support the interpretation of these states as fragments of a collective octupole excitation.^{17,25} It is not immediately clear how the second $\frac{3}{2}$ state at 3.95 MeV would fit into such a simple weak-coupling scheme. The second $\frac{1}{2}^+$ state at 4.02 MeV on the other hand, can possibly be described in terms of positive-parity holes in the sd shell coupled to excited states of ⁴⁰Ca.

The results of the present work combined with the growing body of experimental information on this mass region suggest that no one simple picture is

adequate to explain even the low-lying states. Thus, more experimental work, particularly the measurement of absolute electromagnetic transition probabilities, should provide the detailed information needed to obtain a more complete description of these nuclei in terms of more sophisticated models.

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Proton Polarization in the ${}^{28}Si(d, p)$ ${}^{29}Si$ Reaction*

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The polarizations of protons from the ${}^{28}\text{Si}(d, p){}^{29}\text{Si}$ reactions leading to the ground state and the 1.28-. 2.03-, and 3.62-MeV excited states of 29Si have been measured over the range of laboratory angles from 30° to 147° at a bombarding deuteron energy of 10.8 MeV. A scattering-chamber-polarimeter assembly employing solid-state detectors was used to allow the measurements on all groups to be made simultaneously. To provide additional data for theoretical comparisons, the differential cross sections for the same reactions and for the elastic scattering of deuterons from ²⁸Si have also been measured at the same bombarding energy. For the $l_n=0$ ground-state reaction, the observed polarizations change rapidly from large negative to large positive values near the minima in the cross section at 50° and 90°. The 1.28-MeV excited-state reaction also shows large polarizations, but with no obvious relationship to variations in the cross section. Polarizations of protons corresponding to the 2.03- and 3.62-MeV excited states are smaller (between ± 0.3) and appear to be uncorrelated with cross-section variations. Comparisons are made with published theoretical predictions where available.

I. INTRODUCTION

NOR nearly two decades, the deuteron stripping \mathbf{r} reaction has provided a very useful tool for experimental and theoretical investigations of the states of atomic nuclei and of nuclear-reaction mechanisms. Several theories have enjoyed various degrees of success in describing the observed results of this reaction. The earliest of these, proposed by Butler,¹ was very useful in establishing the orbital angular momentum l_n of the captured particle, but it predicted absolute cross sections which were generally much too large, as well as zero polarizations for the outgoing particles of the stripping reaction. A distinct advance resulted from the development of the distorted-wave Born-approximation (DWBA) theory for this reaction,² which gave improved theoretical fits to the magnitudes and shapes of observed cross sections over a wide range of energy, scattering angle, and target mass. However, difficulties remained in fitting available results on the polarizations

of the outgoing protons in (d, p) reactions.^{3,4} Such measurements are of special interest, since they not only provide information regarding the total angular momentum $j_n = l_n \pm \frac{1}{2}$ of the neutron captured by the target nucleus, but appear to provide the most sensitive tests of models developed to explain the nuclearreaction mechanism. The experiments described in this and the following article⁵ were originally undertaken to provide additional (d, p) polarization data to assist in resolving the difficulties then existing with DWBA fits, as well as for comparison with newer models.

Recent work has been reported using sources of polarized proton and deuteron beams which has added greatly to the experimental information available on stripping reactions. Yule and Haeberli⁶ have carried out measurements of the vector analyzing powers observed in (d, p) reactions initiated by polarized deuterons, and have found good agreement with DWBA predic-

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FIG. 1. A cross section through the median plane of the scattering-chamber-polarimeter assembly used to measure the polarizations of the reaction protons. Letter designations are explained in the text.

tions. A complete test of the reaction ${}^{52}Cr(d, p){}^{53}Cr$ has been reported very recently by Bjorkholm, Haeberli, and Mayer,⁷ in which asymmetry measurements were made on the elastic and reaction channels of both the target nucleus and the residual nucleus of the basic (d, p) reaction using polarized beams of the same energy in the c.m. system. Because of the Q value of the reaction, two different accelerators were required for this purpose. These measurements give a direct comparison of the vector analyzing power and the proton polarization (via the analyzing power of the inverse reaction) for the basic (d, p) reaction. This is apparently the first case where good DWBA fits have been made to all of the cross section and polarization data, although some minor problems remain as discussed by Bjorkholm et al.⁷

One of the more serious objections to the DWBA theory is its treatment of the incident deuteron as a tightly bound or elementary system. The distorted wave depends only on the position of the center of mass of the deuteron, and the effects of its loosely bound structure as it encounters the nuclear surface are ignored. Recently, two theories have been proposed which attempt to resolve this problem by considering the neutron and proton separately, utilizing neutron and proton optical potentials instead of a deuteron optical potential. The "weakly bound projectile" (WBP) model developed initially by Pearson and Coz⁸ and the Butler-Hewitt-McKellar-May (BHMM) model⁹ have generated additional interest in polarization data from stripping reactions because of the sensitivity of the polarization to the details of the models.

The results of Bjorkholm et al.7 discussed above provide (d, p) polarization results for one specific reaction with much better statistical accuracy from the analyzing power for the inverse (p, d) process using polarized protons than is feasible in double-scattering experiments of the type reported here. Nevertheless, the present results are useful because of the different

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nuclei involved, and because for the first time simultaneous measurements are reported of the polarizations of the proton groups leading to several states in the residual nuclei. This will allow theoretical comparisons to be made of the (d, p) polarizations for several reactions with the same target nucleus corresponding to several values of l_n and j_n . The newer technique⁷ which measures the analyzing power of the inverse reaction is presently limited to the ground-state reaction of the forward (d, p) process, and to (d, p) reactions for which the residual nucleus is also stable.

The present paper reports measurements of the polarizations of protons from the reactions ${}^{28}Si(d, p){}^{29}Si$ leading to the ground state and the 1.28-, 2.03-, and 3.62-MeV excited states of ²⁹Si. Previous results at different bombarding energies over a narrower angular range have been reported for the ground-state reaction^{10,11} and the reaction leading to the 1.28-MeV excited state.12

II. EXPERIMENTAL METHOD

A. Apparatus

1. Target Chamber and Polarimeter Assembly

A beam of 10.8-MeV deuterons was produced by the 45-in. cyclotron at Indiana University. In the present experiment a set of water-cooled main slits and a set of air-cooled antiscattering slits situated just upstream from the scattering chamber defined the deuteron beam spot on the natural-silicon target. The Faraday cup was located at the back of a 2-m section of shielded pipe downstream from the scattering chamber.

The scattering-chamber-polarimeter assembly designed by Foster and Maddox is described in detail elsewhere,¹³ and only a brief description will be given here. A cross-sectional view through the median plane of this assembly is shown in Fig. 1. The scattering chamber was a 4-in.-diam cylindrical stainless-steel chamber (U) 9 in. in height. Deuterons from the cyclotron entered through the beam pipe (V) and were incident on the silicon target located at B. The unscattered beam continued through beam pipe V', which was connected to the Faraday pipe previously described. Protons produced in the silicon target passed through a $\frac{1}{2}$ -mil Mylar window at C, a $\frac{1}{4}$ -in. air gap, a $\frac{1}{4}$ -mil aluminized Mylar window at D, and then entered the polarimeter. The center line of the polarimeter could be set to any laboratory angle relative to the beam direction from 26° to 150° in the horizontal plane. In order to determine the zero angle, the entire scatteringchamber-polarimeter assembly was rotated 180° about

the beam axis. The zero angle was measured and found to be $0.5 \pm 0.2^{\circ}$.

Upon entering the polarimeter, the protons were incident upon the carbon second scatterer at I, and could scatter at a mean angle of 45° into the right and left detector systems. In order to measure the instrumental asymmetry of the polarimeter system, it was necessary to exchange the role of the right and left detector systems. This was done by rotating the polarimeter 180° about the line BB'. Collimator dimensions for the polarimeter were chosen to maximize the counting rate, while maintaining the resolution necessary to separate the proton groups under investigation. The horizontal angle of acceptance of the right and left detectors was 3.2°, and the vertical angle of acceptance was 11.3°.

Background measurements were taken by removing the second scatterer from the reaction proton beam and inserting a carbon degrading foil of the same thickness as the second scatterer between positions G and H. This foil could not be seen directly by the detector telescopes.

2. Detectors and Electronics

Detector telescopes on each side of the polarimeter were composed of a thin transmission ΔE detector (K' and K'') and a thick partially depleted E detector (M' and M''). In addition, a detector was placed at position K to provide a monitor counter for polarization runs and a method for measuring cross sections.

In order to separate proton groups exciting nearby states in the polarization spectra, it was necessary to use detectors of the highest available resolution. The Edetectors were partially depleted silicon surface-barrier detectors fabricated by Inskeep at Indiana University. They had thicknesses of about $1100 \ \mu$ and active areas of 150 mm². The ΔE counters were fully depleted Ortec14 transmission detectors with thicknesses of about 400 μ , respectively. Because of the large amount of neutron radiation in the vicinity of the detectors, there was a gradual loss of resolution and an increase in reverse leakage current due to radiation damage. The latter would have resulted in a decrease in detector bias. so the detector bias was monitored during all runs. In order to slow the increase in reverse leakage current, the detectors were cooled by passing alcohol at ice-water temperature through copper tubing soldered to the back plates of the polarimeter.

A block diagram of the electronic equipment is shown in Fig. 2. To record counts from one side of the polarimeter, pulses from the respective ΔE and E detectors were fed into the coincidence units. If the pulses were in fast coincidence, an output pulse opened a linear gate which allowed the pulse from the E detector to be analyzed by a Nuclear Data¹⁵ 1024-channel analyzer.

¹⁰ R. W. Bercaw and F. B. Shull, Phys. Rev. 133, B632 (1964).
¹¹ A. Isoya and M. J. Marrone, Phys. Rev. 138, 800 (1962).
¹² L. H. Reber and J. X. Saladin, Phys. Rev. 133, B1155 (1964).
¹³ C. C. Foster, Ph.D. thesis, Indiana University, 1967 (unpublished), available from University Microfilms, 300 N. Zeeb Road, Ann Arbor, Mich. 48106.

¹⁴ Oak Ridge Technical Enterprises Corporation, P. O. Box C, Oak Ridge, Tenn. 37831.

¹⁵ Nuclear Data, Inc., P. O. Box 1464, Madison, Wisc.



FIG. 2. The detector arrangement and block diagram of the electronics used in the polarization measurements.

FIG. 3. A typical spectrum from the reaction ${}^{28}\mathrm{Si}(d,p){}^{29}\mathrm{Si}$ with a sidecounter telescope mounted in the center position of the polarimeter. Discriminator settings and groups corresponding to excited states of ${}^{29}\mathrm{Si}$ are indicated by the arrows.

60

80

100

120

140

CHANNEL

160

200

180

220

3. Targets

Three types of targets were used in this experiment. For the polarization runs, the first scatterer was a crystalline silicon wafer prepared by Inskeep from a high-purity silicon crystal used to make silicon surfacebarrier detectors. A special lapping and deep-planaretching technique¹⁶ was used to produce 0.001-in.-thick wafers with accurately parallel faces. For the cross-section measurements, a thin evaporated silicon target was used.

The second scatterer was prepared from machined 0.005-in. sheets of high-purity graphite,¹⁷ and had a thickness of 28.6 mg/cm². According to the manufacturer's specifications, the foils were of uniform thickness to within ± 0.0001 in.

B. Experimental Procedures

Most of the specific techniques employed in this experiment were identical with those described in detail in the following paper.⁵ To avoid duplication, only a brief summary of procedures and specific details regarding data analysis for this particular experiment will be discussed here.

The low counting rates obtained in a double-scattering experiment make the setting of the electronics for a polarization run difficult. To produce higher counting rates for this purpose, the side-counter telescopes were moved to the center position. A typical spectrum obtained with a side-counter telescope mounted in the center position is shown in Fig. 3.

Data for the polarization runs were taken with the polarimeter in either of two orientations. The 0° orientation allowed the "right" side (right-side counter telescope and its associated electronics) to measure the particles scattered right-right and the "left" side to measure the particles scattered right-left. The 180° orientation was obtained by rotating the polarimeter 180° about its axis. Polarization runs were taken alternately in the two orientations until the desired number of counts was obtained.

C. Data Analysis and Corrections

The final data for the polarization measurements in the present experiment consisted of 6-30 spectra for each laboratory angle. Spectra for each angle were separated into eight categories according to their run type (signal or background), polarimeter orientation (0° or 180°), and polarimeter side (left or right side). A total spectrum for each category was then obtained by adding the individual spectra channel by channel. Background counts were negligible at the energies of

each proton group except at the energy corresponding to the 3.62-MeV excited state of ²⁹Si, where the background counts were at most a few percent of the signal counts. Figure 4 shows a typical set of four signal spectra, taken in this case at 40° in the laboratory system. At all laboratory angles except 90° every peak in each of the four summed spectra contained at least 150 counts; up to 1500 counts each were obtained at favorable angles.

To extract the total number of counts in each peak, a computer program written by Eckley¹⁸ was used which fitted any number of Gaussian distributions and an exponential background to a given spectrum. The total number of counts in each peak was taken to be the area under the Gaussian distribution fitted to that peak. Final fits obtained in this way for the four polarization spectra at 40° are shown in Fig. 4 by the solid curve.

The product polarization P for protons emitted in a (d, p) reaction at a laboratory angle of θ_p is given by the relation

$$P(\theta_p) = P_1 P_2 = (R - L) / (R + L), \qquad (1)$$

where R is the number of particles scattered right-right and L the number scattered right-left. P_1 is the polarization of the emitted protons, and P_2 the analyzing power of the polarimeter. Equation (1) must be corrected for any intrinsic asymmetry (K) in the polarimeter system, which was determined by taking data with the polarimeter in the 0° and 180° orientations as previously described. In addition, the angle of the first target, the finite beam spots and apertures, and the angular variation of the cross sections can result in a false asymmetry (K_{σ}) which was calculated for each proton group at every angle of the polarization run. Equations showing the applications of K and K_{σ} to the extraction of the corrected product polarization $P^{c}(\theta_{n})$ are given in the following paper.⁵

To determine the polarizations P_1 of the protons emitted in the (d, p) reaction from the corrected product polarization, it is necessary to know the effective analyzing power P_2^{eff} of the polarimeter. If $P_2(E_p, \varphi_p)$ is the polarization of unpolarized protons of energy E_p elastically scattered from carbon through an angle φ_p , then P_2^{eff} must be a weighted average of $P_2(E_p, \varphi_p)$ taken over the solid angle of acceptance of the counter telescopes and over the energy spread due to the finite thickness of the second scatterer. Using polarizations and cross sections from many sources,^{19,20}

¹⁶ C. N. Inskeep and W. W. Eidson, Electronique Nucléaire, Proceedings of the International Symposium Organized by La Société Française des Electroniciens et des Radioélectriciens, Paris, 1963 (Organization for Economic Cooperation and Development, ¹⁷ European Nuclear Energy Agency, Paris, 1963), p. 163. ¹⁷ Poco Graphite, Inc., Garland, Tex.

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27, 41 (1961); P. Huber and K. P. Meyer, Helv. Phys. Acta,
Suppl. VI (1961); Y. Nagahara, J. Phys. Soc. Japan 16, 133 (1961).

²⁰ K. W. Brockman, Jr., Phys. Rev. 110, 163 (1958).



FIG. 4. Typical polarization spectra obtained by summing repeated polarization runs, in this case at a laboratory angle of 40°. R is the spectrum of particles scattered right-right and L the spectrum of particles scattered right-left into the right and left detectors, respectively, with the polarimeter in its normal position. L^{π} and R^{π} are spectra obtained with the same detectors rotated 180°, and correspond to particles scattered right-right and right-left, respectively. The solid curves are computer fits to the data.

the weighted average of $P_2(E_p,\varphi_p)$ was calculated for the incident proton energies of this experiment. The method of determining this weighted average is similar to that given by Reber.²¹

Figure 5^{22,23} shows a plot of the resulting effective analyzing power of the polarimeter versus the proton energy incident on the second scatterer. The final

²¹ L. H. Reber, Ph.D. thesis, University of Pittsburgh, 1963 (unpublished).

²² L. Rosen, P. Darriulat, H. Faraggi, and A. Garin, Nucl. Phys. 33, 458 (1962).
²³ R. I. Brown and W. Haeberli, Phys. Rev. 130, 1163 (1963).



FIG. 5. Plot of the calculated effective analyzing power of the polarimeter, as shown by the solid curve. The data points shown for comparison are those of Brockman *et al.* (Ref. 20), Rosen *et al.* (Ref. 22), and Brown and Haeberli (Ref. 23). Note the ordinate scale.

corrected product polarizations were divided by P_2^{eff} as read from this curve at the appropriate energy for each proton group in order to obtain the final values of P_1 .

D. Differential Cross-Section Measurements

Differential cross sections for elastic deuteron scattering and the reactions ${}^{28}\text{Si}(d, p)$ were measured with the scattering-chamber-polarimeter assembly from 26° to 150° in the laboratory system. For these measurements, the second scatterer and the side telescopes were removed from the polarimeter. A solid-state silicon surface-barrier detector with a 0.187-in. circular collimator was mounted in the center position of the polarimeter.

III. EXPERIMENTAL RESULTS

A. (d, p) Polarization Results

The proton polarizations measured for the reactions ${}^{28}\text{Si}(d, p){}^{29}\text{Si}$ leading to the ground state and the 1.28-, 2.03-, and 3.62-MeV excited states of ${}^{29}\text{Si}$ are listed in Table I and plotted in Figs. 6–9. Total errors listed in Table I were obtained by propagating the standard statistical errors in the peaks to yield statistical errors in the product polarizations, and then dividing by the

analyzing power. No error was assigned to P_2^{eff} because of the method by which it was obtained from a number of sources. Tables listing the c.m. angles, asymmetries, product polarizations, and the values of P_2^{eff} used for each group at each angle are on file with the American Society for Information Science.²⁴

B. Differential Cross-Section Results

Angular distributions of the differential cross sections for the reactions ${}^{28}\text{Si}(d, p){}^{29}\text{Si}$ leading to the ground state and the 1.28-, 2.03-, and 3.62-MeV excited states of ${}^{29}\text{Si}$ and for ${}^{28}\text{Si}(d, d){}^{28}\text{Si}$ elastic scattering are presented in Figs. 6–10. Relative errors in the cross sections (2.5%) are attributed to counting statistics, beam integration for normalization purposes, and multiple scattering in the Mylar windows and the air gap; absolute errors in the cross sections (25%) are due to uncertainties in the target thickness, beam integration, solid angles, and multiple scattering. Tabular results for the measured reaction and elastic cross sections are also available.²⁴

²⁴ Order NAPS Document 00737 from ASIS National Auxiliary Publications Service, c/o CCM Information Sciences, Inc., 22 West 34th Street, New York, N.Y. 10001; remit \$1.00 for microfiche or \$3.00 for photocopy.

θ_{lab}	Proton polarization (P_1)			
(deg)	Ground	1.28 MeV	2.03 MeV	3.62 MeV
29.4	-0.285 ± 0.062	-0.119 ± 0.046	-0.179 ± 0.067	
39.4	-0.095 ± 0.047	$0.128 {\pm} 0.043$	0.049 ± 0.042	-0.008 ± 0.025
49.4	0.531 ± 0.066	-0.288 ± 0.050	-0.195 ± 0.032	0.249 ± 0.030
59.4	0.160 ± 0.048	-0.302 ± 0.069	-0.131 ± 0.055	-0.060 ± 0.083
69.4	0.017 ± 0.042	-0.180 ± 0.073	-0.006 ± 0.062	-0.002 ± 0.046
79.4	-0.148 ± 0.069	0.464 ± 0.078	0.007 ± 0.084	-0.027 ± 0.039
89.4	-0.599 ± 0.093	0.248 ± 0.092	0.122 ± 0.057	0.125 ± 0.026
99.4	0.315 ± 0.074	-0.526 ± 0.064	0.231 ± 0.049	0.132 ± 0.030
109.4	0.365 ± 0.068	-0.255 ± 0.072	-0.195 ± 0.059	0.079 ± 0.041
119.4	-0.238 ± 0.078	0.097 ± 0.089		
129.4	0.043 ± 0.062	0.397 ± 0.056	0.140 ± 0.059	
139.4	-0.173 ± 0.069	0.224 ± 0.057	0.194 ± 0.048	
146.4	-0.099 ± 0.089	0.116 ± 0.071		

TABLE I. Results of proton polarization measurements for the reactions ²⁸Si(d, p) ²⁹Si leading to the ground, 1.28-, 2.03-, and 3.62-MeV states of ²⁹Si at an incident deuteron energy of 10.8 MeV.





FIG. 6. Differential cross section and proton polarization results for the $l_n=0$ reaction $\sum_{i=1}^{28} Si(d, p)^{29}Si$ leading to the ground state of ²⁹Si. Errors shown on the polarizations are statistical errors only. For the cross sections the statistical errors are no larger than the dots. The solid and dashed straight lines connecting the data points serve only as a guide to the eye, and have no other significance.

FIG. 7. Differential cross section and proton polarization results for the $l_n=2$ reaction ${}^{28}\text{Si}(d, p){}^{29}\text{Si}$ leading to the 1.28-MeV excited state of ${}^{29}\text{Si}$. Symbols are explained in the caption of Fig. 6.



FIG. 8. Differential cross section and polarization results for the $l_n=2$ reaction ²⁸Si(d, p)²⁹Si leading to the 2.03-MeV excited state of ²⁹Si. Symbols are explained in the caption of Fig. 6.

IV. DISCUSSION

The proton polarization for the ground-state reaction ²⁸Si(d, p)²⁹Si has been measured previously at deuteron energies of 10 MeV¹⁰ and 15 MeV.¹¹ A comparison of the present data with these measurements is shown in Fig. 11. As can be seen, the agreement over the region of overlap is quite good. In the earlier experiments no measurements were taken between 45° and 55° in the laboratory system, presumably because of the low cross section. With the absence of data in this region, the polarization appears to oscillate rather moderately with no obvious correlation with the cross section. However, it can be seen from the present results near 50°, 90°, 100°, and 110° that the polarization attains much larger values and changes much more rapidly than previously indicated. In Fig. 6, where the differential cross section and the polarization are plotted together for comparison, a correlation between variations in the differential cross section and the polarization is evident. Near the two cross-section minima at 50° and 90°, the polarization suddenly swings from large negative to large positive values. Between these sharp swings, the polarization changes more slowly, thus creating a "saw-tooth" pattern. This pattern has been discussed by Goldfarb²⁵ and observed in other $l_n = 0$ polarization measurements.²⁶

It is also apparent in the recent theoretical work of Gubkin.27

In the present work, no attempt was made to fit theoretical predictions to the experimental data. However, there have been theoretical comparisons made with the ²⁸Si(d, p)²⁹Si ground-state data taken previously. Figure 12 shows the results of the present work along with a DWBA fit (solid curve) by Goldfarb²⁵ to the earlier data at 10 MeV.¹⁰ The large positive polarization observed in the present experiment at 50° is indeed predicted by Goldfarb. Other DWBA fits have been attempted,²⁸ but will not be discussed here.

A recent comparison of the 15-MeV data of Isoya and Marrone¹¹ and the WBP model made by Pearson, Bang, and Pocs²⁹ is shown by the solid curve in Fig. 11. Although there is a bombarding energy difference, the WBP calculation also agrees reasonably well with the present data. Again a pronounced rise is predicted around 50°, as observed in the present results. It is worth noting that data were taken in this and the following experiments⁵ to backward angles, in spite of low cross sections, in a deliberate attempt to provide more severe tests for the theoretical models. Figures 11 and 12 show that it is difficult to choose between the published DWBA and WBP analyses, since they only extend to 80°. It will be interesting to see if a distinc-



FIG. 9. Differential cross section and polarization results for the $l_n = 3$ reaction ²⁸Si(d, p)²⁹Si leading to the 3.62-MeV excited state of ²⁹Si. Symbols are explained in the caption of Fig. 6.

²⁵ L. J. B. Goldfarb, in *Proceedings of the International Symposium on Polarization Phenomena of Nucleons, Karlsruhe, 1965*, edited by P. Huber and H. Schopper (W. Rosch and Co., Bern, 1966).

²⁶ É. J. Ludwig and D. W. Miller, Phys. Rev. 138, B364 (1965).

²⁷ I. A. Gubkin, Nucl. Phys. A126, 355 (1969).

 ²⁸ R. C. Johnson, Nucl. Phys. **35**, 654 (1962).
 ²⁹ C. A. Pearson, J. M. Bang, and L. Pocs, Ann. Phys. (N.Y.) 52, 33 (1969).



FIG. 11. A comparison of the present measurements of the proton-polarization angular distribution for the ground-state reaction ${}^{28}\text{Si}(d, p){}^{29}\text{Si}$ with those of Bercaw and Shull (Ref. 10) and of Isoya and Marrone (Ref. 11). The present work shows large magnitudes for the proton polarization around 50° and 90° which were not evident in the two previous measurements. The solid curve is a WBP comparison with the data of Isoya and Marrone taken from Pearson *et al.* (Ref. 29).



FIG. 13. Comparison of the present results for the polarizations of protons exciting the 1.28-MeV state of 29 Si in the reaction 28 Si $(d, p){}^{29}$ Si with the results of Reber and Saladin (Ref. 12). The solid curve is a WBP prediction made by Pearson *et al.* (Ref. 29) for a bombarding energy of 15 MeV.

tion can be made between the models by comparison with the present back-angle data.

In 1959, Rodberg³⁰ gave a simple discussion of the relation between the differential cross section and the polarization in the optical-model description of elastic scattering. He obtained the approximate result that the polarization of the elastically scattered particles should be given by the "derivative rule":

$$\mathbf{P}(\theta) = \left[\sigma(\theta)\right]^{-1} \left[d\sigma(\theta)/d\theta\right] \langle\beta\rangle \mathbf{n}, \qquad (2)$$

where $\sigma(\theta)$ is the differential cross section, and the constant $\langle \beta \rangle$ is related to the spin-orbit potential in the optical model. Biedenharn and Satchler³¹ extended the derivative rule to $l_n=0$ stripping reactions, and obtained the approximate result for this case that

$$\langle \beta \rangle \approx (mR/2\hbar \mathbf{k}) V_{so}(R),$$
 (3)

where $\hbar \mathbf{k}$ is the momentum of the emitted proton, *m* its reduced mass, and *R* is the nuclear radius. V_{so} is the spin-orbit potential defined in the customary way for DWBA calculations.

Although the validity of this approximate rule is questionable,²⁵ it does seem to fit the $l_n=0$ stripping data at intermediate angles for the reactions ²⁸Si(d, p) and ⁸⁸Sr(d, p).³ This is shown for the present results by the dashed curve in Fig. 12, which was obtained from the cross-section data in Fig. 6 by applying Eq. (2) and adjusting the normalization constant $\langle \beta \rangle$ for best fit. This constant corresponds in Eq. (3) to a value of $V_{so} = +14$ MeV. The agreement is reasonable from about 40° to 150°.

The proton polarization in the reaction ${}^{28}\text{Si}(d, p){}^{29}\text{Si}$ leading to the 1.28-MeV excited state of ${}^{29}\text{Si}$ has also been measured previously, 12 but at an energy of 15 MeV. Figure 13 shows the agreement between the present work and the measurements of that experiment, which were only taken out to 72°. The polarization of this state oscillates between fairly large positive and negative values, but does not seem to show the "saw-tooth" pattern observed for the ground state. In Fig. 7, there appears to be no obvious correlation between variations in the differential cross section and the polarization. Figure 13 also shows a comparison of the polarization results with the prediction of the WBP model obtained by Pearson *et al.*²⁹ for 15-MeV bombarding energy. Again it appears that the results of the present experiment at the backward angles should be very significant for future theoretical comparisons.

Figures 8 and 9 show that reactions leading to the 2.03- and 3.62-MeV states have smaller measured values of the polarization (|P| < 0.30) than reactions leading to the ground and 1.28-MeV states. There is no apparent correlation between the observed differential cross-section variations and the polarizations for these two states.

V. CONCLUSIONS

The present measurements of the proton polarizations in the reactions ${}^{28}\text{Si}(d, p) {}^{29}\text{Si}$ leading to the ground and 1.28-MeV states of ²⁹Si show good agreement in the region of overlap with previous data, but extend the results to backward angles. Results for the 2.03and 3.62-MeV excited states, not previously reported, were obtained simultaneously with the ground and 1.28-MeV states. The measurements thus provide data taken under the same conditions for $l_n=0$, $l_n=2$ $(j_n = \frac{3}{2}^+ \text{ and } \frac{5}{2}^+)$, and $l_n = 3$ reactions with the same target nucleus. Results obtained for the first time on the ground-state reaction near the two cross-section minima add some very interesting features to the polarization angular distributions. The large value for the polarization found around the 50° minimum is predicted by two of the existing theories and the approximate derivative rule. More definitive comparisons between the theoretical models for the reaction ${}^{28}\text{Si}(d, p)$ may now be possible using the present results, particularly at the backward angles.

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³¹ L. C. Biedenharn and G. R. Satchler, Helv. Phys. Acta Suppl. VI (1960).