Nuclear-Structure Calculations for Cr^{52} and Even Ni Isotopes

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Employing Ca^{48} and Ni^{56} as cores for Cr^{52} and Ni^{60}, respectively, the shell-model properties of these nuclei are described using the renormalized reaction matrix elements of the Hamada-Johnston nucleon-nucleon potential. For Cr^{52}, the seniorities 2 and 4 of the $f_{7/2}^4$ configuration are mixed through intermediate states of other configurations in an attempt to explain the branching ratios from $J=6$ and $J=5$ levels to the low-lying $J=4$ levels; and also the effect of increasing the configuration space is explicitly studied. A qualitative agreement between the calculated and experimental spectra is found, and the wave functions thus obtained are not found to be suitable for the description of transition rates. For Ni^{60}, the effect of including the additional $g_{9/2}$ orbital is also investigated. The calculated energy levels agree fairly well with the experimental levels, but the wave functions do not provide a satisfactory explanation for the observed transition rates. The even Ni isotopes are then described in the framework of the modified Tamm-Dancoff approximation (MTDA). The agreement for the energy levels for Ni^{60} between the shell-model and the MTDA results is of the same quality, but the MTDA result indeed shows an improvement over the shell-model results in describing the vibrational characteristic of the second excited 2^+ state. The MTDA results for Ni isotopes appear to be fairly satisfactory.

I. INTRODUCTION

THERE are three general approaches to shell-model calculations. In the conventional approach a detailed residual interaction, involving a number of adjustable parameters, i.e., the range, depth, and the exchange mixtures, is assumed and one then calculates the interaction matrix elements between states of arbitrarily many configurations. A major drawback of the conventional shell model is that a large number of basis states must be used in a problem of any complexity since it is usually impossible to decide *a priori* which states are of negligible importance. It is seldom in such calculations that the nature of the residual interaction is satisfactorily explored, and its origin remains rather obscured. In the second approach, pioneered by Talmi and his co-workers,[1] a very restrictive assumption about the possible configurations is made, and the interaction matrix elements, treated as adjustables, are determined by fitting certain observed nuclear energy levels. This approach has the undesirable feature that one has to decide beforehand on the number of configurations necessary to provide an adequate description of the nuclear states under consideration. In the third approach, pursued by Kuo and Brown,[2] the effective interactions are derived from free two-nucleon interactions. Such studies require a complete knowledge of the nucleon-nucleon interaction and in recent years there has been a considerable advance in our understanding of this problem.

The third approach has been applied previously for the $0d$-$1s$ shell[2] and for the Ni isotopes[3] using the Hamada-Johnston potential, with varying degrees of success. Recently, Kuo and Brown[4] have calculated the renormalized shell-model reaction matrix elements for the $0f$-$1p$ shell with the Hamada-Johnston nucleon-nucleon potential. These matrix elements give rather satisfactory results for some standard nuclear-structure calculations involving the interaction of two valence nucleons. They were further tested in calculations[5] describing the spectra of nuclei having three nucleons outside the Ca^{48} and Ni^{56} cores from the point of view of the configuration mixing, and yielded encouraging results. This paper is based on a continuation of the above work and describes first the results of the shell-model calculations on the properties of nuclei involving the interaction of four nucleons with Z or $N=28$ as the core, i.e., Cr^{52} and Ni^{60}, and then the quasiparticle results for the complete even Ni isotopes.

The low-lying levels of Cr^{52} have been the subject of extensive experimental investigation[6] for many years as they present some interesting problems. For configurations of identical nucleons in $f_{7/2}$, seniority (ν) is a good quantum number, regardless of the form of the residual two-body interaction, and in the jj coupling there will be two states of $J=2$; one with $\nu=2$ and the other with $\nu=4$. The same is true for $J=4$. In fact, the experimental level energies of Cr^{52} show that there exist two low-lying $J=2$ and 4 levels. The shell-model calculations for the energy spectra of Cr^{52} in terms of four $f_{7/2}$ proton configurations were first carried out by Edmonds and Flowers,[7] while Talmi[8] made the seniority assignments to these levels from the consideration of the level energies of $f_{7/2}^2$ and $f_{7/2}^4$ nuclei (Ca^{42} and Cr^{52}). According to his assignment the $J=4$ state with $\nu=4$ is lower

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than the $J=4$ state with $v=2$. This assignment had been shown by Talmi to be consistent with the observed γ-ray transitions.

On the basis of the early experimental data, de-Shalit[9] has performed a self-consistent calculation of the energy levels of the $f_{7/2}^4$ configuration. However, he selected the 3.614-MeV level as his seniority 4, $J=2^+$ level, whereas recent experimental work has shown this to be 5^+. One can also try the rule that as Cr^{52} is in the middle of the proton shell, $E2$ transitions between levels of the same seniority are forbidden,[10] but from the observed branching ratios of the transitions[11] from $J=6$ and 5 states to the two $J=4$ states, one expects that the two $J=4$ states with $v=2$ and 4 should be mixed by an appreciable amount; and, therefore, the states concerned cannot at all be constructed in the pure jj coupling. A seniority mixing calculation introduced by interaction through intermediate states of other configurations had been performed by Komoda[12] for Serber-Rosenfeld force mixtures, with the range parameter λ varying from 1.0 to 1.5. For Cr^{52}, the present calculation is similar to that of Komoda, but we have considered many more configurations, some of which were omitted by Komoda without any justification, and used the two-body reaction matrix elements reported in Ref. 4. Our calculation therefore does not have any adjustable parameters.

The shell-model results on Ni isotopes using Hamada-Johnston potential have already been reported earlier by Lawson *et al.*[3] The renormalized matrix elements used in the present shell-model calculation[13] for Ni^{60} are also for the Hamada-Johnston potential but they are different from those of Ref. 3 in the following sense. In the notation of Kuo and Brown, the bare matrix element is represented by the relation

$$G=G_S+V_L+V_L(Q/e)V_L+2G_S(Q/e)V_L.$$

The calculation of Ref. 3 does not include the contributions due to G_S, while the present matrix elements do. In general, these matrix elements differ slightly from those of Ref. 3, but the difference is appreciable in some cases, especially for $J=0$ states. The differences in the matrix elements influence the energy spectra in an unpredictable fashion. It is therefore difficult to assess the degree of agreement between these two sets of matrix elements simply by inspection; the resulting energy spectrum must be compared. When this comparison is made, it is found that the energy spectrum calculated with the present matrix elements agrees more closely with the observed one.

The quasiparticle calculations for all the even Ni isotopes are carried out in the framework of the MTDA.[14] This method describes the low-lying states of even nuclei as the superposition of zero-, two-, and four-quasiparticle states and uses a complete set of orthonormal and nonredundant quasiparticle basis states. All the working formulas required for this part of the calculation are contained in Ref. 14.

II. NUMERICAL CALCULATIONS, RESULTS, AND DISCUSSION

A. Shell-Model Calculations for Cr^{52}

In all the shell-model calculations, the basic two-body matrix elements of the residual interaction go as the primary input data. The two-body shell-model reaction matrix elements for the Hamada-Johnston potential, used in our Cr^{52} calculation, are listed by Kuo and Brown.[4] In deriving these matrix elements, the authors of Ref. 4 have assumed Ca^{48} as the core with $0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$, and $1p_{1/2}$ as the single-particle orbitals available to the nucleons outside this core, the unperturbed energies for which are 0.0, 5.9, 4.4, and 6.9 MeV, respectively.

In Cr^{52}, the four protons outside the Ca^{48} core can be distributed in the above orbitals in the following possible types of configurations:

$$|j^4J_\nu\rangle,\quad |j_1^3J_1\nu_1,j_2;J\rangle,\quad |j_1^2J_1,j_2^2J_2;J\rangle,$$
$$|j_1^2J_1,j_2j_3J_2;J\rangle,\quad \text{and}\quad |j_1j_2J_1,j_3j_4J_2;J\rangle.$$

In the work of Komoda, only configurations of the types

$$|f_{7/2}^3(J'\nu'),j;J\rangle \quad \text{and} \quad |f_{7/2}^2(J'),j^2(j'');J\rangle \qquad (j=f_{5/2},\ p_{3/2},\ \text{and}\ p_{1/2})$$

have been mixed with the configuration $|f_{7/2}^4J\nu\rangle$. Clearly, many configurations whose unperturbed energies are lower than those considered have been omitted.

The dimensions of the complete matrices involving all the possible four-proton excitations are, respectively, for $J^\pi=0^+$, 28×28; for 1^+, 54×54; for 2^+, 94×94; for 3^+, 91×91; for 4^+, 99×99; for 5^+, 75×75; for 6^+, 59×59; for 8^+, 22×22. Before diagonalizing these matrices as such, a reasonable truncation of our original basis is made by choosing a small unperturbed energy interval in which there is a large number of four-proton configurations, and their effect explicitly studied. In the present case, the truncation is made at unperturbed energies 16 and 18 MeV and will be referred to as approximations A and B, respectively. The dimensions of the matrices for the unperturbed energy (16 MeV)

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[12] T. Komoda, Nucl. Phys. **51**, 234 (1964).
[13] T. T. S. Kuo (private communication).
[14] M. K. Pal, Y. K. Gambhir, and R. Raj, Phys. Rev. **155**, 1144 (1967).

TABLE I. Energy levels (MeV) of Cr^{52} in approximations A and B. All the levels have been calculated with respect to the lowest one. Column 2 lists the dominant configuration common to both the approximations. The two entries α_2 and α_4 denote the percentage admixture of the states with seniority 2 and 4, respectively, of the $f_{7/2}^4$ configuration. The numbers in the parentheses correspond to the percentage admixture of the dominant configuration other than those of seniority 2 and 4, and the entry β is the sum of the percentage admixtures of the states of all other configurations.

J^π	Dominant configuration	Approximation A E (MeV)	α_2	α_4	β	Approximation B E (MeV)	α_2	α_4	β
0^+	$\lvert f_{7/2}^4\nu=0\rangle$	0.000(87.04)			12.96	0.000(86.99)			13.01
2^+	$\lvert f_{7/2}^4\nu=2\rangle$	1.898	88.40	0.11	11.49	1.869	87.18	0.14	12.68
4^+	$\lvert f_{7/2}^4\nu=2\rangle$	2.754	82.46	7.75	9.79	2.740	80.92	8.68	10.40
4^+	$\lvert f_{7/2}^4\nu=4\rangle$	3.056	8.93	81.73	9.34	3.051	10.09	80.42	9.49
6^+	$\lvert f_{7/2}^4\nu=2\rangle$	3.128	92.11		7.89	3.123	91.89		8.11
2^+	$\lvert f_{7/2}^4\nu=4\rangle$	3.659	0.30	90.24	9.46	3.647	0.42	89.53	10.05
5^+	$\lvert f_{7/2}^4\nu=4\rangle$	4.222		91.30	8.70	4.221		91.16	8.84
8^+	$\lvert f_{7/2}^4\nu=4\rangle$	4.975		93.27	6.73	4.975		93.20	6.80
2^+	$\lvert (f_{7/2}^3)_{7/2}, p_{3/2}\rangle$	6.131(76.10)	1.37	0.07	22.46	5.664(79.44)	2.49	0.03	18.04
4^+	$\lvert (f_{7/2}^3)_{7/2}, p_{3/2}\rangle$	6.720(62.62)	0.52	0.16	36.70	6.347(78.80)	0.73	0.12	20.35
1^+	$\lvert (f_{7/2}^3)_{5/2}, p_{3/2}\rangle$	6.794(76.44)			23.56	6.740(77.64)			22.36
3^+	$\lvert (f_{7/2}^3)_{7/2}, p_{3/2}\rangle$	7.013(73.61)			26.39	6.610(83.17)			16.83
0^+	$\lvert (f_{7/2}^3)_{3/2}, p_{3/2}\rangle$	7.089(85.29)			14.71	6.925(83.35)			16.65

are

0^+: 14×14, 1^+: 26×26, 2^+: 49×49,
3^+: 49×49 4^+: 57×57,
5^+: 45×45, 6^+: 39×39,
8^+: 17×17;

and for the unperturbed energy (18 MeV),

0^+: 19×19, 1^+: 38×38,
2^+: 67×67, 3^+: 69×69,
4^+: 77×77, 5^+: 63×63,
6^+: 52×52, 8^+: 21×21,

respectively.

A program is written for the CDC 6400 computer located at the State University of New York at Buffalo, which generates the required four-particle configurations and then sets up the matrix for a given set of single-particle states, total spin, and parity and diagonalizes it. The correctness of the program is tested by reproducing the results of some of the earlier calculations.[15,16] The results for the energy eigenvalues of Cr^{52} corresponding to the unperturbed energies 16 and 18 MeV are shown in the first column under the entries approximation A and approximation B of Table I. In this table, the entries α_2 and α_4 denote the percentage admixture of the seniority 2 and 4 of the $f_{7/2}^4$ configuration, the numbers in the parentheses give the percentage admixture of the dominant configuration (listed in the second column) other than seniority 2 and 4, and the entry β represents the sum of percentage admixture of the states of the other configurations.

The two states ($J=2$ and 4) with different seniority do not interact directly, but are mixed by interaction through intermediate states of other configurations. The lower $J=2$ state is mainly $\nu=2$ and the upper one is mainly $\nu=4$, with a very small component of the other seniority 4 (or 2) and is consistent with the conclusions of Refs. 8, 12. On the other hand the lower $J=4$ state is also dominant with $\nu=2$ and the upper one with $\nu=4$, but now the component of the other seniority 4 (or 2) is a little more ($\simeq$8–9%). This feature for $J=4$ is different from that of Ref. 12, where the seniority mixing is found to be appreciable.

TABLE II. $B(E2)$ values of Cr^{52} in units of $e_{\text{eff}}^2/\alpha^2$. Here α is the oscillator parameter and e_{eff} is the effective charge of the proton. The column headings denote the various $E2$ transitions connecting the ground level (0) and excited states (J). The subscripts on any excited state J have the following meaning: 2_1, 2_2 denote, respectively, the first and second excited states of angular momentum $J=2$.

	$0\text{-}2_1$	$0\text{-}2_2$	$2_2\text{-}2_1$	$4_1\text{-}2_1$	$4_2\text{-}2_1$	$2_2\text{-}4_1$	$2_2\text{-}4_2$	$6\text{-}4_1$	$6\text{-}4_2$	5-6	$5\text{-}4_1$	$5\text{-}4_2$
Approx. A	13.98	0.75×10^{-5}	2.75	0.440	2.58	1.60	0.018	0.214	2.16	1.84	1.88	0.536
Approx. B	14.56	0.17×10^{-3}	2.84	0.525	2.55	1.64	0.015	0.231	2.14	1.83	1.84	0.574

[15] S. Cohen, R. D. Lawson, M. H. MacFarlane, S. P. Pandya, and M. Soga, Phys. Rev. **160**, 903 (1967).
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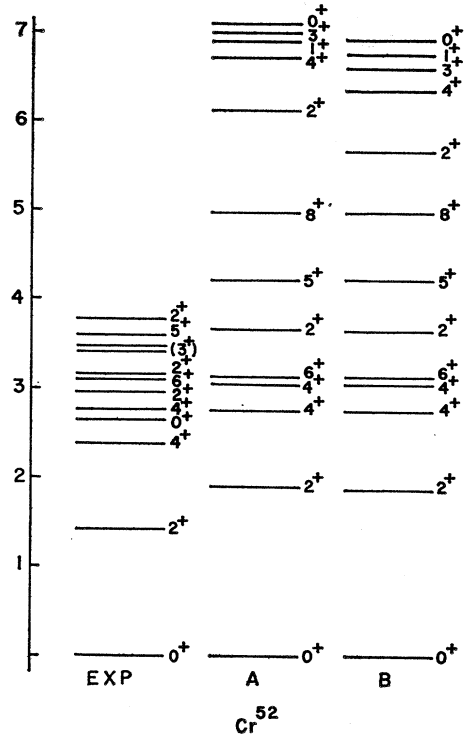

FIG. 1. Comparison of experimental (EXP) and shell-model spectrum of Cr^{52} in approximations A and B.

The admixture of the states of other configurations for $J=2$ and 4 is roughly one-half compared to Ref. 12, except for the second $J=2$ state where it is roughly one-third. Consequently, the transition probability is significantly affected, especially in cases in which one of the $J=4$ states is involved, and will be discussed later.

Figure 1 shows a comparison of the experimental and calculated level spectra for Cr^{52} for both approximations A and B. Most of the low-lying levels have the corresponding experimental levels but their energies lie slightly higher than the observe ones. There are a few discrepancies also; for example, the first excited 0^+ and (3^+) states have been pushed up quite high and one of the excited 2^+ states is missing. On the whole, the results for the approximations A and B differ very slightly below the 8^+ state but have significant differences above it.

The calculated $B(E2)$ and $B(M1)$ values are presented in Table II and III using the wave functions obtained for Cr^{52}. The measured value of the branching ratio[11] $B(E2{:}6\text{-}4_1)/B(E2{:}6\text{-}4_2)$ is 2.2. This suggests that there should be an appreciable amount of seniority mixing introduced by other configurations. This expectation is based on the special selection rule[10] for half-filled shells of identical nucleons, which forbids $E2$ transition between states for which $\Delta\nu=0$. Our calculated value for this ratio is 0.099 and 0.108, respectively, for approximations A and B. This disagreement can be understood in terms of a large admixture of $|f_{7/2}{}^4 J=4, \nu=2\rangle$ and $|f_{7/2}{}^4 J=4, \nu=4\rangle$ configurations in the lower and upper $J=4$ states.

The branching ratio from $J=5$ to the two $J=4$ states has been measured to be 1.7 by Freedman *et al.*[11] and 2.2 by Wilson *et al.*[11] Between these levels an $M1$ as well as an $E2$ transition is allowed. Our calculated value for this ratio is 11.09 and 10.08 for approximations A and B, respectively, and is independent of whether the $M1$ transition is included or not. The $M1$ transition between pure seniority states of identical particles (in this case four protons in $f_{7/2}$) vanishes because of the character of the magnetic moment operator.[10] In our case, the $J=5$ and the two $J=4$ states are more or less pure seniority states, and hence the $M1$ transitions from $J=5$ to the two $J=4$ states are very weak (Table III). This provides an explanation for the above ratio to be independent of $M1$ transition. The mixing ratio of the $E2$ and $M1$ transitions from the $J=5$ to the lower $J=4$ states is measured by Kaplan and Shirley[17] to be 3–23%. This again indicates the existence of configuration mixing in the states involved, otherwise the $M1$ transition would vanish. Our calculated value for this ratio is very poor for the reasons already mentioned. The same argument also holds for the very weak $M1$ transition from the $J=5$ to the $J=6$ states (Table III). The agreement obtained by Komoda[12] is better than ours.

In conclusion, we remark that the reaction matrix elements of Kuo and Brown give a qualitative agreement for the energy spectrum of Cr^{52} but the resulting wave functions are not suitable for the description of the transition rates. The effect of increasing the configurations (i.e., the results of approximation B) indicates that the present results cannot be improved much, at least for the few low-lying states, even if one further increases the configuration space.

B. Shell-Model Calculations for Ni^{60}

In describing the properties of Ni^{60}, the unpublished two-body effective matrix elements of the Hamada-Johnston potential calculated by Kuo and Brown,[13] which have been derived by assuming Ni^{56} to be the

TABLE III. $B(M_1)$ values of Cr^{52} in units of squared nuclear magneton. See Table II for complete explanation.

	$5\text{-}4_1$	$5\text{-}4_2$	5-6
Approx. A	0.07×10^{-6}	0.99×10^{-4}	0.07×10^{-4}
Approx. B	1.18×10^{-6}	0.92×10^{-4}	0.004×10^{-4}

[17] M. Kaplan and D. A. Shirley, Nucl. Phys. **37**, 522 (1962).

TABLE IV. Energy levels (MeV) of Ni^{60} in approximations A′ and B′. All the levels have been calculated with respect to the lowest one. Column *C* lists the results of Ref. 3 using only three orbitals. The entry *v* denotes the percentage admixture of the various seniority components. The seniority *v* of a shell-model configuration $|j_1^{n_1}(\nu_1)j_2^{n_2}(\nu_2)\cdots j_N^{n_N}(\nu_N);J>$ is defined as the sum $v=\sum_{i=1}^{N}\nu_i$.

		Approximation A′				Approximation B′		
J^π	*C*	*E* (MeV)	$v=0$	$v=2$	$v=4$	*E* (MeV)	$v=0$	$v=2$	$v=4$
0^+	0.00	0.00	99.73		0.27	−0.33	99.62		0.38
2^+	2.45	1.68		94.74	5.26	1.68		96.57	3.43
0^+	2.89	2.20	93.46		6.54	2.05	94.75		5.25
1^+		2.28		87.07	12.93	2.38		88.64	11.36
2^+	3.21	2.34		90.57	9.43	2.31		90.79	9.21
4^+	3.42	2.53		98.68	1.32	2.54		98.46	1.54
3^+	3.76	2.71		89.96	10.04	2.79		91.30	8.70
2^+		2.78		93.80	6.20	2.72		93.70	6.30

core with $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$, and $0g_{9/2}$ as the single-particle orbitals available to the nucleon outside the core, have been used. The unperturbed single-particle energies for the orbitals $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ were taken to be 0.0, 0.78, and 1.08 MeV, respectively, from the Ni^{57} spectrum. The unperturbed single-particle energy for $0g_{9/2}$ was taken to be 5.0 MeV, as used by Kuo.

We have performed this calculation for two different cases. In the first case, the four neutrons were distributed in all possible ways among $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbitals. The dimensions of the various matrices for 0^+, 1^+, 2^+, 3^+, and 4^+ are 9×9, 12×12, 21×21, 15×15, and 15×15, respectively. In the second case, the $0g_{9/2}$ orbital was also included and all the four neutrons were now distributed among $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$, and $0g_{9/2}$ orbitals in all possible ways. The configuration of the type $|0g_{9/2}^4 J\nu\rangle$ has not been considered because the unperturbed energy for such a configuration is too high. The dimensions of the matrices are now increased to 19×19, 27×27, 51×51, 43×43, and 51×51 for 0^+, 1^+, 2^+, 3^+, and 4^+, respectively. In the following, these two cases will be referred to as approximations A′ and B′ and sometimes merely as A′ and B′.

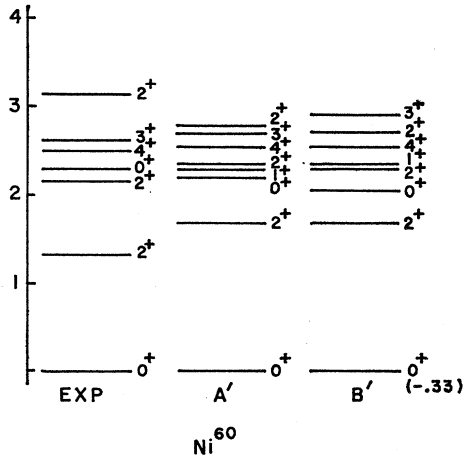

FIG. 2. Comparison of experimental (EXP) and shell-model spectrum of Ni^{60} in approximations A′ and B′.

The energy eigenvalues for Ni^{60} are shown in the first column of Table IV under the entries approximation A′ and approximation B′. The results of Ref. 3 are included under column *C* for comparison. The entries corresponding to the different column for *v* give the percentage seniority decomposition of the wave functions for a few of the low-lying states.

The ground state 0^+ has mainly seniority $\nu=0$ components. The excited states are not as pure as the 0^+ ground state and 4^+ excited state but the lowest-seniority components still predominate; higher-seniority components are of the order of 6%, except in the second 2^+, 1^+, and 3^+ states where it is of the order of 9% or a little more. The results of the two approximations A′ and B′ differ only slightly as regards the mixing of different seniority components are concerned. The few lowest states consist mostly of the lowest-seniority components, but the small higher-seniority components will be important for the calculation of transition probability and will be discussed later.

The ground state in approximation B′ is lowered by an amount 0.33 MeV as compared to the results in approximation A′, if the first 2^+ states in both the cases are reproduced at the same energy. A possible explanation for this may be that the shell-model interaction energies between the $0g_{9/2}$ orbital and the $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbitals are themselves strongly repulsive, or the inclusion of the $0g_{9/2}$ orbital without the excitation of the $0f_{7/2}$ core is not justified as the level at 2.6 MeV in Ni^{57} is strongly excited by $l=3$ pickup reactions and may reasonably be identified as a $0f_{7/2}$ single-hole state. The results under column *C* differ appreciably from those of approximations A′ and B′, and the results for the approximation A′ are much closer to the experimental values (Fig. 2) compared to those of approximation B′. The better agreement in approximation A′ may be because of the reasons mentioned above.

One can write the expression for the BCS wave function for Ni^{60} with the known values of occupation

TABLE V. $B(E2)$ values of Ni^{60} in units of e_{eff}^2 F^4 where $F=10^{-13}$ cm and e_{eff} is the effective charge of the neutron. See Table II for complete explanation. The last column gives the quadrupole moment of 2_1 states in barns ($=10^{-24}$ cm^2).

	2_1-0	2_2-0	2_2-2_1	2_2-0_1	0_1-2_1	4_1-2_1	4_1-2_2	$\langle Q\rangle/e_{eff}$
Approx. A′	42.08	14.12	13.28	18.69	10.74	28.11	0.46	0.092
Approx. B′	50.96	7.69	23.91	14.28	8.89	26.92	2.22	0.070

(nonoccupation) probability of the various single-particle levels which are obtained by solving the usual BCS equations for the chemical potential and energy gap. Then the part of this wavefunction that contains four particles can be projected out in a straightforward manner. The overlap of the projected (normalized) BCS wave function with the present shell-model ground-state wave function was found to be 94.33% for the set of results in approximation A′.

Figure 2 shows a comparison of the experimental and calculated level spectra for Ni^{60} for both approximations A′ and B′. On the whole, the results for A′ agree reasonably well with the experimental values, although the first excited 2^+ state is slightly higher and the third excited 2^+ state is slightly lower in energy compared to the experimental values. The ordering of the first excited 0^+ and the second excited 2^+ states is also reversed. It should be mentioned here that even in the work of Refs. 15, and 16 this ordering and the third excited 2^+ state were not reproduced correctly.

The transition probability and the quadrupole moment provide more crucial tests of the wave functions than the energy spectrum does. Table V gives the $B(E2)$ values for the various $E2$ transitions and the quadrupole moment of the first excited 2^+ state using the wave functions obtained in the present calculation for both the approximations A′ and B′. These are calculated using single-particle harmonic-oscillator wavefunctions whose oscillator parameter $\alpha=M\omega/\hbar$, where M is the nucleon mass, and

$$\hbar\omega=41A^{-1/3}\ \text{MeV}.$$

The effective charges[18] needed to fit the experimental $B(E2)$ value between the ground and first excited 2^+ states are $2.5e$ and $1.95e$ for the approximations A′ and B′, respectively. The branching ratio $B(E2:2_2\text{-}0)/B(E2:2_2\text{-}2_1)$ turns out to be 1.06 and 0.32 for the approximations A′ and B′, respectively, while the experimental value is 0.005. It has been argued that the inhibition of the crossover transition 2_2-0 as compared to the 2_2-2_1 indicates a vibrational characteristic. This selection rule follows from the fact that the transition 2_2-0, according to the vibrational model, is due to a transfer of two phonons, while 2_2-2_1 is a one-phonon transition. Analysis of our wave functions does not support this argument. We have already mentioned that the second excited 2^+ state contains predominantly seniority 2 components, whereas two-phonon states should have large seniority 4 components.

Further, the vibrational model also predicts that the branching ratio $B(E2:3^+_1\text{-}2^+_1)/B(E2:3^+_1\text{-}2_2)$ should be zero as the decay 3^+_1-2^+_1 is a two-phonon transition and is accordingly forbidden. This branching ratio has been measured[19] in Ni^{60} and indeed turns out to be very small (<0.004). Our calculated wave functions give this ratio to be 0.528 and 0.347 for the approximations A′ and B′, respectively. This disagreement can again be understood by remembering that our 3^+_1 state wavefunction is dominant with seniority 2 components. The difference between the results for the approximation B′ has little improvement over the approximation A′, at least for the cases discussed above.

In conclusion, it may be remarked that the matrix elements of Kuo and Brown give fairly good results for the energy spectrum, but do not provide satisfactory agreement for the transition probabilities.

C. Quasiparticle Results for Even Ni Isotopes

The shell-model results for Cr^{52}, described earlier, are not satisfactory, while those of Ni^{60} are fairly good at least for the energy levels. Also the results for the energy levels corresponding to the approximation A′ are in better agreement than those of the approximation B′, when compared with the experimental values. We shall therefore present here only the quasiparticle results for all the even Ni isotopes in approximation A′.

The following quantities, the quasiparticle energy and the occupation (nonoccupation) probability of the various single-particle levels, which enter as one of the basic input data in setting the energy matrix, are calculated with the knowledge of single-particle energies and the two-body reaction matrix elements of the Hamada-Johnston potential. The effect of the spurious 0^+ two-quasiparticle state from the basis wave functions is eliminated, using the recipe of Ref. 14, before the diagonalization of the energy matrix.

[18] It is found that the value of the effective charge, 1.13, quoted in Ref. 16 for Ni^{60} to fit the experimental $B(E2)$ value between the ground and first excited 2^+ states is in error, and should be $1.7e$. Auerbach's value of the effective charge fits only the quadrupole moment of the first excited 2^+ state of Ni^{60}. In order to confirm the correctness of our computer program we reproduced the values of Table VI and VII for Ni^{60} in Ref. 15. A similar error in Ref. 16 has been recently reported by P. Federman and L. Zamick [Phys. Rev. **177**, 1534 (1969)] for Ni^{58}.

[19] D. M. Van Patter and R. K. Mohindra, Phys. Letters **12**, 223 (1964).

TABLE VI. Calculated and experimental energy levels (MeV) of Ni isotopes. The label MTDA(TDA) refers to the quasiparticle results with (without) the mixing of different quasiparticle subspaces. The rows labelled 0qp(%) and 4qp(%) denote percentage admixture of zero- and four-quasiparticle states, and "Exact" denotes the results of shell-model calculations. The superscripts 2 and 4 over the numbers in the row TDA represent the two- and four-quasiparticle states.

A	J^π	0^+	0_1^+	0_2^+	2_1^+	2_2^+	2_3^+	4_1^+	4_2^+
58	Expt	0.00			1.45	2.78	2.90	2.46	
	Exact	0.00	2.36	4.18	1.34	2.21	2.86	2.33	3.80
	TDA	0.00	2.34	3.66	1.44	2.20	2.62	2.15	3.42
60	Expt	0.00	2.29		1.33	2.16	3.12	2.50	
	Exact	0.00	2.20	3.53	1.68	2.34	2.78	2.53	3.48
	TDA	0.00	2.13^2	2.27^4	1.39^2	2.28	2.31^2	2.09^2	2.80^4
	MTDA	0.00	2.38	2.98	1.76	2.36	2.65	2.46	3.15
	4qp (%)	13.37	13.26	47.57	2.83	59.51	6.58	3.50	74.34
	0qp (%)	86.61	0.09	0.02					
62	Expt	0.00	2.05		1.17	2.30		2.34	
	TDA	0.00	2.12^2	2.39^4	1.25^2	1.87^4	2.46^2	2.17^2	2.56^4
	MTDA	0.00	2.60	2.89	1.77	2.37	2.88	2.65	3.13
	4qp (%)	15.51	14.41	51.27	5.93	83.41	35.52	14.90	61.17
	0qp (%)	84.48	0.15	0.88					
64	Expt	0.00	2.28		1.34	2.89		2.62	
	TDA	0.00	2.19^2	2.53^4	1.16^2	1.87^4	2.51^2	2.33^2	2.63^4
	MTDA	0.00	2.67	2.95	1.66	2.37	2.93	2.72	3.27
	4qp (%)	14.57	4.01	59.90	1.68	95.03	22.81	21.37	72.98
	0qp (%)	85.38	0.61	0.16					
66	TDA	0.00	2.33^2	2.42^4	1.20^2	2.54^2	2.56^4	2.50^2	3.11^2
	MTDA	0.00	2.46	2.92	1.46	2.77	2.79	2.71	3.34
	4qp (%)	9.05	20.59	56.52	0.20	86.12	1.72	5.05	3.59
	0qp (%)	90.86	2.45	4.28					

The calculated energy levels for first few states with $J^\pi = 0^+, 2^+, 4^+$ of even Ni isotope are shown in Table VI. The rows labeled "Expt" and "Exact" correspond to the experimental values and the results obtained by the shell-model calculations. The row TDA represents the results obtained by the diagonalization of the matrices in the pure two- and four-quasiparticle subspaces and the superscripts 2 and 4 over the numbers represent the two- and four-quasiparticle states, while the row MTDA corresponds to the results obtained by mixing the zero-, two-, and four-quasiparticle states and the percentage admixtures of zero- and four-quasiparticle states are shown in the rows marked 0qp (%) and 4qp (%), respectively. The number of neutrons outside the closed shells in Ni^{58} being only 2, we restricted ourselves for this nucleus to zero- and two-quasiparticle states only.

From Table VI, one can see that the MTDA results for Ni^{60} are very close to the "exact" results compared to the TDA results. In other words, the mixing of different quasiparticle subspaces is important for achieving the qualitative agreement of the same quality as has been obtained by the shell-model method. From this table, one can also make the following general observations: (1) The ground state is always mainly of the zero-quasiparticle type with the four-quasiparticle component up to $\simeq$16%; (2) the first excited 2^+ state is very predominantly of the two-

TABLE VII. $B(E2)$ values of Ni isotopes in units of $e_{\text{eff}}^2\,F^4$ where $F = 10^{-13}$ cm and e_{eff} is the effective charge of the neutron. See Table V for complete explanation.

A	2_1-0	2_2-0	2_2-2_1	2_2-0_1	0_1-2_1	4_1-2_1	4_1-2_2	$\langle Q\rangle/e_{\text{eff}}$
60	47.79	0.72	23.33	0.09	0.01	4.50	2.46	−0.044
62	48.33	3.43	16.55	1.93	4.25	0.61	0.01	0.017
64	47.56	0.35	8.77	1.94	1.11	8.39	1.34	0.049
66	34.78	0.01	4.52	1.96	2.84	21.06	1.51	0.093

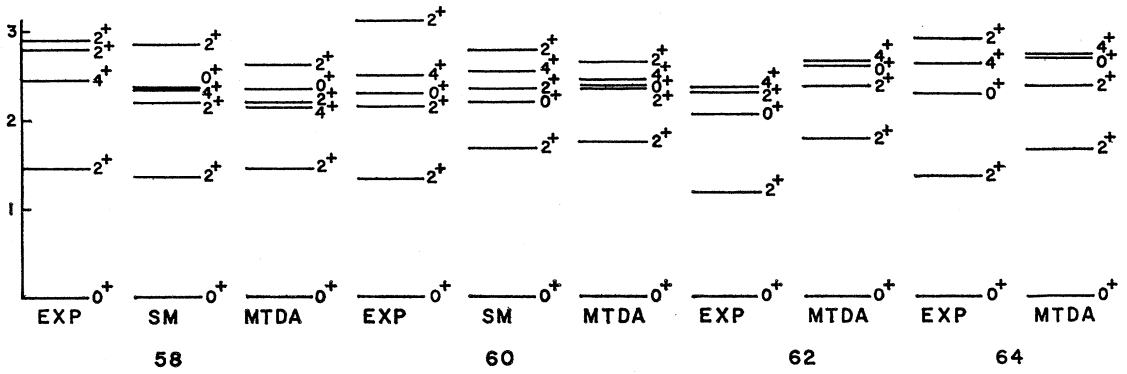

FIG. 3. Comparison of experimental (EXP) and quasiparticle (MTDA) spectrum of even Ni isotopes (58–64). SM: shell-model results.

quasiparticle type with the admixture of four-quasiparticles up to $\simeq 6\%$; (3) the first excited 0^+ and 4^+ states are also of the two-quasiparticle type but the admixture of the four-quasiparticle states now increases to $\simeq 21\%$; (4) the second excited 0^+ state has a four-quasiparticle component nearly equal to the two-quasiparticle component or slightly greater, and the second excited 4^+ state is dominant by four-quasiparticle, except in Ni^{66} where it is very predominantly of the two-quasiparticle type, but still there is a considerable two-quasiparticle component in them; (5) the second excited 2^+ state is also mainly of the four-quasiparticle type, but there is an appreciable two-quasiparticle component in the case of only Ni^{60}; while the third excited 2^+ state is dominantly two-quasiparticle, still Ni^{62} and Ni^{64} have a considerable amount of four-quasiparticle components.

The calculated $B(E2)$ values for the various $E2$ transitions and the quadrupole moment of the first excited 2^+ state using the wave functions obtained from the MTDA calculations are presented in Table VII. These are again calculated using single-particle harmonic-oscillator wave functions and the comparison of the values for Ni^{60} of the MTDA calculations with the corresponding values of the shell-model calculations (Table V) shows a more sensitive dependence on these two types of wave functions. In the case for Ni^{60}, the effective charge needed to fit the experimental $B(E2)$ value between the ground and first excited 2^+ states is $2.01e$ and the branching ratio $B(E2{:}2_2\text{-}0)/B(E2{:}2_2\text{-}2_1)$ is 0.03. There is a considerable improvement for this branching ratio over the shell-model value and the present calculations indeed show the vibrational characteristics while the shell-model calculations do not. The respective values for the above effective charges and the branching ratios for Ni^{62} and Ni^{64} are $1.86e$, $1.91e$, and 0.21, 0.04, respectively. However, this ratio is considerably larger for Ni^{62} but still they indicate the inhibition of the crossover transition 2_2-0 as compared to the transition 2_2-2_1. In other words, the analysis of the MTDA wave functions does show that the first excited 2^+ state has the characteristics of one-phonon vibration having a very strong $E2$ transition to the ground state, and the second excited 2^+ state has the characteristics of two-phonon vibration with a much weaker crossover 2_2-0 transition than for the 2_2-2_1 transition; but the first excited 0^+ and 4^+ levels do not show much of the systematics of two-phonon vibration.

Figure 3 shows a comparison of the experimental and MTDA level spectra for the even Ni isotopes. A similar calculation for the level spectra has been performed by Gambhir[20] using the two-body reaction matrix elements of Ref. 3. The present results for the level spectra show considerable improvement over the results of Ref. 21. It should be mentioned here that the results obtained by other authors[15,16] in this mass region are only tolerably satisfactory and provide the rough trend of the energy spectra.

In conclusion, we remark that the MTDA calculations using the reaction matrix elements of Kuo and Brown give fairly satisfactory results for even Ni isotopes.

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[20] Y. K. Gambhir, Nucl. Phys. **A120**, 193 (1968).