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Spectroscopic Studies of Molybdenum Isotopes with (d, t) Reactions*

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The (d,t) reactions on Mo^{94}, Mo^{96}, and Mo^{100} were studied with 17-MeV incident deuterons. The experimental angular distributions of the triton groups were compared with distorted-wave Born-approximation (DWBA) calculations to determine the l values and the spectroscopic factors. The I^{π} and excitation energy of the levels found in this study are compared with those from (d,p) reactions leading to the same final states. The occupation numbers and single quasiparticle energies are calculated and compared with previous results.

INTRODUCTION

In a recent paper,[1] a study of the odd-A molybdenum isotopes by use of (d,p) reactions was reported. In this paper we report a complementary study of these isotopes by use of (d,t) reactions. This gives a check on I^{π} assignments, and on whether any of the states have been missed because of interference from impurities in the target or, in some cases, because of low cross sections for (d, p) reactions. It determines whether all the major components of each single-quasiparticle (SQP) state have been found. It gives completely independent determinations of occupation numbers and of SQP energies to check those determined from the (d, p) experiments. And finally, it gives information on the "pure-hole" states, those which consist of a hole in a completely filled shell.

Experimental studies of the levels of Mo isotopes by (d, t) reactions have been reported by Hjorth and Cohen[2] and very recently by Ohnuma and Yntema.[3] Their results will be referred to frequently here, but their energy resolutions were much poorer than those in the present experiment.

EXPERIMENTAL

Incident 17-MeV deuterons were obtained from the University of Pittsburgh three-stage Van de Graaff accelerator. The tritons were magnetically analyzed with an Enge split-pole spectrograph and detected with photographic emulsion plates in the focal plane of the spectrograph. Measurements at 5° intervals were taken from 10 to 45°. (A detailed description of the scattering chamber and the spectrograph system is given in Ref. 4.)

The impinging beam was collimated by a 1.0-mm-wide (1.5-mm for Mo^{94}) by 3-mm-high target slit. The antiscattering slit was 3 mm wide by 5 mm high. The Faraday-cup-to-slit current ratio averaged greater than 50:1. The tritons entered the spectrograph thru an entrance aperture of 1.4 msr. The elastically scattered deuterons were monitored by two NaI(Tl) scintillation crystals placed at ±38° with respect to the incident beam. Absolute elastic cross sections at 38° were determined using targets of sufficient thickness that direct target thickness measurements could be made. The monitored elastic deuterons were then used to normalize the differential cross sections for the (d, t) reaction in question.

The Mo targets were prepared by vapor deposition of the Mo isotopes onto 20-μg/cm^2 carbon foils. The thickness and isotopic purity of the targets are listed in Table I.

A typical energy spectrum is shown in Fig. 1. The numbers attached to the peaks are the excitation energies (in MeV). Peaks not so designated

TABLE I. Target thicknesses, and isotopic purities.

Target	Thickness (μg/cm^2)	Isotopic purity (%)
Mo^{100}	71	96
Mo^{96}	48	97
Mo^{94}	26	94

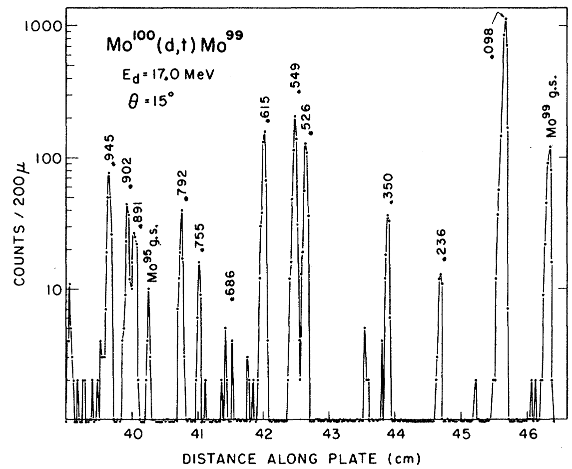

FIG. 1. Typical energy spectrum. Numbers are excitation energies of corresponding states of Mo^{99} in MeV. Peaks without numbers are due to impurities in the target.

are due to impurities in the target. The peaks with assigned energies are found at almost all the angles studied; where they are missing they are obscured by peaks from a low-Z impurity. The energy resolution for this study varied from 7–10 keV.

RESULTS AND ANALYSIS

Figure 2 shows the triton angular distributions

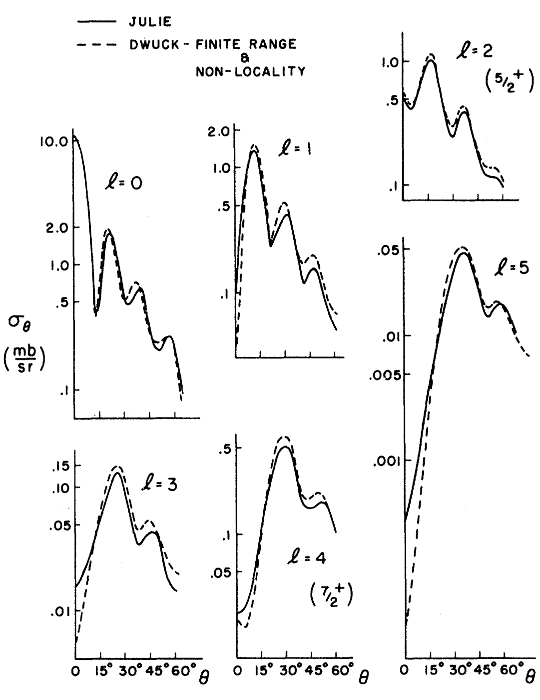

FIG. 2. Angular distributions of tritons from $Mo^{100}(d, t)$ for $Q = -3.0$ MeV as calculated by DWBA.

TABLE II. Optical-model parameters used in DWBA calculations.

	V (MeV)	r_0 (F)	r_C (F)	a (F)	W_S (MeV)	$4W_D$ (MeV)	r_0' (F)	a' (F)	V_{so} (MeV)
Deuteron[5]	95.3	1.15	1.15	0.81	⋯	66.6	1.34	0.68	⋯
Triton[6]	153.0	1.24	1.25	0.69	20.8	⋯	1.42	0.89	⋯
Neutron	⋯	1.25	1.25	0.65	⋯	⋯	⋯	⋯	$\lambda = 25$
Finite-range correction parameter 0.845									
Nonlocality correction parameters $\beta_d = 0.54$, $\beta_t = 0.3$									

predicted by the distorted-wave Born-approximation (DWBA) calculation with the optical-model parameters listed in Table II; those for deuterons are the Perey average parameters,[5] and the triton parameters are from Ref. 6. Code JULIE[7] was used for the standard DWBA calculation and code DWUCK[8] was used when finite-range and nonlocality effects were to be included. Spin-orbit coupling was included in both cases. All angular distributions shown in Fig. 2 are for $Q = -3.0$ MeV. One can see from this figure that the addition of finite range and nonlocality does not change the angular distributions, but it increases the calculated reaction cross sections by 10–20%.

The angular distributions corresponding to an l-value transfer of 0 are forward-peaked with a secondary maximum at 20°. The angular distributions for $l = 1$, 2, 3, 4, and 5 transfers are peaked at 10, 15, 22, 28, and 33°, respectively. The l value of each transition was determined by comparing the experimental angular distribution with those predicted by the DWBA calculation.

For (d, t) reactions, the relationship between the cross section and the spectroscopic factor for a transition with a total angular momentum transfer j is given by[9]

$$(d\sigma/d\Omega)_j^{(i)} = 5 \times \tfrac{2}{3} \times S_j^{(i)}\, \sigma_j(\theta)\,,$$

where $\sigma_j(\theta)$ is the cross section calculated by the appropriate DWBA code, and $S_j^{(i)}$ is the spectroscopic factor for the ith level. The sum of the spectroscopic factors for all transitions of a given j is related to the "fullness" V_j^2 of the quasiparticle state by

$$\sum_i S_j^{(i)} = (2j+1)\, V_j^{\,2}\,.$$

The spin assignments for the $l = 2$ transfers were made by taking the ratio of spectroscopic factors in (d, t) and (d, p) reactions. If the state i includes a fraction f_i of the SQP state,

$$\frac{S_j^{(i)}(d,t)}{S_j^{(i)}(d,p)} \approx \frac{f_i\,(2j+1)V_j^{\,2}}{f_i\,(1-V_j^{\,2})} = \frac{(2j+1)V_j^{\,2}}{1-V_j^{\,2}}\,.$$

Since the $d_{5/2}$ level lies lower and therefore fills before the $d_{3/2}$ level, V_j^2 is larger for the former than for the latter, so the ratio for a transition to a $\frac{5}{2}^+$ state will be larger than for a $\frac{3}{2}^+$ state. Analogous arguments allow a distinction between $g_{7/2}$ and $g_{9/2}$ states; large and small ratios are expected for $g_{9/2}$ and $g_{7/2}$ states, respectively.

The energies of this paper are based on an improved calibration of the Enge split-pole spectrograph. In Tables III, IV, and V, the energy levels seen in the (d, p) experiments at this laboratory (Ref. 1) have been corrected for the new calibration.

$Mo^{94}(d, t)Mo^{93}$

The angular distributions for the $Mo^{94}(d, t)Mo^{92}$ reaction are shown in Fig. 3. Table III summarizes the results and compares them with those from a previous (d, t) study[3] and from the $Mo^{92}(d, p)Mo^{93}$ reaction.[1] Six levels have been found up to 2.0 MeV excitation.

The ground state of Mo^{93} is known to be $\frac{5}{2}^+$, and its measured angular distribution is fitted very well by an $l = 2$ transfer. The $S(d, t)/S(d, p)$ ratio of this level clearly is in agreement with the spin assignment. The first excited state at 0.948 MeV is excited by an $l = 0$ transition and therefore is a $\frac{1}{2}^+$ state. Again we agree with previous work.

The transition to a weakly excited peak at 1.368 MeV was assigned from its angular distribution to be $l = 4$. Ohnuma and Yntema[3] reported indications of this level in their (d, t) study, but could not obtain a satisfactory angular distribution. We assigned its spin to be $\frac{7}{2}^+$, which is consistent with a $Nb^{93}(p, n)Mo^{93}$ study[10] and with the (d, p) work. The fact that it is more strongly excited by the (d, p) reaction indicates the validity of the assignment.

We have resolved the peak that Ohnuma and Yntema reported at 1.50 MeV into two peaks: one at 1.486 MeV and another at 1.500 MeV. The former is clearly excited by an $l = 4$ transition and the latter by an $l = 2$ transition. The 1.486-MeV state corresponds to the $\frac{9}{2}^+$ state observed[11] at 1.477 MeV in the β decay of Tc^{93}. The fact that this level is not excited by the (d, p) reaction confirms that it is a $\frac{9}{2}^+$ state. The 1.500-MeV $l = 2$ state corresponds to the 1.502-MeV state observed by the (d, p) reaction. The ratio of $S(d, t)/S(d, p)$

TABLE III. Data for $Mo^{94}(d,t)Mo^{93}$ reactions.

$Mo^{94}(d,t)Mo^{93}$ Present work							$Mo^{94}(d,t)Mo^{93}$ Ohnuma and Yntema[a]				$Mo^{92}(d,p)Mo^{93}$ Moorhead and Moyer[b]			
Excitation energy (MeV)	σ_{max} (μb/sr)	l	I^π	$\frac{S(d,t)}{S(d,p)}$ –	$S(d,t)$ DWBA	$S(d,t)$ DWUCK	Excitation energy (MeV)	l	I^π	$S(d,t)$	Excitation energy (MeV)	l	I^π	$S(d,p)$
0.0	3990	2	$5/2^+$	1.71	1.491	1.392	0.0	2	$5/2^+$	1.1	0.0	2	$5/2^+$	0.87
0.948	715	0	$1/2^+$	0.32	0.204	0.196	0.95	0	$1/2^+$	0.1	0.950	0	$1/2^+$	0.64
1.368	24	4	$7/2^+$	0.29	0.076	0.075					1.371	4	$7/2^+$	0.26
1.486	109	4	$9/2^+$		0.686	0.666	1.50	4	$9/2^+$	(0.5)				
1.500	246	2	$3/2^+$	0.53	0.267	0.241		2	$3/2^+$	(0.2)	1.502	2	$3/2^+$	0.50
1.529	17	4	$7/2^+$	0.42	0.059	0.058					1.529	4	$7/2^+$	0.14
											1.706	2	$3/2^+$	0.18
											2.157	0	$1/2^+$	0.007
											2.194	2	$3/2^+$	0.053
											2.320	(5)	$11/2^-$	0.33
											2.415	2	$3/2^+$	0.043
							2.44	4	$9/2^+$	3.5				
											2.455	0	$1/2^+$	0.071
											2.555	...	...	...
							2.56	1	$(1/2^-)$	0.9				
											2.688	0	$1/2^+$	0.009
											2.721	0	$1/2^+$	0.32
											2.858	0	$1/2^+$	0.026
											2.899	3	$(7/2^-)$	0.047
											2.991	...	...	...
											3.045	4	$7/2^+$	0.047
											3.086	...	...	...
											3.179	2	$3/2^+$	0.20
											3.230	...	...	...
							3.25	1	$(1/2^-)$	1.0				

[a]See Ref. 3. [b]See Ref. 1.

TABLE IV. Data for $Mo^{96}(d,t)Mo^{95}$ reactions.

$Mo^{96}(d,t)Mo^{95}$ Present work							$Mo^{96}(d,t)Mo^{95}$ Ohnuma and Yntema[c]				$Mo^{94}(d,p)Mo^{95}$ Moorhead and Moyer[b]			
Excitation energy (MeV)	$\left(\frac{\mu b}{sr}\right)$ σ_{max}	l	I^π	$\frac{S(d,t)}{S(d,p)}$	$S(d,t)$ DWBA	$S(d,t)$ DWUCK	Excitation energy (MeV)	l	I^π	$S(d,t)$	Excitation energy (MeV)	l	I^π	$S(d,t)$
0.0	9137	2	$5/2^+$	4.6	2.736	2.536	0.0	2	$5/2^+$	1.7	0.0	2	$5/2^+$	0.59
0.205[a]	178	2	$(5/2)^+$	4.0	0.076	0.067	0.19	2	$3/2^+$	0.1	0.204	(2)	$(3/2)^+$	0.019
0.767[a]	130	4	$7/2^+$	5.5	0.985	0.814					0.769	4	$7/2^+$	0.18
0.787	777	0	$1/2^+$	0.4	0.165	0.152	0.79	{0	$1/2^+$	0.2	0.789	0	$1/2^+$	0.37
0.822	427	2	$(3/2)^+$	1.0 – 1.1[b]	0.240	0.212		2	$(3/2)^+$	0.5	0.823	2	$(5/2)^+$	0.17
0.950	86	4	$7/2^+(9/2)^+$	12	0.705(0.348)	0.582(0.334)	0.94	2	$(5/2)^+$	0.2	0.953	4	$(7/2)^+$	0.060
1.041	884	0	$1/2^+$	1.0	0.208	0.195	1.06	0	$1/2^+$	0.2	1.044	0	$1/2^+$	0.19
1.059	404	2	$(5/2)^+$		0.197	0.186								
											1.310	0	$1/2^+$	0.004
											1.376	2	$3/2^+$	0.030
1.431	27	2	$3/2^+$	0.81	0.021	0.019					1.433	2	$3/2^+$	0.026
1.624	164	2	$3/2^+$	0.93	0.139	0.125	1.67	2	$(5/2)^+$	0.1	1.629	2	$3/2^+$	0.15
1.670	37	2	$(5/2)^+$		0.024	0.023								
1.683	88	4	$(9/2)^+$		0.48	0.46								
											1.707	(0)	$(1/2^+)$	0.006
1.946	24	5	$11/2^-$	1.2	0.32	0.29					1.949	5	$11/2^-$	0.26
1.978	72	2	$5/2^+$	7.0	0.056	0.054					1.981	2	$5/2^+$	0.008
2.056	90	2	$3/2^+$	0.99	0.099	0.090					2.059	2	$(3/2)^+$	0.10
2.067	35	(3)	$(5/2^-)$		0.20	0.18					2.066	0	$1/2^+$	0.097
											2.107	2	$3/2^+$	0.055
											2.136	4	$7/2^+$	0.11
2.181	38	2	$3/2^+$	0.38	0.046	0.042					2.185	2	$3/2^+$	0.12
2.221	50	1	$(1/2)^-$		0.026	0.027								
2.260	34	2	$3/2^+$	0.88	0.044	0.040					2.261	2	$3/2^+$	0.050
2.330	611	1	$(1/2)^-$		0.352	0.366	2.35	1	$(1/2^-)$	0.5				
											2.340	1	$(3/2^-)$	0.006
											2.377	0	$1/2^+$	0.058
											2.383	2	$3/2^+$	0.036
											2.396	2	$3/2^+$	0.040
2.441	233	4	$9/2^+$		1.75	1.82	2.46	4	$(9/2^+)$	2.1				
											2.468	...	...	...
											2.508	2	$(3/2^+)$	0.006
2.539	166	4	$9/2^+$		1.30	1.38	2.57	4	$(9/2^+)$	1.3				
2.558	44	2	$5/2^+$		0.055	0.054					2.545	...	...	...

TABLE IV (*Continued*)

								2.567	(1)	(3/2⁻)	0.023
2.610	...	...	...								
								2.617	0	1/2⁺	0.055
2.680	...	...	...								
								2.694	...	...	...
								2.717	(2)	(3/2⁺)	0.018
								2.747	(2)	(3/2⁺)	0.006
								2.768	(2)	(3/2⁺)	0.025
								2.777	(2)	(3/2⁺)	0.017
				2.78	1	(1/2⁻)	0.3				
								2.855	2	3/2⁺	0.036
								2.874	2	3/2⁺	0.024
								2.943	...	...	...
								2.980	...	...	...
								3.060	2	3/2⁺	0.15
								3.082	0	1/2⁺	0.019
				3.11	1	(1/2⁻)	0.2				
								3.169	2	(3/2⁺)	0.031
								3.183	...	...	...
				3.48	1	(1/2⁻)	0.2				

[a]20% contribution from another isotope. [b]Ratio is taken considering both peaks with same I^{π} value. [c]See Ref. 3. [d]See Ref. 1.

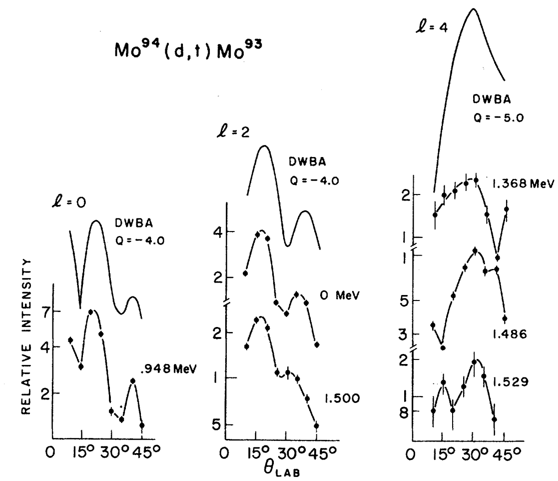

FIG. 3. Measured angular distributions of tritons from $Mo^{94}(d,t)$ grouped according to l values. Top curves (without data points) are from DWBA calculations. Numbers are excitation energies of corresponding states in MeV.

for this level suggests that it is a $\frac{3}{2}^+$ state. The state at 1.529 MeV corresponds to the 1.529-MeV state of the (d,p) work and is an $l=4$ transition. The $S(d,t)/S(d,p)$ ratio here again indicates a $\frac{7}{2}^+$ assignment. These three states in the 1.5-MeV region are also observed by β-decay work. The assignment from this study is in agreement with this experiment.

It is interesting to note that transitions to the $\frac{3}{2}^+$ state at 1.706 MeV do not appear in this work or in the previous (d,t) work. $Nb^{93}(p,n)Mo^{93}$ work assigned a $\frac{5}{2}^+$ level at 1.693 MeV. It is believed by us that this assignment is incorrect due to the fact that we do not observe an $l=2$ level in our work. If it were a $\frac{5}{2}^+$ state, it should be strongly excited by the (d,t) reaction.

$Mo^{96}(d,t)Mo^{95}$

Figure 4 shows the angular distributions for the $Mo^{96}(d,t)Mo^{95}$ reaction. Table IV summarizes the results and compares them with previous (d,t) and (d,p) work.

The ground state of Mo^{95} is known to be a $\frac{5}{2}^+$ state, and the measured angular distribution for transitions to this state agrees well with predictions for an $l=2$ transfer. The $S(d,t)/S(d,p)$ ratio is large, in agreement with the known $\frac{5}{2}^+$ assignment.

The first excited state at 0.205 MeV is weakly excited by an $l=2$ transition in the present work. A (d,t) study on a natural Mo target indicated that there is a 20% contribution to this peak from another isotope. From the $S(d,t)/S(d,p)$ ratio it is

TABLE V. Data from $Mo^{100}(d,t)Mo^{99}$ reactions.

Present work: Excitation energy (MeV)	σ_{max} (μb/sr)	l	I^π	$\frac{S(d,t)}{S(d,p)}$	$S(d,t)$ DWBA	$S(d,t)$ DWUCK	Moorhead and Moyer[b]: Excitation energy (MeV)	l	I^π	$S(d,p)$	Hjorth and Cohen[c]: Excitation energy (MeV)	l	I^π
0.0	2800	0	$1/2^+$	0.57	0.385	0.328	0.0	0	$1/2^+$	0.67	0.0	0	$1/2^+$
0.098	14 090	2	$5/2^+$	15	3.164	2.889	0.098	2	$5/2^+$	0.21	0.100	2	$5/2^+$
0.236	595	4	$7/2^+$	6.6	2.77	2.26	0.236	4	$7/2^+$	0.42	0.222	(3) (4)	$(7/2^-)(7/2^+)$
0.350	466	2	$3/2^+$	1.3	0.147	0.129	0.353	2	$3/2^+$	0.11	0.361	2	$3/2^+$
0.526	3317	0	$1/2^+$	12	0.506	0.442	0.529	0	$1/2^+$	0.042			
0.549	2636	2	$3/2^+$	2.1	0.901	0.789	0.552	2	$3/2^+$	0.43	0.545	2	$3/2^+$
0.615	1855	2	$5/2^+$	28	0.503	0.463	0.619	2	$5/2^+$	0.018			
0.686	180	5	$11/2^+$	7.7	1.08	0.863	0.688	5	$11/2^-$	0.14	0.664	(4) (5)	$(7/2^+)(11/2^-)$
0.755	235	3	$(5/2^-)$		0.538	0.480	0.760	(4)	$(7/2^+)$	0.22			
0.792	465	2	$(3/2^+)$?	2.5[a]	0.176	0.155	0.798	(3)	$(7/2^-)$	0.021	0.774	(2)	$(3/2^+)$
0.891	342	2	$3/2^+$	1.5	0.136	0.120	0.896	2	$3/2^+$	0.092	0.889	2	$3/2^+$
0.906	844	0	$1/2^+$	7.9	0.166	0.148		0	$1/2^+$	0.021			
							0.913						
								(2)	$(3/2^+)$	0.008			
0.945	1001	2	$(5/2^+)$		0.313	0.291	0.952	3	$7/2^-$	0.054			
							1.033	1	$(3/2^-)$	0.048			
							1.261	0	$1/2^+$	0.008			
							1.391	2	$3/2^+$				
							(1.453)						
							1.493	2	$3/2^+$				
							1.548						
							1.672						
							1.722	1	$(3/2^-)$				
							1.755	2	$(3/2^+)$				
							1.812						
							1.845						
							1.930	0	$1/2^+$				
							1.948	0	$1/2^+$				
							1.965	0	$1/2^+$				

[a] $S(d,p)$ taken from Ref. 2.
[b] See Ref. 1.
[c] See Ref. 2.

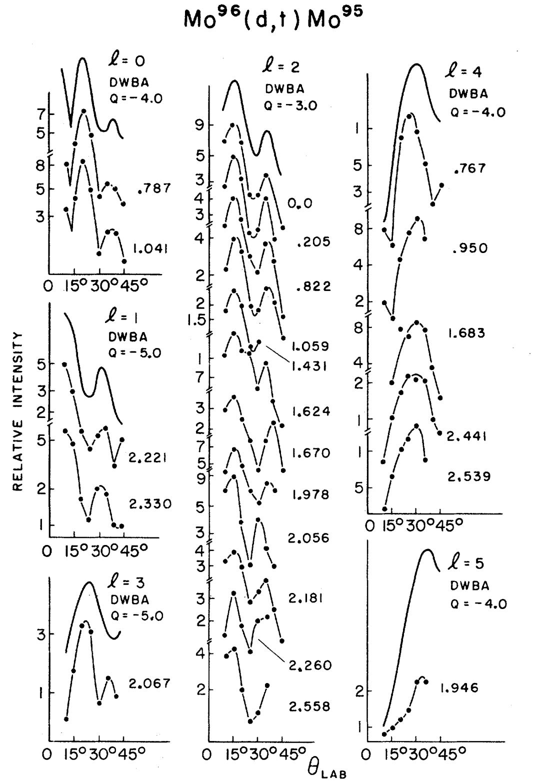

FIG. 4. Measured angular distributions of tritons from $Mo^{96}(d,t)$. See caption for Fig. 3.

assigned tentatively as $\frac{5}{2}^+$. This seemed to be contrary to the (d,p) work done by this group, but upon looking back at the older work, a numerical error was found; after correction the results are in agreement. However γ-ray studies[12] indicate that this level is a $\frac{3}{2}^+$ state. Since it is so weakly excited, and because of the presence of a contribution from another isotope, our assignment is questionable.

The transition to the 0.767-MeV state reported in the (d,p) studies at 0.769 MeV was assigned from its angular distribution as $l=4$. Since it is so weakly excited by (d,t) reactions and is excited more strongly by (d,p), we assigned it as a $\frac{7}{2}^+$ state. This agrees with results from β-decay[13] studies on Nb^{95}.

Ohnuma and Yntema[3] reported that the angular distribution for the 0.79-MeV peak could be fitted either by an $l=0$ or by an $l=2$ pickup. We resolved this peak into a 0.787-MeV $l=0$ state and a 0.822-MeV $l=2$ state. The $S(d,t)/S(d,p)$ ratio is ~1.0–1.1, which would probably indicate that the 0.822 state has a spin of $\frac{3}{2}^+$.

The angular distribution for transitions leading to the 0.950-MeV state indicates that it is $l=4$. This corresponds to the 0.953-MeV $l=4$ transition in $Mo^{94}(d,p)Mo^{95}$. β-decay studies[14] on Tc^{95} indicate that a state at 0.9478 MeV exists with a spin of $\frac{7}{2}^+$ or $\frac{9}{2}^+$. We have assigned it as most probably being a $\frac{7}{2}^+$ state since it is excited by the (d,p) reaction, although we cannot totally exclude the possibility that it is a $\frac{9}{2}^+$ state.

The factor of 2.5 difference between the ratio of $S(d,t)/S(d,p)$ for the $l=0$ transitions at 0.787 and 1.041 MeV is quite surprising and very unexpected for such strongly excited levels. An even greater difference of this type will be discussed

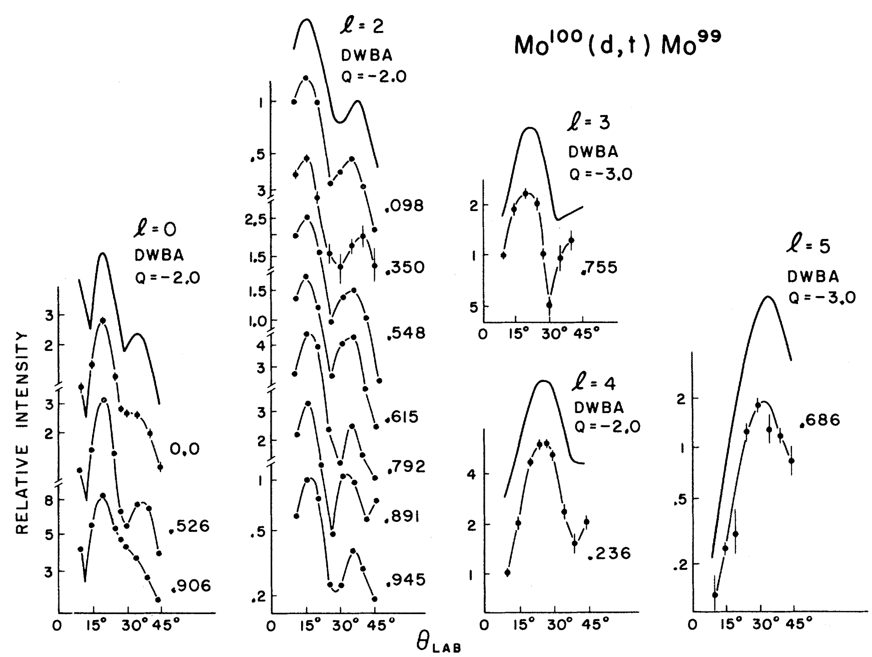

FIG. 5. Measured angular distributions of tritons from $Mo^{100}(d,t)$. See caption for Fig. 3.

in connection with the results from $Mo^{100}(d, t)$.

The $l=2$ transitions at 1.059, 1.670, and 2.558 MeV do not appear in the $Mo^{94}(d, p)Mo^{93}$ reaction and are assigned $\frac{5}{2}^+$ because of their large $S(d, t)/S(d, p)$ ratio. Using similar reasoning we assigned the $l=4$ transition at 1.683 MeV as $\frac{9}{2}^+$.

The only $l=5$ transition excited by either (d, t) or (d, p) was at 1.946 MeV. As expected for the $\frac{11}{2}^-$ state, it has a larger spectroscopic factor for the (d, p) reaction.

The angular distribution of the transition at 2.067-MeV excitation energy is assigned tentatively as $l=3$. Except for the 10° data, it could be $l=0$. Although it is unexpected, the $l=3$ transfer corresponds most likely to a $1f_{5/2}$ neutron pickup. Two $l=1$ transitions are excited by the present study at 2.221 and 2.330 MeV and are tentatively assigned to be $2p_{1/2}$ neutron pickups. Two $l=4$ transitions found at 2.441 and 2.539 MeV were assigned as $\frac{9}{2}^+$ states, because they were strongly excited in this study and not detected in (d, p) studies.

Results from $Mo^{100}(d, t)Mo^{99}$ Reaction

Figure 5 shows the triton angular distributions for $Mo^{100}(d, t)Mo^{99}$ reactions grouped according to l values. Table V summarizes the results and compares them with those from the (d, p) reaction[1,2] on Mo^{98}. The assignments of spin and parity are in agreement for the majority of the states. The differences between the results of the neutron pickup and neutron stripping will now be discussed.

The 0.760-MeV state excited in the (d, p) reaction may correspond to the 0.755-MeV state of this study. The (d, p) study assigned it with some reservations as an $l=4$ transfer. From the present study, its angular distribution is fitted by an $l=3$ transfer. It is surprising to see an $l=3$ transfer in either pickup or stripping reactions at such a low excitation energy. For neutron pickup, $l=3$ would correspond to removal of a neutron from the $1f_{5/2}$ level which is in the shell below the one that is filling, and for stripping, the neutron must be inserted into the $2f_{7/2}$ level which is in the major shell above the one that is filling. The two $l=3$ transfers at 0.798 and 0.952 MeV in the (d, p) reaction correspond to the 0.792- and 0.945-MeV levels in this study. From the triton angular distributions, these states are clearly fitted as $l=2$ transitions. Spin assignments for all $l=2$ transitions which appear in both neutron pickup and neutron stripping are made by taking the ratio of $S(d, t)/S(d, p)$, and assigning the large ratios as $\frac{5}{2}^+$ and the small ratios as $\frac{3}{2}^+$.

The most surprising differences between the stripping and pickup studies appears in the ratio of $S(d, t)/S(d, p)$ for the $l=0$ transitions. There is a factor of 21 difference in this ratio for the two states, which is nearly an order of magnitude larger than for any previously known case. In the (d, p) study, the ground state seems to contain most of the single quasiparticle state, but the (d, t) study suggests that the 0.526-MeV state contains most of it. This is a drastic breakdown in the quasiparticle picture (see Ref. 15 for a more comprehensive discussion).

DISCUSSION

The results for this study of Mo isotopes are summarized in Figs. 6, 7, and 8. Table VI shows a comparison between the summed spectroscopic factors for the three isotopes. For (d, t) reactions

$$\sum_{i} S_j^{(i)} = (2j+1)\, V_j^2 \ .$$

Thus, using

$$U_j^2 + V_j^2 = 1 \ ,$$

the emptiness of the single quasiparticle level U_j^2 can be determined. The U_j^2 from the present study are compared in Table VII with the values obtained by previous neutron stripping studies on the same target nucleus and on isotopes. With exceptions in Mo^{100}, the agreement among these studies is quite good for $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ states. There is also good agreement between the U_j^2 for Mo^{94} and its isotone Nb^{93}, which seems to indicate that the filling of the neutrons is independent of whether the nucleus has 41 or 42 protons. The poor agreement between the $U_{7/2}^2$ values determined by our (d, t) work and previous (d, p) studies is understandable, since many components of the $g_{7/2}$ state are probably missed in both experiments.

In Mo^{94} we see that the (d, p) results indicate

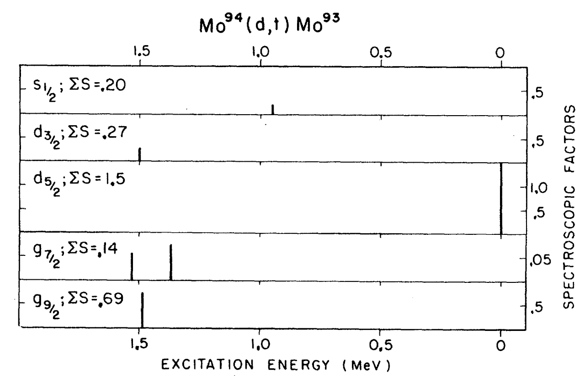

FIG. 6. Distribution of nuclear states of various I^π in Mo^{93} and their spectroscopic factors for excitation in $Mo^{94}(d, t)$ reactions.

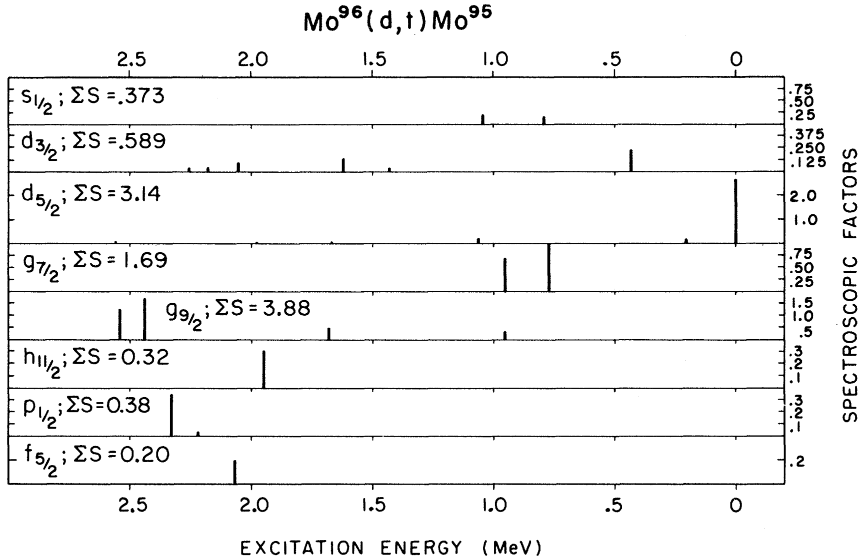

FIG. 7. Distribution of nuclear states of various I^{π} in Mo^{95} and their spectroscopic factors for excitation in $Mo^{96}(d,t)$ reactions.

TABLE VI. $\sum S\,(d,t)$ for Mo isotopes.

Level \ Target	Mo^{94}	Mo^{96}	Mo^{100}
$3s_{1/2}$	0.20	0.37	1.06
$2d_{5/2}$	1.49	3.1	4.0
$2d_{5/2}$	0.26	0.59	1.36
$lg_{7/2}$	0.13	1.69	2.8
$h_{11/2}$	~0	0.32	1.08

that the $\frac{1}{2}^+$ is more empty than the $\frac{11}{2}^-$ state, which is highly unexpected. The present experiment clarifies the situation, since the (d, t) cross sections are proportional to the "fullness" of the levels, whence the absence of $l = 5$ transitions indicates that the $h_{11/2}$ level must be very empty.

In the case of $Mo^{100}(d, p)$ and $Mo^{100}(d, t)$, the U_j^2 do not agree very well. The stripping study could not distinguish between $\frac{5}{2}^+$ and $\frac{3}{2}^+$ states, since the

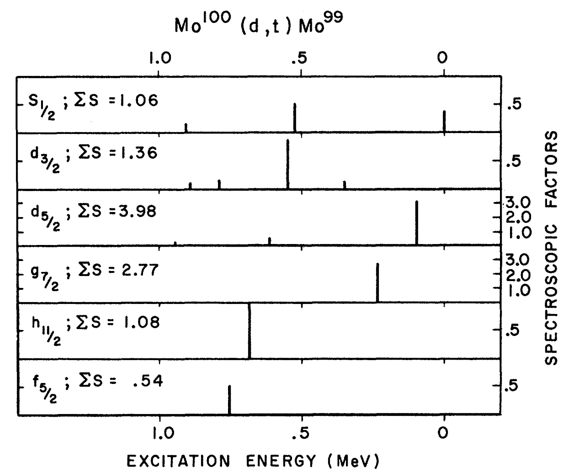

FIG. 8. Distribution of nuclear states of various I^{π} in Mo^{99} and their spectroscopic factors for excitation in $Mo^{100}(d,t)$ reactions.

TABLE VII. U_j^2 from various sources.

Target \ Level	$3s_{1/2}$	$2d_{5/2}$	$2d_{3/2}$	$lg_{7/2}$	$h_{11/2}$	Reference
Mo^{94}	0.90	0.75	0.93	0.98	~1.0	
	0.80	0.76	0.95	0.35	0.26	1
Nb^{93}	0.89	0.82	1.07	0.32	···	1
Mo^{96}	0.81	0.48	0.85	0.79	0.97	
	0.81	0.44	0.91	0.54	···	1
Zr^{94}	1.09	0.30	1.0	0.40	···	16
Mo^{100}	0.47	0.34	0.66	0.65	0.91	
	0.85	0.50	0.50	0.66	···	2

TABLE VIII. Single quasiparticle energies, E_j (MeV).

Target \ Level	$3s_{1/2}$	$2d_{5/2}$	$2d_{3/2}$	Reference
Mo^{93}	0.95	0.0	1.51	
	1.63	0.081	1.97	1
Mo^{95}	0.92	0.16	1.46	
	1.28	0.22	1.98	1
Mo^{99}	0.41	0.23	0.60	
	0.22	0.13	0.64	1

$Mo^{102}(d,t)Mo^{101}$ experiment is not feasible. In the present work, l assignments are made from the complementary stripping reaction, so the values of $U_{3/2}{}^2$ and $U_{5/2}{}^2$ are more reliable. The reason for the difference in the $U_{1/2}{}^2$ is connected with the anomaly previously mentioned.

The difference in the U_j^2 for Zr^{94} and Mo^{96} is a real one. It is due to the fact that as neutrons are added to the closed shell nucleus Zr^{90}, only the $d_{5/2}$ level fills significantly,[16] while in the Mo isotopes several neutron single-particle states are filling simultaneously.[2]

Table VIII compares the single quasiparticle energies $E_j{}'$ determined by this study and by the (d,p) work. The E_j were calculated from

$$E_j = (\sum_i S_j^{(i)} E_i) / \sum_i S_j^{(i)},$$

where E_i is the excitation energy of the ith level.

The energies of SQP levels for Mo^{93} and Mo^{95} are lower in this work than in the previous (d,p) studies. This is mostly due to the fact that pickup reactions are more likely to miss highly excited states than stripping reactions, since DWBA calculations indicate that σ_j decreases with increasing excitation energy for (d,t) while it increases for (d,p). The SQP energies determined in these experiments are therefore somewhat lower than the true values; the determinations from Ref. 1 are therefore generally confirmed. There are, however, two exceptions. In Mo^{99} the SQP ener-

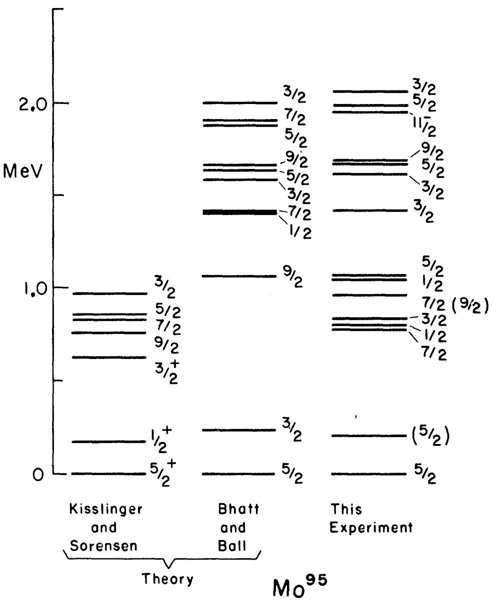

FIG. 10. Energy level spectrum of Mo^{95}. See discussion in text.

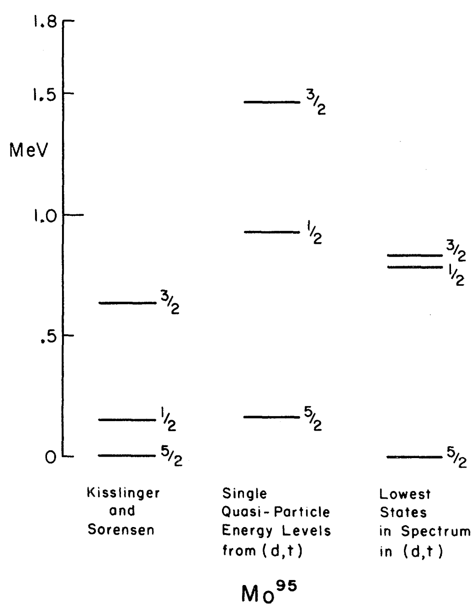

FIG. 9. Energy spectrum of single quasiparticle levels in Mo^{95}. See discussion in text.

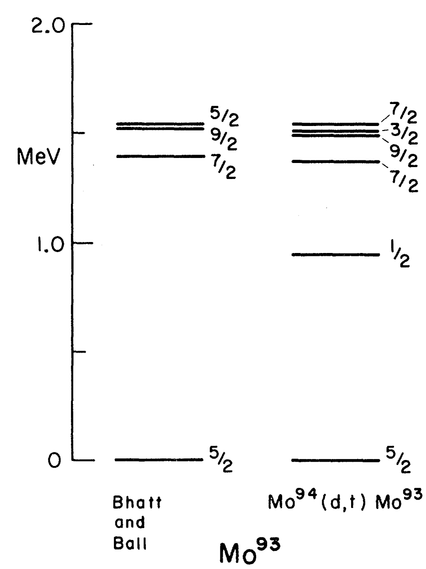

FIG. 11. Energy level spectrum of Mo^{93}. See discussion in text.

gies for $\frac{1}{2}^+$ and $\frac{5}{2}^+$ levels are higher in this work than in previous (d,p) experiments. In the case of the $\frac{5}{2}^+$ level, it is due to the fact that the 0.952-MeV peak is reported here as $\frac{5}{2}^+$, while the (d,p) work assigned it as $(\frac{7}{2}^-)$. The SQP energy for the $\frac{1}{2}^+$ level is higher because of the anomaly mentioned previously.

Figure 9 compares the SQP energies for Mo^{95}, as calculated by Kisslinger and Sorensen,[17] using pairing plus quadrupole interactions with the SQP levels and the lowest states of the same I^π determined in these experiments. The agreement is quite poor, which indicates that their model is inadequate.

The Mo^{95} levels determined by this experiment are compared with the level spectrum calculated by Kisslinger and Sorensen[17] and by Bhatt and Ball[18] (using effective interactions) in Fig. 10. Neither approach successfully predicts the Mo^{95} levels. If the 0.204-MeV state is actually a $\frac{3}{2}^+$ state as indicated by the β-decay work, the effective interaction gives a correct description there. It also seems to predict the three levels around 1.6–1.7 MeV rather well. However, the pairing theory seems to be more successful around 0.8–1.0 MeV. In the case of Mo^{93}, shown in Fig. 11, Bhatt and Ball's calculations[18] agree rather well with the experimental results.

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