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Nuclear Structure of Sc^{43}. II. Gamma-Ray Angular Correlations

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The spin and multipolarity mixing ratios of five resonances and many bound states in Sc^{43} have been determined from angular-correlation and linear-polarization measurements. Unique spin assignments have been made for the following levels: 0.846 MeV ($\frac{5}{2}$), 0.856 MeV ($\frac{1}{2}$), 1.158 MeV ($\frac{3}{2}$), 1.179 MeV ($\frac{3}{2}$), 1.652 MeV ($\frac{5}{2}$), 1.963 MeV ($\frac{5}{2}$), 2.094 MeV ($\frac{3}{2}$), 2.143 MeV ($\frac{7}{2}$), and 2.580 MeV ($\frac{5}{2}$). In addition, limitations on the possible spin assignments have been made for the following levels: 1.885 MeV ($\frac{5}{2}$, $\frac{9}{2}$), 2.383 MeV ($\frac{3}{2}$, $\frac{7}{2}$), 2.986 MeV ($\frac{3}{2}$, $\frac{5}{2}$), 3.289 MeV ($\frac{3}{2}$, $\frac{5}{2}$), and 4.455 MeV ($\frac{5}{2}$, $\frac{9}{2}$). The parity of the 0.846-MeV level was found to be negative from linear-polarization measurements. These results, together with results due to other investigators, are discussed in terms of various nuclear models. It is shown that neither the shell model nor the cluster model can account for the observed properties of Sc^{43}. The Coriolis-coupling model gives reasonable qualitative agreement, but quantitative agreement with experiment will require a modification of the model. Johnstone's model successfully accounts for the observed properties of all levels except for those levels identified as belonging to the $k^{\pi}=\frac{1}{2}^{+}$ band. Specific experiments are suggested to determine the nature of these states.

I. INTRODUCTION

In this report, we present further results in a continuing experimental investigation into the properties of the energy levels of the nucleus Sc^{43}. In an earlier paper[1] (hereafter referred to as I), we reported the results of γ-ray decay scheme studies on six resonances in the $Ca^{42}(p,\gamma)Sc^{43}$ reaction in the proton energy range 1200–2060 keV. Using an 8-in.-diam× 8-in.-long NaI(Tl) and a 40-cm^3 Ge(Li) detector, consistent decay schemes and accurate level energies for 25 bound states below 5 MeV in Sc^{43} were established. We report here the results of double and triple angular-correlation measurements on the same resonance states discussed in I.

Double and triple angular-correlation measurements of radiative capture reactions provide a means of determining the spins of the bound states in the resulting nucleus. Many resonances with sufficient strength to make angular-correlation measurements feasible have been observed in the $Ca^{42}(p,\gamma)Sc^{43}$ reaction (see I). The five resonances at 1235, 1242, 1423, 1808, and 2037 keV were selected for the correlation measurements. The measurements described in this paper lead to unique spin assignments for nine bound states and limitations on possible spin assignments for five bound states.

The ground state of Sc^{43} has $J^{\pi}=\frac{7}{2}^{-}$ as deduced from β-decay[2,3] and magnetic moment[4] measurements. The first excited state at 0.151 MeV is known to have $J^{\pi}=\frac{3}{2}^{+}$ from life-time[3,5], l-value,[6] and K-conversion coefficient[7] measurements. The second excited state at 0.472 MeV has been assigned $J^{\pi}=\frac{3}{2}^{-}$ on the basis of l-value,[6,8] K-conversion coefficient[7], and $Ca^{40}(\alpha, p\gamma)Sc^{43}$ angular-correlation[9] measurements. $Ca^{40}(\alpha, p\gamma)Sc^{43}$ angular-correlation studies[9] have lead to spin assignments of $\frac{5}{2}$ for the 0.880-MeV level and $\frac{7}{2}$ for the 1.336-MeV level. In addition, l-value assignments have been made for certain levels.[6,8,10] Limitations on possible spin assignments for certain levels have been made from studies of the $Ca^{42}(p,\gamma)Sc^{43}$ angular correlations.[11]

From a theoretical standpoint, the study of the Sc^{43} level properties is of interest because of the large number of theoretical predictions of these properties based on various models. Spherical shell-model calculations[12–14] based on a closed Ca^{40} core with the outer shell nucleons in a $1f_{7/2}$ or a mixed $(1f_{7/2}2p_{3/2})$ configuration have met with limited success. In order to account for the low-lying positive-parity states in the $f_{7/2}$ shell nuclei, Dieperink and Brussaard[15] have performed a shell-model calculation in which they lift a $1d_{3/2}$ particle out of the Ca^{40} core, but again they have had very

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limited success. A more successful calculation which reproduces most of the general features of the $f_{7/2}$ shell nuclei is that of Malik and Scholz.[16,17] They have calculated the normal parity spectra for odd-mass $1f_{7/2}$ nuclei using the strong-coupling symmetric-rotator model including the Coriolis coupling between bands. In the case of Sc^{43}, they have calculated[18] the spectra including changed parity states arising from core excitation of the $2s$-$1d$ shell, and a comparison between these results and experiment is presented in Sec. V. The most successful calculation of the level properties of Sc^{43} is that of Flowers, Johnstone, and Payne.[19–22] They have calculated both the positive- and negative-parity states by mixing shell-model states with various positive- and negative-parity rotational bands. A detailed discussion of the excellent agreement between these calculations and experiment is presented in Sec. V.

Sections II and III contain brief descriptions of the experimental procedure and data analysis, respectively. The experimental results are presented in Sec. IV. In Sec. V, we examine the experimental results from the viewpoint of various nuclear models. Section VI contains a summary of the most important aspects of the results.

II. EXPERIMENTAL PROCEDURE

Most of the apparatus and target preparation techniques have been described in Paper I. Target thicknesses were typically 3–5 keV for all angular distribution, triple correlation, and polarization measurements.

All γ-ray angular distributions were measured with a 40-cm^3 Ge(Li) detector. The distributions were derived from spectra recorded at 0°, 35.4°, 55.2°, and 90° relative to the proton beam direction. The finite geometry correction factors Q_k for the Ge(Li) detector were determined by comparing the γ-ray angular distributions derived from the spectra of the 8-in.-diam× 8-in.-long NaI(Tl) and the 40-cm^3 Ge(Li) detectors for selected strong transitions in the (p,γ) reaction on Si^{30} and S^{34}. The correction factors Q_k for the 8-in. NaI(Tl) detector are well known from standard calculations.[23] Corrections for asymmetries in the target-detector system were obtained from measurements of the isotropic 2.37-MeV γ ray from the $C^{12}(p,\gamma)N^{13}$ reaction at $E_p = 459$ keV.

A detailed description of the general experimental techniques used in triple correlation measurements has been given by Harris and Breitenbecher.[24] The specific arrangement for triple correlation measurements used in the present work consisted of two 5-in.-diam× 5-in.-long NaI(Tl) detectors and the 8-in. detector. The 8-in. detector and one of the 5-in. detectors were movable in the horizontal plane through the proton beam axis. The third detector was movable in a vertical plane through the beam axis. This arrangement was used to measure the triple correlation of the cascade Res (2037 keV)→0.846 MeV→0. For all other correlation measurements, the horizontal 5-in. detector was replaced by the 40-cm^3 Ge(Li) detector. Data were usually collected in "geometries" in which two of the detectors were fixed at $\theta = 90°$ relative to the beam direction, while the third was successively placed at selected angles between 0° and 90°. All angular distribution and triple correlation data were recorded in a TMC 4096-channel analyzer. For the triple correlation measurements, an on-line PDP-8 computer was used in conjunction with the analyzer ADC's to set digital windows.

The measurements were usually performed in two stages. Initially, detailed four point angular distribution measurements were made and analyzed. In those cases where a unique solution was not obtained, the angular distribution results were used to select the appropriate geometries for subsequent triple correlation measurements.

The linear polarization measurements were performed using the Compton polarimeter described in detail by Lee and Watson.[25] The apparatus consisted of the two 5-in. NaI(Tl) detectors mounted at a relative aximuthal angle of $\varphi = 90°$ which were used to detect the radiation scattered from a 2-in.-diam×2-in.-long NaI(Tl) detector located with its axis on a line centered on the target and perpendicular to the beam direction. The entire polarimeter can be rotated about this vertical axis, thereby interchanging the role of the two analyzing detectors. This is necessary in order to correct for differing detector efficiencies and for asymmetries of the apparatus. The scattering and absorbing detectors are set for equal gains and the coincident pulses were electronically summed and stored in a multichannel analyzer. The electronic summing is used so that when a γ ray is Compton-scattered from the scattering crystal and absorbed by one of the other detectors, the sum of the two pulses will be proportional to the full γ-ray energy. Thus, the summed pulse height is independent of the scattering angle.

III. ANALYSIS

The ground-state spin of Ca^{42} is 0 so that only resonance formation with channel spin $s = \frac{1}{2}$ is possible.

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Because of the unique channel spin, no formation parameters are involved in the analysis of the angular distribution and correlation data. The magnetic substates of the resonance level which can be populated are restricted to $m=\pm\frac{1}{2}$ and, in addition, symmetry requires that the population parameters P_m be

$$P_{1/2}=P_{-1/2}=0.5.$$

These restrictions on the parameters simplify the form of the angular-correlation function and make the data analysis straightforward.

The general methods of analysis used for angular correlations have been described previously.[23,24,26,27] Briefly, a computer program was used for a least-squares analysis of the data over the allowable range of the mixing ratios δ for all possible choices of level spins. The analysis yields the best set of parameters and the associated value of the goodness-of-fit parameter χ^2 for each spin sequence chosen. The program accepts as input data the measured values (and errors) of the intensity correlation at each angle, or set of angles (θ_1, θ_2, φ) in the case of triple correlations. Angular distributions are treated merely as special cases of the general triple correlations. For all the results presented in this paper, the acceptability of any solution was based on a consideration of the 0.1% confidence limit for the goodness-of-fit parameter χ^2 for each particular set of data.

The analysis of linear polarization data is discussed in detail in Ref. 25. Briefly, if N_0 and N_{90} are the coincident counting rates measured at 0° and 90° angles, then we can write

$$\frac{N_{90}-N_0}{N_{90}+N_0}=Q, \qquad \frac{W(90,90)-W(90,0)}{W(90,90)+W(90,0)}=QP,$$

where $W(\theta,\varphi)$ is the number of reaction γ rays per unit time emitted at an angle θ with respect to the proton beam direction and having their electric vector at an angle φ with respect to the reaction plane. $W(\theta,\varphi)$ depends on the relative parity of the states emitting the radiation. The factor Q accounts for the polarization sensitivity of the compton scattering process and also takes into consideration the finite detector size (geometry) of the polarimeter. The calculation of the polarization correlation function $W(\theta,\varphi)$ and the polarization sensitivity of the compton polarimeter we used is discussed in Ref. 25.

IV. RESULTS

A brief comment on the presentation of our results is in order. Because of the large amount of data taken in this experiment, it would be impractical and indeed unnecessary to present all data since in most cases the analysis is routine. We have therefore chosen to present

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TABLE I. Results of the analysis of angular distribution data for resonance spins. The data were four-point angular distributions. The 0.1% confidence limit is 5.4 in this case.

Resonance	Excitation energy		Assumed spin values for resonance										
			$\frac{9}{2}$		$\frac{7}{2}$		$\frac{5}{2}$		$\frac{3}{2}$		$\frac{1}{2}$		
E_p (keV)	(MeV)	Transition	χ^2	δ	χ^2	δ	χ^2	δ	χ^2	δ	χ^2	δ	Result
1235	6.136	Res→0.151	50		12.0	−0.09	7	−0.36	1.6	0.0	7.0		$\frac{3}{2}$
		Res→0.472	162		88	+0.09	1.8	−0.12	1.1	+0.36	3.0		
1242	6.139	Res→0.151	447		200	−0.0	3.8	−0.27	0.8	0.10	150		$\frac{3}{2}$[a]
		Res→0.472	15		2.3	−0.05	4.2	+0.36	2.0	0.0	15		
1423	6.324	Res→0.151	1872		837	+0.13	1.5	−0.03	1.0	+0.53	291		$\frac{5}{2}$
		Res→0.880[b]	1.7	−0.13	4.6	−0.47	1.0	−0.14	15.6	+0.81	137		
1808	6.691	Res→0.0	0.6	−0.07	7.0	+0.58	0.5	−0.02	0.1	−0.33			$\frac{5}{2}$
		Res→0.151	5010		2227		1.5	+0.14	26.0	+1.33	2.42		
2037	6.920	Res→0.0	0.4	−0.23	0.3	+0.29	0.1	+0.22	0.1	0.0			$\frac{7}{2}$[c]
		Res→0.472	2.0		0.7	−0.04	0.1	−0.36	0.1	0.0	2.7		

[a] Confirmation of $\frac{3}{2}$ assignment comes from elastic scattering data obtained in this laboratory and also that of Browne *et al.* (see Ref. 28).
[b] The 0.880-MeV level is determined to be $\frac{5}{2}$ from 1235-keV resonance AD and TC data and also from Phillips *et al.* (Ref. 9).
[c] This assignment is based on sets of AD and TC data involving the 0.846- and 1.885-MeV levels. See text for details.

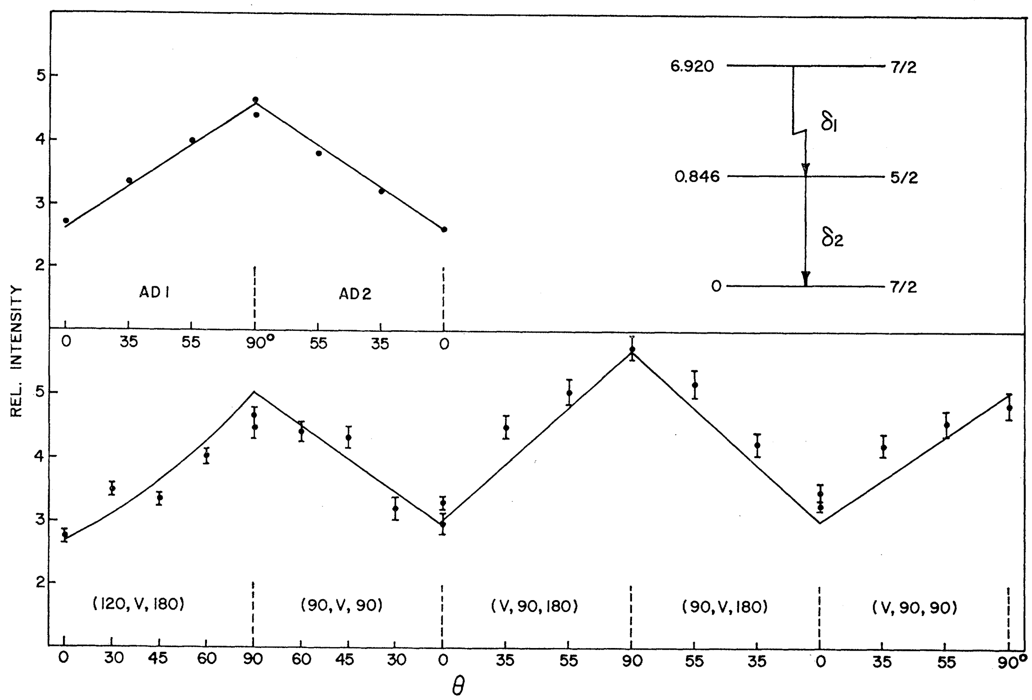

FIG. 1. Combined triple correlation and angular distribution data on the Res→0.846→0 cascade at the E_p=2037-keV resonance. The lines through the data points represent the best least-squares fit for the indicated level spins. Similar fits were obtained for the spin sequences $\frac{3}{2}$-$\frac{5}{2}$-$\frac{7}{2}$ and $\frac{5}{2}$-$\frac{3}{2}$-$\frac{7}{2}$. The horizontal scale is linear in $\cos^2\theta$.

in figure form the data and analysis results for only a selected number of cases. All other results will be presented in table form, however, essential arguments and conclusions will be given when necessary. In some cases, final J^π and mixing ratio values required a synthesis of many different pieces of experimental information, and they will be pointed out.

In the following, we assume no prior knowledge of the spins of the Sc^{43} levels except the assignments $J^\pi = \frac{7}{2}^-$, $\frac{3}{2}^+$, and $\frac{3}{2}^-$ for the ground, first excited, and second excited states, respectively, and the $l=1$ assignment for the 1.179-MeV level. We first present results for resonance spin values and multipolarity mixing ratios determined from a study of the γ-ray angular distributions involving transitions between the resonance states and the ground, first excited, and second excited states, and then discuss the analysis of cascades involving one or more of the other bound levels.

A. Resonance Spins

The results of the analysis of angular distribution data taken at the five resonances studied in this work are presented in Table I. The spin of the 1235-keV resonance is uniquely determined to be $\frac{3}{2}$ by the angular distribution results for the transitions Res→0.151 MeV and Res→0.472 MeV. In the case of the 1242-keV resonance, the analysis of the angular distribution data gave acceptable solutions for a resonance spin of either $\frac{3}{2}$ or $\frac{5}{2}$. In order to resolve this ambiguity, an elastic scattering experiment was performed on this resonance which yielded an $l=1$ distribution. This result has been confirmed by the recent elastic scattering measurements of Browne *et al.*[28] It is clear from these results that the 1242-keV resonance has $J^\pi = \frac{3}{2}^-$. The $J=\frac{5}{2}$ assignment for the 1423-keV resonance required the use of the Res→0.880-MeV angular distribution. An assignment of $J=\frac{5}{2}$ for the 0.880-MeV level has been made on the basis of our work and that of Phillips *et al.*[9] (see Sec. III B 3 for details). A unique assignment of $J=\frac{5}{2}$ for the 1808-keV resonance was made on the basis of the γ-ray angular distributions from the transitions Res→0 and Res→0.151 MeV. In the case of the 2037-keV resonance, the γ-ray angular distributions from the transitions Res→0 and Res→0.472 limited the resonance spin to either $\frac{3}{2}$, $\frac{5}{2}$ or $\frac{7}{2}$ since a $\frac{1}{2}$ or $\frac{9}{2}$ assignment would require octupole transitions. Subsequently, an assignment of $J(\text{Res})=\frac{7}{2}$ was made on the basis of a set of angular distribution and triple correlation measurements involving the 0.846- and 1.885-MeV levels. The data from the cascades Res→0.846→0 and Res→1.885→0 eliminated $\frac{5}{2}$ and $\frac{3}{2}$ spin possibilities, respectively, leaving $J=\frac{7}{2}$ as the only spin assignment consistent with all data (see Sec. III B for details).

B. Bound-State Spins

In the following, we present the conclusions drawn from angular-correlation measurements, and in one

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case, linear-polarization measurements on the bound states of Sc^{43}.

1. 0.846-MeV Level

Angular distribution and triple correlation data taken at the 2037-keV resonance are shown in Fig. 1. Analysis of these data yield $\frac{3}{2}$ or $\frac{5}{2}$ as possible spin assignments for the 0.846-MeV level while the resonance spin was limited to $\frac{5}{2}$ for the $J(0.846)=\frac{3}{2}$ assignment and $\frac{3}{2}$ or $\frac{7}{2}$ for the $J(0.846)=\frac{5}{2}$ assignment. In the case of the $\frac{3}{2}$ assignment for the 0.846-MeV level, a mixing ratio of $\delta(0.846\to 0)=-0.466$ was required to fit the data. Similar measurements made at the $(J^{\pi}=\frac{3}{2}^{-})$ 1242-keV resonance could not distinguish between possible spin assignments of $\frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$ for the 0.846-MeV level. In an attempt to resolve this ambiguity, the linear polarization of the 0.846-MeV γ ray was measured. The predicted polarizations for the cases considered are the following:

$J(0.846)$	Polarization
$\frac{3}{2}$	$+0.064(-0.064)$
$\frac{5}{2}$	$-0.023(+0.023)$,

where the polarization values represent no parity change, and the values between parentheses parity change. The measured polarization is -0.03 ± 0.01. This result eliminates the $\frac{3}{2}$ spin possibility for the 0.846-MeV level since it would require more than a three standard deviation change in the experimental value for agreement. In addition, if one accepts the $\frac{3}{2}$ assignment, it would require a change in parity, so that, one has the situation of an $E3$ transition to the $(\frac{7}{2}^{-})$ ground state being favored over a possible $M1$ transition to the $(J^{\pi}=\frac{3}{2}^{+})$0.151-MeV first excited state. It is clear that the 0.846-MeV level must have $J^{\pi}=\frac{5}{2}^{-}$. This result is consistent with the $\frac{5}{2}$ or $\frac{9}{2}$ assignment reported by Phillips *et al.*[9] The $\frac{5}{2}$ assignment for the 0.846-MeV level limits the possible spin assignments of the 2037-keV resonance to be either $\frac{3}{2}$ or $\frac{7}{2}$.

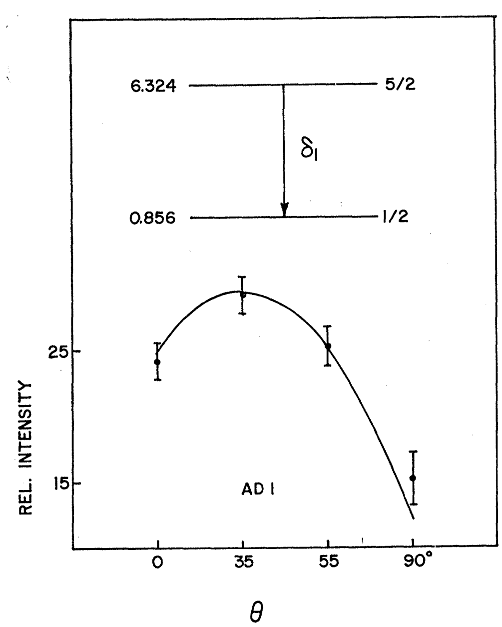

FIG. 2. Angular distribution data on the Res→0.856 transition at the $E_p=1423$-keV resonance. The line through the data points represents the best least-squares fit for the indicated level spins. The horizontal scale is linear in $\cos^2\theta$.

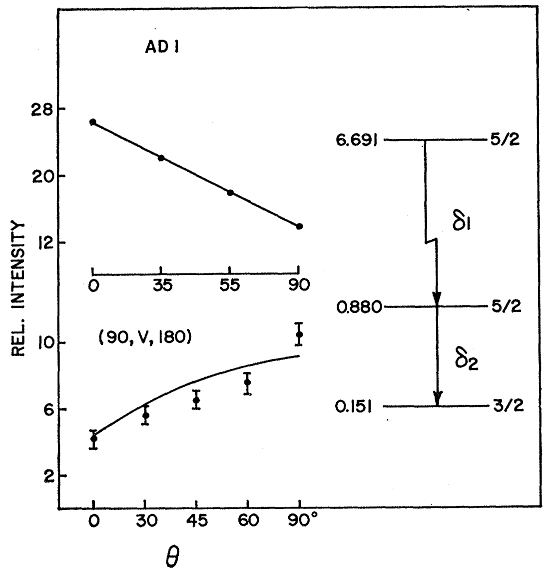

FIG. 3. Combined angular distribution and triple correlation data on the Res→0.880→0.151 cascade at the $E_p=1808$-keV resonance. The lines through the data points represent the best least-squares fit for the indicated level spins. The horizontal scale in linear in $\cos^2\theta$.

2. 0.856-MeV Level

The $l=0$ assignment from the (He^3, d) work of Schwartz *et al.*[6] and the (d,n) work of Grandy *et al.*[8] clearly indicates $J^{\pi}(0.856)=\frac{1}{2}^{+}$. This assignment is confirmed by the angular distribution data shown in Fig. 2. These data were taken at the $(J=\frac{5}{2})$ 1423-keV resonance. The minimum value of χ^2 for $J(0.856)=\frac{1}{2}$ was 1.3; whereas, for any other spin choice χ^2 was 14 or higher. The 0.1% confidence limit for this data is $\chi^2=5.4$.

3. 0.880-MeV Level

Angular distribution and triple correlation data involving transitions to the 0.880-MeV level were taken at the $(J=\frac{3}{2})$ 1235-keV and $(J=\frac{5}{2})$ 1808-keV resonances. The results of the analysis of the data taken at the 1808-keV resonance are shown in Fig. 3. The minimum χ^2 for $\text{Res}(\frac{5}{2})\to 0.880(\frac{5}{2})\to 0.151(\frac{3}{2})$ was 1.81; whereas, for any other possible spin sequence χ^2 was greater than 10. The 0.1% confidence limit for this case is $\chi^2=3.5$. This results confirms the $\frac{5}{2}$ assignment made by Phillips *et al.*[9] based on their $Ca^{40}(\alpha, p\gamma)Sc^{43}$

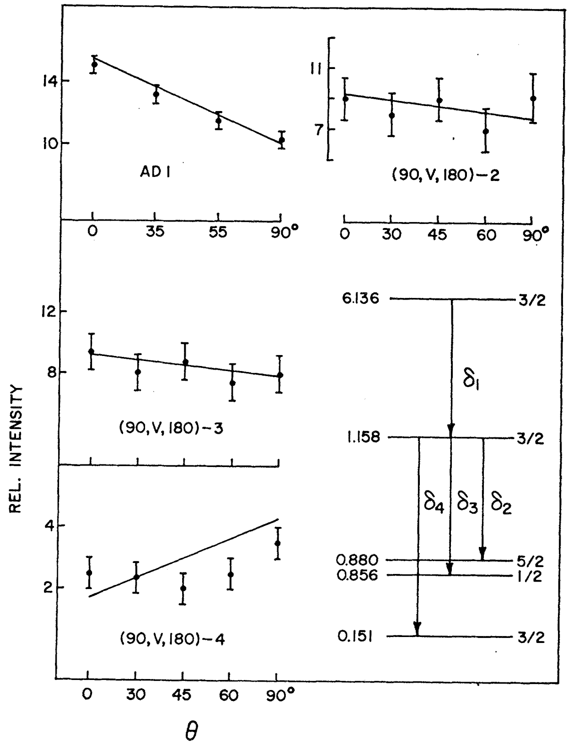

FIG. 4. Combined angular distribution and triple correlation data on the Res→1.158→(0.880, 0.856, 0.151) cascades at the E_p=1235-keV resonance. Geometries labeled 2, 3, and 4 correspond to the transitions from the 1.158 to the 0.880-, 0.856-, and 0.151-MeV levels, respectively. The lines through the data points represent the best least-squares fit for the indicated level spins. The horizontal scale is linear in $\cos^2\theta$.

correlation work. The value of the mixing ratio $\delta(0.880\to0.151)$ obtained in the present work is larger than that reported in Ref. 9. The reason for this discrepancy is unknown.

4. 1.158-MeV Level

Several of the resonance states studied in this work populate the 1.158-MeV level. Angular distributions taken at these resonances limited the possible spin assignments for this level to $\frac{3}{2}$ or $\frac{5}{2}$ in agreement with similar limitations made by Phillips *et al.*[9] In an attempt to resolve the ambiguity in the spin assignment, triple correlation measurements were performed at the $(J=\frac{3}{2})$ 1235-keV resonance and the results of the analysis are shown in Fig. 4. No choice of $J(1.158)$ could be made from these data on the basis of the χ^2 criterion. However, the following results were obtained for $\delta(\text{Res}\to1.158)$ with $J(1.158)=\frac{5}{2}$:

Transition	$\delta(\text{Res}\to1.158)$
Res→1.158→0.151	0.52±0.06
Res→1.158→0.856	1.3±0.2.

For $J(1.158)=\frac{3}{2}$, the value of $\delta(\text{Res}\to1.158)=0.0$ was obtained in each case listed above. It is clear from these results that the $J=\frac{5}{2}$ assignment is inconsistent with the experimental data. We therefore conclude from these results that $J(1.158)=\frac{3}{2}$.

5. 1.179-MeV Level

An $l=1$ assignment has been made for the 1.179-MeV level from the (He^3, d) work of Schwartz and Alford[6] and the (d, n) work of Grandy *et al.*[8] The $l=1$ assignment restricts the possible $J(1.179)$ values to either $\frac{1}{2}$ or $\frac{3}{2}$. A $\frac{1}{2}$ assignment would be characterized by isotropic angular distribution of the γ rays depopulating this level. Angular distributions taken at the 1235-keV resonance are shown in Fig. 5. From the observed anisotropy of these data, it is clear that the level spin is not $\frac{1}{2}$. The conclusion is that $J(1.179)=\frac{3}{2}$. This result is in disagreement with the $\frac{1}{2}$ assignment made by Broman *et al.*[11]

6. 1.652-MeV Level

Angular distribution data taken at the $(J=\frac{3}{2})$ 1235-keV and the $(J=\frac{5}{2})$ 1808-keV resonances limited the possible spin assignments to $J(1.652)=\frac{3}{2}$ or $\frac{5}{2}$. The triple correlation data taken at both resonances are shown in Fig. 6. For $J(1.652)=\frac{5}{2}$, the minimum values of χ^2 for these data were <1.0; whereas, for $J(1.652)=\frac{3}{2}$ the minimum values of χ^2 were >5.0. The 0.1% confidence limit is $\chi^2=3.5$. Therefore, we conclude that $J(1.652)=\frac{5}{2}$. The parity of this level is most likely positive since it only decays to positive-parity levels.

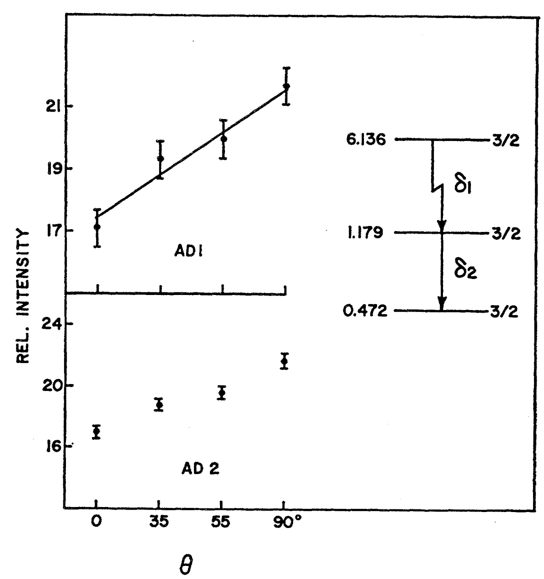

FIG. 5. Angular distribution data on the Res→1.179→0.472 cascade at the E_p=1235-keV resonance. The solid line through the AD1 data is the best fit for $J(1.179)=\frac{3}{2}$. No analysis of AD2 was possible because this level is fed by other higher lying bound states; however, the data are given to show the anisotropy of the transition. The horizontal scale is linear in $\cos^2\theta$.

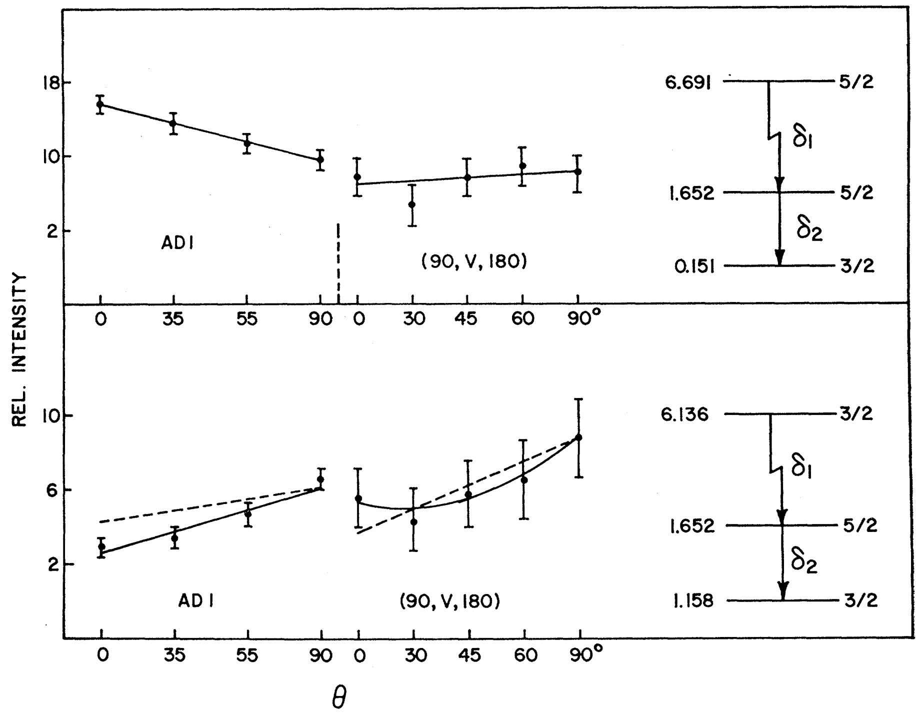

FIG. 6. Combined angular distribution and triple correlation data on the cascades Res→1.652→1.158 and Res→1.652→0.151 at the $E_p=$ 1235-keV and $E_p=1808$-keV resonances, respectively. The upper part of the figure shows data taken at the 1808-keV resonance and the lower part shows the data taken at the 1235-keV resonance. The solid lines through the data points represent the best least-squares fit for the indicated level spins. The dashed lines represent the best fit for $J(1.652)=\frac{3}{2}$. The horizontal scale is linear in $\cos^2\theta$.

7. 1.885-MeV Level

The 1.885-MeV is only populated by one of the resonances studied in this work, namely, the 2037-keV resonance. The possible spin assignments for the 2037-keV resonance were limited to either $\frac{3}{2}$ or $\frac{7}{2}$ by the angular correlation and polarization measurements on the cascade Res→0.846→0. The angular distribution and triple correlation data taken at the 2037-keV resonance for the cascade Res→1.885→0 are shown in Fig. 7. For the value $J(\text{Res})=\frac{3}{2}$, no choice of $J(1.885)$ lead to a minimum χ^2 value less than 4.0. The 0.1% confidence limit for this case is $\chi^2=2.8$. This clearly establishes the 2037-keV resonance spin to be $\frac{7}{2}$. With the resonance spin fixed at $\frac{7}{2}$, only the spin assignments $J(1.885)=\frac{5}{2}$ or $\frac{9}{2}$ yield minimum χ^2 values below the 0.1% confidence limit stated above.

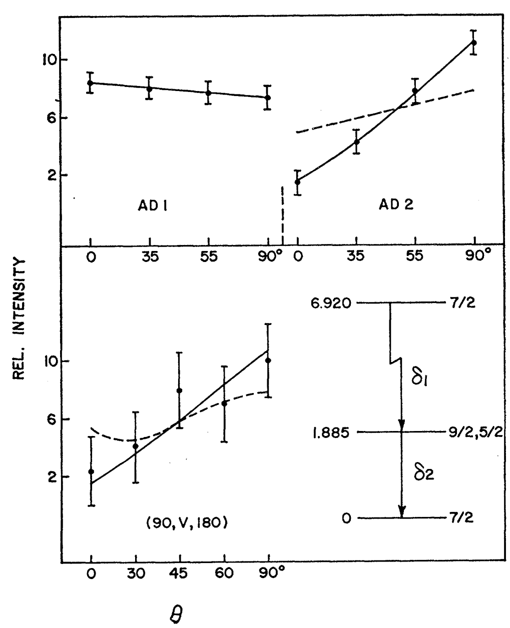

FIG. 7. Combined angular distribution and triple correlation data on the Res→1.885→0 cascade at the 2037-keV resonance. The solid lines through the data points represent the best least-squares fit for the indicated level spins. The dashed lines represent the best fit for the spin sequence $\frac{3}{2}$-$\frac{7}{2}$-$\frac{7}{2}$. Similar fits were obtained for other choices of $J(1.885)$. The horizontal scale is linear in $\cos^2\theta$.

8. 1.963-MeV Level

Angular distribution data taken at the $(J=\frac{3}{2})$ 1235- and 1242-keV resonances limit the possible spin assignments to $J(1.963)=\frac{3}{2}$ or $\frac{5}{2}$. The triple correlation data for the cascade Res→1.963→1.179 taken at the 1235-keV resonance is shown in Fig. 8. The minimum χ^2 values obtained for $J=\frac{3}{2}$ and $\frac{5}{2}$ were 7.89 and 0.52, respectively. The 0.1% confidence limit for these data is $\chi^2=3.5$.

Conclusion: $J(1.963)=\frac{5}{2}$.

9. 2.094-MeV Level

The angular distribution data taken at the $(J=\frac{3}{2})$ 1235-keV resonance is shown in Fig. 9. Similar data was also obtained at the 1242-keV resonance. For the assignment $J(2.094)=\frac{3}{2}$, the minimum value of χ^2 obtained was 1.2; whereas, for any of the other possible choices of $J(2.094)$ the minimum value of χ^2 obtained was >4. The 0.1% confidence limit is $\chi^2=3.5$. We con-

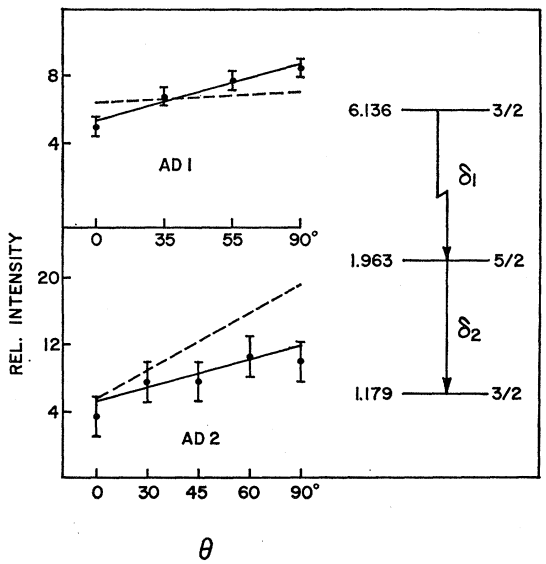

FIG. 8. Combined angular distribution and triple correlation data on the Res→1.963→1.179 cascade at the E_p=1235-keV resonance. The solid lines through the data points represent the best least-squares fit for the indicated level spins. The dashed lines represent the best fit for $J(1.963)=\frac{3}{2}$. The horizontal scale is linear in $\cos^2\theta$.

clude from these results that $J(2.094)=\frac{3}{2}$. This result is in agreement with the $l=1$ assignment of Schwartz and Alford[6] based on their (He^3, d) work.

10. 2.143-MeV Level

Angular distribution data were taken at both the $(J=\frac{3}{2})$ 1235-keV and the $(J=\frac{5}{2})$ 1423-keV resonances. The data from the 1423-keV run are shown in Fig. 10. For an assignment of $J(2.143)=\frac{7}{2}$, the minimum value of χ^2 obtained for these data was 1.0; whereas, for any other choice the minimum value of χ^2 was >15. With a 0.1% confidence limit of 3.5, these results clearly establish $J(2.143)=\frac{7}{2}$.

11. 2.580-MeV Level

Angular distribution data taken at the $(J=\frac{3}{2})$ 1235-keV resonance for the cascade Res→2.580→1.179 limited the possible spin assignments to $J(2.580)=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$. The angular distribution data taken at the $(J=\frac{7}{2})$ 2037-keV resonance together with the data taken at the 1235-keV resonance are shown in Fig. 11. For an assignment of $J(2.580)=\frac{5}{2}$, the minimum χ^2 obtained for the 2037-keV data was 1.6; whereas, for $J(2.580)=\frac{1}{2}$ or $\frac{3}{2}$, the minimum value of χ^2 was >20. We therefore conclude that $J(2.580)=\frac{5}{2}$.

12. Other Levels

In the following, we first present results for the spin limitations of several levels obtained from the angular distribution and triple correlation measurements of the present work, and second, summarize all available information for those levels not observed in the present work.

a. Spin limitations. Angular distribution and triple correlation data taken at the various resonances studied in the present work lead to limitations on the possible spin assignments for the following levels: 2.383 MeV $(\frac{3}{2},\frac{7}{2})$, 2.670 MeV $(\leq\frac{7}{2})$, 2.986 MeV $(\frac{3}{2},\frac{5}{2})$, 3.289 MeV $(\frac{3}{2},\frac{5}{2},\frac{7}{2})$, 3.808 MeV $(\frac{3}{2},\frac{5}{2},\frac{7}{2})$, and 4.455 MeV $(\frac{5}{2},\frac{9}{2})$. In the case of the $J(3.289)=\frac{7}{2}$ assignment, angular distribution data taken at the $(J=\frac{3}{2})$ 1242-keV resonance requires a large value for δ(Res→3.289). This implies an $M3$ transition rate enhancement of about 1×10^7 W.u. (Weisskopf units) (1×10^5 W.u. for $E3$). We conclude that the spin of the 3.289-MeV level is most probably either $\frac{3}{2}$ or $\frac{5}{2}$. Finally, all measurements made involving transitions to or from the 3.808-MeV level favor a $\frac{5}{2}$ spin assignment although the $\frac{3}{2}$ or $\frac{7}{2}$ possibilities cannot be completely ruled out.

b. Summary of published results. The 1.336-MeV level has been assigned $J=\frac{7}{2}$ by Phillips *et al.*[9] from (α, $p\gamma$) studies. From this same work, Phillips *et al.* conclude that the 1.410-MeV level is most likely $J=\frac{7}{2}$, although they cannot completely rule out possible $\frac{5}{2}$ or $\frac{9}{2}$ assignments. The level at 1.810 MeV is either $J=\frac{1}{2}$ or $\frac{3}{2}$ based on the (He^3, d) work of Schwartz and Alford[6] and the (d, n) work of Grandy *et al.*[8] Two $l=5$ levels at 1.827 and 2.620 MeV have been observed by Bernstein.[29] An $l=5$ assignment requires either spin $\frac{9}{2}$ or $\frac{11}{2}$ for these levels. Recent work[30] on the (α, $p\gamma$) reaction shows that the 1.827-MeV level decays completely to the ground state and the particle-γ correlations limited

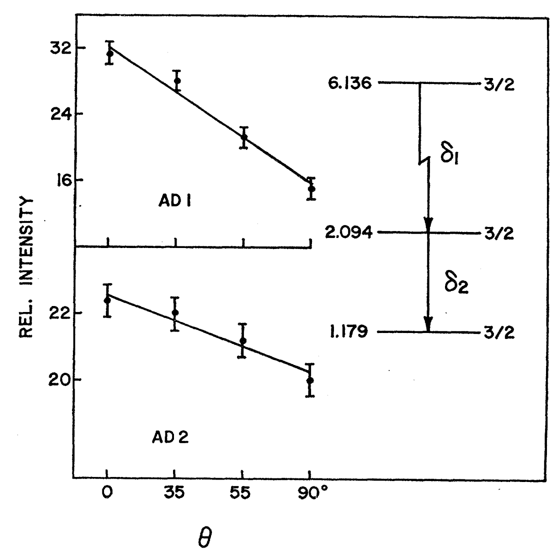

FIG. 9. Angular distribution data on the Res.→2.094→1.179 cascade at the E_p=1235-keV resonance. The lines through the data points represent the best fit for the indicated level spins. The horizontal scale in linear in $\cos^2\theta$.

[29] A. M. Bernstein (private communication quoted in Ref. 34).
[30] G. C. Ball (private communication).

the spin to $J(1.827)=\frac{7}{2}$ or $\frac{11}{2}$. If the l-value assignment is correct, then $J^{\pi}(1.827)=\frac{11}{2}^{-}$. A level at 1.930 MeV has been observed in the $(\alpha, p\gamma)$ work of Phillips[31] and Forster *et al.*[32] and has been assigned $J=\frac{9}{2}$ by both groups. An $l=0(J=\frac{1}{2})$ level at 1.947-MeV has been observed by Grandy *et al.*[8] in their (d, n) work. However, it should be noted that this level has not been observed in any other work on Sc^{43}. Additional $l=3(J=\frac{5}{2}$ or $\frac{7}{2})$ levels at 2.294, 3.677, and 4.238 MeV have been observed in the (He^3, d) reaction work of Schwartz and Alford.[6] The level observed by Forster *et al.*[32] at 2.551 MeV decays to the 1.930- and 1.336-MeV levels.[30]

C. Multipolarity Mixing Ratios

The multipolarity mixing ratios for many γ-ray transitions have been measured. For clarity, we present the results in two tables. Table II contains γ-ray mixing ratios for transitions from the resonance states and Table III contains the γ-ray mixing ratios for transitions from bound states. In those cases where a unique spin assignment is not available, the mixing ratios for all allowed spins are given. In some cases, more than one measurement is available, and in those cases each result is presented together with the weighted average.

V. DISCUSSION

A summary of the spin-parity assignments and the electromagnetic decay properties of all bound levels in Sc^{43} is presented in Fig. 12. The summary contains all results obtained in this laboratory as well as results obtained by other investigators. The decay schemes and branching ratios for the levels at 1.336, 1.930, and 2.551 MeV were taken from the work of Forster *et al.*,[32] and Phillips.[31] There is evidence in the $(\alpha, p\gamma)$ coincidence work of these two groups that the 1.336-MeV level has a weak branch to the 0.880-MeV level. We did not observe this branch in our decay-scheme studies.[33] The level at 1.947 MeV has only been observed in the $Ca^{42}(d, n)Sc^{43}$ work of Grandy *et al.*,[8] and they find $l=0$ for this level. It is difficult to understand why a $J^{\pi}=\frac{1}{2}^{+}$ level is not populated by at least one of the low-spin (p, γ) resonances we have studied.

Tables II and III contain a summary of all multipolarity mixing ratios measured in this work. In the following, we compare the experimental results with the predictions of various nuclear models.

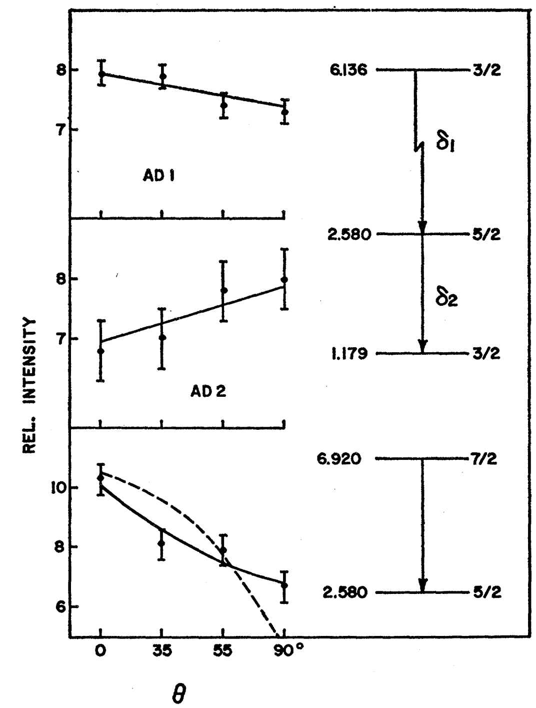

FIG. 11. Angular distribution data on the Res→2.580→1.179 cascade at the $E_p=1235$-keV resonance and angular distribution data on the Res→2.580 transition at the $E_p=2037$-keV resonance. The solid lines represent the best least-squares fit for the indicated level spins. The dashed line through the 2037-keV angular distribution data is for $J(2.580)=\frac{3}{2}$. The horizontal scale is linear in $\cos^2\theta$.

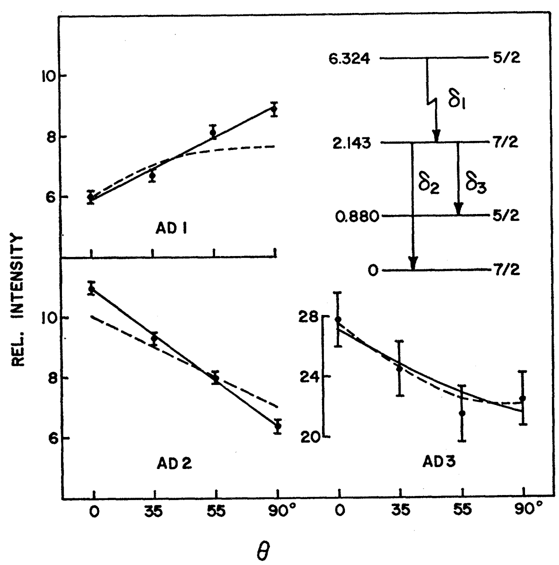

FIG. 10. Angular distribution data on the cascades Res→2.143→0 and Res→2.143→0.880 at the $E_p=1423$-keV resonance. The solid lines through the points represent the best least-squares fit for the indicated level spins. The dashed lines represent the best fit for $J(2.143)=\frac{5}{2}$. The horizontal scale is linear in $\cos^2\theta$.

A. Shell-Model Calculations

It is clear from the many experimental results on $f_{7/2}$ shell nuclei that the shell-model calculation of

[31] W. R. Phillips (private communication).

[32] J. S. Forster, G. C. Ball, F. Ingebretsen, and C. F. Monahan, Bull. Am. Phys. Soc. **14**, 601 (1969).

[33] The 1.336-MeV level was weakly populated by the (p, γ) resonances that we investigated, and therefore it is quite possible that the 0.456-MeV γ ray could not be seen because of the high background which existed in that region of the Ge(Li) spectrum.

TABLE II. γ-ray mixing ratios for transitions from the resonance states studied in this work.

E_p (keV)	Transition (MeV)	E_γ (MeV)	J_R-J_f	Mixing ratio δ
1235	6.136→0.151	5.985	$\frac{3}{2}\to\frac{3}{2}$	0.00 ± 0.02 or -3.73 ± 0.50
	→0.472	5.664	$\frac{3}{2}\to\frac{3}{2}$	0.36 ± 0.02 or $7.60_{-2.00}^{+3.80}$
	→0.880	5.256	$\frac{3}{2}\to\frac{5}{2}$	0.05 ± 0.03
	→1.158	4.978	$\frac{3}{2}\to\frac{3}{2}$	0.05 ± 0.03 or $-4.70_{-0.70}^{+2.00}$
	→1.179	4.957	$\frac{3}{2}\to\frac{3}{2}$	0.36 ± 0.06 or $9.50_{-3.00}^{+7.00}$
	→1.652	4.484	$\frac{3}{2}\to\frac{5}{2}$	-0.36 ± 0.02 or $-7.60_{-4.80}^{+\infty}$
	→1.963	4.173	$\frac{3}{2}\to\frac{5}{2}$	-0.14 ± 0.10 or $19.1_{-13.0}^{+\infty}$
	→2.094	4.042	$\frac{3}{2}\to\frac{3}{2}$	-0.07 ± 0.05 or $-2.70_{-0.6}^{+1.0}$
	→2.580	3.556	$\frac{3}{2}\to\frac{5}{2}$	0.14 ± 0.07 or $2.60_{-0.50}^{+0.70}$
1242	6.139→0.151	5.988	$\frac{3}{2}\to\frac{3}{2}$	0.10 ± 0.03 or $-8.80_{-2.50}^{+6.50}$
	→0.472	5.667	$\frac{3}{2}\to\frac{3}{2}$	0.00 ± 0.03 or $-3.70_{-0.50}^{+0.80}$
	→1.179	4.960	$\frac{3}{2}\to\frac{3}{2}$	0.17 ± 0.04 or $-19.1_{-8.0}^{+28.0}$
	→1.963	4.176	$\frac{3}{2}\to\frac{5}{2}$	-0.06 ± 0.05
	→2.094	4.045	$\frac{3}{2}\to\frac{3}{2}$	-0.13 ± 0.07 or -2.4 ± 0.5
	→2.986	3.153	$\frac{3}{2}\to\frac{3}{2}$	-0.11 ± 0.12 or $-2.7_{-0.5}^{+1.3}$
			$\frac{3}{2}\to\frac{5}{2}$	0.78 ± 0.40
	→3.289	2.850	$\frac{3}{2}\to\frac{3}{2}$	0.00 ± 0.06 or $-3.7_{-0.9}^{+1.5}$
			$\frac{3}{2}\to\frac{5}{2}$	0.81 ± 0.20
1423	6.324→0.151	6.173	$\frac{5}{2}\to\frac{3}{2}$	-0.03 ± 0.03
	→0.856	5.468	$\frac{5}{2}\to\frac{1}{2}$	-0.01 ± 0.03 or 3.1 ± 0.5
	→0.880	5.440	$\frac{5}{2}\to\frac{5}{2}$	-0.14 ± 0.05
	→1.179	5.145	$\frac{5}{2}\to\frac{3}{2}$	0.00 ± 0.03
	→2.143	4.181	$\frac{5}{2}\to\frac{7}{2}$	-0.07 ± 0.06
	→2.383	3.941	$\frac{5}{2}\to\frac{7}{2}$	-0.45 ± 0.08 or $-2.7_{-0.5}^{+0.8}$
			$\frac{5}{2}\to\frac{3}{2}$	0.18 ± 0.08
	→2.986	3.338	$\frac{5}{2}\to\frac{3}{2}$	0.02 ± 0.04
			$\frac{5}{2}\to\frac{5}{2}$	0.81 ± 0.12
1808	6.691→0.000	6.691	$\frac{5}{2}\to\frac{7}{2}$	-0.02 ± 0.04
	→0.151	6.540	$\frac{5}{2}\to\frac{3}{2}$	0.14 ± 0.04
	→0.472	6.219	$\frac{5}{2}\to\frac{3}{2}$	-0.22 ± 0.05
	→0.846	5.845	$\frac{5}{2}\to\frac{5}{2}$	0.27 ± 0.10
	→0.880	5.811	$\frac{5}{2}\to\frac{5}{2}$	-0.03 ± 0.03
	→1.336	5.355	$\frac{5}{2}\to\frac{7}{2}$	-0.14 ± 0.06 or $-22.9_{-11.5}^{+\infty}$
	→1.652	5.039	$\frac{5}{2}\to\frac{5}{2}$	0.07 ± 0.07
	→1.963	4.728	$\frac{5}{2}\to\frac{5}{2}$	0.47 ± 0.08
	→2.383	4.308	$\frac{5}{2}\to\frac{7}{2}$	0.13 ± 0.10 or $4.2_{-1.0}^{+1.5}$
			$\frac{5}{2}\to\frac{3}{2}$	-0.20 ± 0.10
2037	6.920→0.000	6.920	$\frac{7}{2}\to\frac{7}{2}$	0.29 ± 0.08
	→0.472	6.448	$\frac{7}{2}\to\frac{3}{2}$	-0.04 ± 0.04
	→0.846	6.074	$\frac{7}{2}\to\frac{5}{2}$	0.00 ± 0.02
	→1.410	5.510	$\frac{7}{2}\to\frac{7}{2}$	0.04 ± 0.04
	→1.885	5.035	$\frac{7}{2}\to\frac{9}{2}$	0.18 ± 0.16 or $5.7_{-2.0}^{+6.0}$
			$\frac{7}{2}\to\frac{5}{2}$	-0.22 ± 0.16
	→2.580	4.340	$\frac{7}{2}\to\frac{5}{2}$	-0.32 ± 0.10
	→4.455	2.465	$\frac{7}{2}\to\frac{9}{2}$	-0.18 ± 0.06
			$\frac{7}{2}\to\frac{5}{2}$	0.04 ± 0.06

McCullen, Bayman, and Zamik[12] which allows only particles in the $f_{7/2}$ shell is inadequate. Dieperink and Brussaard[15] have extended these calculations to include positive-parity states by including a $1d_{3/2}$ hole state. In these calculations, the effective interaction of the $f_{7/2}$ nucleons was determined by least-squares fitting a selected number of levels with known spin and parity throughout the mass region $40<A<48$. In the case of Sc^{43}, they used the $J^\pi=\frac{3}{2}^+$ level at 0.151 MeV in their fitting procedure and were able to predict a $J^\pi=\frac{1}{2}^+$ level at 0.640 MeV in reasonable agreement with the known $J^\pi=\frac{1}{2}^+$ level at 0.856 MeV. However, they predict a $J^\pi=\frac{5}{2}^+$ level at 1.81 MeV and a second $J^\pi=\frac{3}{2}^+$ level at 2.30 MeV which is in poor agreement with the experimental findings of two $J^\pi=\frac{5}{2}^+$ levels at 0.880 and 1.652 MeV and a $J^\pi=\frac{3}{2}^+$ level at 1.158 MeV. This discrepancy can probably be removed if one includes a larger configuration space in the calculations. However,

TABLE III. γ-ray mixing ratios for transitions from the bound states in Sc^{43} observed in the present work.

Resonance (keV)	Transition (MeV)	E_γ (MeV)	$J_i \rightarrow J_f$	Mixing ratio δ
2037	0.846→0.000	0.846	$\frac{5}{2}\rightarrow\frac{7}{2}$	-0.18 ± 0.02
1235	0.880→0.152	0.729	$\frac{5}{2}\rightarrow\frac{3}{2}$	0.64 ± 0.18 (av. 0.54 ± 0.09)
1808	0.880→0.152	0.729	$\frac{5}{2}\rightarrow\frac{3}{2}$	0.49 ± 0.08
1235	1.158→0.880	0.278	$\frac{3}{2}\rightarrow\frac{5}{2}$	-0.23 ± 0.20, $-(22.9_{-19.2}^{+\infty})$ or >5.7
	1.158→0.856	0.302	$\frac{3}{2}\rightarrow\frac{1}{2}$	-0.19 ± 0.20 or $2.9_{-1.3}^{+8.5}$
	1.158→0.151	1.007	$\frac{3}{2}\rightarrow\frac{3}{2}$	1.30 ± 0.50 or -1.50 ± 1.50
1808	1.652→1.158	0.494	$\frac{5}{2}\rightarrow\frac{3}{2}$	0.00 ± 0.20 or $2.4_{-1.2}^{+5.0}$
1808	1.652→0.151	1.501	$\frac{5}{2}\rightarrow\frac{3}{2}$	0.05 ± 0.18
2037	1.885→0.000	1.885	$\frac{9}{2}\rightarrow\frac{7}{2}$	$0.42_{-0.20}^{+1.13}$
			$\frac{5}{2}\rightarrow\frac{7}{2}$	$-(1.10_{-0.60}^{+1.30})$
1235	1.963→1.179	0.784	$\frac{5}{2}\rightarrow\frac{3}{2}$	0.04 ± 0.25 or $-(1.50_{-1.0}^{+\infty})$
1235	2.094→1.179	0.915	$\frac{3}{2}\rightarrow\frac{3}{2}$	0.00 ± 0.10, $-(3.7_{-1.0}^{+2.5})$ or $9.5_{-4.0}^{+48.0}$
1423	2.143→0.880	1.263	$\frac{7}{2}\rightarrow\frac{5}{2}$	-0.27 ± 0.10 or $23.0_{-12.0}^{+\infty}$
	2.143→0.000	2.143	$\frac{7}{2}\rightarrow\frac{7}{2}$	0.00 ± 0.04
1235	2.580→1.179	1.401	$\frac{5}{2}\rightarrow\frac{3}{2}$	-0.11 ± 0.10 or $5.7_{-2.0}^{+8.0}$
1423	2.986→0.880	2.106	$\frac{5}{2}\rightarrow\frac{5}{2}$	0.95 ± 0.50
			$\frac{3}{2}\rightarrow\frac{5}{2}$	-0.13 ± 0.11 or $11.4_{-7.0}^{+\infty}$
	2.986→0.151	2.835	$\frac{5}{2}\rightarrow\frac{3}{2}$	$-(0.66_{-0.30}^{+0.60})$
			$\frac{3}{2}\rightarrow\frac{3}{2}$	0.00 ± 0.09 or $-(4.5_{-1.3}^{+3.0})$
2037	4.455→0.000	4.455	$\frac{9}{2}\rightarrow\frac{7}{2}$	-0.13 ± 0.05
			$\frac{5}{2}\rightarrow\frac{7}{2}$	0.05 ± 0.05 or $5.7_{-1.4}^{+3.5}$

as they correctly point out, this will introduce the serious complication of spurious states. One wonders whether a conventional shell-model approach to $f_{7/2}$ shell nuclei is possible.

B. Cluster-Model Calculations

The cluster model was applied to the Sc^{43} nucleus by Broman[34] because of the success this model had in predicting the positive- and negative-parity states in F^{19}. In this model, the interactions between the unexcited Ca^{40} core and H^3 clusters and a K^{39} core and He^4 clusters gives rise to the positive- and negative-parity states, respectively. The agreement between the predictions of this model and experiment for the negative-parity states is very poor. A predicted low-lying $J^\pi=\frac{3}{2}^-$ level can possibly be identified with the 0.472-MeV level, but there is no experimental evidence for the low-lying $J^\pi=\frac{11}{2}^-$ (~300 keV) or $J^\pi=\frac{1}{2}^-$ (~1 MeV) states predicted by the model. The predictions of the model for the positive-parity states is in reasonable qualitative agreement with experiment. The predictions of a low-lying $J^\pi=\frac{3}{2}^+$ state and a set of states with $J^\pi=\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{7}{2}^+$ having a center of gravity at approximately 1.15 MeV are in agreement with experimental results. However, the inability of the model to account for the normal negative-parity states indicates that the cluster model cannot provide a general understanding of the nuclei in this mass region.

[34] L. Broman, Arkiv Fysik **35**, 371 (1967).

C. Coriolis-Coupling Model

As indicated earlier, the Coriolis-coupling model of Malik and Scholz[16–18] has been reasonably successful in predicting the general qualitative features of the nuclei in this mass region. The normal parity spectra are calculated using the strong-coupling symmetric-rotation model with Coriolis coupling between bands, and the changed-parity spectra are calculated from core excitation in the $2s$-$1d$ shell. The model correctly predicts all the ground-state spins of the light odd-A nuclei, and, in addition, the magnetic moments computed with the model are in reasonable agreement with experiment. Almost all magnetic dipole and electric quadrupole moments together with the enhanced and inhibited electric quadrupole transitions are accounted for without using effective charge. The prediction of a ground-state triplet in Ti^{45} has also been experimentally verified.[35]

The results of Coriolis-coupling calculations on Sc^{43} are shown in Fig. 13. The free parameters used in these calculations are the deformation β and the rotational constant $A=\hbar^2/2I$. The best fit to the experimental data was obtained with the parameter values $\beta=0.26$ and $A=130$ keV. The agreement between theory and experiment is good for the positive-parity states but very poor for the negative-parity states. The $J^\pi=\frac{3}{2}^-$ state at 1.179 MeV and the probable $J^\pi=\frac{7}{2}^-$ state at 1.410 MeV are not predicted by the model. Further-

[35] J. H. Jett, G. D. Jones, and R. A. Ristinen, Phys. Letters **28B**, 111 (1968).

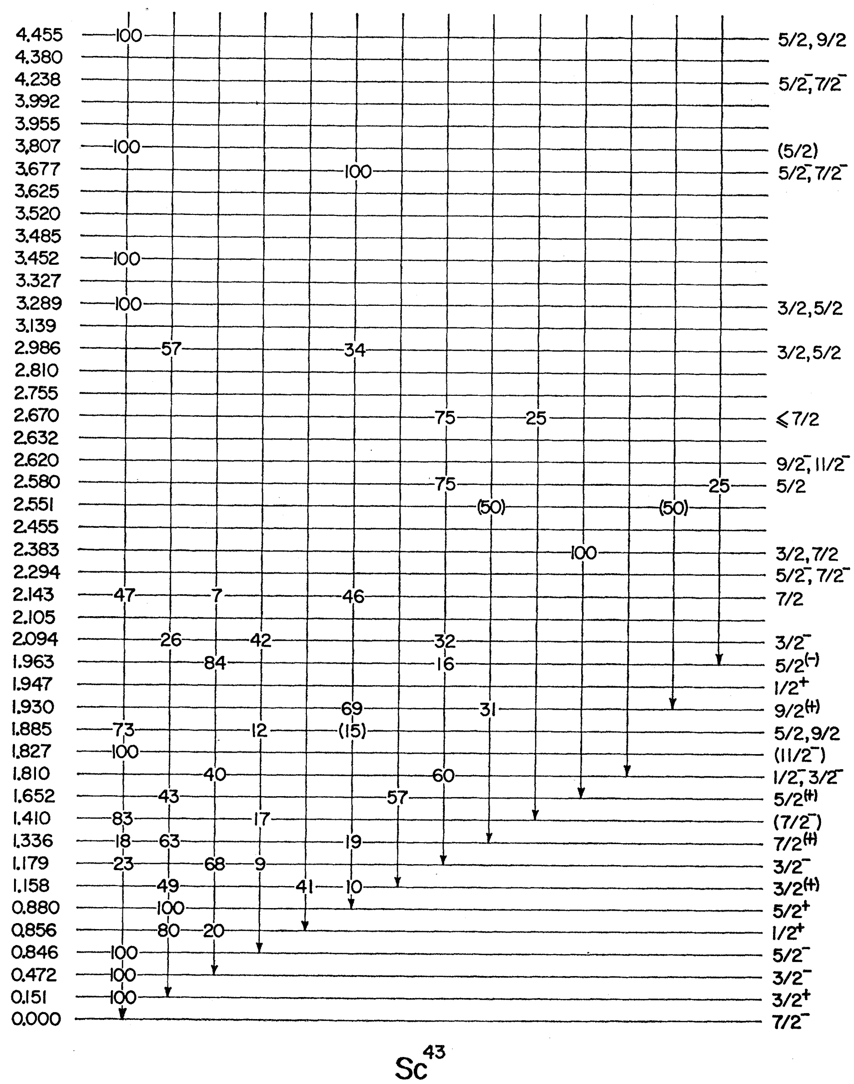

FIG. 12. A summary of the spin-parity assignments and electromagnetic decay properties of all bound levels in Sc^{43} below 4.45 MeV. This figure contains all results obtained in the present work, results reported in Paper I, and all results obtained by other investigators. The decay schemes of the 1.336- and 1.930-MeV levels were taken from the work of Forster *et al.* (Ref. 32) and Phillips (Ref. 31). The 1.885-MeV branch to the 0.880-MeV level is questionable (see Paper I). The branching ratios shown for the 2.551-MeV level are given only to indicate that these transitions have been observed, but no branching ratios have been measured (Ref. 30). The spins and parities given in parentheses indicate most probable assignments.

more, a predicted $J^\pi=\frac{11}{2}^-$ state comes approximately 1 MeV too low in energy to be identified with the $J^\pi=\frac{11}{2}^-$ state at 1.827 MeV. It is not clear that a different choice of parameters will remove this discrepancy because the same parameters will have to be used in calculating the positive-parity states where agreement between theory and experiment is already quite good. However, it is possible that these discrepancies may be removed if one includes pairing effects in the calculations. (There is some preliminary evidence that this is indeed the case[36] and calculations along these lines are in progress.) A calculation of the electromagnetic decay properties of those levels identified as belonging to the model would be useful in determining the validity of the model. For example, the model must be able to explain the almost complete lack of $E1$ transitions in Sc^{43}. Such calculations, however, are not available at the present time.

[36] F. B. Malik (private communication).

D. Johnstone-Model Calculations

The Johnstone model is essentially a unified model in which the negative-parity states are assumed to arise from the mixture of pure $(fp)^3$ and 5-particle–2-hole (hereafter referred to as 5p-2h) $k^\pi=\frac{3}{2}^-$ band states, and the positive-parity states are the 4-particle–1-hole (hereafter referred to as 4p-1h) $k^\pi=\frac{3}{2}^+$ and $\frac{1}{2}^+$ band states. In the following, we discuss the calculations of level positions and decay properties separately.

1. Level Positions

The results of the calculation for the energy levels in Sc^{43} are given in Fig. 13. The pure $(fp)^3$ negative-parity states shown in Fig. 13 were calculated by Johnstone[37] using the Kuo and Brown[38] realistic interaction. These states are the ground state $(\frac{7}{2}^-)$, 1.06 MeV

[37] I. P. Johnstone (private communication).
[38] T. T. S. Kuo and G. E. Brown, Nucl. Phys. **A114**, 246 (1968).

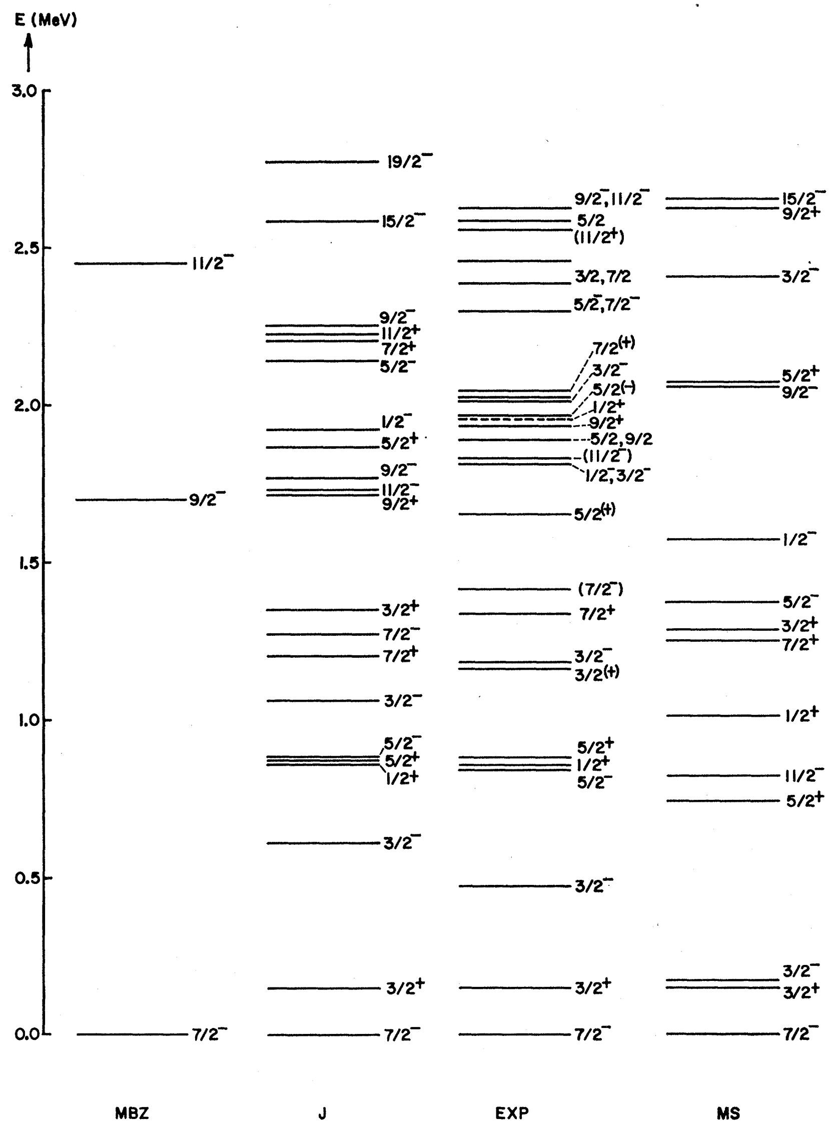

FIG. 13. Comparison of experimental and theoretical energy levels calculated in various models. MBZ stands for the shell-model calculations of McCullen, Bayman, and Zamik, J for the Johnstone model calculations, and MS for the Malik and Scholz calculations.

($\frac{3}{2}^-$), 1.75 MeV ($\frac{11}{2}^-$), 1.92 MeV ($\frac{1}{2}^-$), 2.14 MeV ($\frac{5}{2}^-$), 2.27 MeV ($\frac{9}{2}^-$), 2.60 MeV ($15/2^-$), and 2.79 MeV ($19/2^-$). These states can probably be identified with the experimentally observed levels at 1.179 MeV ($\frac{3}{2}^-$), 1.810 MeV ($\frac{1}{2}^-$, $\frac{3}{2}^-$), 1.827 MeV ($\frac{11}{2}^-$), 2.294 MeV ($\frac{5}{2}^-$, $\frac{7}{2}^-$), and 2.620 MeV ($\frac{9}{2}^-$, $\frac{11}{2}^-$). The predicted $J^\pi = 15/2^-$, $19/2^-$ states have not been observed experimentally. The positions of the unperturbed 5p-2h states shown in Fig. 13 are not calculated positions, but instead are deduced from the experimental data. The measured spectroscopic factors were used to deduce the degree of mixing between pure $(fp)^3$ states and 5p-2h states in the observed levels. Knowing the degree of mixing and the observed excitation energies, the unperturbed positions of the 5p-2h states were deduced. In the case of the lowest $\frac{5}{2}^-$ state, the spectroscopic factor was not known, so that the position of this state was calculated assuming that the unperturbed 5p-2h band has a rotational spectrum. The 5p-2h states are 0.608 MeV ($\frac{3}{2}^-$), 0.884 MeV ($\frac{5}{2}^-$), 1.271 MeV ($\frac{7}{2}^-$), and 1.767 MeV ($\frac{9}{2}^-$). These states can probably be identified with the observed levels at 0.472 MeV ($\frac{3}{2}^-$), 0.846 MeV ($\frac{5}{2}^-$), 1.410 MeV ($\frac{7}{2}^-$), and 1.885 MeV ($\frac{5}{2}^-$, $\frac{9}{2}^-$). The experimentally observed $J^\pi = \frac{5}{2}^{(-)}$ and $\frac{3}{2}^-$ states at 1.963 and 2.093 MeV, respectively, are probably members of a higher 5p-2h band.

The positive-parity states have been calculated using a $d_{3/2}-s_{1/2}$ gap of just under 1 MeV, using a Serber interaction whose strength was chosen to give the correct Sc^{43} binding energy relative to Ca^{40}. The level separations within the $k^\pi = \frac{3}{2}^+$ band are insensitive to changes in the $d_{3/2}-s_{1/2}$ gap, however, the $k^\pi = \frac{1}{2}^+$ band position is very sensitive to the value one uses (see Ref. 21). The value of the $d_{3/2}-s_{1/2}$ gap deduced from

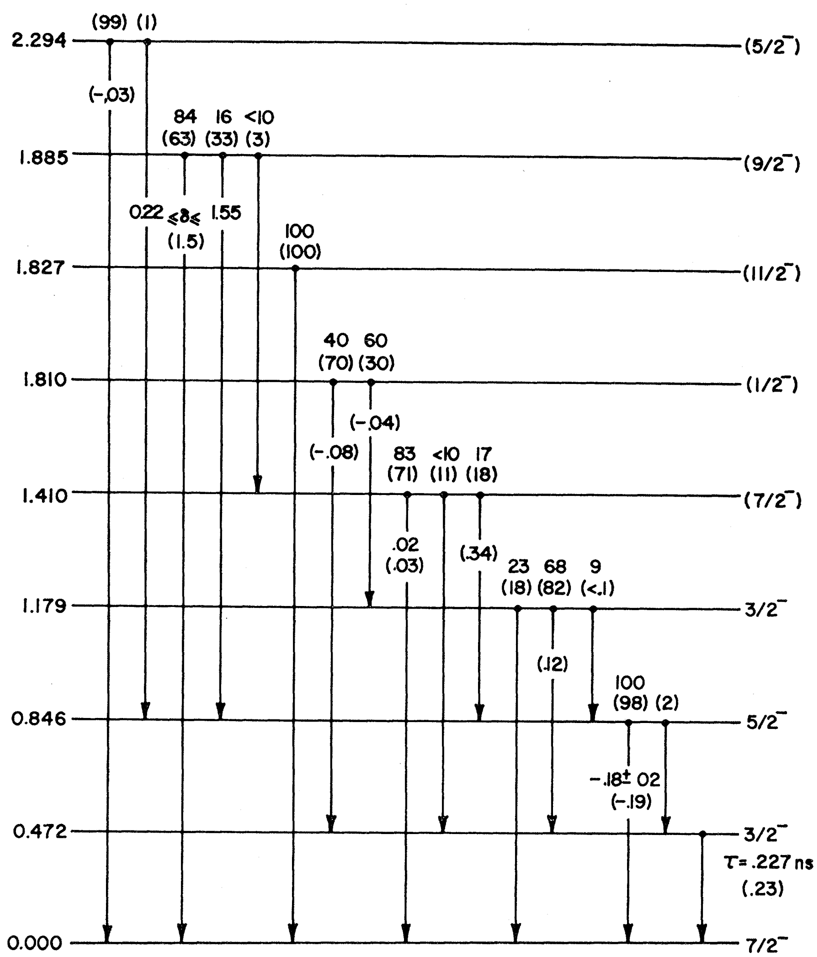

FIG. 14. Comparison of experimental and theoretical electromagnetic decay properties for the negative-parity levels. The 1.885-MeV level was assumed to decay only to the ground and 0.846-MeV level and the branching ratios have been adjusted accordingly. The branching ratios, mixing ratios, and lifetime given in parentheses are theoretical predictions. In these calculations, the following spin-parity assignments were assumed; 1.410($\frac{7}{2}^-$), 1.810($\frac{1}{2}^-$), 1.827(11/2⁻), 1.885-($\frac{9}{2}^-$), 2.294($\frac{5}{2}^-$). There is experimental evidence to support these assignments (see Sec. IV). The value of $\delta(1.410\to 0)$ was taken from the work of Phillips *et al.* (Ref. 9).

the experimentally observed K^{39} spectra is 2.54 MeV; whereas, in the case of Sc^{43}, the gap must be lowered to about 1 MeV to obtain agreement between theory and experiment. The $k^\pi=\frac{3}{2}^+$ band states are 0.15 MeV ($\frac{3}{2}^+$), 0.87 MeV ($\frac{5}{2}^+$), 1.20 MeV ($\frac{7}{2}^+$), 1.71 MeV ($\frac{9}{2}^+$), and 2.22 MeV ($\frac{11}{2}^+$). These states can be identified with the experimentally observed levels at 0.151 MeV ($\frac{3}{2}^+$), 0.880 MeV ($\frac{5}{2}^+$), 1.336 MeV ($\frac{7}{2}^+$), 1.930 MeV ($\frac{9}{2}^+$), and the probable $J^\pi=\frac{11}{2}^+$ level at 2.551 MeV. The $k^\pi=\frac{1}{2}^+$ band states 0.86 MeV ($\frac{1}{2}^+$), 1.35 MeV ($\frac{3}{2}^+$), 1.86 MeV ($\frac{5}{2}^+$), and 2.20 MeV ($\frac{7}{2}^+$) may be identified with the observed levels at 0.856 MeV ($\frac{1}{2}^+$), 1.158 MeV ($\frac{3}{2}^+$), 1.652 MeV ($\frac{5}{2}^+$), and 2.143 MeV ($\frac{7}{2}^+$). It is clear that, in general, there is good agreement between theory and experiment for levels below 2 MeV in excitation energy.

2. *Decay Properties*

As mentioned previously, the validity of any model depends on its ability to predict the electromagnetic decay properties of those states described by the model. In the following, we examine the calculated decay properties of the positive- and negative-parity states separately.

Figure 14 shows the calculated electromagnetic decay properties of the negative-parity levels identified in the model together with the available experimental data. The quantities given in parenthesis are the theoretical predictions. In these calculations, the following assignments were assumed: 1.410 MeV ($\frac{7}{2}^-$), 1.810 MeV ($\frac{1}{2}^-$), 1.885 MeV ($\frac{9}{2}^-$), and 2.294 MeV ($\frac{5}{2}^-$). The agreement between theory and the available experimental data is good. The only serious discrepancy between theory and experiment is in the predicted and observed values of the branching ratio for the 1.179→0.846-MeV transition which are 0.1 and 9%, respectively. A measurement of the mixing ratio for the 1.179→0.472 transition and a lifetime measurement of the 1.179-MeV level could indicate whether the theoretical description of this state is correct. The apparent disagreement between theory and experiment for the decay properties of the 1.810- and 1.885-MeV levels is probably not serious because these states were assumed to be pure $(fp)^3$ and 5p-2h states, respectively, in the present

TABLE IV. Comparison between experiment and theory for the $k^\pi=\frac{1}{2}^+$ 4p-1h bands. All theoretical quantities are calculated using the Serber interaction with a $d_{3/2}-s_{1/2}$ gap of 1 MeV and an effective charge of 0.5.

Level (MeV)	J^π	Transition	$J_i^\pi \to J_f^\pi$	Branching ratio Expt	Branching ratio Theor	Mixing ratio Expt	Mixing ratio Theor
0.856	$\frac{1}{2}^+$	0.856→0.151	$\frac{1}{2}^+\to\frac{3}{2}^+$	80	100		−1.07
		0.856→0.472	$\frac{1}{2}^+\to\frac{3}{2}^-$	20			
0.880	$\frac{5}{2}^+$	0.880→0.151	$\frac{5}{2}^+\to\frac{3}{2}^+$	100	100	0.54±0.09	0.62
1.158	$\frac{3}{2}^{(+)}$	1.158→0.151	$\frac{3}{2}^+\to\frac{3}{2}^+$	49	83	1.30±0.5 or −1.50±1.50	−0.28
		1.158→0.856	$\frac{3}{2}^+\to\frac{1}{2}^+$	41	0.7	−0.19±0.20	
		1.158→0.880	$\frac{3}{2}^+\to\frac{5}{2}^+$	10	16	−0.23±0.20	0.04
1.336	$\frac{7}{2}^+$	1.336→0.000	$\frac{7}{2}^+\to\frac{7}{2}^-$	18			
		1.336→0.151	$\frac{7}{2}^+\to\frac{3}{2}^+$	63	78	*E*2	*E*2
		1.336→0.880	$\frac{7}{2}^+\to\frac{5}{2}^+$	19	22		0.20
1.652	$\frac{5}{2}^{(+)}$	1.652→0.151	$\frac{5}{2}^+\to\frac{3}{2}^+$	43	72	0.05±0.18	0.12
		1.652→0.856	$\frac{5}{2}^+\to\frac{1}{2}^+$	<10	1		
		1.652→0.880	$\frac{5}{2}^+\to\frac{5}{2}^+$	<10	5		
		1.652→1.158	$\frac{5}{2}^+\to\frac{3}{2}^+$	57	23	0.00±0.28	−0.04
1.930	$\frac{9}{2}^{(+)}$	1.930→0.880	$\frac{9}{2}^+\to\frac{5}{2}^+$	69	78		
		1.930→1.336	$\frac{9}{2}^+\to\frac{7}{2}^+$	31	22	0.14	0.31
2.143	$\frac{7}{2}^{(+)}$	2.143→0.000	$\frac{7}{2}^+\to\frac{7}{2}^-$	47			
		2.143→0.151	$\frac{7}{2}^+\to\frac{3}{2}^+$	<10	1		
		2.143→0.472	$\frac{7}{2}^+\to\frac{3}{2}^-$	7			
		2.143→0.880	$\frac{7}{2}^+\to\frac{5}{2}^+$	46	17	−0.27±0.10	0.39
		2.143→1.158	$\frac{7}{2}^+\to\frac{3}{2}^+$	<10	24		
		2.143→1.336	$\frac{7}{2}^+\to\frac{7}{2}^+$	<10	56		0.02
		2.143→1.652	$\frac{7}{2}^+\to\frac{5}{2}^+$	<10	2		

calculations. For example, if a small amount of $(fp)^3$ is mixed into the $J^\pi=\frac{9}{2}^-$ 5p-2h state at 1.885 MeV, the effect would be to reduce the $E2/M1$ mixing and the theoretical branching ratio for the 1.885→0.846-MeV transition bringing both quantities in closer agreement with experiment. A measurement of the decay properties of the 1.810- and 2.294-MeV levels, together with the other unmeasured negative-parity states would be helpful in determining the nature of these states and the amount of mixing between $(fp)^3$ and 5p-2h states.

The positive-parity states are more complicated and not as easily explained as the negative-parity states. As reported in Ref. 21, Johnstone has taken into account the coupling of a $d_{3/2}$, $s_{1/2}$, or $d_{5/2}$ proton hole to the lowest Ti^{44} levels; that is, those states projected from the lowest $k=0$ Hartree-Fock configuration. Recently, Benson and Flowers[39] successfully calculated the odd-parity levels in F^{19} with a similar model; however, in the case of Sc^{43}, the theoretical gap between the $k=\frac{3}{2}$ and $k=\frac{1}{2}$ band is much too large. The lowest $J^\pi=\frac{1}{2}^+$ state is calculated to be at about 2 MeV for any reasonable interaction (Kuo and Brown, Kallio and Kolltveit, Serber, etc.). In order to bring the $\frac{1}{2}^+$ state down in energy, it was necessary to reduce the $d_{3/2}-s_{1/2}$ hole gap to approximately 1 MeV. With this value for the gap energy the spectrum is in reasonable agreement with experiment (see Fig. 13); however, the calculated decay scheme of the $k=\frac{1}{2}$ band is in poor agreement with experimental observations. (See Table IV.) For example, the second $J^\pi=\frac{3}{2}^+$ state at 1.158 MeV is calculated to have a branch of <1% to the $J^\pi=\frac{1}{2}^+$ state; whereas, experimentally one finds a 41% branch.

The failure of the model to account for the low-lying $k^\pi=\frac{1}{2}^+$ band states is probably due to the inadequate representation of the Ti^{44} spectrum. Although the normal-parity spectrum of Ne^{20} up to 7 MeV is well described by the $k=0$ band,[39] recent experimental work[40] has shown that this is certainly not the case for Ti^{44}. In addition to the $J=0, 2, 4, \ldots$ levels arising from the $k=0$ band, there are other levels including a $J^\pi=2^+$ state at 2.65 MeV which is somewhat lower in energy than predicted by McGrory and Bhatt[41] from a $(fp)^4$ shell-model calculation. Johnstone[37] has recalculated the $k^\pi=\frac{1}{2}^+$ band taking this state into account, but its effect is to lower the $J^\pi=\frac{1}{2}^+$ state by only 0.5 MeV. It may be that the $k^\pi=\frac{1}{2}^+$ states must be described by a 6p-3h configuration.

Even though the $k^\pi=\frac{1}{2}^+$ band states are poorly de-

[39] H. G. Benson and B. H. Flowers, Nucl. Phys. **A126**, 305 (1969).

[40] D. G. Madland and N. M. Hintz, Bull. Am. Phys. Soc. **14**, 602 (1969).

[41] J. B. McGrory and K. H. Bhatt, Bull. Am. Phys. Soc. **14**, 605 (1969).

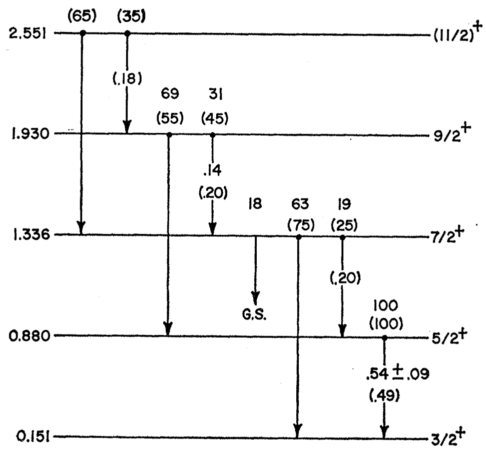

FIG. 15. Comparison of experimental and theoretical electromagnetic decay properties for those positive-parity states identified as belonging to the $k^\pi=\frac{3}{2}^+$ band in the Johnstone model. The branching ratios and mixing ratios given in parentheses are theoretical predictions. The experiment value for $\delta(1.930\rightarrow1.336)$ was taken from the work of Forster *et al.* (Ref. 32). In these calculations, the 2.551-MeV level was assumed to be $11/2^+$.

scribed by the model, it might still be reasonable to assume that the $k^\pi=\frac{3}{2}^+$ band states are good. Figure 15 shows the predicted decay properties[42] of the $k^\pi=\frac{3}{2}^+$ band states (calculated with a $d_{3/2}-s_{1/2}$ gap of 2.5[4] MeV and an effective charge of 0.5) together with all available experimental data. The agreement between theory and experiment is good. In the case of the 1.336-MeV level, the relevant quantity is the ratio of the branchings 1.336→0.151/1.336→0.880, since $E1$ transitions are not predicted in the model (see below). Both experiment and theory find a value of 3 for this ratio. It would be a good confirmation of the model if the 2.551-MeV level is shown to have the predicted spin and decay properties. Also, lifetime measurement of all levels would be useful.

An interesting characteristic of the levels in Sc^{43} is the almost complete absence of $E1$ transitions. Benson and Flowers[39] have proposed an explanation for this effect based on c.m. arguments which, in effect, states that $(fp)^3$ states will not proceed by $E1$ transitions to 4p-1h states. Johnstone[37] has extended these arguments to show that this effect also applies to transitions between 4p-1h states and mixed $(fp)^3$–5p-2h states provided that the 5p-2h states have good T quantum numbers for particles and holes, separately. (This has been assumed in all the Sc^{43} calculations.) This rule is expected to breakdown when one moves to higher excitation energies. The fact that the 0.472-MeV $(\frac{3}{2}^-)$ state decays $>97\%$ to the ground $(\frac{7}{2}^-)$ and has a lifetime of 0.23 nsec[43] indicates that the $\frac{3}{2}^-\rightarrow\frac{3}{2}^+$ $E1$ transition rate is $<3\times10^{-6}$ W.u. If one measures the lifetime of those levels having a known partial $E1$ decay (e.g., the 0.856-MeV $J^\pi=\frac{1}{2}^+$ state), then the degree of $E1$ retardation can be given more precisely. The 20% branch of the $\frac{1}{2}^+$ state to the $\frac{3}{2}^-$ state suggests a lifetime of >1 nsec if the $E1$ rate is retarded by as much as the $E1$ decay of the $\frac{3}{2}^-$ state (0.472 keV); this would require a very small $\frac{1}{2}^+\rightarrow\frac{3}{2}^+$ $M1$ transition rate of $<10^{-4}$ W.u. implying that the $k^\pi=\frac{1}{2}^+$ and $\frac{3}{2}^+$ bands have totally different structure.

VI. CONCLUSION

In the present investigation, definite spin assignments have been made for nine bound levels and, in addition, limitations have been placed on the possible spin assignments of several other levels. The results obtained in this investigation have been combined with the results reported in Paper I and by other investigators to test the predictions of various models of Sc^{43}. It was found that neither the conventional shell model nor the cluster model could account for the observed properties of Sc^{43}. The Coriolis-coupling model gives qualitative agreement with the experimental observations, but quantitative agreement will probably require inclusion of pairing forces in the calculations. The most successful model is that due to Johnstone and co-workers. Their model gives excellent agreement with the experimentally observed properties of the negative-parity states. The predicted properties of the $k^\pi=\frac{3}{2}^+$ band states also agree with experimental measurements; however, the predicted $k^\pi=\frac{1}{2}^+$ band states were found to be in disagreement with experiment. The complete understanding of the nature of these states requires additional theoretical and experimental work. The results of the experiments proposed throughout the discussions of Sec. V could lead to a clearer understanding of the Sc^{43} nucleus.

ACKNOWLEDGMENTS

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[42] An error in one of the computer routines caused the $E2$ transition rates quoted in Ref. 21 to be incorrect.

[43] J. C. Merdinger, N. Schulz, and R. W. Kavanagh, Nucl. Phys. **A114**, 204 (1968).