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Prolate-Oblate Difference and its Effect on Energy Levels and Quadrupole Moments

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From the microscopic calculation for the W, Os, and Pt nuclei, we obtain semiquantitative relations among the prolate-oblate difference of the potential energy of deformation, the 2_γ^+ and 4^+ level splitting, and the static quadrupole moment Q_{2^+}. These relations are applied to ^{24}Mg, ^{56}Fe, ^{110}Cd, and ^{152}Sm. The important role of the β-γ dependence of the mass parameters is demonstrated.

If there were no prolate-oblate difference (POD) and the nuclear motion were γ-independent, rotational spectra would not exist. Such a nucleus would be soft against γ vibrations; hence the γ-vibrational states would be low lying and strongly mixed with the rotational states. Furthermore, the static quadrupole moments would vanish since the nuclear states would be either even or odd in the magnitude of deformation β.

Thus, the existence of a rotational spectrum means not only that the nucleus has a permanently deformed shape, but also that it has a strong preference for prolate over oblate shapes (or vice versa, depending on the sign of the quadrupole moment). Moreover, nuclei considered previously to be spherical and good vibrators are found to have large static quadrupole moments.[1,2] It has been suggested[3] that such quadrupole moments can be attributed to the POD caused by the cubic term in deformation. Even in a nucleus whose equilibrium shape is spherical, the cubic term can lead to a "rotational" value (denoted in what follows by Q_R) for the static quadrupole moment Q_{2^+}. The POD is directly responsible[3,4] for the splitting of 2_γ^+ and 4^+ states.[5] Such considerations lead us to believe that the role of POD (within the context of intrinsic or deformed representation) cannot be overemphasized.

As is well known, the γ dependence complicates the microscopic treatment since "m" is not a good quantum number, and also the collective treatment since the Schrödinger equation in β and γ is nonlinear. With the advent of modern computers, it has become possible to tackle both of these problems.[6] But it must be admitted that some of the beauty and the accessibility of the Bohr-Mottelson model is lost.

In order to rectify this situation somewhat, we take a closer look at the numerical results[6] for the W, Os, and Pt region and obtain simple, semiquantitative relations among POD, $E_{2_\gamma^+}-E_{4^+}$, and Q_{2^+}/Q_R. These relations and their applicability to other nuclei are discussed below.

A convenient measure of POD is V_{PO}, the energy difference between the oblate and prolate minima[7] of the potential energy of deformation (V_{PO} is defined here to be positive for prolate nuclei). The calculated[6] and observed[8] splittings of 2_γ^+ and 4^+ levels are plotted against the calculated V_{PO} in Fig. 1. The straight line corresponds to the relation

$$S=E_{2_\gamma^+}-E_{4^+}=\tfrac{1}{2}V_{PO} \qquad (-0.6<V_{PO}<1.5 \text{ MeV}). \tag{1}$$

This straight line passes through the origin in accord-

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† Operated by Union Carbide Corporation for the U.S. Atomic Energy Commission.

[1] J. de Boer and J. Eichler, *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum Press, Inc., New York, 1968), Vol. 1, p. 1.

[2] D. Cline, Bull. Am. Phys. Soc. **14**, 726 (1969).

[3] K. Kumar, in *Nuclear Structure*: *Dubna Symposium* (International Atomic Energy Agency, Vienna, 1968), p. 419.

[4] L. Wilets and M. Jean, Phys. Rev. **102**, 788 (1956).

[5] The symbol 2_γ^+ denotes a 2^+ state which is mostly γ vibrational ($K=2$) or $N=2$ phonon. Criteria for the identification of such a state are given later in the text.

[6] K. Kumar and M. Baranger, Nucl. Phys. **A122**, 273 (1968); Phys. Rev. Letters **17**, 1146 (1966).

[7] Note that (1) a local minimum in β may not be a minimum in the γ direction, and (2) if $\beta_{\min}=0$, the relevant quantity is $V_{PO}=V(-\beta_{\text{rms}})-V(\beta_{\text{rms}})$, where $\beta_{\text{rms}}\sim[B(E2;0\to2)]^{1/2}$.

[8] C. M. Lederer, J. M. Hollander, and I Perlman, in *Table of Isotopes* (John Wiley & Sons, Inc., New York, 1967), 6th ed.

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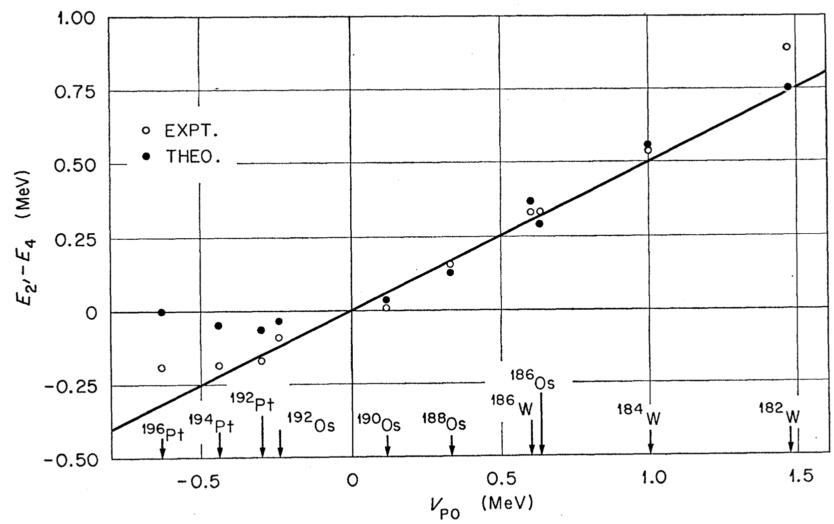

FIG. 1. The 2_γ^+ and 4^+ level splitting and the prolate-oblate difference. The straight line has been drawn for convenience, but it certainly does not give the correct behavior at large $|V_{PO}|$.

ance with the Wilets-Jean rule[4] that the 2_γ^+ and 4^+ levels in a γ-independent potential well are degenerate (no matter how large or small the magnitude of $\beta_{\min}$ may be). However, if the mass parameters were independent of deformation, we would have expected the splitting to be independent of the sign of deformation. The change of sign of S in Fig. 1 is a direct consequence of the β-γ dependence of the mass parameters.

The calculated[6] and observed[9] values of Q_{2^+}/Q_R are plotted against V_{PO} in Fig. 2. The smooth curve corresponds to the relation

$$Q_{2^+}/Q_R=1-0.45\exp(-2.6V_{PO}) \quad (-0.6<V_{PO}<1.5\text{ MeV}). \qquad (2)$$

This curve also deviates from the expected behavior. The net quadrupole moment is nonzero even when V_{PO} equals zero! The reason is that the mass parameters favor prolate shapes and hence increase the domain of "prolate"-type quadrupole moments.

For checking the applicability of relations (1) and (2) to other nuclei, three criteria are used. (1) The experimental value of Q_{2^+}/Q_R is known. (2) In order to distinguish a 2_γ^+ state from a β-vibrational or a two-quasiparticle state, it is necessary to locate a 3^+ state. Then we are guided by the following relations of the rotational model (R) and the phonon model (P):

$$(E_{3^+}-E_{2_\gamma^+})/E_{2^+}=1.0(\text{R}), \qquad 1.0(\text{P}), \qquad (3)$$

$$B(E2;3^+\to2_\gamma^+)/B(E2;2^+\to0^+)=(25/14)(\text{R}), \quad (15/7)(\text{P}). \qquad (4)$$

(3) The third criterion is that the experimental value of S lies within the range of validity of Eqs. (1) and (2).

These three criteria are satisfied by four nuclei out-

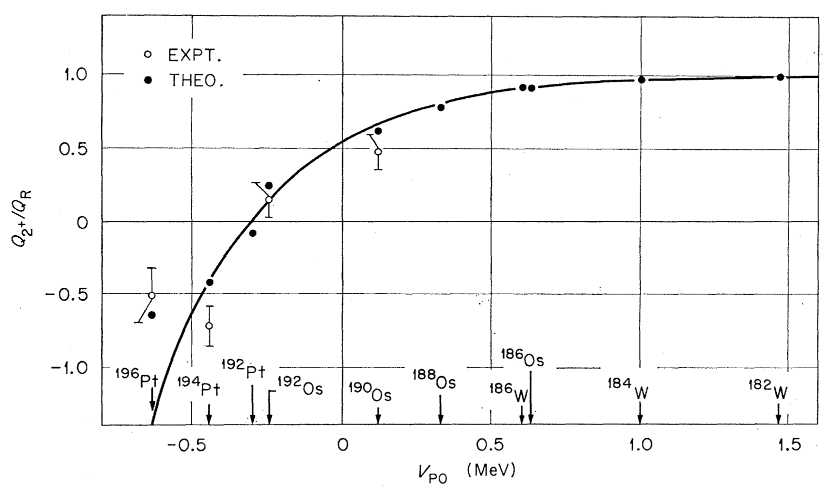

FIG. 2. The quadrupole moment ratio and the prolate-oblate difference. The smooth curve has been drawn for convenience, but it certainly does not give the correct behavior at $V_{PO}\ll0$.

[9] J. X. Saladin and R. J. Pryor (private communications). See also Ref. 2.

side the W, Os, Pt region already discussed. Table I shows that the estimated Q_{2^+}/Q_R values are somewhat too low in magnitude, but Eqs. (1) and (2) do give the correct trends. The large discrepancy in ^{24}Mg probably reflects the fact that the asymmetric nature of this nucleus[10] is not properly taken into account in the above semiquantitative relations.

Let us summarize the conclusions. For a complete treatment of collective quadrupole motion in the intrinsic system, the γ dependence of the potential function and the mass parameters must be taken into account. However, as an intermediate step we can use the POD as a reasonably reliable, additional test of a calculation of equilibrium deformations. The usual calculation for prolate shapes only can be easily extended to oblate shapes (without even a modification of computer codes). The POD that is required by the experimental spectra and quadrupole moments can be estimated from Eqs. (1) and (2).

The above results also indicate that the crossing of 2_γ^+ and 4^+ levels rather than the change of sign of Q_{2^+} is a better indicator of a prolate-oblate transition. This conclusion has been checked by extending[11] the W, Os, Pt region calculation to ^{186}Pt and comparing with the level spectra obtained by the ISOLDE (CERN) groups,[12] but it needs to be checked further. These results also demonstrate the important role of the β-γ dependence of the mass parameters.

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TABLE I. The static quadrupole moments and the $2_\gamma^+ - 4^+$ level splitting.

Nucleus	$E_{2_\gamma^+} - E_{4^+}$ MeV[a]	Q_{2^+}/Q_R Est[b]	Q_{2^+}/Q_R Expt[c]
^{24}Mg	0.11	0.75	1.3±0.2
^{56}Fe	0.573	0.98	1.3±0.3
^{110}Cd	−0.0665	0.36	0.7±0.4
^{152}Sm	0.72	0.99	1.1±0.4

[a] See Ref. 8.
[b] The estimated values have been obtained by using Eqs. (1) and (2), and values in column 2 of this Table.
[c] See Ref. 2 or Ref. 9.

[10] G. Ripka, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum Press, Inc., New York, 1968), Vol. 1, p. 183.
[11] K. Kumar, International Conference on Properties of Nuclear States, Montreal, Canada, 1969 (unpublished).
[12] R. Foucher *et al.*, International Conference on Properties of Nuclear States, Montreal, Canada, 1969 (unpublished).

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Neutron-Proton Bremsstrahlung Calculations* †

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The neutron-proton bremsstrahlung cross section between 14 and 208 MeV is calculated by including the external radiation terms to all orders in the momentum of the photon, K, and by using the low-energy theorem to compute the internal scattering contribution to order K^0. These calculations, which employ a simple off-shell extrapolation of the elastic phase shifts, are in good agreement with the data at 208 MeV. These off-shell calculations are also compared to calculations using the corresponding on-shell phase shifts in order to determine the strength of the off-shell terms. In coplanar scattering where $\theta_p+\theta_n \leq \sim 60°$, these off-shell effects constitute as much as 50% or more of the cross section, while for $60° \leq \theta_p+\theta_n \leq 90°$, the off-shell contributions are much smaller.

INTRODUCTION

THE proton-proton and neutron-proton bremsstrahlung processes,

$$p+p \to p+p+\gamma,$$
$$n+p \to n+p+\gamma, \qquad (1)$$

have been studied in the past several years in the hope of finding useful information about the off-energy-shell (OES) behavior of the strong interaction. After some initial difficulties, it has been shown that the limited amount of $pp\gamma$ data[1] is in generally good agreement with various calculations.[2] Moreover, it has

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[2] V. R. Brown, Phys. Letters **25B**, 506 (1967); P. Signell and D. Marker, *ibid.* **28B**, 79 (1968); D. Drechsel and L. C. Maximon, Ann. Phys. (N.Y.) **49**, 403 (1968). Also, Refs. 7–10 below.