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Absorption, Charge-Exchange, and Double-Charge-Exchange Reactions of π Mesons with Complex Nuclei: Comparison of Theoretical Predictions with Experimental Results*

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The two-step intranuclear-cascade evaporation model is used in the calculation of nucleon spectra, particle multiplicities, and radiochemical cross sections following stopped-π^- absorptions by complex nuclei. The nucleon spectra and particle multiplicities from the pion absorption by light elements are, in general, predicted quite well. The theoretical particle yields from the heavy elements are overestimated and the particle spectra are too soft. These discrepancies are attributed to an insufficient number of pion absorptions near the edge of the nucleus for the heavy elements in the theoretical model. The magnitudes of the theoretical radiochemical cross sections from slow-π^- absorption by iodine are in fair agreement with those from experiments, but the peak in the cross-section distribution versus isotope mass for some of the isotopes appears at lower mass values than the measured ones. This discrepancy is consistent with those above. The model does well in predicting the change-exchange cross section, but does poorly in estimating the double-charge-exchange cross section for pions with energies below 200 MeV. These cross sections represent about 1% or less of the total interaction cross section, and the theoretical model does not accurately reproduce such events, in general.

INTRODUCTION

EXPERIMENTAL data on pion absorption, charge-exchange, and double-charge-exchange reactions have been generated recently, and comparison of these data with the theoretical results of a two-step cascade-evaporation calculation is deemed appropriate. Earlier attempts[1,2] to evaluate the theoretical model for incident pions were unsatisfactory because of the dearth of experimental data. Even now the situation is not ideal for all of the quantitative pion reaction data that can be predicted from the model, but there are sufficient data, particularly on pion-absorption reactions, to warrant an evaluation. Comparisons with the charge-exchange reactions are made because experimental data exist, and an interesting trend is visible.

A paper on the application of the theoretical model to pertinent reactions in cancer radiotherapy has been published elsewhere.[3]

THEORY

The cascade phase of the two-step model is assumed to occur in the following manner: When an incident pion enters the nucleus, it can either scatter (with or without charge exchange) from the individual nucleons that are in motion but bound within the nucleus, it can be absorbed by a pair of the nucleons, or it can pass through the nucleus. In the event of a scattering or absorption, the collision products are transported through the nucleus, and they in turn can interact with the bound nucleons in a similar manner with the reaction products again being transported, etc., thus generating a cascade. The life history of each particle is followed until it either escapes or its energy falls below an arbitrary cutoff energy, which, in general, is taken to be half the Coulomb barrier at the surface of the nucleus. A pion can therefore disappear when it is absorbed by a nucleon-nucleon pair or when its energy falls below the cutoff energy, and it is thereby assumed to be trapped within the nucleus. In the latter case, the process is equivalent to an absorption by the nucleus as a whole.

Many incident particles are made to impinge on the nucleus, and the cascade development for each is calculated using the Monte Carlo method. All of the possible charge combinations of the particle-particle reactions are taken into account. The probabilities that scattering or absorption reactions will occur are calculated from the free-particle-scattering cross sections[2] and from the pion-absorption cross section.[4] The cross section for pion absorption in nuclear matter by a two-nucleon cluster is the same as that used by Metropolis *et al.*,[1] where the absorption cross section is associated with the free-deuteron cross section in the manner described by Frank, Gammel, and Watson.[5] The same absorption cross section is used for all charge states of the pion. The absorptions of pions by single nucleons are not considered.

An evaporation calculation, the second step in the two-step process, follows the calculation of each cascade. The program used is a modification[6] of the one by Dres-

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[1] N. Metropolis *et al.*, Phys. Rev. **110**, 204 (1958).

[2] H. W. Bertini, Phys. Rev. **131**, 1801 (1963); **138**, AB2 (1965).

[3] M. P. Guthrie, R. G. Alsmiller, Jr., and H. W. Bertini, Nucl. Instr. Methods **66**, 29 (1968).

[4] C. D. Zerby, R. B. Curtis, and H. W. Bertini, in Oak Ridge National Laboratory Report No. ORNL-CF-61-7-20, 1961 (unpublished).

[5] R. M. Frank, J. L. Gammel, and K. M. Watson, Phys. Rev. **101**, 891 (1956).

[6] EVAP-2, described by M. P. Guthrie, in Oak Ridge National Laboratory Report No. ORNL-4379, 1969 (unpublished).

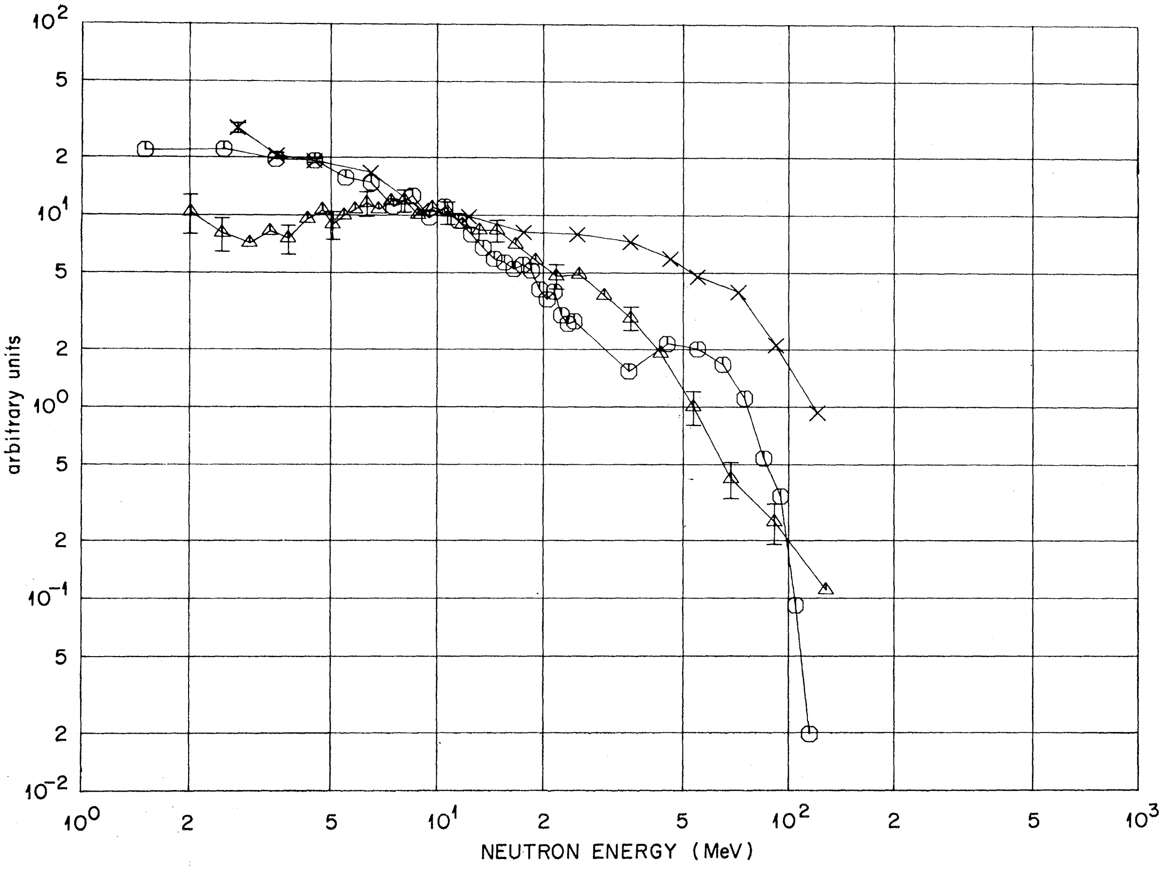

FIG. 1. Energy spectra of emitted neutrons from π^--meson capture in carbon. Open circles: theoretical spectrum; triangles: experimental spectrum of H. L. Anderson *et al.*, Phys. Rev. **133**, B392 (1964); ×'s: experimental spectrum of P. M. Hattersley, H. Muirhead, and J. N. Woulds, Nucl. Phys. **67**, 309 (1965). The lines connecting the points are drawn to guide the eye. All the data are arbitrarily normalized to Anderson's results at 9.5 MeV.

ner[7] who incorporated the work of Dostrovsky, Fraenkel, and Friedlander.[8]

Given that a pion is absorbed by a nucleon pair, the probability that the absorption will take place with the various charge combinations of the nucleons is assumed to be given by the number of pairs of these combinations in the nucleus. Only those pairs that are consistent with charge conservation are permitted to act as absorbers. For example, a π^+ can be absorbed by an n-p or n-n pair, and the probability that the absorption will occur with one or the other is given by the ratio of either the number of n-p or n-n pairs to the total number of n-p and n-n pairs in the nucleus. The effects of a change in this ratio are discussed in the comparisons with experiment. In the calculation of the kinematics for an absorption, the nucleon pair (acting as a single entity) is assumed to have a momentum equal to the momentum of the center of mass of a two-nucleon system where the momentum of each nucleon has been selected from its momentum distribution inside the nucleus. Pion absorption is the only reaction with "clusters" that is taken into account in the calculation.

The details of the model of the nucleus and the free-particle cross sections that were used are described elsewhere.[2] It is sufficient to say that the diffuse surface of the nucleus, the exclusion principle, the motion of the bound nucleons, and a potential for pions (equal to that for the nucleons) are taken into account.

The validity of the general approach is dependent upon the fact that at high energies the deBroglie wavelengths of the incident and cascading particles are small compared to the internucleon distances. A large fraction of the comparisons in this paper are at energies that strain this condition (pion energies less than about 100 MeV), a fact that must be kept in mind in judging the comparisons.

Incident pions with 1-MeV energy are used in the calculation of the pion-absorption reactions. And although at this energy the deBroglie wavelength is about the same size as the internucleon distance, ab-

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[7] L. Dresner, in Oak Ridge National Laboratory Report No. ORNL-CF-61-12-30, 1961 (unpublished).

[8] I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. **116**, 683 (1959).

TABLE I. Experimental and theoretical neutron multiplicities and neutron energies associated with π^- absorption in several targets.

Target	Average value of total number of emitted neutrons per pion absorption		Average value of the number of "direct" neutrons emitted per pion absorption		Total kinetic energy carried away by all the neutrons (MeV)		Kinetic energy carried away by the evaporation neutrons (MeV)		Average excitation energy prior to evaporation (MeV)	
	Expt	Theoret[a]	Expt	Theoret[a]	Expt	Theoret[a]	Expt	Theoret[a]	Expt	Theoret[a]
C	2.92±0.36[b]	2.7	1.83±0.23[b]	1.2	110±11[b]	63	5.84±0.7[b]	9		63
	2.9±0.3[c]		2.4[d]		68[d]					
O	2.77±0.34[b]	3.0	1.74±0.24[b]	1.2	105±12[b]	61	5.62±0.8[b]	15		68
Al	3.07±0.42[b]	2.9	1.60±0.23[b]	1.1	100±12[b]	51	7.70±1.0[b]	11		78
		3.0		1.2		53		12		77
	3.6±0.3[c]	2.9	2.2[d]	1.2	74[d]	51		11		74
Cu	5.03±0.62[b]	5.7	2.19±0.3[b]	0.95	123±12[b]	51	12.4±1.5[b]	20	89±5[e]	91
	7.4±0.4[e]		2.1[e]				30±3[e]			
$_{48}^{112}$Cd	4.4±0.4[c]	8.1	1.8[d]	0.78	80[d]	51		25	95±5[e]	100
$_{50}^{119}$Sn	8.5±0.5[e]		2.1[e]				33±3[e]			
Pb	4.98±0.58[b]	11	1.81±0.23[b]	0.67	100±12[b]	50	13.2±1.5[b]	26	88±6[e]	107
		11		0.68		50		26		107
	5.0±0.5[c]	11	1.9[d]	1.0	69[d]	49		25		104
	9.4±0.5[e]		2.6[e]				41±4[e]			
U	6.9±0.7[c]	13	2.2[d]	0.64	100[d]	52		30		109

[a] Theoretical values listed for Al and Pb are for nuclear models involving different ratios of *n-p* to *p-p* absorption and different cutoff energies. The details are described in the text.

[b] P. M. Hattersley, H. Muirhead, and J. N. Woulds, Nucl. Phys. **67**, 309 (1965).

[c] These numbers were taken from P. H. Fowler and V. M. Moyes [Proc. Phys. Soc. (London) **92**, 377 (1967)], who extrapolated the spectra of Anderson *et al.* (Ref. d) to zero energy.

[d] H. L. Anderson, E. P. Hincks, C. S. Johnson, C. Rey, and A. M. Segar, Phys. Rev. **133**, B392 (1964).

[e] G. Campos Venuti, G. Fronterotta, and G. Matthiae, Nuovo Cimento **34**, 1446 (1964).

sorption reactions are the only reactions in the calculation that have a significant probability of occurring. If the absorption is by a nucleon-nucleon cluster, then the kinetic energy of each nucleon emanating from the point of absorption is approximately equal to half the pion rest mass, and hence for these nucleons there is a better justification for the cascade approach. If the absorption is by the nucleus as a whole, then a highly excited compound nucleus is formed with subsequent deexcitation by particle evaporation.

COMPARISON WITH EXPERIMENT

Spectra and Yields from Slow-π^- Absorptions

Spectral comparisons of emitted neutrons following π^--meson capture by light- to heavy-weight elements are illustrated in Figs. 1–4. The data in these figures have been arbitrarily normalized, as described in each figure caption, to facilitate comparisons of the shapes of the curves. Other calculated and experimental quantities are compared in Table I. The total number of neutrons emitted per absorption is compared in columns 2 and 3 and the number of direct or fast neutrons is compared in columns 4 and 5.

The agreement in the shapes of the spectra and the neutron multiplicities is quite good for the light elements, but discrepancies appear with the element copper and persist through the heavy elements. The theoretical spectra appear too soft, which is confirmed by the data in Table I. The theoretical fast-particle multiplicities for the medium- to heavy-weight elements are underestimated by factors of 2–3, while the predicted total number of neutrons is considerably larger than that obtained experimentally.

The nuclear model was altered in two ways in an attempt to alleviate these discrepancies. First, the ratio of *n-p* to *p-p* absorptions was changed to 5, to correspond to early experimental results,[9] although the neutron and proton cascade cutoff energies remained the same. Second, the *n-p* to *p-p* absorption ratio remained the same, but the neutron cutoff energy was set to zero while the full Coulomb barrier was used as the cutoff energy for the protons. The results for the unaltered model and the first and the second changes are given, respectively, in the theoretical columns of Table I for aluminum and lead targets, and the neutron spectra in all directions for these cases are illustrated in absolute units in Figs. 5 and 6. As can be seen, the results are quite insensitive to the ratio of *n-p* to *p-p* absorptions and to relatively small changes in the height of the barrier. The units on these figures, that is, mb/sr MeV, are proportional to the yield per absorption and arise from the fact that a 1-MeV incident pion beam is not distinguished in the calculation from one at higher energies where these units are appropriate.

Part of the discrepancy for the heavier-weight ele-

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[9] S. Ozaki *et al.*, Phys. Rev. Letters **4**, 533 (1960).

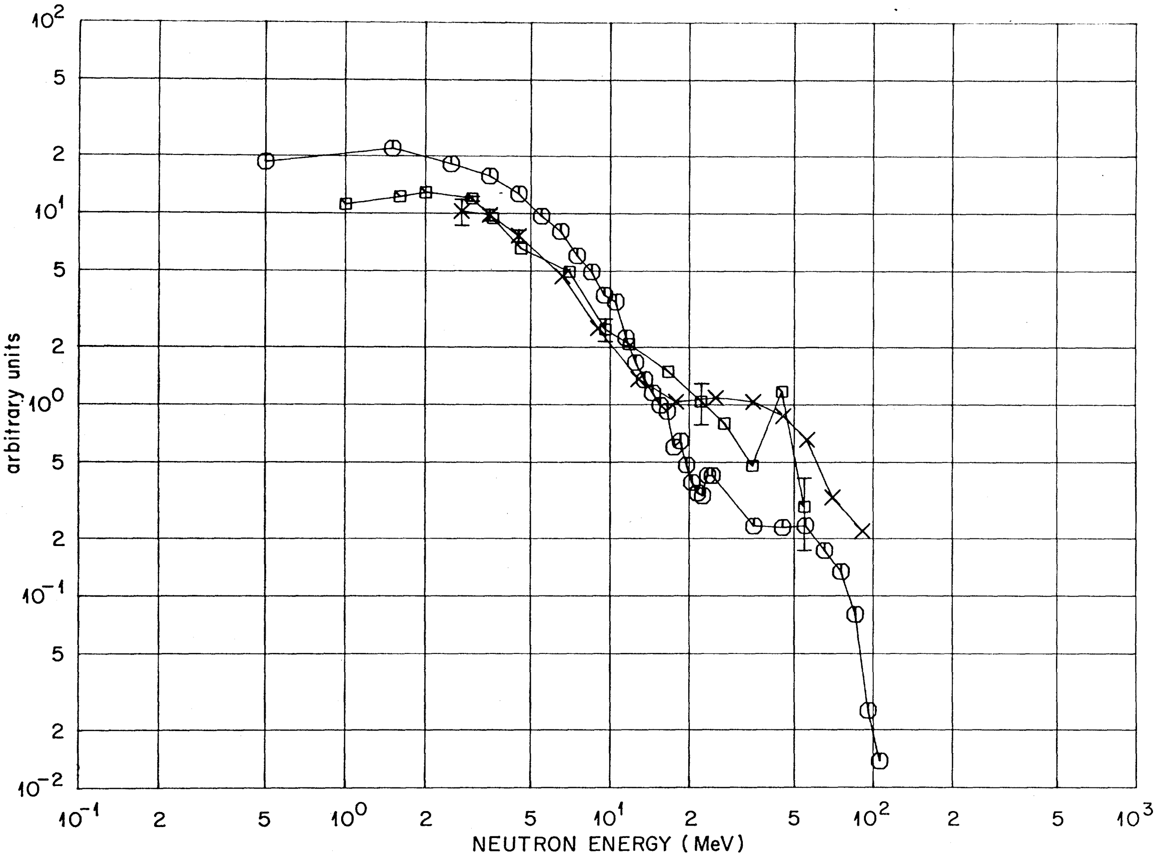

FIG. 2. Energy spectra of emitted neutrons from π^--meson capture in copper. Open squares: experimental spectrum of G. Campos Venuti, G. Fronterotta, and G. Matthiae, Nuovo Cimento **34**, 1446 (1964). The other symbols are defined in Fig. 1. The experimental data are normalized to give the same area from 3 to 55 MeV as the theoretical curve.

ments could be accounted for if absorptions of the pions took place from the higher-energy states of the π-mesic atom. Absorptions from these states imply that the process is taking place near the nuclear surface, in which case the absorbing nucleons can more readily escape without collision and thereby maintain most of their initial energy. A plausibility argument for the existence of such events follows: K-series x rays from π-mesic atoms for elements ranging from lithium to sodium have been measured, and the yield per pion absorbed is a decreasing function of the mass and becomes vanishingly small for sodium.[10] L-series x rays from π-mesic atoms for elements from boron to arsenic have been measured, and the yield of these x rays per absorbed pion has a peak near aluminum and then decreases to very small values as the mass increases.[11] Since the K- and L-series x rays come from transitions to the $n=1$ and $n=2$ states, respectively ($n\equiv$the principal quantum number), the absence of K x rays and the presence of L x rays imply pion absorption from

TABLE II. Fraction of initial events for 1-MeV incident π^- and fractional $2p$ overlap for the three regions of the nucleus.

	Core or inner region		Intermediate region		Outermost region	
Nucleus	Fraction of initial events	Overlap	Fraction of initial events	Overlap	Fraction of initial events	Overlap
Oxygen	0.13	0.02	0.66	0.56	0.21	0.42
Aluminum	0.19	0.04	0.64	0.62	0.17	0.34
Lead	0.44	0.49	0.48	0.45	0.08	0.06

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[10] M. Camac *et al.*, Phys. Rev. 99, 897 (1955); M. Stearns and M. B. Stearns, *ibid.* **107**, 1709 (1957).

[11] M. Camac, M. L. Halbert, and J. B. Platt, Phys. Rev. **99**, 905 (1955); M. B. Stearns, M. Stearns, and L. Leipuner, *ibid.* **108**, 445 (1957).

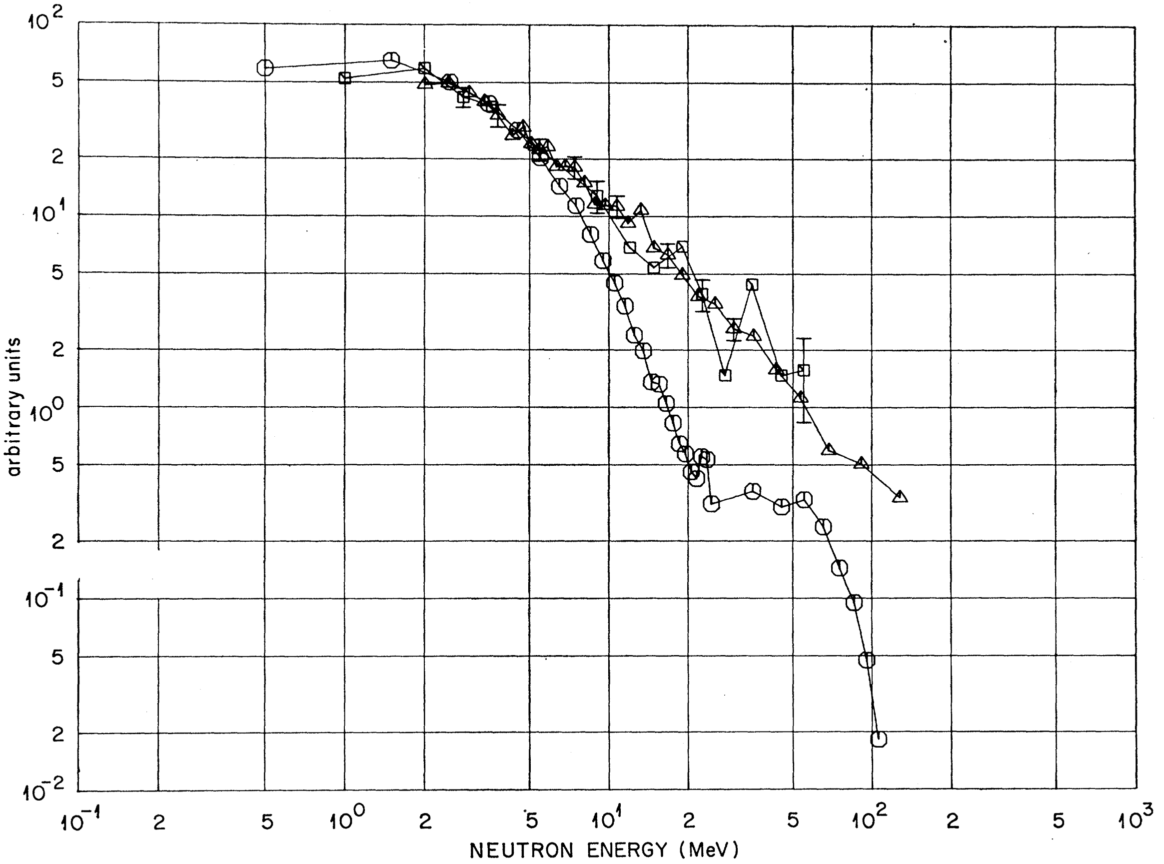

FIG. 3. Energy spectra of emitted neutrons from π^--meson capture in cadmium. The symbols are defined in Fig. 1. The experimental data have been arbitrarily normalized to the theoretical spectrum at 5 MeV. The data of Campos Venuti *et al.* represent measurements on tin.

the $n=2$ state, which would preclude transitions to the $n=1$ state. Similarly, the absence of both K and L x rays implies absorptions from states with $n>2$. The *yield* of x rays per pion absorbed has not been measured for heavier-weight elements. However, π-mesic x-ray measurements on these elements have been made in order to determine various linewidths and level shifts.[12] Only $4f$-$3d$ transitions for elements from $Z=49$ to 59 are reported, and for $Z=73$ to 82 only $5g$-$4f$ results are given; that is, there are no data for K- and L-series x rays for the medium-weight elements, and no data for K-, L-, and M-series x rays for the heavier-weight elements. It seems reasonable to assume that the same trend that exists in the lighter-weight elements continues for the heavier-weight elements, and hence one concludes that pion absorption takes place from the higher and higher-energy states of the π-mesic atom as the nuclear mass increases.

It is not a simple matter to change the distribution of absorptions within the calculation to correspond to the capture by higher-energy states because the calculation was not exclusively designed to handle absorption

TABLE III. Number of emitted particles of charge 1 per absorption of a π^- meson by oxygen and silver bromine.

Target	Theoretical Protons	Deuterons	Tritons	Total	Experimental[a] (total)
O	1.32	0.25	0.07	1.64	0.95±0.05
AgBr	0.66[b]	0.08[b]	0.02[b]	0.76[b]	0.52±0.02

[a] P. H. Fowler and V. M. Mayes, Proc. Phys. Soc. (London) **92**, 377 (1967).

[b] These values are for a $_{48}{}^{112}$Cd target.

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[12] H. Schmitt *et al.*, Phys. Letters **27B**, 530 (1968).

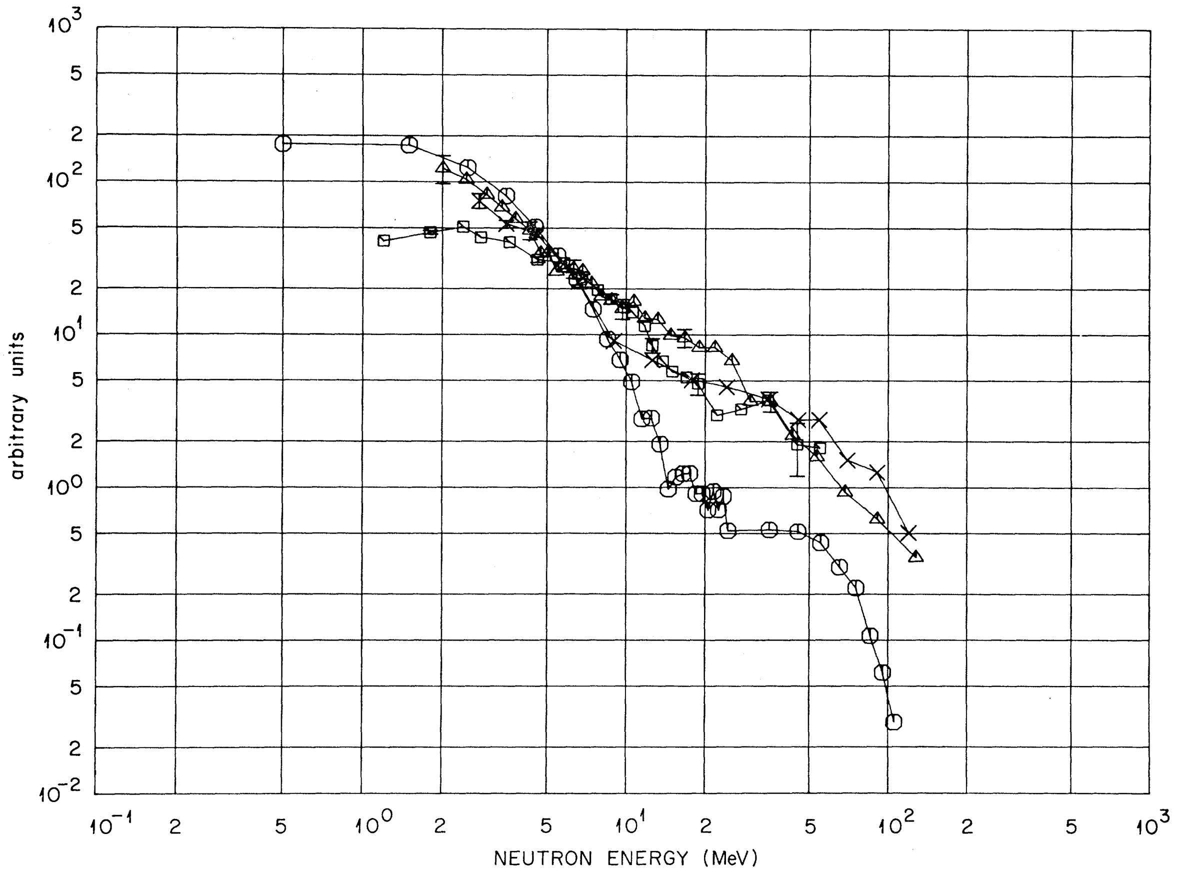

FIG. 4. Energy spectra of emitted neutrons from π^--meson capture in lead. The symbols are defined in Figs. 1 and 2. The experimental spectra have been arbitrarily normalized to the theoretical spectra at 6 MeV.

reactions. In order to ascertain a correspondence between the energy states of the π-mesic atom from which absorptions take place and the distribution of absorptions that is generated by the calculation, a rather crude approach is used. It is assumed that the wave function of the π-mesic atom can be represented by the solution to the Schrödinger equation for a charged meson in a Coulomb field. For purposes of comparison, the $2p$ Coulomb wave function was selected, and its radial part is given by[13]

$$\psi_{2p}(r)=(Z/2a_0)^{3/2}(Zr/a_0\sqrt{3})\exp[-Zr/2a_0],$$

where Z is the charge of the central field and a_0 is the radius of the first Bohr orbit for the meson. a_0 is given by $\hbar^2/\mu e^2$ where μ is the reduced mass of the system, and for a heavy nucleus, $a_0=194$ F. The model of the nucleus used in the calculation consists of three regions: a dense central core surrounded by two spherical annuli, each with diminishing density.[2] The distribution of absorption events for a π-mesic atom in the $2p$ state is estimated to be given by the overlap of the square of the $2p$ wave function with the density distribution. In fractional form the overlap in region i for a π meson in the $2p$ state is given by

$$f_i=\rho_i\int_{\text{region } i}\psi_{2p}^2r^2dr\Big/\sum_{i=1}\rho_i\int_i\psi_{2p}^2r^2dr.$$

In Table II, the fractional overlap for each region is compared to the fraction of initial events as generated by the calculation for a few elements. (The initial events are those reactions that occur when the incident 1-MeV pion undergoes its first collision within the nucleus. About 70% of the time the initial event is an absorption by a two-nucleon cluster.) Indications from experiments are that the absorptions from the $n=2$ and higher states dominate[10,11] the absorption process for oxygen and aluminum, while for lead the absorption very likely proceeds from even higher-energy states, as discussed above. On the other hand, the distribution of initial events generated by the calculation somewhat underestimates the absorptions that would take place in the outer region of the nucleus from the $2p$ state for oxygen and aluminum, and, since there is a good match

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[13] Leonard I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Co., New York, 1955), 2nd ed., p. 85.

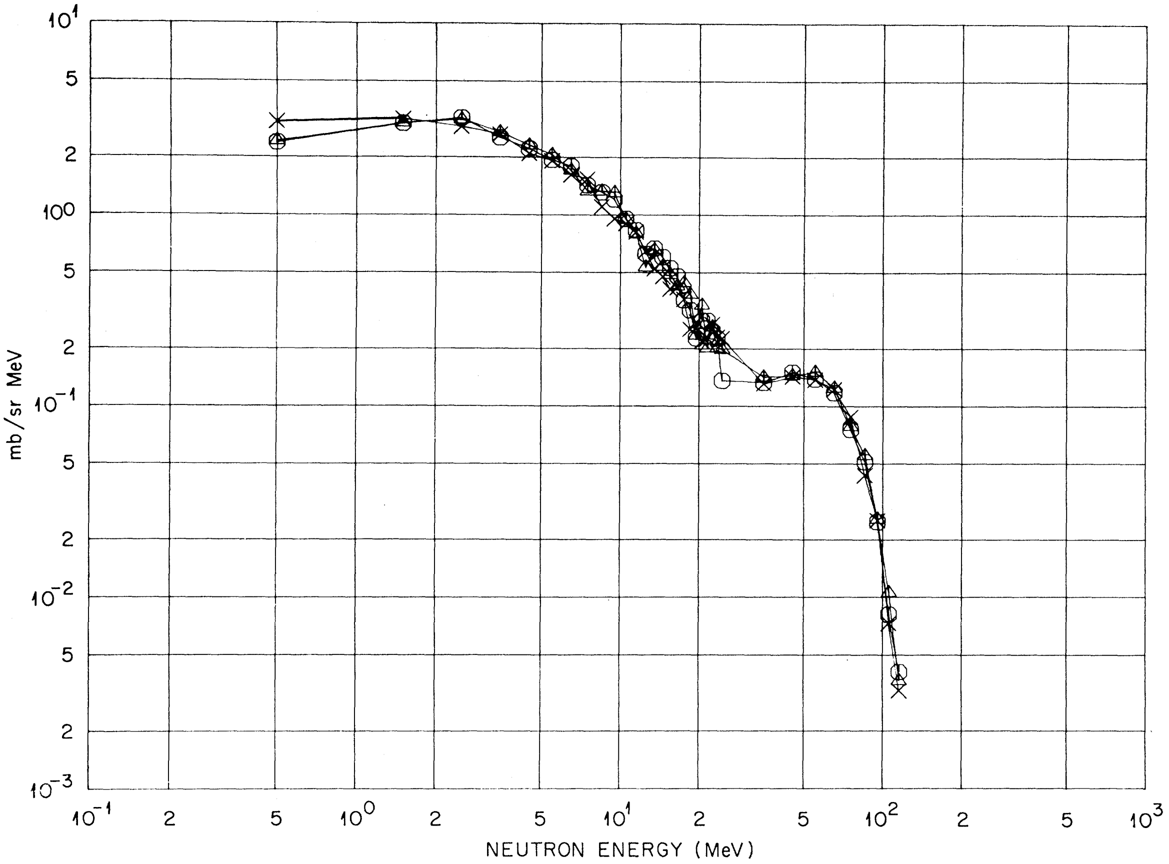

FIG. 5. Theoretical energy spectra of emitted neutrons from 1-MeV π^- mesons on aluminum. Open circles: unaltered model; ×'s: cutoff energy altered as described in text; triangles: ratio of n-p to p-p absorptions altered as described in text.

for the $2p$ absorption distribution in lead, the calculation likely underestimates the actual absorption rate that takes place near the nuclear surface for the heavy elements. This could readily account for the discrepancy in the hardness of the neutron spectra and in the number of "direct" neutrons emitted from the medium- to heavy-weight elements.

Comparisons of the calculated and experimental yields of particles with one unit of charge for slow-π^- absorption are shown in Table III. All experimental data referred to as proton data in the paper by Fowler and Mayes[14] actually represent data from protons, deuterons, and tritons.[15] The theoretical values for the total yields are about 75% too high for a light element, a discrepancy that is difficult to explain because it is not consistent with the comparisons for the neutron yields.

The shape of the theoretical deuteron and proton spectrum from oxygen compares favorably with the experimental spectrum for protons, deuterons, and tritons, as shown in Fig. 7. Tritons were not included in the calculated spectrum because of their small contribution. It should be recalled that in the calculation the only way deuterons and tritons are permitted to emerge is through the evaporation process. The calculated proton spectrum from ^{112}Cd is compared to the experimental spectrum from heavy emulsion nuclei in Fig. 8. At high energies the calculated spectrum exhibits the same characteristics, but not as pronounced, as the high-energy neutron spectra from heavy elements. Again, deuterons and tritons were not included in the theoretical spectrum because of their small contribution.

The low-energy part of the calculated spectra drops toward zero for energies below the Coulomb barrier, while the experimental data do not exhibit such severe changes. This discrepancy, which has been noted elsewhere,[16] is probably due to the approximate manner in which the barrier penetration factor is handled in the

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[14] P. H. Fowler and V. M. Mayes, Proc. Phys. Soc. (London) **92**, 377 (1967).

[15] P. H. Fowler (private communication).

[16] F. E. Bertrand *et al.*, in Oak Ridge National Laboratory Report No. ORNL-4274, 1968 (unpublished).

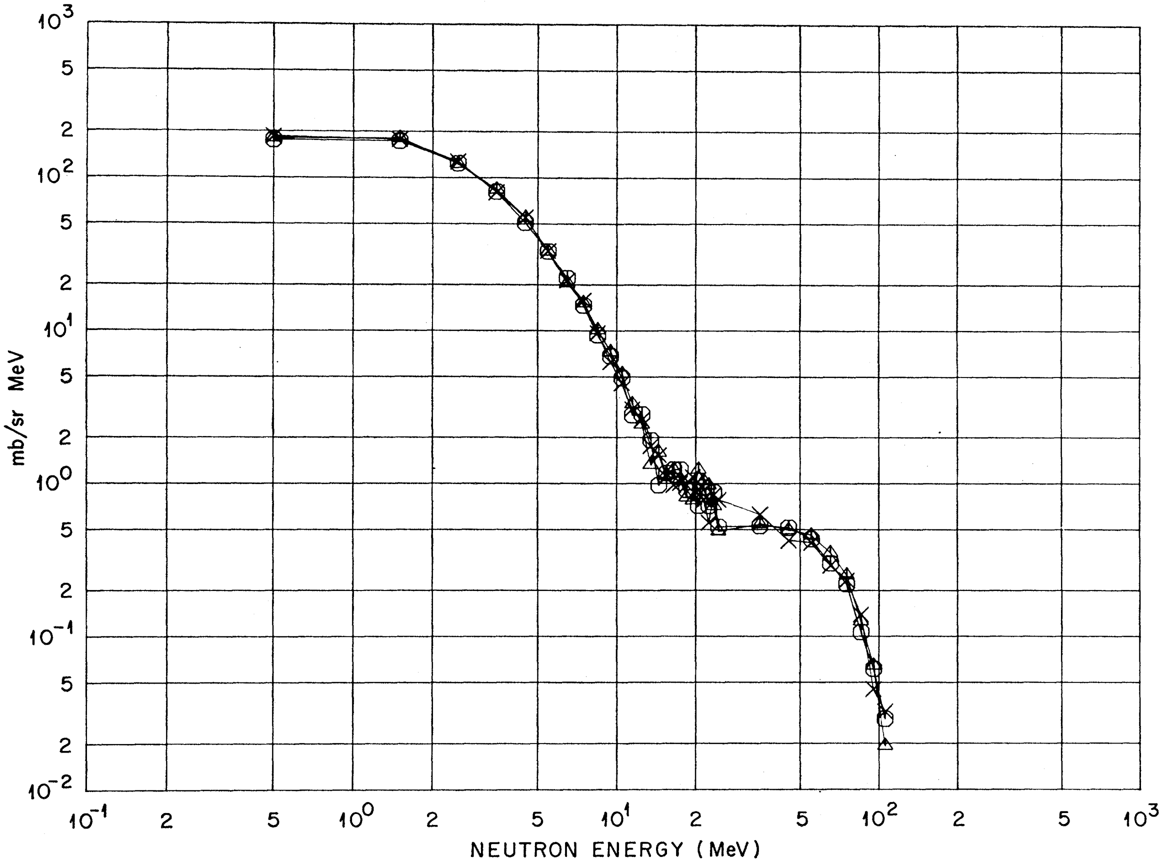

FIG. 6. Theoretical energy spectra of emitted neutrons from 1-MeV π^- mesons on lead. The same notation applies as in Fig. 5.

evaporation calculation; that is, the probability is zero that a charged particle will be emitted with an energy below some fraction of the Coulomb barrier.[8]

Correlated Emissions Following Slow-π^- Absorptions

If the pion-absorption process is, in fact, dominated by that from two-nucleon clusters, then information on the correlation of two nucleons within the nucleus can be obtained by detailed examination of the data from pion-absorption experiments. If a pion is absorbed by two nucleons, the nucleons will move out from the point of absorption with relatively high energy and with momenta that are roughly equal and oppositely directed. If these particles escape without subsequent collisions, their angular distribution with respect to the angle between them will exhibit a peak at 180°. The distribution would be a spike at 180° if the absorbing clusters were stationary. Experiments aimed at detecting correlations have been carried out, and peaks in the distributions at 180° have indeed been observed.[9,17,18]

In two of these experiments,[9,18] attempts were made to deduce the ratio of absorptions by *n-p* pairs as opposed to *p-p* pairs by counting particles emitted in coincidence with each other. It was pointed out in another paper[19] that the ratios obtained in this manner could be in considerable error unless measures were taken to ensure that the particles had not collided prior to their escape. If the particles that absorb the pion undergo collisions, then one cannot tell whether the escaping particles have been knocked out of the nucleus or whether they were the original absorbing pair. Furthermore, collisions tend to extend the distribution to smaller angles, as is illustrated in Figs. 9 and 10 for slow-π^- capture by oxygen. The measured distributions of Nordberg, Kinsey, and Burman,[18] arbitrarily normalized, are illustrated for comparison.

To make the theoretical results in these figures compatible with the measured data, only those nucleons with energies above 20 MeV were counted. This corresponds to the detector cutoffs for these particles in the experiment. The histograms that pertain to the particles without regard to their collision history were constructed as follows: Since there may be several

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[17] J. Laberrigul, M. P. Balandine, and S. J. Otvinovski, J. Phys. Radium **21**, 54 (1960).

[18] M. E. Nordberg, Jr., K. F. Kinsey, and R. L. Burman, Phys. Rev. **165**, 1096 (1968).

[19] H. W. Bertini, Phys. Letters **30B**, 300 (1969).

TABLE IV. Yields from the π^- interaction with $_{53}^{127}$I.

Nucleons emitted (n=nucleon, p=proton)	Nuclide	Yield percent Expt[a]	Theoret[b] 1-MeV π^-	40-MeV π^-
0	^{127}Te	1.13[c]	0.041	0.62
$4n$	104-day ^{123m}Te	7.3	1.4	1.1
$6n$	150-day ^{121m}Te	10.9	3.1	1.9
$9n$	6-day ^{118}Te	3.2	4.6	1.9
$10n$	2.2-h ^{117}Te	0.77	5.3	1.6
$11n$	^{116}Te		8.8	1.4
$12n$	^{115}Te		0.95	0.5
$13n$	^{114}Te		0.0	0.07
pn	2.7-yr ^{125}Sb	~0.1	0.37	0.52
$p2n$	60-day ^{124}Sb	0.55	0.41	0.30
$p4n$	2.8-day ^{122}Sb	2.8	0.78	0.61
$p6n$	^{120}Sb	4.5[c]	2.4	1.2
$p7n$	39-h ^{119}Sb	4.5	4.4	1.7
$p8n$	5.1-h ^{118}Sb	2.2	4.1	1.7
$p10n$	60-min ^{116}Sb	0.24	13.8	3.0
$p11n$	^{115}Sb		7.9	3.7
$p12n$	^{114}Sb		0.78	5.3
$p13n$	^{113}Sb		0.0	3.0
$p14n$	^{112}Sb		0.0	0.9
$2p4n$	27-h ^{121}Sn	0.09	0.21	0.25
$2p8n$	14-day ^{117m}Sn	1.9	2.6	0.59
$3p7n$ $3p8n$	^{116}In ^{117}In	0.5	0.25	0.09
$3p9n$	4.5-h ^{115m}In	0.10	0.21	0.18
$3p10n$	49-day ^{114m}In	0.55	0.17	0.50
$3p13n$	2.8-day ^{111}In	0.03	0.0	0.63
$4p6n$	2.9-h ^{117}Cd	0.0048	0.0	0.0
$4p8n$	^{115}Cd	0.043[c]	0.0	0.0
$4p16n$	6.7-h ^{107}Cd	<0.0007	0.0	0.0

[a] Lester Winsberg, Phys. Rev. **95**, 198 (1954).

[b] The theoretical results are the sum of the cross sections for transitions to all isomeric states, where they exist.

[c] This represents the sum of the cross sections for two measured isomeric states.

nucleons that emerge after an absorption event, all pair combinations from each absorption contribute to the distribution. However, the contribution from each pair combination is the inverse of the number of such pairs in that event. This corresponds closely to the experimentally measured distributions where only two of all the nucleons that emerge simultaneously from an absorption event are counted; that is, only one pair combination out of several that may occur is counted for each absorption. This, of course, is because some of the nucleons can emerge at angles and in planes that are different from those in which the coincidence detectors are placed.

TABLE V. Charge-exchange cross sections for 180-MeV incident π^+.

^{13}C$(\pi^+, \pi^0)^{13}$N Expt[a]	Theoret	^{18}O$(\pi^+, \pi^0)^{18}$F Expt[a]	Theoret
3.8 mb	2.3 mb	3.1 mb	3.4 mb

[a] B. W. Allardice *et al.* (private communication). Quoted by C. Zupancic, in *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, 1967), p. 171.

The shapes of the calculated distributions for which all escaping particles contribute correspond more closely to those of the measured ones. The distributions from the nucleons that have escaped free of collision show no "background" and do not extend into the forward hemisphere.

Radiochemical Cross Sections

Winsberg[20] has measured the yield of several radioactive nuclei following the absorption of slow-π^- mesons

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[20] Lester Winsberg, Phys. Rev. **95**, 198 (1954).

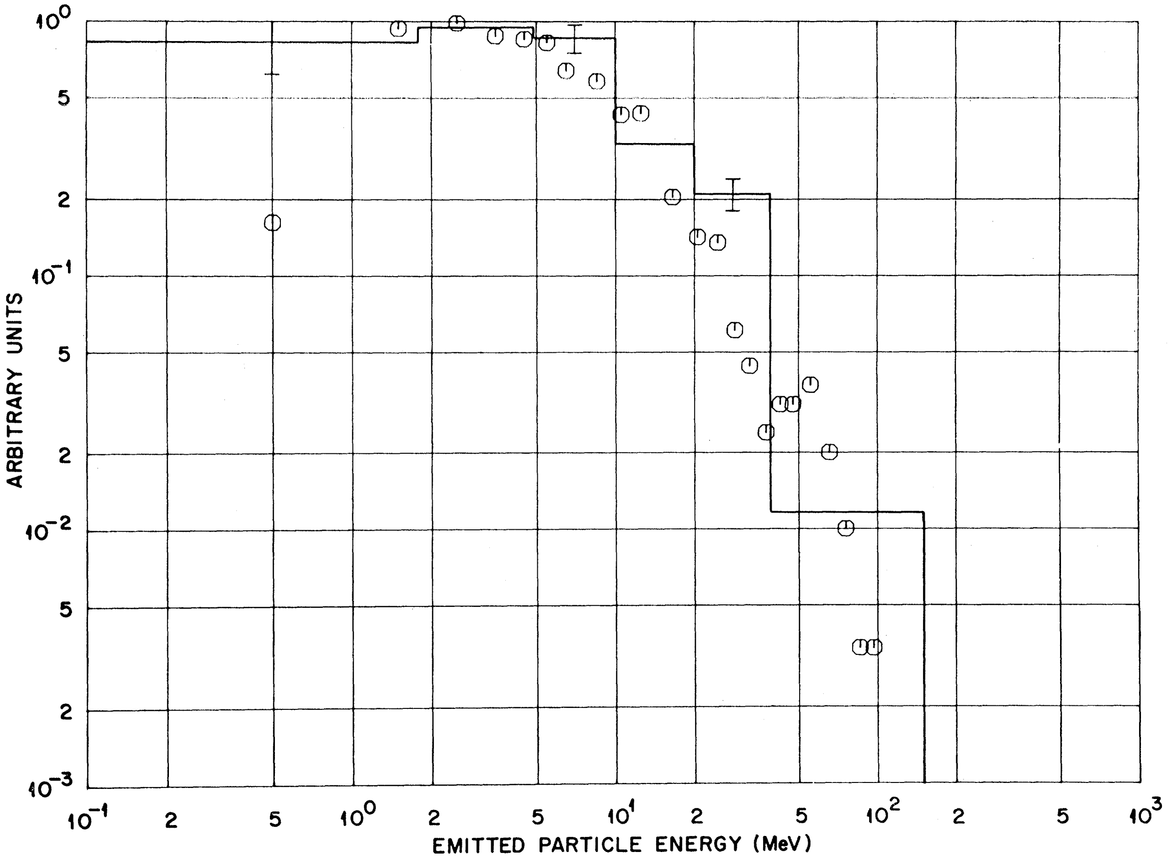

FIG. 7. Energy spectra of emitted protons, deuterons, and tritons from π^--meson capture in oxygen. Open circles: theoretical spectrum for protons and deuterons; histogram: experimental spectrum of Fowler and Mayes (Ref. 14). The published experimental values, that is, the number of protons in each energy interval, were divided by the interval and the resulting spectrum was arbitrarily normalized to the calculated data near the peak.

by iodine. His results are compared to those from the calculation in Table IV. The magnitudes of the experimental cross sections are predicted with only fair accuracy for 1-MeV negative pions. The peaks in the theoretical cross sections for the isotopes of tellurium and antimony are at lower mass values than the measured peaks. Since the experimental pion beam was slowed down from about 70 MeV inside the iodine absorber, there is a possibility that the pions may have been absorbed before they lost all of their energy. To see if absorptions at higher energies would improve the agreement between the cross-section distributions, a calculation was done for 40-MeV incident negative pions, and the results are given in the last column of Table IV. The peak in the distribution is shifted to lower mass values for tellurium but to higher mass values for antimony. Hence, this does not appear to be the cause of the discrepancy, which may again be due to the fact that the theoretical model does not allow a sufficient number of absorptions to take place near the

TABLE VI. Double-charge-exchange cross sections for 80-MeV π^+ and 140-MeV π^- on various nuclei.

	80-MeV π^+		140-MeV π^-	
Target	Expt[a]	Theoret	Expt[a]	Theoret
Be	0.1±0.03	1.5±0.3	0.14±0.04	2.7±0.4
C			0.15±0.08	5.4±0.7
Al	1±0.3	3.9±0.8		
Pb	4±1	16±2	0.8±0.4	12±2

[a] Yu. A. Batusov *et al.*, Yadern. Fiz. **3**, 309 (1966) [English transl.: Soviet J. Nucl. Phys. **3**, 223 (1966)].

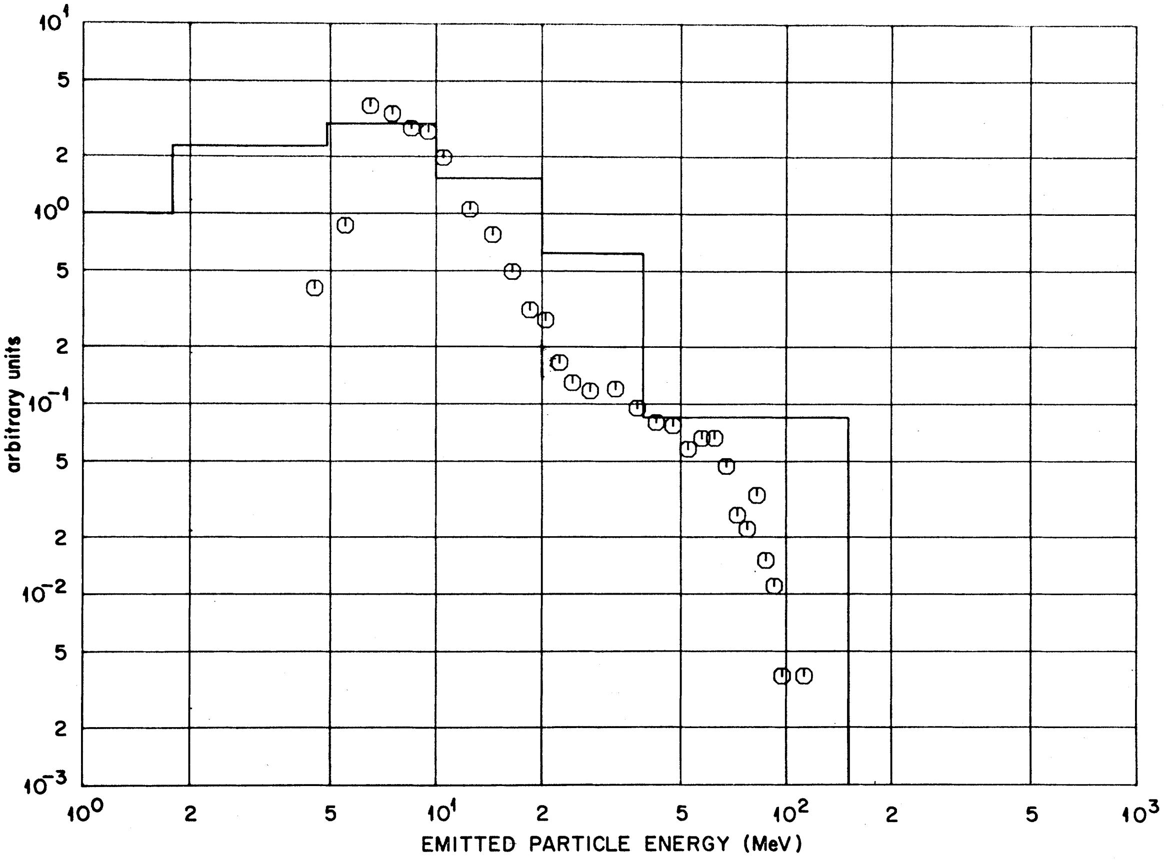

FIG. 8. Energy spectrum of emitted protons, deuterons, and tritons from π^--meson capture in heavy emulsion nuclei. Open circles: theoretical spectrum for protons using a ^{112}Cd target; histogram: same as in Fig. 7.

surface for heavy elements. Under the present conditions, the initial absorbing pair of nucleons transfers a larger fraction of their energy to the nucleus. This will shift the final residual nuclei to lower mass values since then a greater number of nucleons will be emitted in the cascade and evaporation processes.

Charge-Exchange and Double-Charge-Exchange Reactions

There are very little experimental data on the charge-exchange reactions. The data illustrated and compared with the predictions in Table V are all that a literature

TABLE VII. Double-charge-exchange cross sections for various reactions of pions on nuclei.

	Incident particle	Incident particle energy (MeV)	Target	Double-charge-exchange cross sections (mb)
Expt[a]	π^+	30–80	Emulsion nuclei with $\langle Z\rangle=21$	0.4±0.1
Theoret	π^+	55	$_{21}{}^{45}$Sc	2.7±0.5
Expt[b]	π^-	40–87	Probably emulsion nuclei with $\langle Z\rangle=21$[c]	0.09±0.03
Theoret	π^-	55	$_{21}{}^{45}$Sc	2.1±0.4
		65	$_{21}{}^{45}$Sc	2.9±0.6

[a] Yu, A. Batusov *et al.*, Zh. Eksperim. i Teor. Fiz. **46,** 817 (1964) [English transl.: JETP **19,** 557 (1964)].

[b] Yu. A. Batusov *et al.*, Yadern. Fiz. **1,** 383 (1965) [English transl.: Soviet J. Nucl. Phys. **1,** 271 (1965)].

[c] Not specified, but a description of the experiment is very similar to that given in reference a by the same authors.

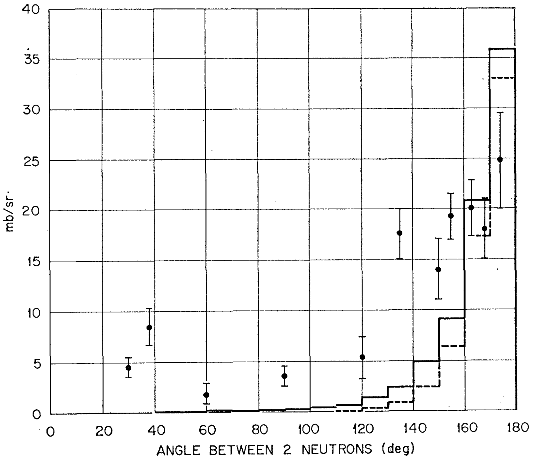

FIG. 9. Angular distribution of *n-n* pairs as a function of the angle between them for π^- absorption by oxygen. Solid-line histogram: theoretical values for all escaping neutron pairs; dashed-line histogram: theoretical values for two neutrons escaping without collision; dots: experimental data of Nordberg *et al.* (Ref. 18) with arbitrary normalization.

search revealed. The comparisons are quite reasonable, although experience has shown that estimates from the model are not accurate when the reactions represent a small fraction of the total interaction cross section.[21]

Comparisons with experimental data for a few double-charge-exchange cross sections are shown in Table VI. The predictions are higher than the measured values. It is difficult to understand why the discrepancy is so large, unless one is below the threshold of validity of the model. It would be very helpful to have more data in this energy region.

A breakdown of charge symmetry in the pion double-charge-exchange reactions is indicated by the measurements of Batusov *et al.*[22] Charge symmetry is built into the theoretical model, and the experimental and theoretical results are compared in Table VII. This appears to be another fruitful area for experimental research.

EMISSION OF π^0'S

A phenomenon is predicted by the theory that has yet to be verified. It is the emission of neutral pions following stopped-π^- absorption. The prediction is that a slow π^0 will be emitted 0.5–2% of the time in π^--absorption reactions. The production mode of the π^0's in the model may be somewhat unrealistic in that the π^0's emerge following a charge-exchange reaction inside the nucleus. But charge-exchange events may occur in proximity to the nuclear surface, while the π-mesic atoms are in low-energy states. The π^0's, being then free of the Coulomb attraction, may emerge from the vicinity of the nucleus and decay. Additional impetus is given to the π^0 for its escape, even if it is within the range of the nuclear forces, by its gain in kinetic energy of about 4.5 MeV due to the mass difference between the π^- and π^0 mesons.

SUMMARY AND CONCLUSIONS

The method of intranuclear cascades has been used to calculate slow pion reactions with nuclei.

The spectrum and multiplicity of neutrons following stopped-π^- absorptions by light elements are predicted reasonably well by this method. However, the theoretical neutron spectrum is not hard enough and the multiplicity is too high for the heavy elements. This discrepancy may be due to the fact that the distribution of π^- absorptions within the nucleus does not correspond to the distribution one might expect from absorptions out of high-energy states of the π-mesic atom; that is, the model may not allow for a sufficient number of absorptions to take place near the nuclear surface of the heavy elements.

The predicted spectra and multiplicity of the emitted particles with one unit of charge are consistent with the data for the neutrons, but the theory overestimates the multiplicity from light elements. This inconsistency with the data for neutrons is difficult to explain.

The magnitude of the cross section for the production of several radioactive nuclides following slow-π^- absorption by iodine is predicted with only fair accuracy, as might be expected, since these yields represent only a small fraction of the total. The peaks in the cross section for the production of two of the isotopes from

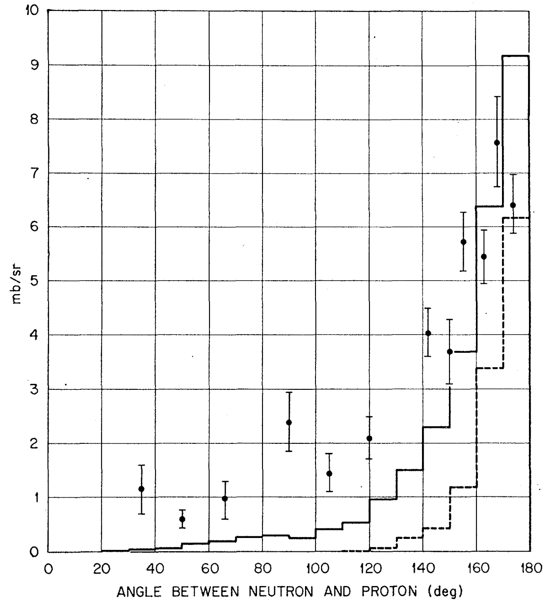

FIG. 10. Angular distribution of *n-p* pairs as a function of the angle between them for π^- absorption by oxygen. Same notation as in Fig. 9 for *n-p* pairs rather than *n-n* pairs.

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[21] H. W. Bertini, Phys. Rev. **171**, 1261 (1968).

[22] Yu. A. Batusov *et al.*, Zh. Eksperim. i Teor. Fiz. **46**, 817 (1964) [English transl.: Soviet Phys.—JETP **19**, 557 (1964)]; Yadern. Fiz. **1**, 383 (1965) [English transl.: Soviet J. Nucl. Phys. **1**, 271 (1965)].

this reaction occur at lower mass values than those measured. This again can be attributed to an insufficient number of absorptions near the nuclear surface.

Indications from the model are that π^0's are produced about 1% of the time from π^--absorption events. There have been no experiments performed to detect this reaction to date.

The theoretical predictions of the charge-exchange cross section for 180-MeV π^+ on carbon and oxygen are in good agreement with experiment although there appear to be data for only two reactions.

The model fails to accurately predict the double-charge-exchange cross sections for relatively low-energy pions. These reaction cross sections are quite small, and hence the predictions are known to be inaccurate, but the model may also be beyond the limits of its validity at these energies.

There are measurements that indicate that there is a breakdown of charge symmetry for the double-charge-exchange reactions. These results are in disagreement with those from the model because the assumption of charge symmetry has been incorporated therein.

There are a great many interesting experiments that can be performed, to both verify some of the existing discrepancies and test the validity of the model in its ability to predict other quantities. To name a few, measurements of the energy spectra of the nucleons and pions following the inelastic interaction of pions with nuclei, their angular distribution, particle multiplicities, etc., would be very useful.

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Cross Sections of Li, Be, and B Emitted in 125-MeV p and 90-MeV α-Particle Interactions with C and N—Application to Nucleosynthesis

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The production cross sections for Li, Be, and B fragments emitted in medium-energy proton and α-particle reactions with the C, N, and O nuclei of a nuclear emulsion have been measured. The corresponding results for the p-induced and α-induced reactions have been compared. Since helium abundance in the universe is about 10%, these reactions must be taken into account when considering the nucleosynthesis of Li, Be, and B.

I. EXPERIMENTAL TECHNIQUE

Two experiments were carried out, both involving Ilford K0-K5 emulsion stacks exposed to a flux of 10^5 particles/cm^2. For the proton beam, we used the Orsay synchrotron at an incident energy of 138.7±0.3 MeV. α particles of 98.5±2.5 MeV [1] came from the Karlsruhe synchrotron.

After chemical processing, the emulsions were scanned to find the interaction stars, and all useful geometric parameters were measured. These data were transmitted to an IBM 360-65 computer, where an appropriate program interpreted the track lengths in terms of the energy, and permitted us to look for possible reactions on carbon, nitrogen, and oxygen targets, taking into account energy and momentum conservation.[2]

II. EXPERIMENTAL RESULTS

A. Study of the Reactions with 125-MeV Incident Protons

Altogether, 256 interaction stars were measured. For some of them the number of acceptable solutions was too large, and the reactions could not be identified; those interactions generally had a great number of emitted prongs and corresponded to an oxygen target. For that reason, we could not obtain meaningful cross sections for reactions with oxygen.

Table I. Cross sections (in mb) and isotopic ratios of different elements emitted in p+C and p+N reactions at 126 MeV.

Fragments	p+C reactions	p+N reactions
^{6}He	<0.4	6.4±3.7
^{6}Li	8.3±2.1	12.7±4.9
^{7}Li	6.5±1.9	9.5±4.2
^{7}Li/^{6}Li	0.78±0.24	0.75±0.40
^{7}Be	4.0±1.4	1.6±1.6
(^{7}Li+^{7}Be)/^{6}Li	1.26±0.35	0.86±0.44
(^{7}Li+^{7}Be)/(^{6}Li+^{6}He)	1.26±0.35	0.58±0.30
^{9}Be	2.5±1.0	4.8±2.8
^{10}Be	<0.4	1.6±1.6

The different cross-section values and isotopic ratios for the emitted fragments are given in Table I. For the carbon target, we did not find any ^{6}He or ^{10}Be fragments. According to our statistics, the corresponding cross sections have an upper limit of 0.4 mb.

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[1] C. Kern, University of Strasbourg Report D.E.A., 1967 (unpublished).

[2] C. Jacquot, thesis, University of Strasbourg, 1965 (unpublished).