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Modified Alaga Rules for Allowed Beta Decay of Deformed Nuclei

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Considerations based on the projection-operator method indicate the need for modifications of Alaga's asymptotic quantum-number selection rules for allowed beta transitions of deformed nuclei. Certain transitions previously classified as allowed-hindered can be expected to have beta moments only slightly smaller than those of allowed-unhindered transitions. The distribution of logft values for the $150 \leq A \leq 190$ region seems to show such an effect, and detailed calculations for several cases give results in agreement with the available data.

For the evaluation of nuclear matrix elements a method of approximation, which involves the commutator of the nuclear Hamiltonian with the transition operator, has previously been developed.[1] In the case of Gamow-Teller β^- transitions, $|i\rangle \rightarrow |f\rangle$, the original transition operator, $\tau_-\vec{\sigma}$ is replaced by the effective operator

$$(\tau_-\vec{\sigma})_{\text{eff}} \cong \frac{Q}{(E_f - E_i - \Delta)} 2\kappa\tau_-(i\vec{1}\times\vec{\sigma}), \tag{1}$$

in which

$$Q = 1 - \frac{(\tau_-\vec{\sigma})|i\rangle\langle i|(\tau_-\vec{\sigma})^\dagger}{\langle i|(\tau_-\vec{\sigma})^\dagger(\tau_-\vec{\sigma})|i\rangle}, \tag{2a}$$

$$E_i + \Delta = \frac{\langle i|(\tau_-\vec{\sigma})^\dagger H(\tau_-\vec{\sigma})|i\rangle}{\langle i|(\tau_-\vec{\sigma})^\dagger(\tau_-\vec{\sigma})|i\rangle}, \tag{2b}$$

where E_i is the energy of the initial nuclear state $|i\rangle$, for which a suitable model wave function is substituted, and the approximation has been made that the only term in the commutator $[H, \tau_-\vec{\sigma}]$ which has important matrix elements between $|i\rangle$ and $|f\rangle$ is due to the one-body spin-orbit part[2] of H: $H_{s-o} = -2\kappa\vec{1}\cdot\vec{s}$. In short, the approximation corresponds to first-order perturbation theory starting from the collective state $\tau_-\vec{\sigma}|i\rangle$, whose decay is prescribed by

$$\frac{d}{dt}(\tau_-\vec{\sigma}) = i[H, \tau_-\vec{\sigma}].$$

It has been noted by Mottelson[3] that the operator $(\tau_-\vec{\sigma})_{\text{eff}}$ of Eq. (1) has different asymptotic quantum-number selection rules in the deformed regime, from those given by Alaga[4] for the original operator, $\tau_-\vec{\sigma}$:

$$\Delta N = \Delta n_z = \Delta\Lambda = 0. \tag{3a}$$

The selection rules for $i(\vec{1}\times\vec{\sigma})$ are:

$$\Delta N = 0, \quad |\Delta n_z| = |\Delta\Lambda| = \pm 1 \text{ or } 0, \tag{3b}$$

and include rules (3a) as a special case. Allowed Gamow-Teller transitions in heavy nuclei, however, always involve proton orbitals with $N = n_z + \Lambda$, $\Sigma = \uparrow$, e.g., in the $150 \leq A \leq 190$ region, the proton orbitals [532↑], [523↑], [514↑], etc. For $|\Delta K| = 1$ transitions involving these proton orbitals, the largest matrix elements of $i(\vec{1}\times\vec{\sigma})$ are found between states where the original rules (3a) are obeyed. For example,

$$\frac{\langle \pi N n_z \Lambda\uparrow|\tau_-(i\vec{1}\times\vec{\sigma})_{+1}|\nu N n_z \Lambda\downarrow\rangle}{\langle \pi N n_z \Lambda\uparrow|\tau_-(i\vec{1}\times\vec{\sigma})_{+1}|\nu N, n_z-1, \Lambda-1, \uparrow\rangle} = -\Lambda\left(\frac{2}{n_z}\right)^{1/2}. \tag{4}$$

Thus transitions allowed under rules (3a) may still be expected to be less retarded, by something like an order of magnitude, than the additional transitions allowed under rules (3b).

The amount of experimental data on logft values of allowed transitions in heavy nuclei has increased considerably in recent years. In the figure is collected, from a compilation of Artna-Cohen and Gove,[5] a histogram of logft values for allowed transitions in the region $150 \leq A \leq 190$.[6] The asymptotic quantum-number assignments are not known for many of these cases, but the simple argument above seems consistent with the interpretation that transitions obeying rules (3a) correspond to the first peak (log$ft \approx 4.9$), and those obeying rules (3b) [but not rules (3a)] correspond to the second peak (log$ft \approx 5.9$) in the distribution. The large gap in earlier distribution plots[4] between allowed-unhindered and allowed-hindered transitions is no longer evident.

More detailed calculations have been carried out for three examples, for which the asymptotic quantum-number assignments are fairly clear cut, and the results are given in the table. In obtaining these results, the deformation parameter δ was assumed to have the value 0.30; the results, howev-

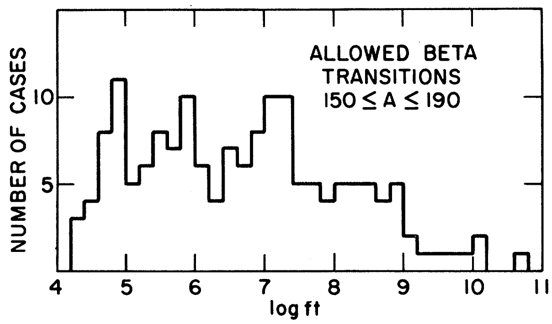

FIG. 1. A histogram of $\log ft$ values for allowed beta transitions in the rare-earth deformed region, as compiled by Artna-Cohen and Gove (Ref. 5).

er, are insensitive to small changes in δ. The parameters of the Nilsson Hamiltonian, as well as the pairing-energy gaps, were the same as in the work of Nilsson and Prior.[7] The BCS equations were solved for the eight levels nearest the Fermi energy. The pairing corrections to the matrix elements must be considered uncertain by about a factor of 2 (± 0.3 in $\log ft$). The energy shift, Δ, in the denominator of Eq. (1) was approximated by the Coulomb shift alone, $\Delta = (1.32 ZA^{-1/3} - 1.29)$ MeV.

When these results are compared with those[8] for transitions obeying the original Alaga rules, one concludes that, even with complete Nilsson wave functions, pairing, etc., there still remains some discrimination in ft value between those transitions corresponding to Eqs. (3a) and the additional transitions allowed by Eqs. (3b). The boundary, however, may not be sharp. Detailed examination of more individual cases is highly desirable.

It should be remarked that, if one adopts conventional perturbation theory starting from asymptotic wave functions, one still obtains ratios quite similar to Eq. (4), and selection rules similar to Eq. (3b), because of the mixing among the asymptotic states due to the spin-orbit interaction. In that picture, however, one must introduce a phenomenological effective coupling constant,[9,10] which is assumed to be approximately independent of A.

TABLE I. Calculated values of $\log ft$ for some β-decay transitions in the rare-earth deformed region are compared with experimental values. The experimental values are taken from the compilation of Artna-Cohen and Gove (Ref. 5). The present results, obtained with the transition operator $(\tau_\pm \vec{\sigma})_{\rm eff}$ of Eq. (1), are given in the last column under $(\log ft)_{\rm calc}$. References to experimental papers and discussions of related cases are given in the comments below.

Transitions	Asymptotic quantum no. assignments	$(\log ft)_{\rm expt}$	$(\log ft)_{\rm calc}$ $\tau_\pm \vec{\sigma}$ without pairing	$\tau_\pm \vec{\sigma}$ with pairing	$(\tau_\pm \vec{\sigma})_{\rm eff}$ with pairing	Comments
$_{66}Dy^{157}_{91} \xrightarrow{\rm E.C.} {}_{65}Tb^{157}_{92}$	$[521\uparrow] \rightarrow [532\uparrow]$	5.45 ± 0.20	4.5	4.8	5.4	a
$_{68}Er^{163}_{95} \xrightarrow{\rm E.C.} {}_{67}Ho^{163}_{96}$	$[523\downarrow] \rightarrow [532\uparrow]$	5.35 ± 0.20	4.1	4.3	5.0	b
$_{67}Ho^{169}_{102} \xrightarrow{\beta^-} {}_{68}Er^{169}_{101}$	$[523\uparrow] \rightarrow [512\uparrow]$	6.03	4.8	5.5	6.3	c

[a]The most recent comprehensive investigation of Dy^{157} decay is by P. H. Blichert-Toft, E. G. Funk, and J. W. Mihelich, Nucl. Phys. A100, 369 (1967). Transitions between the same one-quasiparticle states are seen in the decay of $_{68}Er^{161}_{93}$ to $_{67}Ho^{161}_{94}$, where $\log ft$ is 5.4 (Ref. 5), and in the decay of $_{64}Gd^{159}_{95}$ to $_{65}Tb^{159}_{94}$, where $\log ft$ is 6.6 (Ref. 5). The proton $[532\uparrow]$ state is a hole state in the Tb and Ho isotopes, and one expects that the pairing-correlation effects should not be very different in the even-jumping Dy^{157} and Er^{161} cases, but that $\log ft$ should be larger by about 0.8 in the odd-jumping Gd^{159} case. That the neutron $[521\uparrow]$ state persists as the ground state for $N = 91$, $N = 93$, and some $N = 95$ systems, seems consistent with the (d,p) and (d,t) results of P. O. Tjøm and B. Elbek, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 36, No. 8 (1967). The nuclear spin of Er^{161} has recently been determined to be $\frac{3}{2}$ [see C. Ekström, T. Noreland, M. Olmsats, and B. Wannberg, Nucl. Phys. A135, 289 (1969)].

[b]For the experimental work on Er^{163} decay see L. Funke, H. Graber, K.-H. Kaun, H. Sodan, and J. Frâna, Nucl. Phys. 84, 471 (1966). A transition between these same two Nilsson states is seen in $_{64}Gd^{161}$ β^- decay, where its $\log ft$ is 6.3 (Ref. 5). A detailed discussion of the Coriolis mixing between the $[523\uparrow]$ and $[532\uparrow]$ odd-proton final states in that decay is given by J. Zylicz, P. G. Hansen, H. L. Nielsen, and K. Wilsky, Nucl. Phys. 84, 13 (1966). Since the $[532\uparrow]$ state is a hole state, one would expect the $\log ft$ value in Gd^{161} decay to be larger by about 0.85 than in Er^{163} decay.

[c]The states in Er^{169} to which Ho^{169} decays are discussed by L. Funke, H. Graber, H.-H. Kaun, H. Sodan, and J. Frâna, Nucl. Phys. 86, 345 (1966), and by R. A. Harlan and R. K. Sheline, Phys. Rev. 168, 1373 (1968). The same states are involved in the most intense β group in $_{68}Er^{171}$ decay to $_{69}Tm^{171}$ [F. P. Cranston, Jr., M. E. Bunker, and J. W. Starner, Phys. Rev. 110, 1427 (1958); A. Artna and M. W. Johns, Can. J. Phys. 39, 1817 (1961)], where the $\log ft$ value is 6.4 (Ref. 5). One expects pairing effects and the different statistical factor to increase $\log ft$ by about 0.2 in Er^{171} decay, with respect to Ho^{169} decay.

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[1]J. I. Fujita, Phys. Rev. 172, 1047 (1968); H. Ejiri, K. Ikeda, and J. I. Fujita, Phys. Rev. 176, 1277 (1968); J. I. Fujita, Y. Futami, and K. Ikeda, Progr. Theoret. Phys. (Kyoto) 38, 107 (1967). References to earlier work are given in these papers.

[2]Equation (1) takes into account only the breakdown of supermultiplet symmetry due to the one-body spin-orbit interaction. Additional effects may arise from various residual spin-dependent two-body forces; these effects are probably smaller and are omitted for simplicity.

[3]B. R. Mottelson, J. Phys. Soc. Japan Suppl. 24, 441 (1968).

[4]G. Alaga, Phys. Rev. 100, 432 (1955); B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Skrifter 1, No. 8 (1959).

[5]A. Artna-Cohen and N. B. Gove, private communication; we thank these authors, and Dr. W. B. Ewbank, for access to this compilation.

[6]A plot showing similar features, with a more restricted selection of data, has been presented by K. Ya. Gromov, in Structure of Complex Nuclei, edited by N. N. Bogolyubov (Consultants Bureau, New York, 1969), p. 185 ff.

[7]S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 32, No. 16 (1960).

[8]J. I. Fujita, Y. Futami, and K. Ikeda, J. Phys. Soc. Japan Suppl. 24, 437 (1968).

[9]V. G. Soloviev, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Skrifter 1, No. 11 (1961); J. Zylicz, P. G. Hansen, H. L. Nielsen, and K. Wilsky, Arkiv Fysik 36, 643 (1967).

[10]A general discussion of effective coupling constants has been given by A. Winther, On the Theory of Nuclear β-Decay (Munksgaard, Copenhagen, 1962). In the framework of the random-phase approximation, the Gamow-Teller β decays of spherical nuclei have been studied in detail by J. A. Halbleib, Sr., and R. A. Sorensen, Nucl. Phys. A98, 542 (1967).

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γ-γ Directional Correlations in the Decay of Sb^{125} †

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γ-γ directional correlations have been measured for two cascades in the decay of 2.7-yr Sb^{125}: 320–177 keV, $A_2=-0.15(2)$, $A_4=-0.04(4)$, $\delta(177)=0.9(+0.9,\ -0.4)$; 203–177 keV, $A_2=-0.52(5)$, $A_4=0.02(5)$, $\delta(203)=1.0(+1.1,\ -0.4)$ for spin $\frac{11}{2}$ assigned to the 525-keV level in Te^{125}. The present results are inconsistent with spin $\frac{9}{2}$ for this level.

INTRODUCTION

Many features of the low-lying levels in Te^{125} have been established.[1-7] The main features of the level scheme are shown in Fig. 1. This system of levels has been the subject of interesting theoretical studies[8,9] based on a treatment of pairing forces by the quasiparticle concept as developed in the superconductivity problem. The present investigation is aimed at further clarification of a few points which may be studied advantageously by the technique of directional correlation measurements with good energy resolution in one channel through use of a germanium detector. We are concerned here with the spin of the 525-keV level and the multipole mixing parameters of the 203- and 177-keV transitions. The properties of the latter transition have, in fact, been studied[1,6] but in the present work it is essential to have a consistent set of measurements for both transitions. Furthermore, agreement between different investigations in areas where they overlap lends credence to those conclusions which depend upon a combination of the experimental results. A preliminary report of these results has been made.[10]

EXPERIMENTAL PROCEDURE AND RESULTS

The 2.7-yr Sb^{125} was obtained from the Oak Ridge National Laboratory. Except for replacement of a multichannel analyzer by a computer core memory, the general features of the experimental and data reduction procedures are similar to those employed previously in a study of Ir^{192}.[11] The output of the fixed 30-cm^3 lithium-drifted germanium detector was routed to analog-to-digital converters (ADC) interfaced to a Digital Equipment Corporation PDP-8 computer which was operated on-line during the experiment. The movable detector was a 3-in.-diam×3-in.-high NaI(Tl) crystal. The output went to a single-channel pulse-height analyzer set to the 177-keV γ photopeak which gated the ADC. The region of the coincidence spectrum of interest here is shown in Fig. 2. Data accumulated during a run (usually of 1-h duration) at a particular angle, if consistent with certain accept-