search was performed first: U was varied from 120 to 150 MeV in steps of 3 MeV; W_P from 0 to 100 MeV, $\Delta W_P=10$ MeV; σ from 1 to 10, $\Delta\sigma=2$; and ρ from 0.1 to 20, $\Delta\rho=2$ and from 20 to 200 with $\Delta\rho=10$.

Values of U, W_P, σ, and ρ near minima of χ^2 were then scanned using a fine grid search: U was varied in steps of 0.5 MeV; W_P in 0.1 MeV; σ and ρ in 0.1 units.

Figures 1 and 2 show the resulting best fits to the energy-averaged cross sections in the bombarding

[11] R. A. Chatwin, J. S. Eck, A. Richter, and D. Robson, in *Proceedings of the International Conference on Nuclear Reactions Induced by Heavy Ions, Heidelberg* (North-Holland Publishing Co., Amsterdam, to be published).

[12] R. A. Chatwin, J. S. Eck, D. Robson, and A. Richter, Phys. Rev. (to be published).

[13] J. S. Eck, R. A. LaSalle, and D. Robson Phys. Rev. **186**, 1132 (1969).

[14] J. S. Eck, R. A. Chatwin, K. A. Eberhard, R. A. LaSalle, A. Tichter, and D. Robson Ref. 11.

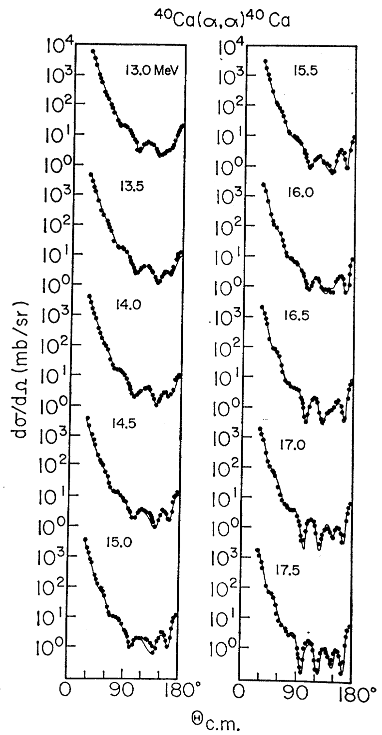

FIG. 2. Same as in Fig. 1, but for bombarding energies 13.0–17.5 MeV.

energy range 5.5–17.5 MeV. The dots represent the experimental energy-averaged angular distributions and the solid lines represent the fits obtained from the incoherent sum of cross sections calculated with the l-dependent optical model and the Hauser-Feshbach term Eq. (8). Values of χ^2 ranged from 2 at low energies to 9 at high energies with an average value of about 6.

Figure 3 shows the optical-model and the Hauser-Feshbach parameters as well as a compound-elastic-scattering cross section obtained by fitting the energy-averaged angular distributions. The reaction cross section has been calculated using these parameters and is shown as a function of energy in Fig. 4. The solid line represents the sum of the reaction cross section and the compound elastic one shown in Fig. 3.

IV. ANALYSIS OF EXCITATION FUNCTIONS

The statistical analysis of cross-section fluctuations in excitation function introduced by Ericson[15] and by Brink and Stephen[16] can be used to estimate (i) the relative amount of the direct-reaction contribution Y_d and (ii) the coherence width Γ of the compound nucleus. According to the theory, the quantities Y_d and Γ are determined from a comparison of the autocorrelation function

$$C(\epsilon)=[\langle\sigma(E)\sigma(E+\epsilon)\rangle/\langle\sigma(E)\rangle\langle\sigma(E+\epsilon)\rangle]-1, \quad (11)$$

and its theoretical prediction

$$C^{\text{theor}}(\epsilon)=(1-Y_d^2)\Gamma^2/(\Gamma^2+\epsilon^2). \quad (12)$$

Equation (12) is valid for a reaction where only zero-spin particles are involved as is the case for the reaction studied here. The differential cross sections $\sigma(E)$ and $\sigma(E+\epsilon)$ are measured at energies E and $E+\epsilon$, respectively, where ϵ is the experimental energy step size or a multiple of it. The averages $\langle\ \rangle$ are taken over energy. The quantity $Y_d=\sigma_d/\langle\sigma(E)\rangle$ is the relative fraction of the direct-reaction cross section σ_d in the reaction, which is simply given in this case by the normalized variance $C(0)$:

$$C(0)=1-Y_d^2. \quad (13)$$

The above equations hold for ideal conditions, i.e., excitation functions over an infinite long-energy interval, perfect energy resolution, etc. Otherwise corrections have to be made for practical cases, especially if these conditions are met only poorly. In this analysis, corrections are made due to the energy step

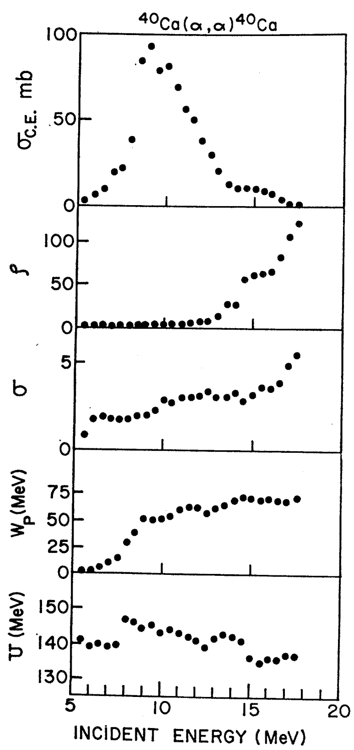

FIG. 3. Plot of the energy-dependent parameters used in the extended optical-model calculations shown in Figs. 1 and 2. The quantities U and W_P are the real and the surface imaginary potential; σ and ρ are statistical-model parameters describing the compound elastic scattering. The calculated angle-integrated compound elastic cross section $\sigma_{\text{c.e.}}$ is displayed at the top of the figure.

[15] T. Ericson, Advan. Phys. **9**, 425 (1960); Ann. Phys. (N.Y.) **23**, 390 (1963); Phys. Letters **4**, 258 (1963).

[16] D. M. Brink and R. O. Stephen, Phys. Letters **5**, 77 (1963).

size ϵ and the energy resolution ΔE. The effect of a finite-energy resolution ΔE on Γ and $C(0)$ has been studied by various authors.[17–20] The following corrections have been used here:

$$\Gamma^2 = \Gamma_{\text{expt}}^2 - (\Delta E)^2, \tag{14}$$

$$C(0) = C(0)_{\text{expt}}(\Delta E/\pi\Gamma + 1), \tag{15}$$

where Γ_{expt} and and $C(0)_{\text{expt}}$ are the uncorrected values, and ΔE is the energy spread in the entrance channel (which was assumed to be equal to the target thicknesses plus 2-keV energy spread of the α-particle beam).

Since there do not exist simple correction formulas like Eqs. (14) and (15) for finite-energy step size corrections, an excitation function of the $^{24}Mg(\alpha, \alpha_0)^{24}Mg$ reaction[21] measured with good energy resolution ($\epsilon/\Gamma \approx \frac{1}{3}$) was used as a test case to estimate the effect of increasing ϵ/Γ on the $C(0)$ value. It was found that $C(0)$ decreases by about 30% if ϵ/Γ is increased from $\epsilon/\Gamma = \frac{1}{3}$ to 2. The values of ϵ/Γ found here are between 1 and 2, so that the actual values for $C(0)$ are probably 10–30% higher than those determined from Eqs. (11) and (12) and corrected by Eq. (15). Because of the small magnitude and somewhat tenuous determination of this correction, it has not been applied to the values of Γ and $C(0)$ obtained by the autocorrelation method.

Another, completely independent, method of estimating the coherence width Γ, first proposed by Brink and Stephen,[16] is to count the number of maxima, n, in an excitation function measured over energy interval I. This number is simply related to Γ by

$$n = 0.5bI/\Gamma, \tag{16}$$

where the factor b is a coefficient which has been

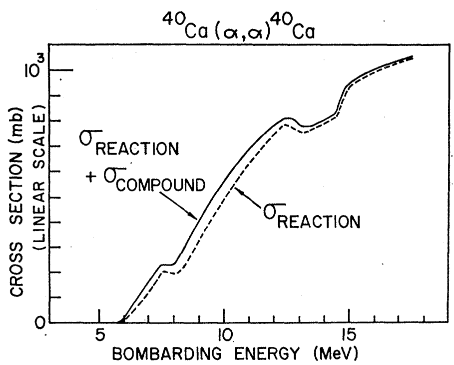

FIG. 4. Reaction cross section and compound elastic cross section obtained from fitting the energy-averaged data.

[17] H. L. Acker, Max-Planck Institut für Kernphysik, Tandem Laboratory Report No. V/3, Heidelberg, 1964 (unpublished).
[18] W. R. Gibbs, Los Alamos Scientific Laboratory Report No. LA-3266, 1965 (unpublished); P. Fessenden, W. R. Gibbs, and R. B. Leachman, Phys. Rev. Letters **15**, 796 (1965).
[19] D. W. Lang, Bull. Am. Phys. Soc. **9**, 461 (1964).
[20] A. van der Woude, Nucl. Phys. **80**, 14 (1966).
[21] K. A. Eberhard and C. Mayer-Boricke Nucl. Phys. (to be published).

calculated under statistical assumptions by various authors.[16,22–24] In this paper, b is taken from Ref. 24, where b is given as a function of ϵ/Γ. Consequently, the value of Γ from Eq. (16) already includes a correction for the finite-energy step size ϵ. In addition, all Γ values derived by the peak-counting method have also been corrected for the finite-energy resolution ΔE by Eq. (14).

The procedure used to determine Γ and $C(0)$ was the following. First, all excitation functions were corrected for the gross energy dependence of the mean cross section. Gross structure, due to the opening of new reaction channels as the energy is increased and to direct reaction effects, was removed by dividing each experimental point $\sigma_{\text{expt}}(E)$ by the averaged cross section $\langle\sigma(E)\rangle$ of Sec. II. The resulting cross section $\sigma(E) = \sigma_{\text{expt}}(E)/\langle\sigma(E)\rangle$ shows the fluctuations about the average cross section. Figure 5 illustrates the difference between the measured excitation function and the trend-reduced one at 158.1° (c.m.). The solid line represents the trend and the dashed line was obtained by the Hauser-Feshbach formula Eq. (8) using optical-model transmission coefficients from Sec. III and computed values for σ and ρ as given in Sec. V.

The trend-reduced excitation functions were divided into overlapping 1-MeV intervals centered 0.5 MeV apart for bombarding energies between 5.5 and 17.5 MeV for each of the 16 angles between 26.7° and 176.1° c.m.). The quantities Γ and $C(0)$ were determined for every interval and angle from the autocorrelation function.

The peak-counting method was used to extract the coherence width Γ at 7.5, 9.5, and 11.5 MeV, counting over an energy interval of $I = 2$ MeV and, for better statistics, at 13.5 and 16.5 MeV over $I = 3$ MeV. The coherence widths obtained at a given energy but different angles were averaged to give a more statistically meaningful value.

The results for Γ are shown in Fig. 6. The values obtained by the peak-counting method are slightly larger on an average than those obtained by the autocorrelation method, which behavior is in agreement with a theoretical study of this effect by van der Woude.[20] The solid and the dashed curves are calculated results discussed in Sec. V. The vertical bars represent the finite range-of-data errors.[8]

V. SIGNIFICANCE OF PARAMETERS

A. Quantities σ, ρ, and Γ

For comparison with the extracted results, the magnitudes and the energy dependencies of the "fit-

[22] P. J. Dallimore and I. Hall, Phys. Letters **18**, 138 (1965).
[23] P. G. Bizzeti and P. R. Maurenzig, Nuovo Cimento **47B**, 29 (1967).
[24] J. D. A. Roeders, Nuovo Cimento **54B**, 151 (1968).

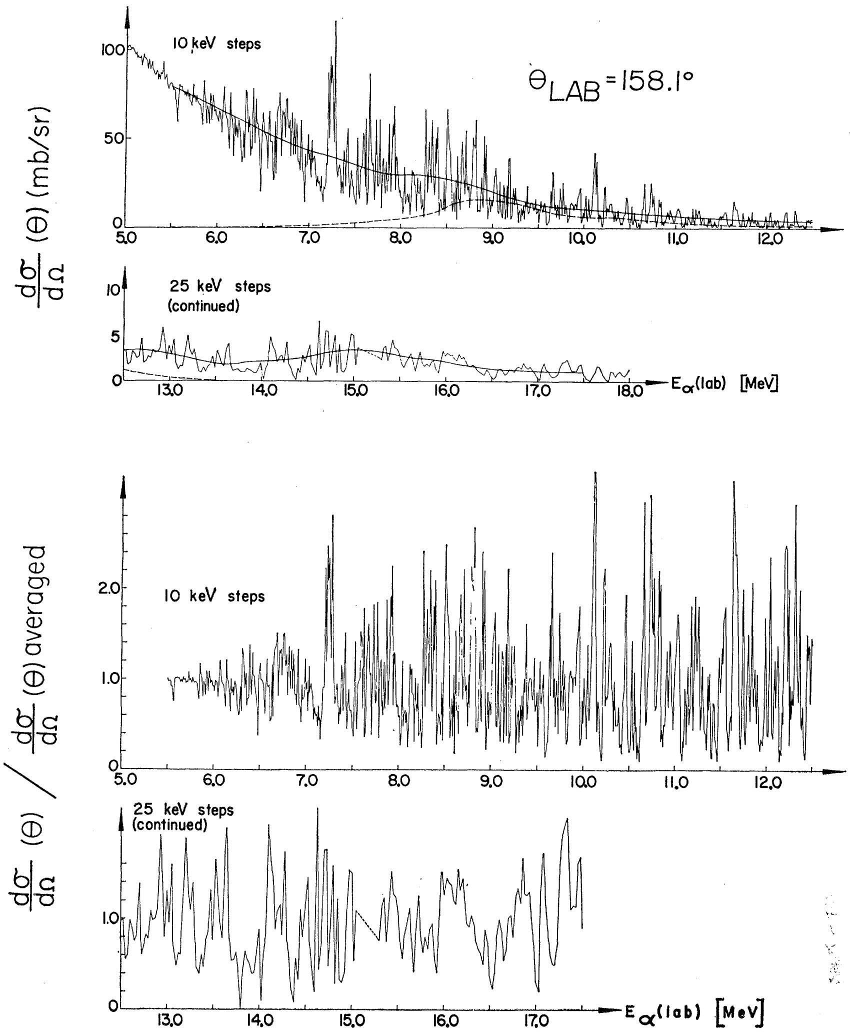

FIG. 5. Experimental and trend-reduced excitation function for $\theta_{c.m.}=158.1°$. The trend is represented by the solid line and the dashed line is a Hauser-Reshbach calculation for the compound elastic cross section.

ting parameters" σ and ρ and of the coherence width Γ have been calculated using the statistical-model formulas recently given by Eberhard *et al.*[10] In addition, the values for the moment of inertia, on which σ depends, have been compared with superconductor model calculations by Vonach *et al.*[25]

Several input quantities must be fixed in order to carry out the calculations. The assumed expression for the level-density parameter a is $a=A/9.5$ MeV^{-1}, where A is the atomic mass number. This value is consistent with the recent work by Gadioli and Zetta.[26] The mass-radius parameter r_0 is 1.2 F and the interaction-radius parameter r_ν is 1.5 F. Effective excitation energies in the compound nucleus E_{CN} and in the residual nuclei E_{res}^ν, formed by the decay of particle ν from the compound nucleus, have been taken as

$$E_{CN}=E_\alpha(\text{c.m.})+Q, \tag{17}$$

$$E_{res}^\nu=E_{CN}-B_\nu-B_{Coul}^\nu-\Delta_{res}^\nu+\Delta_{CN}. \tag{18}$$

The energy E_α(c.m.) is the center-of-mass bombarding energy and Q is the (α, γ) Q value for ^{40}Ca; the quantity B_ν is the binding energy of particle ν in the compound nucleus and Δ_{res}^ν and Δ_{CN} are the pairing energies for the residual and the compound nucleus, respectively, and were taken from Gilbert and Cameron.[27] Coulomb barriers (B_{Coul}^ν) for particle ν in the compound nucleus of 2 MeV for protons and 6 MeV for α particles have been used.

The effective excitation energies introduced above are somewhat arbitrary, but are in agreement with

[25] H. K. Vonach, R. Vandenbosch, and J. R. Huizenga, Nucl. Phys. **60**, 70 (1964).
[26] E. Gadioli and L. Zetta, Phys. Rev. **167**, 1016 (1968).

[27] A. Gilbert and A. G. W. Cameron, Can. J. Phys. **43**, 1446 (1965).

similar expressions used and discussed by other authors.[26,28,29]

Figure 7 shows the results for σ in terms of the nuclear moment of inertia τ/τ_{rig}, where $\tau_{rig}=\frac{2}{5}MR^2$ is the moment of inertia of a spherical rigid body. The relevant relation between σ and τ is

$$\tau=\sigma^2(\hbar^2/T)(1+\omega_{res})^{-1}, \tag{19}$$

based on Eq. (6) and on $\sigma_{res}{}^2=\tau T/\hbar^2$, where $T=(E_{res}/a)^{1/2}$ is the nuclear temperature. Equation (19) is calculated for the residual nucleus ^{43}Sc which is populated by the dominant proton decay of the compound nucleus. The vertical lines in Fig. 7 indicate the change in τ/τ_{rig} when the cross sections were varied within the experimental error bars of $\pm 10\%$ in the optical-model analysis. Above 11 MeV, the experimental values are exatly those for a rigid body, and below 11 MeV they decrease smoothly with decreasing energy. Below 8 MeV they increase again; but here Eq. (19) is no longer valid, so that no significance can be attached to this behavior. The solid line represents a calculation by Vonach *et al.*[25] based on the superconductor model. In excellent agreement with the experimental results, τ remains constant from 18 to 12 MeV and decreases below 12 MeV. Lower than 7 MeV, where the solid line ends, no predictions were made from the superconductor model.

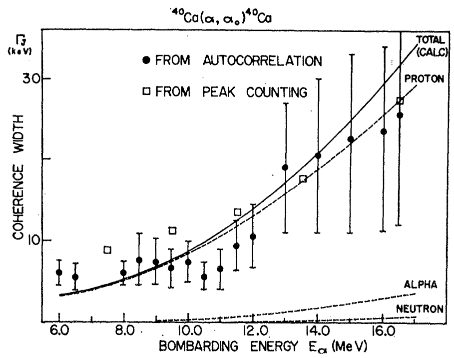

FIG. 6. The coherence width extracted from the excitation functions using the autocorrelation method and the peak-counting method. The solid line is the sum of the proton-, neutron-, and α-particle widths (dashed lines) which were calculated from the statistical model.

In a similar presentation, Fig. 8 shows the values for the ρ parameter and a theoretical curve calculated from $\rho=\frac{1}{4}(2\pi\Gamma_0/D_0)$. The factor $\frac{1}{4}$ comes from adjusting the calculated curve to the experimental points and implies along with Eq. (7) a width-fluctuation factor $W_{cc}=4$. Although no experimental value for W_{cc} has yet been determined for α scattering, $W_{cc}=4$ seems to be rather large. A recent calculation by Ullah,[30] however, revealed that in formulas like Eq. (8) the cross section is overestimated in cases of large transmission coefficients by a factor up to about $K=2$. Taking into account this correction we find $W_{cc}=4/K<4$. The ratio D_0/Γ_0 was calculated from Ref. 10 using the parameter values given at the beginning of this section and the calculated spin cutoff parameter values of Fig. 7. As seen in Fig. 8, the behavior of ρ with energy is in excellent agreement with the calculated one.

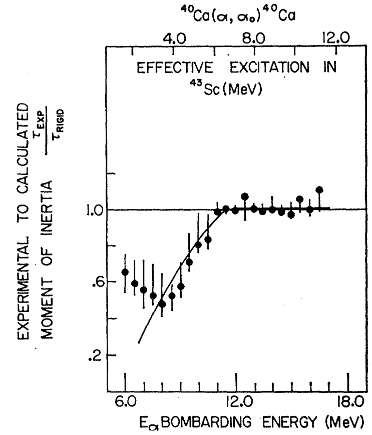

FIG. 7. The parameter σ in terms of the moment of inertia τ/τ_{rig} as a function of energy. The solid line represents a calculation based on the superconductor model.

Neutron-, proton-, and α-particle widths have been calculated from the Γ formula given by Eberhard

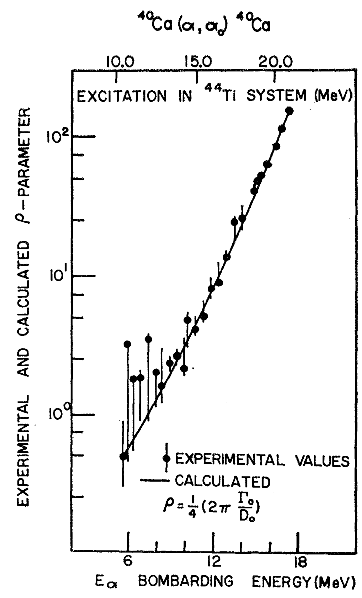

FIG. 8. Experimental (solid circles) and calculated (solid line) ρ parameter as a function of energy.

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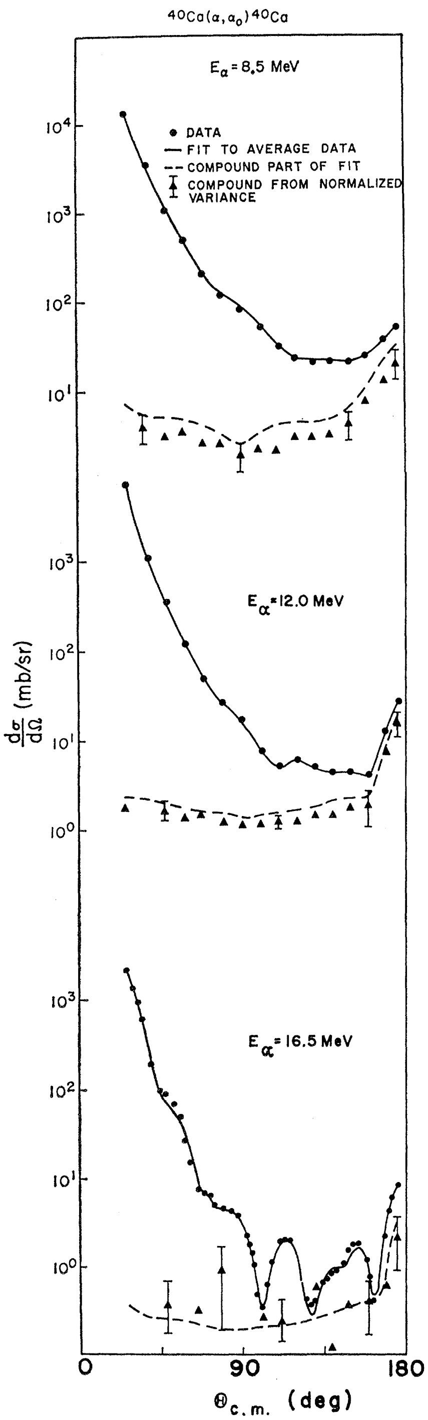

FIG. 9. Comparison of the compound elastic cross section at 8.5, 12.0, and 16.5 MeV obtained from fitting the energy-averaged angular distributions (dashed line) and from a fluctuation analysis of the excitation functions (triangles). The vertical bars indicate the errors corresponding to the values of the normalized variance. The solid line represents the fit to the data as indicated in the figure.

et al.[10] The results are shown by the dashed curves in Fig. 6. The solid line is the sum of the particle widths.

Since Γ depends on J, the spin of the compound-nucleus levels, the Γ values, presented in Fig. 6, were calculated for a comparison with the experimental coherence widths at $J=k_\nu R_\nu$, where k_ν is the wave number of particle ν and R_ν is the interaction radius between ν and the corresponding residual nucleus. As seen in Fig. 6, the proton partial width predominates largely in the decay of the compound nucleus. Both the magnitude and the energy dependence of the experimental data are well described by the calculated curve.

B. Compound Elastic Cross Section

Figure 9 shows the correspondence between the compound elastic cross sections obtained from fitting the energy-averaged angular distributions and those obtained from a fluctuation analysis of the fine-energy-resolution excitation functions in the energy regions around 8.5, 12.0, and 16.5 MeV. As seen in Fig. 9, the compound elastic angular distributions at 8.5 and 12 MeV from both methods are very similar in shape; they differ, however, by 10–30% in magnitude. Since this difference is within the error limits of both methods, no significance is attached to it.

At 16.5 MeV the comparison is not as convincing, since the errors are larger because of the larger energy steps in which the data were taken, and because of the increased sensitivity to the values of the normalized variance due to the small compound elastic contribution at this energy (see Fig. 3).

At 8.5, 12.0, and 16.5 MeV, the values of ϵ/Γ are approximately 2, 1, and 1, respectively. These particular energies were chosen to show the typical correspondence obtained as a function of bombarding energy.

VI. DISCUSSION OF RESULTS

The incoherent sum of calculated cross sections obtained by an optical model with an l-dependent imaginary part and a Hauser-Feshbach compound-elastic-scattering term adequately parametrizes the energy-averaged angular distributions for $^{40}Ca(\alpha,\alpha_0)^{40}Ca$ scattering in the bombarding energy range 5.5–17.5 MeV. The l-dependent absorption is interpreted as the consequence of the angular momentum mismatch between the elastic and other open exit channels, which in particular gives rise to an enhancement of the optical-model cross sections at backward angles.

The compound elastic parameters σ and ρ extracted from fitting the energy-averaged angular distributions are more than fitting parameters. Except for very low bombarding energies ($E_\alpha < 8$ MeV), they are well represented by the combined use of the superconductor model and the Fermi gas model. No particular significance can be attributed to the values of σ and ρ

obtained at low bombarding energies, since the formulas based on the statistical model are no longer valid.

The compound elastic cross sections obtained from fitting the energy-averaged angular distributions are to within the error bars the same as those extracted by a statistical-model analysis of the fine-energy-resolution excitation functions.

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Electron Scattering from Deformed Nuclei

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A general phenomenological theory is developed for the calculation of both elastic and inelastic transverse and longitudinal form factors for electron scattering from nuclei whose surfaces are deformed according to the expression $R+(4\pi)^{1/2}\Sigma_{lm}\,\hat{a}_{lm}Y_{lm}(\hat{r})$. Here, the $\hat{a}_{lm}$ are operators in the nuclear Hilbert space, whose detailed dynamical character is left unspecified. The effects of deformation on the scattering, to any desired accuracy, are contained in reduced matrix elements of tensor products of the $\hat{a}_{lm}$. These are then treated as phenomenological parameters whose values are to be fixed from an analysis of the scattering cross sections, particularly at high momentum transfer. All the static and transition multipole moments of the nucleus are thereby determined, at least in principle. The Helm model and its generalization to transverse scattering are obtained in the small-deformation limit. The theory is used to analyze the Coulomb elastic and inelastic (to the 1.3-MeV $\frac{3}{2}^+$ level) scattering of electrons from ^{59}Co, as well as the magnetic elastic and inelastic scattering from ^{10}B. It is shown that the contribution of the deformation to elastic monopole scattering may be interpreted as due to an effective, oscillatory, spherically symmetric "modulating-charge" distribution, superimposed upon the more usual smeared uniform distribution. It is therefore suggested that the oscillations in the charge distribution which seem to be required in order to fit the high-energy elastic scattering from ^{40}Ca and other nuclei may be due to deformations.

I. INTRODUCTION

IT is well known that electron scattering is a versatile tool for investigating nuclear structure, since it provides a way of determining the momentum-transfer dependence of the relevant nuclear form factors.[1] The reproduction of this dependence in detail is a test for any particular nuclear model used to describe the elastic and inelastic scattering. However, calculations with detailed models, such as the shell model, are laborious, often yield only semiquantitative agreement with experiment, and provide little physical insight into the results. On the other hand, much of the gross structure of the cross sections does not depend upon the details of the nuclear model, but rather on the general shape of the nuclear charge, current, and magnetization densities that enter into the scattering process. Accordingly, there is much value in a more phenomenological approach which makes use of broadly chosen assumptions allowing one to easily extract and interpret useful information, such as transition multipolarities, from the experimental data, and which would relegate the effects due to the details of the nuclear structure to a small number of parameters. These parameters, when adjusted to yield good fits to the cross sections, then determine the static multipole moments, radiative widths of excited states, etc.

Such a model was proposed by Helm[2] for the case of Coulomb electroexcitation, and has recently been extended by Überall and the authors[3] to describe

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