Faddeev Treatment of ^{6}Li with a Separable Potential

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(Received 7 May 1969)

The nucleus ^{6}Li, together with its first excited state ^{6}Li*, is treated as a three-body problem consisting of an α particle, a proton, and a neutron, interacting through separable potentials in pairs. The N-N potential includes a tensor force in addition to a central (S-wave) term of the Yamaguchi type. The p-wave part of the α-N potential is parametrized in terms of the sum of a central and a small spin-orbit term which fits the experimental $P_{3/2}$ and $P_{1/2}$ α-N phase shifts quite accurately. The smallness of the spin-orbit part justifies its omission from the full three-body equations. The binding energies and the wave functions for the ground ($J^P=1^+$) and excited ($J^P=0^+$) states of ^{6}Li are calculated by solving the coupled Faddeev equations numerically. The calculated binding energy for the 1^+ state is a little higher than, and that for the 0^+ state a little lower than, the corresponding experimental values, corrected for Coulomb effects.

1. INTRODUCTION

THE unusually stable structure of the α particle has led to many investigations of light nuclei in which this object is taken as a single structureless entity. ^{6}Li and ^{6}He are particularly well suited for such investigations, since their structure consists of only two nucleons in addition to the α particle. Indeed, quite encouraging results have been obtained for the energy levels of ^{6}Li in a shell-model spirit,[1] with L-S coupling configurations for the two nucleons outside the α particle which is assumed to be an inert core. An apparently more "dynamical" model which regarded this nucleus as an α-d system[2] did not, however, work so well, since it gave a wrong sign for the quadrupole moment.

The recent advances in three-body techniques through the Faddeev formalism[3] have given rise to a renewed interest in the problem of ^{6}Li-like nuclei looked upon as three-body structures. A variational treatment of ^{6}Li as a three-body problem is already available in the work of Wackman and Austern, and in the more recent work of Basella *et al.*[4] The emphasis in the Faddeev technique is, however, on an *exact* rather than a variational solution of the problem, the price for "exactness" being a truncation of the two-body reaction matrix by one or more pole terms (always a finite number). However, this approximation can be defended on theoretical grounds.[5]

A related approach which leads to the same equations, and which is particularly convenient for bound-state problems, is provided through a direct parametrization of the input potentials in a separable form, and their use in conjunction with a three-body Schrödinger equation.[6,7] Such an approach has been extensively used in connection with the three-nucleon problem involving both ^{3}H and n-d states,[8] and appears equally suited to the ^{6}Li problem. Similar methods have also been used for light hypernuclei like $_\Lambda{}^3$H,[9] $_\Lambda{}^9$Be,[10] and $_{\Lambda\Lambda}{}^6$He,[11] all of which involve unequal mass particles. An important difference between ^{6}Li and these hypernuclei is in the nature of the configurations. While the Λ particle is not prevented by the Pauli principle from having mainly S-wave interactions with the other entities in these hypernuclei, the nucleons in ^{6}Li are constrained by the same principle to interact with the α particle mainly through the P wave. This renders the system more delicate; and hence numerically more difficult. Nevertheless, the feasibility of such a program was recognized several years ago[6] and a separable form of the α-N interaction was devised for the purpose.[12] In the meantime, Hebach *et al.*[13] have given a similar treatment of ^{6}He using a different form of the α-N interaction. The present treatment is, however, designed to cover both ^{6}Li and ^{6}Li* (the latter state being the counterpart of the ground state of ^{6}He, apart from Coulomb effects). We also take the opportunity to clarify the role of the (P-wave) α-N interaction as well as to point out certain differences of our results

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[1] V. L. Narsimham *et al.*, Nucl. Phys. **33**, 529 (1962).
[2] J. E. Young and P. R. Stein, Nucl. Phys. **26**, 97 (1962).
[3] L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. **39**, 1459 (1960) [English transl.: Soviet Phys.—JETP **12**, 1014 (1961)].
[4] P. H. Wackman and N. Austern, Nucl. Phys. **30**, 529 (1962); B. Barsella, L. Lovith, and S. Rosati, *ibid.* **A117**, 638 (1968).
[5] C. Lovelace, Phys. Rev. **B135**, 1225 (1964).
[6] A. N. Mitra, Nucl. Phys. **32**, 529 (1962).
[7] A. Sitenko and V. Khachenko, Nucl. Phys. **49**, 15 (1963).
[8] For a complete formulation as well as an extensive set of references on this subject, see A. N. Mitra, G. L. Schrenk, and V. S. Bhasin, Ann. Phys. (N.Y.) **40**, 357 (1966).
[9] J. Hetherington and L. Schick, Phys. Rev. **156**, 1602 (1967); S. K. Monga, Nuovo Cimento **41A**, 164 (1966).
[10] S. K. Monga, Phys. Rev. **160**, 846 (1967).
[11] S. K. Monga and A. N. Mitra, Nuovo Cimento **42A**, 1024 (1966); J. Dabrowski *et al.*, Report "P" No. 831/VII/PI, Institute of Nuclear Research, Warsaw (unpublished).
[12] A. N. Mitra, V. S. Bhasin, and B. S. Bhakar, Nucl. Phys. **38**, 316 (1962).
[13] H. Hebach, P. Hennenberg, and H. Kummel, Phys. Letters **24B**, 134 (1967); hereafter referred to as HHK.

TABLE I. The parameters for the triplet potential are as given by Yamaguchi (Ref. 14) and by Naqvi (Ref. 15). For notation see the text.

Force	$\beta_t(\mathrm{F}^{-1})$	$\lambda_t(\mathrm{F}^{-3})$	t	$\gamma_t(\mathrm{F}^{-1})$
C_Y^{eff}	1.4487	0.414	...	...
$(C+T)_Y$	1.3338	0.249	1.784	1.5682
$(C+T)_N$	1.3433	0.289	0.9519	1.3433

from those of Hebach, Hennenberg, and Kummel (HHK).[14] In Sec. 2 we discuss the input forces for the problem, and in Sec. 3 we outline the essential steps for the three-body treatment of ^{6}Li. Section 4 gives the numerical results, a comparison with those of HHK, and a discussion of the three-body wave function.

2. INPUT FORCES

The main ingredients for the three-body treatment of ^{6}Li are the α-N and N-N interactions. For the ground state of ^{6}Li (1^+), the relevant N-N interaction is the triplet one, while for the excited state ^{6}Li* (0^+) the corresponding force acts in the singlet ($^1S_0$) state. These forces were given earlier by Yamaguchi,[15] and were then modified by Naqvi[16] to give a detailed fit to the N-N system. For the case of the singlet state, we use the attractive S-term only, viz.,

$$M\langle \mathbf{q} \mid v_s \mid \mathbf{q}' \rangle = -\lambda_s (q^2+\beta_s^2)^{-1}(q'^2+\beta_s^2)^{-1}, \quad (2.1)$$

where

$$\lambda_s = 0.291\ \mathrm{F}^{-3}, \qquad \beta_s = 1.4487\ \mathrm{F}^{-1}, \quad (2.2)$$

and M is the nucleon mass. It may be noted that in this potential the short-range repulsion is not included, since it contributes not more than 10–15% in the case of a three-nucleon system[17] and would make computations exceedingly difficult. For the triplet case the force is

$$M\langle \mathbf{q} \mid v_t \mid \mathbf{q}' \rangle = -\lambda_t g(\mathbf{q}) g(\mathbf{q}'), \quad (2.3)$$

where

$$g(\mathbf{q}) = C(q) + (1/\sqrt{8})\, T(q)\, S_{12}(\hat{q}),$$
$$S_{12}(\hat{q}) = 3\sigma_1\cdot\hat{q}\sigma_2\cdot\hat{q} - \sigma_1\cdot\sigma_2, \quad (2.4)$$
$$C(q) = (q^2+\beta_t^2)^{-1}, \qquad T(q) = -tq^2(q^2+\gamma_t^2)^{-2}.$$

The values of the various constants are given in Table I. C_Y^{eff} denotes the effective pure central force given by Yamaguchi.[15] It is known from the study of the three-nucleon system that this force is over attractive. $(C+T)_Y$ is the complete Yamaguchi potential, whereas $(C+T)_N$ is the Naqvi potential, which is an incomplete one in the sense that the original potential also contained a short-range spin-orbit term, which in our treatment is dropped for simplicity.

As for the α-N force, it consists of an S-wave part and a P-wave part. The S-wave part which is relevant mainly for the α-N scattering problem is, however, prevented by the Pauli principle from entering into the ^{6}Li calculations, and will henceforth be neglected.[18] The P-wave part was shown in Ref. 12 to consist of two terms: (1) a dominant central term and (ii) a small spin-orbit term (of positive sign), which together fitted the $P_{1/2}$ and $P_{3/2}$ phase shifts of Miller and Phillips.[19] In their treatment of ^{6}He, however, HHK have considered only an effective central P-wave force tuned exclusively to the $P_{3/2}$ data,[20] thus ignoring the $P_{1/2}$ phase shifts. HHK justified this parametrization on the ground that in the energy region important for ^{6}He, the partial cross sections for $P_{1/2}$ are smaller than the $P_{3/2}$ cross sections. While this is true, we believe that the larger $P_{3/2}$ phase shifts should be obtained through an attractive spin-orbit term in conjunction with a central term, as in Ref. 12, rather than through a single effective term as in HHK. This is particularly important in view of the recognition that the effectiveness of a noncentral force (like spin-orbit) in three-body binding is much more limited than that of the central force, in much the same way as a central+tensor force in the triplet N-N state gives less binding for ^{3}H than does an effective central force. At this stage, we only incorporate the central part of the P-wave term in the three-body Schrödinger equation, and the spin-orbit part is left to be considered at most perturbatively at a later stage.

[14] We note in this connection that the validity of the three-body approach to ^{6}Li at positive energies has been demonstrated by the work of P. E. Shanley [Phys. Rev. Letters **21**, 627 (1968)] who has obtained rather good fits to the data on α-d scattering.

[15] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954); **95**, 1635 (1954).

[16] J. Naqvi, Nucl. Phys. **36**, 578 (1962).

[17] G. L. Schrenk and A. N. Mitra, Phys. Rev. Letters **19**, 530 (1967).

[18] It might be thought at first sight that the neglect of the (repulsive) S-wave force between α and N is questionable, since it is really the Pauli principle that brings it about in the first place. The important point however is to decide on systems where this force plays an important role, and those *where* it should be neglected. Certainly the S-wave α-N force plays a very important role in α-N scattering where it makes an important contribution to the cross section. On the other hand, the Pauli principle also tells us at the same time that in a *bound* system like ^{6}Li or ^{6}He, the n and/or p must reside in a *P shell* with respect to the core (i.e., the α particle), so that the S-wave part of the α-N force does not, so to say, get a chance to play its role in such a system. While a completely satisfactory explanation of this apparent anomaly must depend on a six-body treatment of ^{6}Li or ^{6}He, it looks physically plausible enough that the neglect of the S-wave α-N force in the ^{6}Li calculation must really also be justified by the Pauli principle.

[19] P. D. Miller and G. C. Phillips, Phys. Rev. **112**, 2043 (1958); C. L. Chritchfield and D. C. Dodder, *ibid.* **76**, 602 (1949).

[20] W. Pearce and P. Swan, Nucl. Phys. **78**, 433 (1966).

As in Ref. 12, we take the P-wave α-N interaction (still a matrix in spin space) of the form

$$-2M_R\langle \mathbf{q} \mid v_{\alpha-N} \mid \mathbf{q}'\rangle = 4\pi\lambda_\alpha v_1(q)v_1(q') \times \sum_m Y_{1m}^*(\hat{q})\{1+(\mathbf{L}\cdot\boldsymbol{\sigma}/t')\}Y_{1m}(\hat{q}'), \quad (2.5)$$

where

$$v_1(q) = q(q^2+\beta_\alpha^2)^{-1} \quad (2.6)$$

with

$$\lambda_\alpha = 0.0884 \text{ F}^{-1}, \quad \beta_\alpha = 0.9727 \text{ F}^{-1}, \quad t' = 21.6. \quad (2.7)$$

M_R is the reduced mass of the α-N system, given by

$$M_R = MM_\alpha/(M+M_\alpha) = \eta M \quad \text{with } a = M/M_\alpha,\ \eta = (1+a)^{-1}. \quad (2.8)$$

This gives an effective strength parameter $\lambda_{\alpha\frac{3}{2}}$ for the $P_{3/2}$ state as $\lambda_{\alpha\frac{3}{2}} = 0.0925$ F^{-1}. These values may be compared with the corresponding parameters of HHK, normalized to the expression (2.5)

$$\lambda_{\alpha\frac{3}{2}} = 0.0773 \text{ F}^{-1}, \quad \beta_\alpha = 0.8 \text{ F}^{-1}. \quad (2.9)$$

To estimate the strength of the P-wave force of HHK, let us compare the volume strength parameter which is related to λ_α/β. This quantity turns out to be 0.0967 for HHK parameters, whereas our parameters yields the value 0.0909. Even with our effective-strength parameter $\lambda_{\alpha\frac{3}{2}}$, this value is 0.0951. The P-wave force in the HHK calculation appears, therefore, to be somewhat over attractive. In Sec. 3 we give the essential formulation of the three-body problem for the ground state (1^+).

3. THREE-BODY EQUATIONS

We consider the nucleus in its c.m. system and choose the momenta of the nucleons as $\mathbf{P}_1$ and $\mathbf{P}_2$, so that the momentum of the α particle $\mathbf{P}_3$ is $-(\mathbf{P}_1+\mathbf{P}_2) = -\mathbf{P}$. The relative momenta $\mathbf{q}_{13}$ and $\mathbf{q}_{23}$ for the α-N pair are

$$\mathbf{q}_{13,23} = \{M_\alpha \mathbf{P}_{1,2} + M(\mathbf{P}_1+\mathbf{P}_2)\}/(M+M_\alpha) = \mathbf{P}_{1,2} + a\eta\mathbf{P}_{2,1}. \quad (3.1)$$

The potentials in the momentum space of $\mathbf{P}_1$ and $\mathbf{P}_2$ are normalized according to

$$\langle \mathbf{P}_1\mathbf{P}_2 \mid V_{n-p} \mid \mathbf{P}_1'\mathbf{P}_2'\rangle = \langle \mathbf{q}_{12} \mid v_{n-p} \mid \mathbf{q}_{12}'\rangle \delta(\mathbf{P}_1+\mathbf{P}_2-\mathbf{P}_1'-\mathbf{P}_2') \quad (3.2)$$

with

$$\mathbf{q}_{12} = \tfrac{1}{2}(\mathbf{P}_1-\mathbf{P}_2), \quad (3.3)$$

$$\langle \mathbf{P}_1\mathbf{P}_3 \mid V_{\alpha-N} \mid \mathbf{P}_1'\mathbf{P}_3'\rangle = \langle \mathbf{q}_{13} \mid v_{\alpha-N} \mid \mathbf{q}_{13}'\rangle \delta(\mathbf{P}_2-\mathbf{P}_2'), \quad (3.4)$$

where the reduced elements in Eqs. (3.2) and (3.4) are given by Eqs. (2.3)–(2.4) and (2.5)–(2.6), respectively. The three-particle Schrödinger equation now reads

$$D_E\Psi(\mathbf{P}_1\mathbf{P}_2) = -M(V_{n-p}+V_{\alpha-n}+V_{\alpha-p})\Psi(\mathbf{P}_1\mathbf{P}_2), \quad (3.5)$$

where

$$D_E = \tfrac{1}{2}P_1^2+\tfrac{1}{2}P_2^2+\tfrac{1}{2}aP_3^2-ME = (2\eta)^{-1}(P_1^2+P_2^2)+a\mathbf{P}_1\cdot\mathbf{P}_2+\alpha_T^2 \quad (3.6)$$

with

$$E \equiv -E_b = -\alpha_T^2/M. \quad (3.7)$$

Substitution of the factorable shapes of the potentials in (3.5) leads, in the standard manner,[6] to the following expressions for the wave function in terms of two spectator functions $\mathcal{G}$ and F describing, respectively, the associations $[\alpha, (NN)]$ and $[(\alpha N), N]$

$$\Psi(\mathbf{P}_1, \mathbf{P}_2) = D_E^{-1}\{g(\mathbf{q}_{12})\mathcal{G}(\mathbf{P}) + v(q_{13})\mathbf{q}_{13}\cdot\mathbf{P}_2F(P_2) + v(q_{23})\mathbf{q}_{23}\cdot\mathbf{P}_1F(P_1)\} \mid \chi_{m_s}{}^1\rangle, \quad (3.8)$$

where

$$\mathcal{G}(\mathbf{P}) = G(P)+\sqrt{8}^{-1}H(P)S_{12}(\hat{P}) \quad (3.9)$$

and

$$v(q) = q^{-1}v_1(q). \quad (3.10)$$

$\mid \chi_{m_s}{}^1\rangle$ is the spin triplet wave function. For convenience, we have explicitly exhibited in (3.8) the vector structures of the α-N wave function $D_E^{-1}\mathbf{q}_{13}v(q_{13})$ and the $[(\alpha N), N]$ spectator wave function $\mathbf{P}_2F(P_2)$ in which $F(P_2)$ is merely a scalar function of its argument. We have taken only the central part of the α-N interaction as mentioned in Sec. 2. The integral equations for the spectator functions for the ground state are

$$\{\lambda_t^{-1}-h_t(p)\}G(p) = (-2\eta)\int d^3qF(q)A(\mathbf{qp}), \quad (3.11)$$

$$\{\lambda_t^{-1}-h_t(p)\}H(p) = (-2\eta)\int d^3qF(q)B(\mathbf{qp}), \quad (3.12)$$

and

$$\tfrac{1}{3}\{\lambda_\alpha^{-1}-h_\alpha(p)\}p^2F(p) = \int d^3qF(q)K(\mathbf{qp}), \quad (3.13)$$

TABLE II. The comparison of the binding energies in MeV of ^{6}Li and ^{6}Li* with different potentials, calculated with our parameters and those of HHK. The Coulomb-corrected binding energy $E_{\text{expt}}(1^+) = 3.7+(0.83) = 4.53$ MeV, and $E_{\text{expt}}(0^+) = 0.13+(0.83) = 0.96$ MeV.

State	V_{N-N} potential	$V_{\alpha-N}$ potential	E_b (MeV)
$J^P = 1^+$	$C_Y{}^{\text{eff}}$	Ours	6.011
		HHK	8.7
	$(C+T)_Y$	Ours	4.974
	$(C+T)_N$	Ours	4.560
$J^P = 0^+$	Eqs. (2.1) and (2.2)	Ours	0.2
		HHK	2.1

where

$$K(\mathbf{qp}) = K(\mathbf{pq}) = \frac{v(\mathbf{p}+a\eta\mathbf{q})v(\mathbf{q}+a\eta\mathbf{p})\{a^2\eta^2p^2q^2+a\eta(p^2+q^2)\mathbf{p}\cdot\mathbf{q}+(\mathbf{p}\cdot\mathbf{q})^2\}}{\{(p^2+q^2)+2\eta\alpha_T{}^2+2a\eta\mathbf{p}\cdot\mathbf{q}\}} + \eta^{-1}\int d^3x\{\lambda_t{}^{-1}-h_t(x)\}^{-1}\{A(\mathbf{px})A(\mathbf{qx})+B(\mathbf{px})B(\mathbf{qx})\} \quad (3.14)$$

with

$$A(\mathbf{xy}) = \frac{C(\mathbf{x}+\tfrac{1}{2}\mathbf{y})v(\mathbf{y}+\eta\mathbf{x})\{\eta x^2+\mathbf{x}\cdot\mathbf{y}\}}{\{x^2+(2\eta)^{-1}y^2+\alpha_T{}^2+\mathbf{x}\cdot\mathbf{y}\}}, \quad (3.15)$$

$$B(\mathbf{xy}) = \frac{T(\mathbf{x}+\tfrac{1}{2}\mathbf{y})P_2(\mathbf{u}_{\mathbf{x}+\frac{1}{2}\mathbf{y}}\cdot\hat{y})v(\mathbf{y}+\eta\mathbf{x})\{\eta x^2+\mathbf{x}\cdot\mathbf{y}\}}{\{x^2+(2\eta)^{-1}y^2+\alpha_T{}^2+\mathbf{x}\cdot\mathbf{y}\}}, \quad (3.16)$$

where $\mathbf{u}_{\mathbf{x}+\frac{1}{2}\mathbf{y}} = (\mathbf{x}+\frac{1}{2}\mathbf{y})/|\mathbf{x}+\frac{1}{2}\mathbf{y}|$,

$$h_t(x) = \int d^3y\{C^2(y)+T^2(y)\}/\{y^2+\tfrac{1}{4}(1+2a)x^2+\alpha_T{}^2\}, \quad (3.17)$$

$$h_\alpha(x) = \int d^3y v_1{}^2(y)/\{y^2+(1+2a)\eta^2x^2+2\eta\alpha_T{}^2\}. \quad (3.18)$$

$P_2(\mu)$ is the Legendre polynomial of order two. For the excited state 0^+, the same formulation is utilized with $t=0$ and λ_t, β_t, and $|\chi_{m_s}{}^1\rangle$, replaced by λ_s, β_s, and $|\chi_{m_s}{}^0\rangle$, respectively.

4. RESULTS AND DISCUSSION

The solution of Eqs. (3.11)–(3.13) for the eigenvalue problem of determining the binding energy parameter $\alpha_T{}^2$ proceeds on the familiar lines of first *assuming* an input value of this parameter and calculating the quantity λ_α by the standard determinantal method. This "three-body" determination of λ_α should, of course, match the "two-body" value (2.7) or (2.9), as the case may be. Hence, the input value of $\alpha_T{}^2$ is adjusted, until the matching is sufficiently accurate. With our parameters (2.7) for $V_{\alpha-N}$ as well as those of HHK (2.9), we have used this method to evaluate the binding energy of (1^+) state for the N-N potential as given in Table I and the binding energy of (0^+) state with the parameters (2.2). The results are given in Table II which also includes the Coulomb-corrected experimental values.[21] Table II bears out the fact that the effective central potential $C_Y{}^{\text{eff}}$ is over attractive for the three-body system. The complete Yamaguchi potential including central as well as tensor force does give a better result. The very good agreement for the binding energy with Naqvi's set need not be taken too seriously as it is an incomplete force, though one must recognize that the neglected part, being a short-range noncentral force, may well be unimportant. Thus for the ground state (1^+) the binding energy is somewhat larger compared to the (Coulomb-corrected) experimental one. Table II also bears out our earlier assertion of an over-attractive $(P_{3/2})$ potential used by HHK which seems to give too large a binding energy with $C_Y{}^{\text{eff}}$ for the ground state. For the ^{6}Li* state, we get a lower value for the binding energy, although the experimental number itself is quite small in this case. Our estimate of the same quantity with HHK parameters appears appreciably larger. On the other hand, the value quoted by HHK, obtained with the same parameters, is only 1.06^{+}_{-}MeV, compared to our estimate of 2.1 MeV. We also note the decisive role of the N-N tensor force which was not considered earlier.

The eigenvectors of Eqs. (3.11)–(3.13), or the spectator functions in the ground-state wave function have

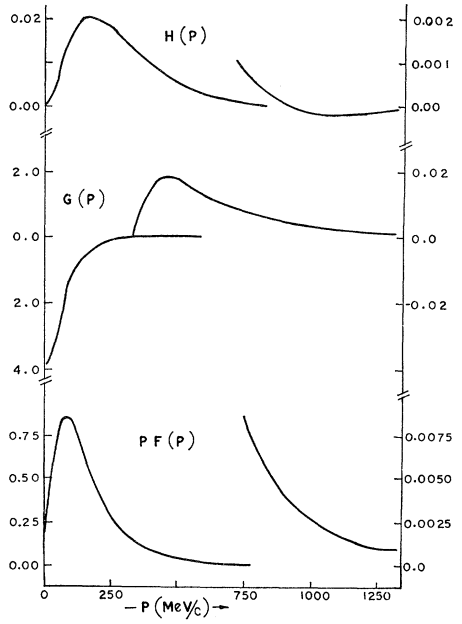

FIG. 1. Spectator functions P $F(P)$, $G(P)$, and $H(P)$ are plotted against momentum P for the parameter $\alpha_T{}^2=0.12$ F^{-2}, i.e., $E_b=4.947$ MeV. The right-hand scale and curves refer to the magnified plot of the spectator functions for larger values of P.

[21] T. Lauritsen and F. Ajzenberg-Selove, Nucl. Phys. **78**, 1 (1966).

also been evaluated, and are shown in Fig. 1. It may be noted that $G(P)$ has a negative sign for low values of P, and eventually becomes positive (small) for moderately large momenta. Its dominant part is essentially negative. This merely reflects the fact that in the "polarization" α-NN of the ^{6}Li system which $G(P)$ represents, one has an effective *repulsion*. This repulsion is, of course, more than offset by the attraction in the other two pairs N-αN as represented by the spectator function $PF(P)$, which is clearly seen to be positive. It is also worth noticing that the spectator functions $G(P)$ and $H(P)$ corresponding to the central part and tensor part, respectively, have opposite phases in most of momentum space, in contrast with their behavior in the three-nucleon system.[22] This is possibly due to the other pairs's being in the P state in the case of ^{6}Li. The important fact about the spectator function $PF(P)$, which more or less behaves like the form factors $v_1(P)$ of $C(P)$, is that it has sizable components for momenta above 300 MeV/c, whereas the harmonic oscillator wave function fitted to the gross structure of ^{6}Li goes practically to zero for large momenta. This feature of the "exact" wave function obtained with reasonably short-range potentials tuned to the α-N scattering data should have important consequences in other areas of investigations requiring fairly accurate ^{6}Li wave function. One such calculation on the pion capture in ^{6}Li is now in progress.[23]

The over-all normalization

$$\sum \iint d^3P_1 d^3P_2 \mid \Psi(\mathbf{P}_1\mathbf{P}_2) \mid^2 = 1$$

tells directly about the probability of the D state contained in the ground-state wave function. This probability is now no more a free parameter to be fitted to the observed data. It is a dynamical output, once the spectator functions are calculated by fixing the potential. As we are working in the total c.m. frame, it should be clarified that the D state which we are talking about is of the N-N system explicitly coming from the tensor force in the N-N system and not from the P-wave structure of the α-N system. This D-state probability turns out to be 2.66 and 1.44% for the $(C+T)_Y$ and $(C+T)_N$ parameters, respectively. This value of D-state probability does not fit in with the quadrupole moment of ^{6}Li, which is negative. However, a fuller comparison of the quadrupole moment would require a more careful treatment of the $\mathbf{L}\cdot\boldsymbol{\sigma}$ term in the $V_{\alpha-N}$ potential which is known to be necessary to give the negative sign for the quadrupole moment. To incorporate the effect of $\mathbf{L}\cdot\boldsymbol{\sigma}$ term in the binding energy, we have calculated it in a perturbative manner. In the first-order perturbation, this correction is zero for the excited state (0^+). For the ground state it is again zero for the wave function obtained with the C_Y^{eff} potential. The correction is proportional to the D state, and hence we expect it to be very small. The first-order correction to the binding energy is only 5 keV. Thus, we expect only a very small value of this contribution to the binding energy even in a more adequate treatment.[24] We believe that this accounts largely for the difference of our results from those of HHK who parametrize the $\mathbf{L}\cdot\boldsymbol{\sigma}$ term through an effective ($P_{3/2}$) force and overestimate the three-body attraction. A second (but smaller) reason for the difference may be that HHK have fitted the α-N resonance to the 0.95 MeV (c.m.) values of Pierce and Swan,[20] while we use the value 2.35 MeV (c.m.) given by Miller and Phillips.[19]

So far we have not been able to test the effects of short-range repulsion in the N-N potential, since this cannot be taken into account perturbatively. There also remains the question of the extent to which the composite structure of the α particle could possibly affect the calculations, though one expects such effects to be extremely small. In conclusion, the results of our calculations show that this three-body model of ^{6}Li works satisfactorily for the calculation of binding energies.

ACKNOWLEDGMENT

We are grateful to Professor S. P. Pandya for helpful comments.

[22] B. S. Bhakar, Ph.D. thesis, Delhi University, 1965 (unpublished).

[23] M. S. Shah (to be published).

[24] However the effect of the $\mathbf{L}\cdot\boldsymbol{\sigma}$ term on the ^{6}Li wave function could be more important; and hence also on its quadrupole moment. This last calculation has not been possible so far, because of the excessive amount of work involved (algebraic and numerical).