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Two-Nucleon Transfer Reactions*

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Assuming isospin conservation, we perform an analysis of the two-nucleon transfer reactions (p, t) and $(p, {}^3\mathrm{He})$ on a target with isospin T_i going to analog final states with $T_f=T_i$. Formulas for the relevant cross sections are given which explicitly show the isospin dependence of the transferred pair. When restricted to $T=1$ pair transfer, our result is more general than that recently derived by Hardy, Brunnader, and Cerny.

RECENTLY, a new technique has been proposed for investigating the parentage of nuclear states using two-nucleon transfer reactions.[1] The method involves simultaneous observation of (p, t) and $(p, {}^3\mathrm{He})$ reactions on a target with isospin T_i to analog final states with $T_f=T_i$. If one begins with a 0^+ target, then J^π which characterizes the final nuclear state must also be the quantum numbers of the two transferred nucleons. If these two nucleons originate from the same j shell of the target, then for $J=$ even (an antisymmetric space-spin state) the pair must have $T=1$ in order to obey the Pauli principle. (Conversely, for $J=$ odd only $T=0$ pairs would be involved.) This case is treated in HBC with the result

$$R=\frac{d\sigma(p,t)/d\Omega}{d\sigma(p,{}^3\mathrm{He})/d\Omega}=(k_t/k_{{}^3\mathrm{He}})(2/T_i). \tag{1}$$

Equation (1) is obtained from the distorted-wave Born approximation (DWBA)[2] after making suitable assumptions which are equivalent to imposing isospin conservation. We will derive a more general expression similar to (1) assuming only isospin conservation. Experimentally, the effects of isospin nonconservation seem to be small as shown in HBC. Our result is essentially model-independent. When restricted to $T=1$ pair transfer, our result differs somewhat from (1).

We begin by noting that a proton ($T=\frac{1}{2}$, $T_3=-\frac{1}{2}$) and a target nucleus ($T=T_3=T_i$) can interact in either of two isospin channels, $T_\pm=T_i\pm\frac{1}{2}$. Both channels have $T_3=T_i-\frac{1}{2}$. The reaction products are a triton or ^{3}He (both with $T=\frac{1}{2}$, $T_3=+\frac{1}{2}$ or $-\frac{1}{2}$, respectively) and a nucleus with $T_f=T_i$. These products can be formed in either of two isospin channels $T_\pm$. Both initial and final states may be expanded in states of pure total isospin

$$\Psi_i=\sum_T C(T_i,\tfrac{1}{2},T;T_i,-\tfrac{1}{2})\,|\,T,T_i-\tfrac{1}{2}\rangle_i, \tag{2a}$$

$$\Psi_f(t)=\sum_T C(T_i,\tfrac{1}{2},T;T_i-1,+\tfrac{1}{2})\,|\,T,T_i-\tfrac{1}{2}\rangle_f, \tag{2b}$$

and

$$\Psi_f({}^3\mathrm{He})=\sum_T C(T_i,\tfrac{1}{2},T;T_i,-\tfrac{1}{2})\,|\,T,T_i-\tfrac{1}{2}\rangle_f. \tag{2c}$$

The expansion coefficients are the usual Clebsch-Gordan (CG) coefficients. In the above sums, T is limited to the two possible values $T_\pm$.

For the reactions of interest, the amplitudes are given by the matrix elements between initial and final states

$$A(t)=\langle\Psi_f(t)\,|\,\mathcal{O}\,|\,\Psi_i\rangle \tag{3a}$$

and

$$A({}^3\mathrm{He})=\langle\Psi_f({}^3\mathrm{He})\,|\,\mathcal{O}\,|\,\Psi_i\rangle, \tag{3b}$$

where $\mathcal{O}$ is the interaction responsible for the transition. Since $\mathcal{O}$ is assumed to be an isoscalar, matrix elements

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[1] J. C. Hardy, H. Brunnader, and J. Cerny, Phys. Rev. Letters **22**, 1439 (1969). This paper will be referred to as HBC.

[2] N. K. Glendenning, Phys. Rev. **137**, B102 (1965).

between states with different isospins will vanish. Substituting (2) into (3), and evaluating the CG, we obtain[3]

$$A(t)=(2T_i+1)^{-1}(2T_i)^{1/2}(A_+-A_-) \tag{4a}$$

and

$$A(^3\mathrm{He})=(2T_i+1)^{-1}(A_++2T_iA_-), \tag{4b}$$

where

$$A_\pm={}_f\langle T_i\pm\tfrac{1}{2},\, T_i-\tfrac{1}{2}\mid \mathcal{O}\mid T_i\pm\tfrac{1}{2},\, T_i-\tfrac{1}{2}\rangle_i. \tag{5}$$

Since the cross section is proportional to the square of the amplitude, we obtain

$$R=(k_t/k_{^3\mathrm{He}})(2T_i)\,|A_+-A_-|^2/|A_++2T_iA_-|^2. \tag{6}$$

The momentum ratio in (6) arises from phase-space considerations for the final states.[4]

To further evaluate $A_\pm$, we must look at the reaction more closely. We assume a pickup-type reaction where an incident proton strips a pair of nucleons from the target forming an outgoing $A=3$ nucleus. The transferred pair can, in general, have $T=0$ or $T=1$. We perform an isospin decomposition of the target

$$|T_i,T_i\rangle_{\mathrm{target}}=\alpha\,|(T_i)_{\mathrm{core}},(1)_{\mathrm{pair}};T_i,T_i\rangle + \beta\,|(T_i)_{\mathrm{core}},(0)_{\mathrm{pair}};T_i,T_i\rangle. \tag{7}$$

This decomposition implies no assumption about the target wave function, since such an expansion is always possible. The first term on the right-hand side of (7) corresponds to $T=1$ for the pair to be transferred; the second corresponds to a $T=0$ pair. α and β are the amplitudes of the two parts ($|\alpha|^2+|\beta|^2=1$). The subscript "core" applies to the isospin of the remaining nucleons in the target.[5] Vector coupling of the isospins of core and pair to give $T=T_3=T_i$ for the target is implied by the notation in (7).

The channel wave function for the input state is constructed by isospin coupling of the target and incident nucleon wave functions

$$|T_i\pm\tfrac{1}{2},T_i-\tfrac{1}{2}\rangle_i=|(T_i)_{\mathrm{target}},(\tfrac{1}{2})_{\mathrm{nucleon}};T_i\pm\tfrac{1}{2},T_i-\tfrac{1}{2}\rangle_i. \tag{8}$$

The explicit pair dependence can be displayed in the input channel wave function by substituting (7) into (8).

A decomposition similar to (7) is performed for the outgoing $A=3$ nucleus,

$$|\tfrac{1}{2},t\rangle_{A=3}=\gamma\,|(1)_{\mathrm{pair}},(\tfrac{1}{2})_{\mathrm{nucleon}};\tfrac{1}{2},t\rangle + \lambda\,|(0)_{\mathrm{pair}},(\tfrac{1}{2})_{\mathrm{nucleon}};\tfrac{1}{2},t\rangle. \tag{9}$$

The channel wave function for the final state may be formed analogous to (8),

$$|T_i\pm\tfrac{1}{2},T_i-\tfrac{1}{2}\rangle_f=|(T_i)_{\mathrm{core}},(\tfrac{1}{2})_{A=3};T_i\pm\tfrac{1}{2},T_i-\tfrac{1}{2}\rangle_f. \tag{10}$$

The explicit pair dependence for the final state can be obtained by substituting (9) into (10).

Now that the isospin structure of the channel wave functions has been specified, the isospin dependence of the transition amplitudes may be calculated. Since we have assumed an isoscalar interaction $\mathcal{O}$, we may write

$$\mathcal{O}=V_0+V_{NC}\mathbf{T}_\mathrm{N}\cdot\mathbf{T}_\mathrm{C}+V_{PC}\mathbf{T}_\mathrm{P}\cdot\mathbf{T}_\mathrm{C}+V_{NP}\mathbf{T}_\mathrm{N}\cdot\mathbf{T}_\mathrm{P}. \tag{11}$$

This is the most general form we can specify if we neglect three-body interactions. V_0 is the isospin-independent part of the interactions. The subscripts N, P, and C refer to the incident nucleon, transferred pair, and core, respectively. (In a DWBA calculation, the interaction between N and P would be omitted since this part of the Hamiltonian would have already been used in obtaining the state of the final $A=3$ nucleus.)

We evaluate the transition amplitudes by combining (5) and (7)–(11). The result, after much manipulation of Racah algebra, is

$$A_+=\lambda^*\beta\langle V_0+\tfrac{1}{2}T_iV_{NC}\rangle_0-\gamma^*\alpha[T_i/3(T_i+1)]^{1/2} \times\langle V_0-V_{NP}-V_{PC}+\tfrac{1}{2}(T_i+2)V_{NC}\rangle_1 \tag{12}$$

and

$$A_-=\lambda^*\beta\langle V_0-\tfrac{1}{2}(T_i+1)V_{NC}\rangle_0+\gamma^*\alpha[(T_i+1)/3T_i]^{1/2} \times\langle V_0-V_{NP}-V_{PC}-\tfrac{1}{2}(T_i-1)V_{NC}\rangle_1. \tag{13}$$

$\langle\cdots\rangle_0$ and $\langle\cdots\rangle_1$ are matrix elements between states which contain $T=0$ and $T=1$ pairs, respectively. Finally, using (6), we obtain

$$R=\frac{k_t/k_{^3\mathrm{He}}}{2/T_i}\left|\frac{\gamma^*\alpha\langle V_0-V_{NP}-V_{PC}+\frac{1}{2}V_{NC}\rangle_1-\frac{1}{2}[3T_i(T_i+1)]^{1/2}\lambda^*\beta\langle V_{NC}\rangle_0}{\gamma^*\alpha\langle V_0-V_{NP}-V_{PC}-\frac{1}{2}T_iV_{NC}\rangle_1+[3(T_i+1)/T_i]^{1/2}\lambda^*\beta\langle V_0-\frac{1}{2}T_iV_{NC}\rangle_0}\right|^2. \tag{14}$$

[3] The same result was previously obtained for quasielastic nucleon scattering where both incident and outgoing particles also have $T=\frac{1}{2}$. A. M. Lane, Phys. Rev. Letters 8, 171 (1962).

[4] When one considers similar reactions to final nuclear states with $T_f=T_i+1$, the reaction can proceed only through the $T_+=T_i+\frac{1}{2}$ channel since the reaction products cannot be combined to yield T_-. The result of a similar calculation for this case yields $R=(k_t/k_{^3\mathrm{He}})(2/2T_f-1)$ in agreement with Eq. (3) of HBC.

[5] The introduction of the term "core" should not be misleading. We are not working with a traditional "core plus valence nucleon" model.

If only $T=1$ pairs are transferred and $\langle V_{NC}\rangle_1=0$, then we would obtain

$$R \propto 2/T_i$$

in agreement with (1). Since we would expect nonzero V_{NC} [in general, the (p, n) reaction is observed], deviations from this simple ratio should occur. These deviations will be small as long as V_{NC} is much smaller than $(V_0-V_{NP}-V_{PC})$. If this is the case, then the ratio of A_+/A_- is uniquely determined:

$$A_+/A_- = -(T_i/T_i+1). \qquad (15)$$

Thus deviations from (15) and, consequently, (1) may be attributed to either $T=0$ pair transfer or V_{NC}, or both.

To estimate the effect of V_{NC} on $T=1$ pair transfer, we ignore V_{NP} and V_{PC}. In the analysis of quasielastic (p, n) scattering, a potential frequently used is $U=U_0+U_1\mathbf{t}\cdot\mathbf{T}/A$. In the region $A=45$–93, it is found that $U_0\sim-50$ MeV and $U_1\sim100$ MeV assuming the same form factor for the two terms.[6] Setting $V_0\sim-50$ MeV and $V_{NC}\sim3$–5 MeV for the nuclei considered by HBC ($A=20$–36), we would expect R to be reduced from the value predicted by (1) by 11–18%. The reduction is even more dramatic if cases with $T_i=T_f>1$ are considered. The presence of V_{NP} and V_{PC} would surely alter this result (V_0 would also change), but the effect may be large—even an increase in R is possible.[7]

It should be noted that $T=0$ pair transfer contributes to both (p, t) and $(p, {}^3\mathrm{He})$ reactions. The contribution to the first occurs via V_{NC} only. Although a proton cannot pick up a $T=0$ pair to form a triton, it can do so if it simultaneously undergoes charge exchange with the core via the interaction proportional to V_{NC}.

From (6) or (14), it seems that the ratio R can attain almost any value depending upon the reaction under consideration. HBC states that the value given in (1) is the maximum which can be achieved in any case. From the data which they quote it can be seen that R is less than or within one standard deviation of the maximum value given by (1) in all but one case.[8] It would be of great interest to see if this exception is real or merely a statistical fluctuation.

It is our contention that one must be exceedingly careful in interpreting the results of two-nucleon transfer reactions in terms of simple parentage concepts. Experimental observation of (1) need not automatically imply paired nucleon transfer. Conversely, paired nucleon transfer does not unambiguously lead to (1).

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[6] D. Robson, Ann. Rev. Nucl. Sci. **16**, 119 (1966).

[7] It has been previously noted that the (p, t) transition is occasionally much stronger relative to the $(p, {}^3\mathrm{He})$ transition than would be expected from the DWBA theory. D. G. Fleming, J. Cerny, and N. K. Glendenning, Phys. Rev. **165**, 1153 (1968).

[8] Using a ^{26}Mg target, the authors of HBC find $R_{\mathrm{expt}}=2.50\pm0.30$ which should be compared with $R_{\max}=1.86$.