Statistical-Model Calculation of the Angular Distributions of (α, n) Reaction Products*

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A calculation of the angular distribution of recoil products resulting from compound nuclear (α, n) reactions has been programmed on the basis of the spin-dependent statistical theory of nuclear reactions. The program, which is based on the Monte Carlo technique, considers the emission of neutrons, protons, α particles, and γ rays, and uses either the Fermi-gas or the superconductor model of the nucleus. The computed angular distributions have been compared with the experimental results for various (α, n) reactions and, in agreement with experiment, exhibit a smooth dropoff from a peak close to 0°. This contrasts with the marked disagreement between the experimental results and previous spin-independent calculations, which predicted a peak at large angles to the beam. The difference is ascribed to the effect of competitive γ-ray emission, which is not adequately considered in spin-independent calculations. The present calculation predicts a faster decrease with angle in the magnitude of the differential cross section than is observed experimentally. This result is independent of the specific parametrization of the calculation and may reflect a small contribution from direct interactions.

I. INTRODUCTION

DETAILED information concerning the nature of intermediate-energy nuclear reactions can be obtained from a study of the recoil properties of the products of these reactions. It is frequently possible to distinguish between compound-nuclear and direct-interaction processes from measurements of the average or differential ranges of recoil nuclei. Angular-distribution data provide information about the details of the evaporation process if the reaction is compound nuclear.

In earlier publications from this laboratory, the results of average and differential-range measurements,[1–3] as well as angular-distribution studies,[4–6] for various (α, xn), (He^3, xn), (α, αn), and (He^3, α) reactions of Cu^{63} and Cu^{65} have been reported. Porile and Saha analyzed the angular distributions on the basis of the spin-independent statistical theory of nuclear reactions, and found that theory and experiment were in satisfactory agreement, except in the case of the Cu^{63}(α, n) reaction.[4] The spin-independent calculation appeared to be in contradiction with the results for this reaction because it predicted a sharp peak in the differential cross section at an angle of approximately 10°, whereas the experimental curve displayed a broad peak near 0°. Since recoil-range measurements were consistent with a compound-nuclear mechanism, the discrepancy was probably caused by approximations made in the calculation.

A similar discrepancy has been previously observed by Morton and Harvey.[7] These workers measured the angular distribution of Po^{210} recoil nuclei from the Pb^{207}(α, n) reaction and compared the results with a spin-independent statistical-model calculation based on the constant-temperature approximation. Their experimental angular distributions displayed broad maxima at angles below 5°, while their calculated curves showed peaks above 10°. They suggested that the discrepancy was evidence that the reaction proceeded by a direct-interaction mechanism. An alternative explanation, offered by Grover and Nagle,[8] suggested that this discrepancy was ascribable to the effect of competitive γ-ray emission above the Pb^{207}(α, $2n$) reaction threshold, an effect neglected by the calculation.

A qualitative understanding of the shortcomings of previous angular-distribution calculations may be obtained from a consideration of the competition between the (α, n) and (α, $2n$) reactions. This is illustrated in Fig. 1, which is a plot of the calculated energy spectrum of the first neutron evaporated from a Ga^{67} compound nucleus under two different circumstances. On the left-hand side of the figure, the maximum excitation energy of the residual nucleus following neutron evaporation, $E_{R\max}$, is smaller than S_{n_2}, the neutron separation energy of Ga^{66}. Consequently, only one particle can be emitted and the cross section of the (α, n) reaction is proportional to the entire integrated neutron energy spectrum. The right-hand side of the figure illustrates the situation when $E_{R\max}$ is larger than S_{n_2}. In a calculation in which competitive γ-ray emission is neglected, a second neutron is emitted whenever energetically possible, and so only the shaded portion of the spectrum contributes to the (α, n) reaction. If a neutron is emitted with an energy

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lying in the unshaded portion of the spectrum, the excitation energy of the residual nucleus is sufficiently large to permit further neutron evaporation. If the bombarding energy is such that the dividing line falls at or above the peak in the neutron energy spectrum, then it is apparent that the neutrons leading to the (α, n) reaction are nearly monoenergetic. The reaction then involves the formation of two particles (neutron and Ga^{66}) of unique energies and the application of two-body kinematics predicts a peak in the angular distribution at the maximum laboratory recoil angle. Since the emitted neutron is not exactly monoenergetic, the expected peak is, in fact, obtained at a somewhat smaller angle. This analysis of the situation is supported by the fact that the large-angle peak in the calculated angular distribution of the $Cu^{63}(\alpha, n)$ reaction product disappears when the incident energy is assumed to be smaller than the $(\alpha, 2n)$ reaction threshold.[4]

The situation can be different if γ-ray emission is allowed to compete with neutron evaporation. Low-energy neutrons can now, presumably, also lead to the (α, n) reaction, even though the emission of a second neutron is energetically possible. If the probability of competitive γ-ray emission is sufficiently large, the energy spectrum of neutrons emitted in the (α, n) reaction will be much broader than before, and the large-angle peak will no longer be obtained.

This qualitative analysis also indicates why a discrepancy between experiment and the spin-independent calculation is not observed for the (α, xn) reactions when $x>1$, even though competitive γ-ray emission is, presumably, of equal importance. The emission of more than one neutron necessarily leads to a spectrum of recoil energies and this violates the conditions required for the occurrence of a peak at large angles.

The present work is an attempt to obtain quantitative confirmation of the above notions by means of a spin-dependent calculation of the angular distribution of the recoil products of (α, n) reactions. In recent years, a number of attempts have been made to include angular momentum effects in statistical-model calculations.[9] Spin-dependent calculations of excitation functions[8–16] and evaporation spectra[17–21] have been reported in the literature. Our calculation follows along the lines of several of these reports and is based on the use of the Monte Carlo technique. The theoretical formalism is discussed in Sec. II, and the method of calculation is described in Sec. III. In Sec. IV the results are presented and discussed, and conclusions are given in Sec. V.

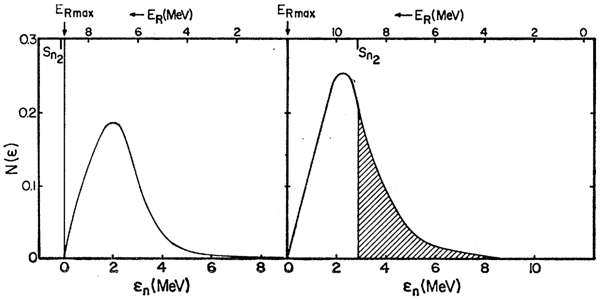

FIG. 1. Calculated neutron energy spectrum illustrating the competition between (α, n) and $(\alpha, 2n)$ reactions on Cu^{63}. On the left, the maximum excitation energy of the residual nucleus $E_{R\max}$ is less than the energy required to separate a neutron from the residual nucleus S_{n2}. On the right, $E_{R\max}$ is greater than S_{n2}. The calculation is that described in the text.

II. THEORY

The cross section for a nuclear reaction can be expressed in the formalism of the statistical model as the product of the cross section for the formation of a compound nucleus and the branching ratio for its decay by a particular deexcitation mode:

$$\sigma(\alpha, i)=\sum_{J_c}\sigma_c(\epsilon_\alpha, J_c)\Gamma_i(\epsilon_i, J_c)/\sum_j\Gamma_j(\epsilon_j, J_c), \quad (1)$$

where $\sigma_c(\epsilon_\alpha, J_c)$ denotes the cross section for the formation of a compound nucleus with angular momentum J_c, when the target interacts with a projectile α, the entrance-channel energy being ϵ_α. The second term is the branching ratio for decay of the compound nucleus by a particular reaction channel, and is expressed in terms of the emission width Γ.

In the channel-spin coupling scheme, the capture cross section is given by[22]

$$\sigma_c(\epsilon_\alpha, J_c)=\pi\lambda\!\!\!^{-2}\sum_{S=|I-s|}^{I+s}\sum_{l=|J_c-S|}^{J_c+S}\frac{2J_c+1}{(2s+1)(2I+1)}T_l(\epsilon). \quad (2)$$

In this equation, $\lambda\!\!\!^{-}$ is the Broglie wavelength of the projectile, I denotes the target spin, s represents the projectile spin, S is the channel spin, and $T_l(\epsilon)$ is the transmission coefficient for a projectile with energy ϵ and orbital angular momentum l.

The transmission coefficients were calculated from the optical model using real and imaginary well radii,

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surface-diffuseness parameters, well depths, and spin-orbit well depths given by Hodgson.[23] The transmission coefficients of the lth partial wave with projectile spin J are given by $T_{l,J}$, and the average transmission coefficient is defined for neutrons and protons by

$$T_l(\epsilon)=(l+1)/(2l+1)T_{(l,J=l+1/2)}(\epsilon)+l/(2l+1)T_{(l,J=l-1/2)}(\epsilon). \quad (3)$$

For α particles, it follows that $T_l(\epsilon)=T_{l,J}(\epsilon)$.

A compound nucleus that has been formed in a state with excitation energy E_c and angular momentum J_c will deexcite by emitting one or more particles or photons until all the excitation energy is dissipated. At each stage in the evaporation cascade, the rate of emission of particles that gives a product nucleus with excitation energy E_R and spin J_R is given by

$$R(J_c,E_c;J_R,s_n,E_R)dE_R=\frac{dE_R\Omega(E_R,J_R)}{h\Omega(E_c,J_c)}\sum_{S=|J_R-s_n|}^{J_R+s_n}\sum_{l=|J_c-S|}^{J_c+S}T_l(\epsilon), \quad (4)$$

where s_n is the spin of the emitted particle. The level densities Ω are obtained from the Fermi-gas model according to the relation of Gilbert and Cameron[24]

$$\Omega(E,J)=\frac{\sqrt{\pi}}{12}\frac{\exp[2(aE)^{1/2}]}{a^{1/4}E^{5/4}}\times\frac{(2J+1)\exp[-(J+\frac{1}{2})^2/2\sigma^2]}{2(2\pi)^{1/2}\sigma^3}, \quad (5)$$

where the spin cutoff parameter σ^2 can be related to the moment of inertia and the temperature t by

$$\sigma^2=\mathcal{I}t/\hbar^2. \quad (6)$$

The temperature is, in turn, obtained from the equation of state

$$t=(E/a)^{1/2}, \quad (7)$$

and the level-density parameter a is given by

$$a=\tfrac{1}{6}\pi^2 g, \quad (8)$$

where g represents the density of single-particle levels at the Fermi level.

The Fermi-gas model predicts that the moment of inertia is that of a rigid body, which for an infinite square well is

$$\mathcal{I}_r=\tfrac{2}{5}MR^2, \quad (9)$$

where M is the mass of the nucleus and R is the nuclear radius. While this equation is expected to be valid above approximately 10 MeV, a smaller value of $\mathcal{I}$ is appropriate at lower energies because of pairing correlations.[25] We have performed the present calculation with several energy independent values of $\mathcal{I}$, as well as with an energy-dependent value, as predicted by the superconductor model of the nucleus. The procedure outlined by Vonach *et al.*[25] was employed to determine the value $\mathcal{I}$ at different excitation energies.

The excitation energy appearing in the level-density formula is determined with reference to a characteristic level that is displaced upward from the actual ground state. This characteristic level is the ground state in the absence of the enhanced stability due to pairing. Consequently, the effective excitation energy employed in Eq. (5) is related to the ground-state excitation energy by[26]

$$\begin{aligned} E'&=E && \text{for odd-odd nuclei}\\ &=E-\delta_{(n\text{ or }p)} && \text{for odd-}A\text{ nuclei}\\ &=E-\delta_n-\delta_p && \text{for even-even nuclei,}\end{aligned} \quad (10)$$

where δ denotes the energy associated with the pairing of a single nucleon. Values of δ from the papers of Gilbert and Cameron were used[24,27] in this calculation.

In the derivation of the dependence of the level density on angular momentum [Eq. (5)], the distribution of the projections of the total angular momentum on a space-fixed axis was assumed to be Gaussian. If the number of nucleons is large, or if the excitation energy is high, this is a valid approximation. However, for a small number of nucleons, or for a small excitation energy, the Gaussian approximation breaks down.[9] This breakdown can be expressed[9,28] in terms of a cutoff energy E_J defined by Eq. (11), below which no levels exist with spin larger than or equal to J:

$$E_J=(\hbar^2/2\mathcal{I})J(J+1). \quad (11)$$

The use of a spin cutoff energy in the formalism also provides a useful way to introduce competitive γ-ray emission. It has been shown[10] that Γ_γ becomes larger than Γ_n in an evaporation sequence when the excitation energy of a particular nucleus is sufficiently low that the emission of a neutron of low angular momentum leads to a product with excitation energy below E_J. The condition for the emission of just one neutron from a compound nucleus can therefore be written as

$$E_c-S_{n1}-\epsilon_1-S_{n2}-E_{J2}<0, \quad (12)$$

where E_c is the excitation energy of the compound

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nucleus, S_n and S_{n2} are the separation energies of the first and second neutrons, respectively, ϵ_1 is the kinetic energy of the first emitted neutron, and the spin cutoff energy E_{J2} refers to the nucleus formed after the emission of two neutrons. In order to account for the much less likely, but still possible, effect of competitive γ-ray emission from the compound nucleus, the following condition has to be satisfied in addition:

$$E_c - S_{n1} - E_{J1} > 0, \tag{13}$$

where E_{J1} refers to the nucleus formed after the emission of one neutron.

The angular distribution of the neutrons emitted in the (α, n) reaction was determined on the basis of the semiclassical theory of Ericson and Strutinski.[29,30] The angular distribution is related to J_c, $\boldsymbol{\sigma}^2$, and the orbital angular momentum of the emitted neutron, l, by the relation

$$W(\theta) = 1 + (\langle J_c^2 \rangle \langle l^2 \rangle \cos^2\theta)/8\boldsymbol{\sigma}^4, \tag{14}$$

which is applicable when the angular momentum is not too large, i.e., $J_c l/\sigma^2 \ll 1$. In evaluating Eq. (14) for a particular cascade, the average quantities $\langle J_c^2 \rangle$ and $\langle l^2 \rangle$ may be replaced by the corresponding values obtained for the iteration in question.

III. METHOD OF CALCULATION

The calculation was programmed for the Purdue CDC 6500 computer and was divided into eight overlay links using disk storage for arrays. Typically, the program required about 12 min for 12 000 iterations. The calculation was limited to the emission of neutrons, protons, α particles, and γ rays, the dominant deexcitation modes at intermediate energies. The atomic masses used in the calculation were obtained from Mattauch *et al.*[31] The nuclear radius parameter was taken as 1.2 F.

In the first stage of the calculation, transmission coefficients were generated for the incoming projectile and for the three kinds of emitted particles at each possible exit-channel energy from 0.5 MeV to the maximum in 1-MeV intervals, and for all possible values of the orbital angular momentum. The transmission coefficients were calculated by an adaptation of the ABACUS-2 code.[32] This information was stored in arrays on the disk.

In the second stage, the spin distribution of the compound nucleus was computed on the basis of Eq. (2), normalized to unity and stored in arrays for future use. The particle emission rates were computed by means of Eq. (4) for all possible values of l at 1-MeV intervals in the exit-channel energy. The rates were normalized so that the sum of the rates of all allowed decay processes was equal to unity. The total rates of neutron, proton, and α emission were obtained by summation of the partial rates.

In the next stage of the calculation, reaction events were selected by the Monte Carlo technique. Random numbers were used to select the spin of the compound nucleus and the type of particle emitted. If the chosen particle was of the desired kind, i.e., a neutron, the exit-channel energy and orbital angular momentum were selected by means of additional random numbers. These quantities were used to determine the range of possible values of J_R according to

$$(\mathbf{J}_c + \mathbf{s}_n + \mathbf{l}_n)_{\min} \le J_R \le J_c + s_n + l_n. \tag{15}$$

The value of J_R was determined by a random number on the basis of the calculated probabilities.

The possible occurrence of competitive γ-ray emission was determined next by means of the test embodied in Eqs. (12) and (13). Since the calculation was restricted to the emission of a single particle, the angular momentum of the residual nucleus following the emission of a second neutron J_{R2} was determined by an approximate procedure. At the bombarding energies for which the calculation was performed, the kinetic energy of a second emitted neutron is sufficiently low that its orbital angular momentum is effectively restricted to $l \le 3$. This fact was ascertained by a calculation of the appropriate transmission coefficients. A value of J_{R2} was chosen by random number selection out of the allowed range of J_{R2} values on the assumption that the orbital angular momentum of the second neutron was equal to 2. The calculation is not at all sensitive to this assumption.

Since the tables of normalized emission rates were compiled for values of the exit-channel energy in 1-MeV steps, the resulting neutron energy spectrum had the undesirable feature of being discrete. In order to avoid the resulting perturbation of the angular distribution of the residual nucleus, the discrete spectrum was converted to a continuous one. Since each value of the exit-channel energy corresponds to a 1-MeV interval, values were chosen at random out of these intervals using a weighting function based on linear interpolation between the emission rates for adjacent neutron energies.

The angular distribution of the evaporated neutron in the center-of-mass system was determined by random choice of the cosine of the angle with respect to the beam, weighted by the distribution in Eq. (14). The velocity of the residual nucleus was then determined by conservation of momentum. The velocity of the product in the laboratory system and the recoil

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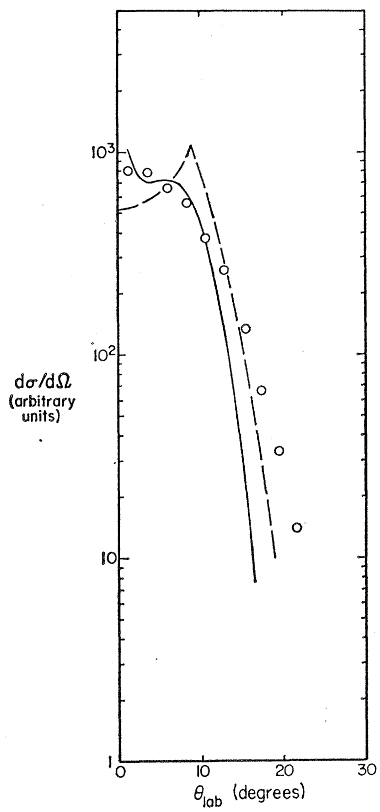

FIG. 2. Comparison of experimental and calculated angular distributions for the $Cu^{63}(\alpha, n)$ reaction at 20.8 MeV. The dashed line represents the spin-independent calculation (Ref. 4) and the solid line the present calculation. Experimental points (Ref. 4) are given by the open circles. All curves are normalized to each other in area.

angle were determined by vectorial addition of this velocity to that of the compound nucleus.

Twelve thousand iterations were usually performed for a given set of initial conditions. At the end of the calculation, the differential cross section and its standard deviation were computed in 1-deg intervals. Additional information that could be obtained when desired included the energy spectrum of the emitted neutron, and the relative number of neutrons, protons, α particles, competitive γ-rays, and two-particle reactions.

IV. RESULTS

The angular distribution of the (α, n) reaction product was computed for comparison with the experimental data of Porile and Saha[4] for the Cu^{63} target, and with that of Morton and Harvey[7] for Pb^{207}. The calculation was performed using a value of the level-density parameter appropriate to each target, $a=8.25$ for Cu^{63} and $a=6.88$ for Pb^{207} as given by Gilbert and Cameron.[24] The moment of inertia was regarded as an adjustable parameter, and its value reflects the approximations inherent in the treatment of competitive γ-ray emission. As discussed below, the value $\mathcal{I}=0.5\,\mathcal{I}_r$ gave the best fit to the data.

The comparison with the experimental data is shown in Figs. 2–4. Included for comparison are the results of the previous spin-independent calculations.[4,7] The three figures show that the present calculation correctly predicts the occurrence of a peak close to 0°, and, unlike the spin-independent analysis, shows no evidence of a spurious peak at large angles. However, the calculation does significantly underestimate the width of the angular distributions, and gives poorer agreement in this respect than the spin-independent calculation. In order to understand the source of this discrepancy, we performed the calculation for $\mathcal{I}=10^4\,\mathcal{I}_r$. In this case, the effects of angular momentum vanish, both because the spin-dependent exponential in the level density becomes unity, and because E_J becomes zero. The results for the $Cu^{63}(\alpha, n)$ reaction are shown in Fig. 5. As expected, the calculated angular distribution is very similar to that obtained by Porile and Saha[4] by the spin-independent formulation. The difference between the two formulations in the differential cross sections at large angles is thus entirely ascribable to angular-momentum effects.

In order to investigate this discrepancy in greater detail, the energy spectrum of the emitted neutron was examined. A comparison for $\mathcal{I}=0.5\,\mathcal{I}_r$ and $\mathcal{I}=10^4\,\mathcal{I}_r$ reveals that the relative number of high-energy

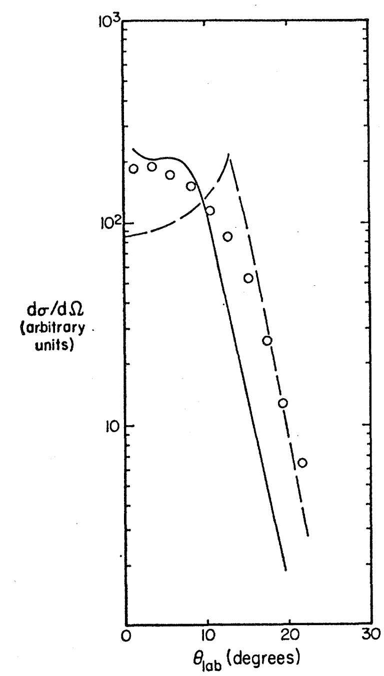

FIG. 3. Comparison of experimental and calculated angular distributions for the $Cu^{63}(\alpha, n)$ reaction at 24.5 MeV. See Fig. 2 for details.

neutrons is much larger in the latter case. It is precisely these high-energy neutrons that give rise to large-angle recoils. Further analysis of the problem disclosed that when $\mathscr{I}=0.5\,\mathscr{I}_r$, high-energy neutrons are preferentially emitted from compound nuclei with low angular momentum, whose formation probability tends to be small. This can be understood in terms of the effect of the spin-cutoff energy E_J in reducing the probability that compound nuclei with large J_c will emit high-energy neutrons. The calculation was repeated for various values of the moment of inertia, including the energy-dependent value predicted by the superconductor model. The value of the level-density parameter was also varied. In addition, the effect of competitive γ-ray emission from the compound nucleus was determined by setting E_J, in Eq. (13), equal to zero. None of these factors had a significant effect on the discrepancy observed at the largest angles.

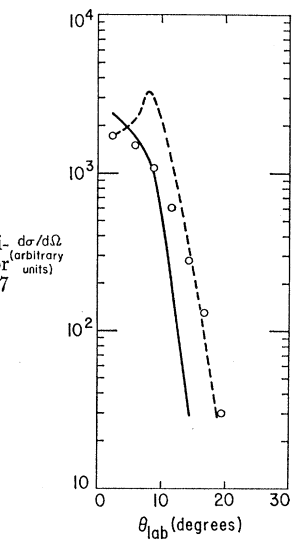

FIG. 4. Comparison of experiment (Ref. 7) and calculation for the $Pb^{207}(\alpha, n)$ reaction at 21.7 MeV. See Fig. 2 for details.

Another explanation of the difference between experiment and calculation lies in the possible contribution of a direct interaction process to the (α, n) reaction. Although the average range of the reaction product was found to be consistent with the value expected for compound nucleus formation,[1] this agreement does not preclude a significant contribution from a direct process. This follows from the fact that the expected difference in range for the two mechanisms is only 25%. The angular distribution of the recoil product of a (He^3, n) reaction, proceeding via a stripping process, has been calculated by the distorted-wave theory.[6] It was found that the angular distribution was nearly independent of angle up to the kinematic limit. A qualitatively similar result may be expected for a direct (α, n) reaction. A small contribution from this mechanism would thus result in a

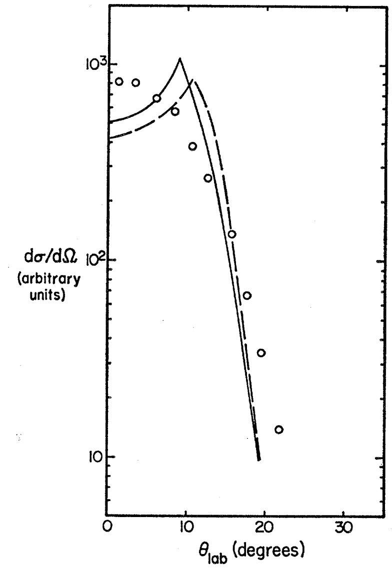

FIG. 5. Angular distribution of the $Cu^{63}(\alpha, n)$ product at 20.8 MeV in the limit of large $\mathscr{I}$. The solid line represents the present calculation with $\mathscr{I}=10^4\,\mathscr{I}_r$; the dashed line is the spin-independent calculation (Ref. 4). The experimental points are also shown.

sizeable discrepancy at large angles, since the compound nuclear yield decreases very rapidly beyond 10°.

In spite of the discrepancy at large angles, it is clear that the spin-dependent calculation reproduces the qualitative features of the angular distribution.

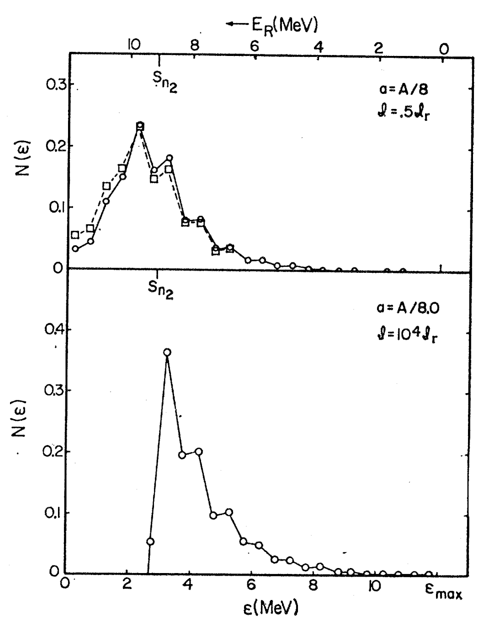

FIG. 6. Effect of the moment of inertia on the neutron energy spectrum for the $Cu^{63}(\alpha, n)$ reaction at 20.8 MeV. The top panel gives the energy spectrum of the first emitted neutron (dashed line) and that of the (α, n) reaction neutron (solid line) for $\mathscr{I}=0.5\,\mathscr{I}_r$. At the bottom, the energy spectrum of the (α, n) neutrons is shown for $\mathscr{I}=10^4\,\mathscr{I}_r$.

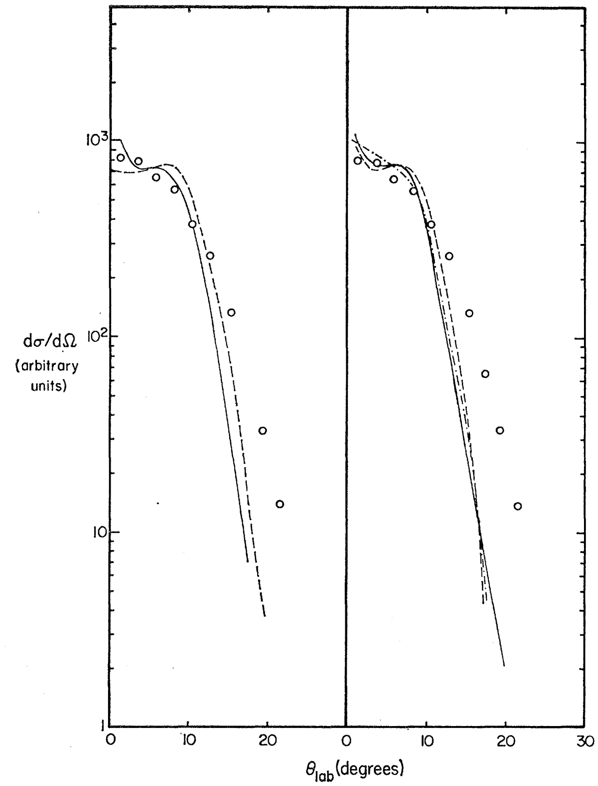

FIG. 7. Effects of the level-density parameter a and the moment of inertia, on the angular distribution of the $Cu^{63}(\alpha, n)$ reaction product at 20.8 MeV. Left panel: $\mathcal{I}=0.5\mathcal{I}_r$ and $a=\frac{1}{8}A$ (—), $a=1/12A$ (---). Right panel: $a=\frac{1}{8}A$, $\mathcal{I}=\mathcal{I}_r$ (---); $\mathcal{I}=0.1\,\mathcal{I}_r$ (—•—•); $\mathcal{I}$ from superconductor model (—).

The explanation discussed in the Introduction has thus been confirmed by a detailed calculation. The occurrence of a peak in the recoil angular distribution at 0° arises from the effect of competitive γ-ray emission in allowing the evaporation of low-energy neutrons to lead to the (α, n), as well as the $(\alpha, 2n)$ reaction. This fact is seen clearly from an examination of the calculated energy spectrum of the neutron emitted in the (α, n) reaction, shown in Fig. 6 for both $\mathcal{I}=0.5\,\mathcal{I}_r$ and $\mathcal{I}=10^4\,\mathcal{I}_r$. When competitive γ-ray emission is eliminated by making $\mathcal{I}$ very large, the (α, n) reaction neutrons must have an exit-channel energy greater than 3 MeV, the excitation energy of the compound nucleus minus the separation energy of two neutrons. If the exit-channel energy is less than 3 MeV, a second particle would be emitted. On the other hand, competitive γ-ray emission permits Ga^{66} to be formed even when much lower-energy neutrons are emitted and, therefore, considerably broadens the (α, n) neutron energy spectrum. Also shown in Fig. 6 is the energy spectrum of the first emitted neutron for $\mathcal{I}=0.5\,\mathcal{I}_r$. The small difference between this spectrum and the (α, n) spectrum reflects the fact that competitive γ-ray emission is sufficiently important to prevent the $(\alpha, 2n)$ reaction from occurring about 80% of the time that it is an energetically possible reaction, for an incident energy of 20.8 MeV.

The effect of varying some of the parameters of the calculation has been explored. In Fig. 7 the effect on the angular distribution of changing the level-density parameter a while leaving the moment of inertia is shown on the left. The graph on the right shows the effect of changing $\mathcal{I}$ while holding the parameter a constant. The observed differences tend to be small and are perhaps more easily understood in terms of the effect of these parameters on the average angle (Fig. 8). The increase in $\langle\theta_L\rangle$ with $\mathcal{I}$ can be understood in terms of the corresponding increase of the average kinetic energy of the emitted neutrons. This increase results from the corresponding decrease of the rotational energy E_J. In the lower half of the figure, an increase in the parameter a results in a more forward-peaked angular distribution because of the inverse dependence of the kinetic energy of the evaporated particles on the value of the level-density parameter and the direct relation between the former and the width of the angular distribution.

The calculation can also be used to obtain the cross section of the (α, n) reaction, and its dependence on the above parameters is shown in Fig. 9. It is seen that σ increases with a for large $\mathcal{I}$, but shows the opposite trend for small $\mathcal{I}$. Evidently, there are two opposing effects at work here. A higher value

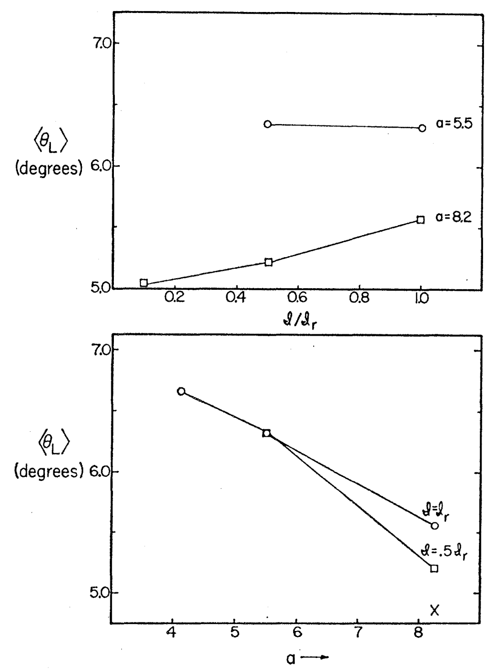

FIG. 8. Dependence of the average recoil angle, $\langle\theta_L\rangle$ on $\mathcal{I}$ (top panel) and a (bottom panel) for the $Cu^{63}(\alpha, n)$ reaction at 20.8 MeV. The superconductor model gives a value denoted by ×.

of a gives rise to a lower exit-channel energy, which reduces the probability of protons and α's being emitted due to the Coulomb barrier and, consequently, makes neutron emission more probable. On the other hand, a lower exit-channel energy also increases the cross sections of the $(\alpha, 2n)$ reaction, thereby reducing that of the (α, n) reaction. The top part of the figure shows that the cross section for the (α, n) reactions increases as $\mathscr{I}$ approaches $\mathscr{I}_r$. The horizontal lines in each figure represent the experimental values.[33] As can be seen, good agreement is obtained for $a=8.25$ when $\mathscr{I}=0.5\,\mathscr{I}_r$. The cross section calculated on the basis of the superconductor model is seen to be slightly larger than that obtained from the Fermi-gas model for $\mathscr{I}=\mathscr{I}_r$. On the other hand, the average recoil angle predicted by this model is somewhat smaller than that obtained for $\mathscr{I}=0.5\,\mathscr{I}_r$. In neither instance is the agreement with experiment improved by use of the superconductor model.

The energy dependence of the calculated cross section and average angle has been obtained between 20.8 and 24.5 MeV. Using the parameters that give the best fit at 20.8 MeV, $a=8.25$ and $\mathscr{I}=0.5\,\mathscr{I}_r$, σ is found to decrease from 0.42 to 0.28 b, whereas the experimental value decreases from 0.42 to 0.21 b. The calculated average angle increases by 8% between these two energies, as does the experimental value.

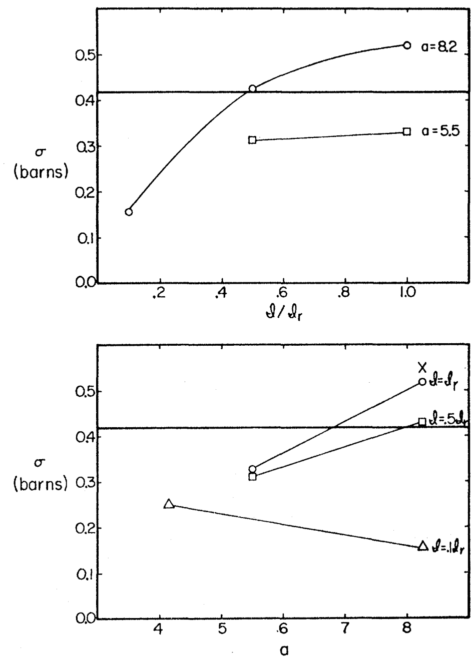

FIG. 9. Dependence of the cross section of the $Cu^{63}(\alpha, n)$ reaction at 20.8 MeV on $\mathscr{I}$ (top panel) and a (bottom panel). The superconductor model gives a value denoted by $\times$. The horizontal line gives the experimental value (Ref. 33).

V. CONCLUSIONS

The angular distribution of the recoil nuclei from (α, n) reactions has been calculated on the basis of the spin-dependent statistical theory. The calculation correctly predicts a peak at 0° in the differential cross section in the laboratory system, in marked contrast to spin-independent calculations which predict a sharp spike at approximately 10°. The difference is shown to result from the effect of competitive γ-ray emission in broadening the energy spectrum of (α, n) reaction neutrons, thereby washing out the large-angle peak expected from the kinematics for two-body reactions with unique final states. The discrepancy between experiment and calculation in the magnitude of the differential cross section at large angles cannot be removed by reasonable variation in the values of the various parameters in the theory, and may reflect a small contribution from a direct-interaction process.

[33] N. T. Porile and D. L. Morrison, Phys. Rev. **116**, 1193 (1959).