0^+, $T = 2$ States in $A = 40$ Nuclei*

H. P. Jolly†
Physics Department, Southeastern Massachusetts University, North Dartmouth, Massachusetts
and
L. B. Hubbard
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830, and Knoxville College, Knoxville, Tennessee
(Received 28 January 1970)

The 0^+, $T=2$ states of $A=40$ nuclei were calculated by diagonalizing representative nuclear forces in the complete $2\hbar\omega$ basis of 80 states. Both a phenomenological force and a realistic force were employed. When the single-particle energies were corrected in a manner suggested by previous calculations, the realistic force made a good fit to the experimental data. The results for the phenomenological force and the traditional single-particle energies did not fit the data. Many more low-lying 0^+ states in ^{40}Ar are predicted than have been seen experimentally.

INTRODUCTION

The 0^+, $T=2$ excited states of closed-shell nuclei are of particular interest for configuration-mixing calculations, because the spin and isospin coupling requirements restrict the possible many-particle states to a reasonable number. This allows the use of a more complete basis for these calculations than would be possible for other groups of states. For $A=40$, for example, one can include, in a basis of only 80 states, all of the many-particle states with a gross energy of two oscillator quanta. It is also advantageous that these states have no components of the spurious states, involving motion of the center of mass.

Experimentally, the $T=2$ excitations are of special interest, because isospin selection rules make their widths very small, even though they are higher than the threshold for particle emission. They can therefore be observed, and they have been observed in ^{42}Ca(p, t) reactions by Garvey and Cerny.[1] They are also seen as states of ^{40}Ar.

The purpose of the calculation is to see to what extent the theoretical calculations agree with experiment, and to test the effect of different choices for the force parameters and single-particle energies.

II. CALCULATION

The basis for the calculation contained all two-particle, two-hole states with oscillator energy $2\hbar\omega$ relative to the $A=40$ closed shell. This included all configurations with two holes in the s-d shell and two particles in the p-f shell, a total of 80 basis states. A list of those basis states which are important contributors to the lowest eigenstates is included in Table I. All components of spurious states are excluded, since $T=2$ is the maximum isospin possible for a $2\hbar\omega$ configuration, and the corresponding spurious states must have $T=1$ in an harmonic-oscillator basis.

We have employed two different forces in this calculation. As an example of a phenomenological force, we have used the Gaussian interaction of Gillet and Sanderson,[2] chosen to give a good fit to the odd-parity excited states of ^{40}Ca in the random-phase approximation (RPA). As an example of an interaction which fits the two-body scattering data, we have used the Tabakin[3] potential with a single-particle, harmonic-oscillator well parameter $m\omega/2\hbar=3.5$ F^{-2}. While the use of these two particular forces is arbitrary, they are representative of both phenomenological and realistic forces.

It has not been customary in shell-model calculations to compute the interaction of the valence nucleons with the core with the same force used to compute the interaction between valence nucleons. Instead, they have been taken "from experiment," from the spectra and binding energies of nuclei having one particle or one hole in addition to closed shells. This extraction is based on the unsound premise that these near-magic nuclei have no configuration mixing, and we have shown elsewhere[4] that the single-particle energies found in this manner are substantially in error. In this work, we have used both the "traditional" single-particle energies taken from experiment and our estimates for the correct single-particle energies.

In our prescription, based on configuration-mixing calculations for $A=40$, $A=39$, and $A=41$ nuclei,[4] the $1d_{3/2}$ single-particle level is elevated 1

MeV above the traditional value, while the $1f_{7/2}$ level is lowered an equal amount. The single-particle splittings are shown in Table II.

The matrix elements were calculated with the program PLEXUS[5] and its associated programs, and the resulting 80×80 matrix was diagonalized.

TABLE I. The wave functions of low-lying levels calculated using the corrected single-particle splittings. The Gaussian force is above, the Tabakin below. Only the ten most important configurations are shown.

Level number	1	2	3	4
Energy in MeV relative to the closed shell	10.10	13.90	14.68	15.47
	11.21	13.18	14.16	15.31
Configuration				
$d_{3/2}{}^{-2}(0)f_{7/2}{}^2$	0.83	−0.38	−0.13	−0.19
	0.79	−0.12	0.20	0.12
$d_{3/2}{}^{-2}(2)f_{7/2}{}^2$	−0.44	−0.72	−0.33	0.17
	−0.44	−0.48	0.62	0.05
$d_{3/2}{}^{-2}(2)f_{7/2}p_{3/2}$	0.09	−0.09	0.64	0.20
	−0.11	0.61	0.38	0.52
$s_{1/2}{}^{-1}d_{3/2}{}^{-1}(2)f_{7/2}{}^2$	0.21	0.36	−0.42	0.49
	0.19	0.30	0.42	−0.51
$d_{3/2}{}^{-2}(0)p_{3/2}{}^2$	0.10	−0.26	0.33	0.55
	0.15	−0.40	−0.05	0.28
$s_{1/2}{}^{-2}(0)f_{7/2}{}^2$	0.11	0.07	−0.28	0.38
	0.13	0.02	0.28	−0.09
$d_{5/2}{}^{-2}(0)f_{7/2}{}^2$	0.12	−0.09	−0.06	0.00
	0.20	−0.05	0.11	0.05
$d_{5/2}{}^{-1}d_{3/2}{}^{-1}(2)f_{7/2}{}^2$	−0.05	0.11	0.10	−0.13
	0.02	−0.14	0.31	−0.19
$d_{5/2}{}^{-1}d_{3/2}{}^{-1}(4)f_{7/2}{}^2$	−0.06	0.24	0.02	0.23
	0.09	−0.04	0.01	0.26
$d_{3/2}{}^{-2}(2)p_{3/2}{}^2$	−0.02	0.05	−0.14	−0.22
	0.01	0.03	−0.05	−0.22

TABLE II. Single-particle splittings in MeV between s-d and p-f shells.

Configuration	Traditional	Corrected
$(1d_{3/2})^{-1}(1f_{7/2})$	7.0	5.0
$(1d_{3/2})^{-1}(2p_{3/2})$	8.9	7.9
$(2s_{1/2})^{-1}(1f_{7/2})$	9.8	8.8
$(1d_{3/2})^{-1}(2p_{1/2})$	11.0	10.0
$(2s_{1/2})^{-1}(2p_{3/2})$	11.7	11.7
$(1d_{5/2})^{-1}(1f_{7/2})$	12.9	11.9
$(1d_{3/2})^{-1}(1f_{5/2})$	13.4	12.4
$(2s_{1/2})^{-1}(2p_{1/2})$	13.8	13.8
$(1d_{5/2})^{-1}(2p_{3/2})$	14.8	14.8
$(2s_{1/2})^{-1}(1f_{5/2})$	16.2	16.2
$(1d_{5/2})^{-1}(2p_{1/2})$	16.9	16.9
$(1d_{5/2})^{-1}(1f_{5/2})$	19.3	19.3

III. RESULTS

Results for the energy levels using the Tabakin force for both traditional and corrected single-particle energies are shown in Fig. 1. Results for the Gaussian force are shown in Fig. 2. The effect of the Coulomb interaction on the experimental levels has been taken into account by aligning the levels at the lowest 0^+, $T=2$ state. This is the ground state of ^{40}Ar, and it has also been observed by Garvey and Cerny[1] in ^{40}Ca as an excited state at 11.97 MeV. A second 0^+, $T=2$ state is known in ^{40}Ar at 2.13 MeV. Thus, we have two experimental numbers with which to compare.

Both of these numbers are fitted very well by the Tabakin force and corrected single-particle energies. In fact, it seems likely that they could be fitted exactly with a reasonable parameter choice, perhaps with only a slight adjustment of the d-f single-particle energy. We have made no attempt to do this, because with only two experimental numbers it would not mean very much. We should like, of course, to find a force which fits the entire spectrum of $A=40$ and neighboring nuclei at the same time.

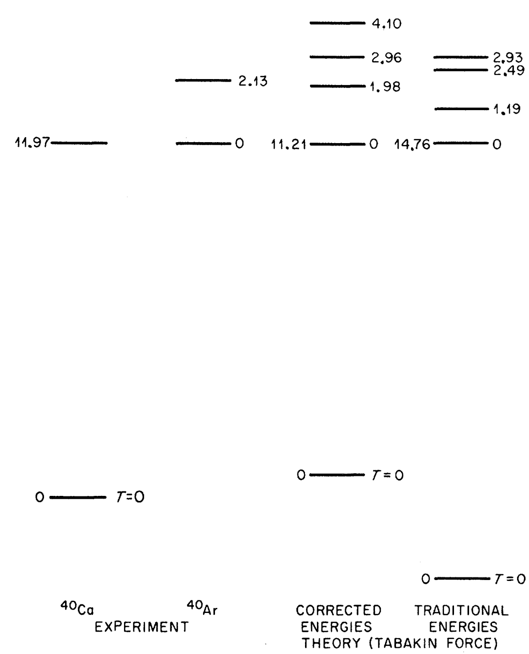

FIG. 1. Comparison of the experimental results with the calculations using the Tabakin potential. The lowest 0^+, $T=2$ levels of both the experimental and theoretical results are set at the same position on the graph. The energies relative to the lowest 0^+, $T=0$ level, i.e., the ground state of ^{40}Ca, are shown to the left. The energies relative to the lowest 0^+, $T=2$ level, i.e., the ground state of ^{40}Ar, are shown to the right.

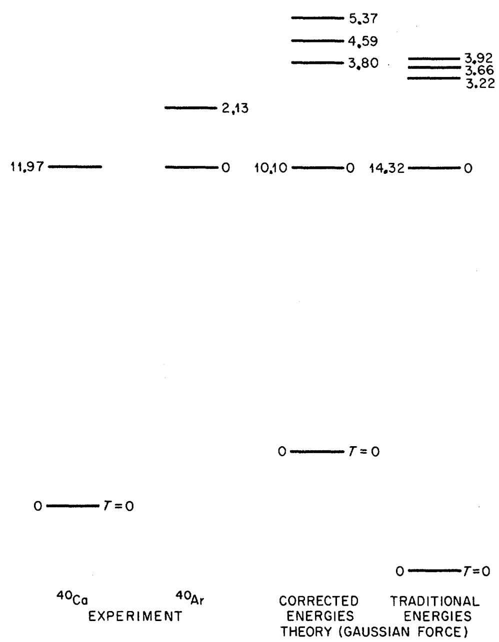

FIG. 2. Comparison of the calculations using the Gaussian potential with the experimental results. The plotting conventions are the same as used in Fig. 1.

Additional low-lying 0^+, $T=2$ states are predicted in ^{40}Ar, and it would be of interest to find more of these experimentally.

The Gaussian force does not fit as well. Unlike the Tabakin case, the corrected single-particle energies do not give a significantly better fit than the traditional single-particle energies.

The wave functions are presented in Table I. Only the most important components of the four lowest states are shown. The results for the lowest state are quite similar for the two forces, indicating a 65% contribution for the lowest two-particle, two-hole state with a pair of $d_{3/2}$ holes and a pair of $f_{7/2}$ particles, each coupled to zero spin. There are many other states with large components, however, and the higher states are even more strongly mixed. Four of these ten most important states consist of zero-coupled pairs excited from the s-d to the f-p shell. The large components of the other three states imply, however, that the pairing model alone is not an adequate description of even the lowest state.

---

*Work supported in part by the National Science Foundation and in part by the U. S. Atomic Energy Commission by funds provided under Contract No. AT(30-1)-2098. A preliminary report of this work was presented at the Washington meeting of the American Physical Society, April, 1969.

†Guest of the Laboratory for Nuclear Science, Massachusetts Institute of Technology. Present address: Health Sciences Center, State University of New York, Stony Brook, New York.

[1]G. T. Garvey and J. Cerny, to be published; J. Cerny, Ann. Rev. Nucl. Sci. 18, 39 (1968).

[2]V. Gillet and E. A. Sanderson, Nucl. Phys. A91, 292 (1967).

[3]F. Tabakin, Ann. Phys. (N.Y.) 30, 51 (1965).

[4]L. B. Hubbard and H. P. Jolly, Phys. Rev. 178, 1783 (1969).

[5]H. P. Jolly and L. B. Hubbard, Laboratory for Nuclear Science and Center for Theoretical Physics, Massachusetts Institute of Technology Report No. CTP 29, 1968 (unpublished).