Unified Reaction Formulation with Two Particles in the Continuum*

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By means of a Faddeev-like method, a formulation of nuclear reactions leading to three-fragment final channels is presented. It is a shell-model theory of nuclear reactions in which one can, in principle, deal with two particles in the continuum in the intermediate states. The amplitude also includes as one term the distorted-wave t-matrix approximation (DWTA). An approximate treatment of the resonance-type term leads to a two-step process for $(p, 2p)$ reactions. The importance of such a mechanism, which proceeds via the 1p-1h or 2p-2h states of the target nucleus, is presented for reactions which heretofore have been analyzed by means of the DWTA.

I. INTRODUCTION

Recently, the theory of nuclear reactions of the form $(p, 2p)$ and (p, pn) has received quite a bit of attention.[1-5] Analysis of data is presently done in two ways. One method treats the reaction as a one-step or direct interaction process. The other method treats the reaction as a two-step process in which the incoming particle undergoes a direct inelastic collision leaving the target nucleus in an excited state which eventually decays by particle emission.

For incident proton energies greater than 100 MeV the distorted-wave t-matrix approximation (DWTA)[2,3] or various high-energy approximations to the DWTA have, in general, provided a consistent method of analysis. At lower energies the reaction is generally a sequential one. The work of Detenbeck[1] for the reaction ${}^{14}N(p, 2p){}^{13}C$ at 19 MeV clearly shows the existence of the two-step $(p, 2p)$ reaction.

However, recent work by Pugh *et al.*[6] and Clegg *et al.*[7] indicates that there are two-step processes occurring in the reaction ${}^{12}C(p, 2p){}^{11}B$ at incident proton energies of 50 MeV and 120–150 MeV, respectively. This conclusion is based on the fact that, if one believes the structure calculations for the ground state of ${}^{12}C$, there is a comparatively large production of final states in ${}^{11}B$ which are forbidden by the direct process. This also implies that in the case of an allowed direct transition there is also a contribution to the amplitude from the two-step process.

In Sec. II, we obtain a unified formalism which will allow the simultaneous treatment of both the one-step and two-step processes. The formalism will be found to contain a direct term which corresponds to the DWTA and a resonance-type term in which it is possible to treat in the intermediate states, as well as in the final state, one or *two* bodies in the continuum. In Sec. III approaches and approximations to calculation are discussed. In Sec. IV we show the importance of the two-step process when it is calculated in terms of discrete intermediate states; namely that it is this process which allows us to study the excited 1p-1h and 2p-2h states of the target. This study will be done in the spirit of nuclear-reaction investigations[8-11] that so far have been carried out for elastic and inelastic scattering to unite nuclear-structure theory, in particular shell-model theory, with reaction theory.

II. DERIVATION OF THE UNIFIED AMPLITUDE

We will consider reactions of the type (p, pn) and $(p, p\alpha)$. The interaction between the incident particle and the target will be denoted by $V_1 + v$, where v is the potential between the incident particle and the particle which will be emitted. Then V_1 is the interaction between the incident particle and the residual nucleus. The interactions in the target will be written as $V_2 + U$, where V_2 is the potential between struck particle and the residual nucleus and U is the sum of interactions within the residual nucleus.

The transition operator T, in prior form, is given by[12]

$$T = V_1 + v + (V_1 + V_2 + v)G(V_1 + v), \tag{1}$$

where the total Green function $G = (E^{(+)} - K - U - V_1 - V_2 - v)^{-1}$, and K is the kinetic-energy operator for the entire system. When we expand G according to $G = G_M + G_M v G$, where $G_M = (E^{(+)} - K - U - V_1 - V_2)^{-1}$, Eq. (1) can be written

$$T = [1 + (V_1 + V_2)G_M][V_1 + v + vG(V_1 + v)].$$

When G is again expanded as $G = G_M + GvG_M$, we obtain

$$T = \Omega_{12}[V_1 + (v + vGv)\Omega_1], \tag{2}$$

where $\Omega_{12} = 1 + (V_1 + V_2)G_M$ and $\Omega_1 = 1 + G_M V_1$.

When the cross section is calculated, matrix elements of T are taken between the initial state (incident plane wave plus target wave function) and the final state (two plane waves plus residual-nucleus wave function). The operator Ω_1 distorts the incident state into a state $|\chi_I^{(+)}\rangle$ which is composed of a distorted wave for the scattering of particle 1 from the residual nucleus and the true target wave function. It is a scattering eigenfunction of the operator $H_M = K + U + V_1 + V_2$ with outgoing waves. The operator Ω_{12} distorts the outgoing state into the state $\langle\chi_F^{(-)}|$ which consists of two outgoing particles both distorted by the interaction with the residual nucleus and the true residual nucleus wave function. The wave function $|\chi_F^{(-)}\rangle$ is also an incoming wave eigenfunction of H_M.

To compute the cross-section it will be necessary to evaluate τ, where

$$\tau = v + vGv = v + \tau G_M v. \tag{3}$$

However, as the expression now stands, difficulties will arise. The interaction v does not affect the residual nucleus which is assumed to remain bound. Whether the residual nucleus is assumed to be an inert core or a bound system with structure, the evaluation of Eq. (3) will introduce disconnected diagrams in the motion of the center of mass of the two particles with respect to the residual nucleus. The solution of Eq. (3) cannot then be calculated by any convergent scheme.[13] These problems were essentially eliminated in three-body scattering by methods mainly due to Faddeev.[14] Greider and Dodd[15] also made a study of expressions of this form in the distorted-wave formalism and showed that they diverge. To be able to do calculations with two bodies in the continuum Eq. (3) must be modified.[14]

With $G_0 = (E - K - U)^{-1}$ we expand G_M as

$$G_M = G_0 + G_M(V_1 + V_2)G_0. \tag{4}$$

Multiplying Eq. (3) on the right by $(1 - G_0 v)^{-1}$, and using Eq. (4) we have

$$\tau = t + \tau G_M(V_1 + V_2)G_0 t, \tag{5}$$

where $t = v(1 - G_0 v)^{-1}$. The kernel of Eq. (5) is now compact with respect to a basis that includes two bodies in the continuum. This multiplier method of obtaining an equation with a compact kernel was introduced by Sugar and Blankenbecler.[16] It is equivalent to the methods of Faddeev and others.[17] Equation (5) and that which follows are exact for the three-body model which consists of the incident and struck particles and the inert residual nucleus. However, it is possible to treat all excitations in the intermediate states which do not have more than two particles in the continuum, since no formal difficulties are encountered by allowing the residual nucleus to become excited.

When Eq. (5) is rearranged and placed in Eq. (2) the final expression for T is

$$T = \Omega_{12}\{t + t[E - H_M - (V_1 + V_2)G_0 t]^{-1} \times (V_1 + V_2)G_0 t\}\Omega_1 + \Omega_{12}V_1 . \tag{6}$$

In Ref. 2 an integral equation with a compact kernel was found for T and special attention was given to the Born term. In Eq. (6) we show the solution of the equation and explicitly display the resonance term. The result of obtaining a formally calculable expression is the appearance of the compact "residual" operator $(V_1 + V_2)G_0 t$ in the resonance term and of t instead of v in the direct term.

If we neglect all but the first term in Eq. (6) we have

$$T \simeq \Omega_{12} t \Omega_1 , \tag{7}$$

which is the DWTA and has been used by McCarthy.[3,4] The second term which may lead to resonance reactions will be discussed in the next section. The third term is a recoil type of term, wherein the incoming particle interacts only with the residual nucleus with the struck particle acting as a spectator. It can lead to direct formation of final states which may be forbidden by the DWTA. When antisymmetric wave functions are used, this term is proportional to $A^{-1/2}$ and is usually neglected. To correspond to the physically realizable situation, the scattering amplitude should be antisymmetrized. This is done in the Appendix as it does not effect the discussion which follows.

III. DISCUSSION OF TERMS IN THE AMPLITUDE

In full generality, Eq. (6) is still a very complicated many-body expression. We will now consider some reasonable approximations and approaches to calculation of the $(p, 2p)$ amplitude with the use of Eq. (6).

How to treat the interactions V_1 and V_2 is the first question that arises. To arrive at any distorted-wave formalism, the distorting potentials are taken to be actual interactions, as was done to obtain Eq. (2). Then to calculate distorted waves, the distorting potentials (in this case V_1 and V_2 when they appear in the distorted-wave operators Ω_{12} and Ω_1) are usually taken to be complex central potentials. This means that the many-body aspects of V_1 and V_2 are built up in the distorted waves by the complex potential. In all that follows we will assume that V_1 and V_2, when they appear in Ω_{12} and Ω_1, are to be taken as optical potentials which represent the scattering of a proton from the residual nucleus.

In the resonance term the potentials V_1 and V_2

can be treated in two ways. If they are assumed to be central (single-particle) potentials, then there can only be coupling to intermediate states in which the residual nucleus is in the same state as in the entrance channel. In this case, the residual nucleus can be treated as an inert core ($U=0$) and we have a three-body model for the problem. On the other hand, if V_1 and V_2 are sums of two-body potentials, there is coupling to intermediate states in which the residual nucleus is excited with respect to the entrance channel.

In either case, H_M in the denominator of the resonance term can be approximated by the shell-model Hamiltonian. This entails making the standard shell-model assumption that the central parts of V_1 and V_2 are taken with respect to the center of mass of the residual nucleus. In the case of the three-body model H_M is a single-particle Hamiltonian for two particles; whereas, in the many-body case H_M includes also the residual interactions between protons 1 and 2 and the residual nucleus. The shell-model assumption is consistent with the separation of coordinates of protons 1 and 2 in Ω_{12} so that each distortion can be calculated separately.

A calculation that includes two particles in the continuum could in principle be carried out by extending the Weinberg[18] method (which rigorously treats the continuum-continuum coupling) to two continua.[19] Whether such an attempt is practical is, to say the least, very much in doubt. However, the importance of contributions from the resonance term can be investigated by truncating the set of intermediate states to include only the excited states of the target that are of interest combined with a scattering state of the incident particle at a discrete energy which is correspondingly reduced from the entrance channel energy. This is consistent with the two-step process in which the incident particle undergoes a direct inelastic collision. The inverse operator in the resonance term is then calculated by inverting a finite matrix. An even more simple (and more approximate) approach is to treat the matrix by the weak coupling (diagonal) approximation. In the next section we present evidence that mechanisms other than the direct DWTA are required.

IV. APPLICATIONS

We now consider through what kind of states the two-step process can go. If V_1 and V_2 are sums of two-body interactions, the expression $(V_1+V_2)G_0t$ can couple to intermediate states no more complicated than 3p-2h. If one of the particles is in the continuum, the intermediate states consist of 2p-2h or 1p-1h states of the target. The continuum particle can be interpreted as the incident proton participating in the reaction inelastically. If that proton carries off enough energy, the intermediate state of the target can be in a region of distinct resonances even if the incident proton energy is in the 50-MeV range. The excited state of the target can then proton decay via t to a variety of states of the residual nucleus.

In the reaction $^{12}\mathrm{C}(p,2p)^{11}\mathrm{B}$ mentioned in the Introduction, it was observed that the $\frac{5}{2}^-$ level at 4.46 MeV and the $\frac{7}{2}^-$, 6.76-MeV level in $^{11}\mathrm{B}$ could not be reached by a direct transition.[6] According to structure calculations, there is not enough of the needed component in the ground state of $^{12}\mathrm{C}$ for this to occur. Thus, for these transitions to occur they must proceed through the second or third term of Eq. (6). Other studies[20,21] clearly show that in the excitation region 16–35 MeV there are resonances, including the giant dipole resonance at 23-MeV excitation, which are of the 1p-1h type. Contributions from the 2p-2h states in this same excitation region have also been found.[22] The multiplier method we have introduced automatically gives this particle-core excitation by means of the interactions V_1 and V_2 which appear in the numerator of the resonance-type term.

We recall that in the three-body model $U=0$ and V_1 and V_2 are central potentials. If this model is to be reasonable, it is required that the target state be well represented by a single-particle state outside an inert core. The intermediate states that t or $(V_1+V_2)G_0t$ can couple to consist of two particles in single-particle states outside the core. If one of the particles is in the continuum, the target in intermediate states is in an excited single-particle state.

The single-particle description is valid for a $^{89}\mathrm{Y}$ target.[23] The $^{89}\mathrm{Y}(p,2p)^{88}\mathrm{Sr}$ reaction has been investigated by McCarthy[4] in the DWTA using a phenomenological t. He finds that the DWTA description is inadequate in that the angular correlation cross section decreases too fast at larger angles. However, it should be remembered that the three-body aspects of the problem are not exhausted by the use of the DWTA. The two-step process can again be simply (but approximately) included and could very well account for discrepancies between the DWTA and experiment.

V. SUMMARY

We have obtained an expression for reactions that lead to three-fragment final channels which contains a direct term which corresponds to the DWTA and a resonance term in which it is possible to treat two bodies in the continuum. After discussing approximate ways of dealing with the resonance term, we have shown its importance to reactions which have only been analyzed with the DWTA term.

We have begun calculations to obtain a good approximation of the contribution of the other direct terms; namely the recoil term and direct-exchange term. In the future we hope to include the two-step process also.

APPENDIX

To complete the formulation we obtain an antisymmetrized $(p, 2p)$ amplitude. We assume the wave functions for the target and residual nuclei are properly antisymmetric. If N is the number of relevant protons in the target, the antisymmetrized amplitude is given by [24]

$$\langle f|T|i\rangle=(\tfrac{1}{2}N)^{1/2}\langle[\chi_1(p_1)\chi_2(p_2)-\chi_1(p_2)\chi_2(p_1)]\phi_R|T|\chi(p_1)\phi_T\rangle-(N-1)\,(\tfrac{1}{2}N)^{1/2}\langle[\chi_1(p_r)\chi_2(p_2) -\chi_1(p_2)\chi_2(p_r)]\phi_R|T_{\mathrm{ex}}|\chi(p_1)\phi_T\rangle, \qquad \text{(A1)}$$

where p_1 is the incoming proton, p_2 is the proton emitted, p_r is one of N-1 protons exchanged. Also ϕ_T and ϕ_R are the target-and residual-nucleus wave functions, respectively.

The first matrix element on the right side of Eq. (A1) is evaluated by using the T operator given in Eq. (6). The second matrix element represents the exchange amplitude. The T operator for exchange is

$$T_{\mathrm{ex}}=V_1+v+(V_2+V_r+v_{2r})G(V_1+v),$$

where G is the total Green function, and V_1 and v are defined as before. The exchanged particle r that is emitted interacts with the other emitted particle through v_{2r} and with the residual nucleus through V_r. When we extract the distorting operators and rearrange, as was done previously to obtain Eq. (6), we arrive at

$$T_{\mathrm{ex}}=\Omega_{2r}\{t_{\mathrm{ex}}+t_{\mathrm{ex}}[E-H_M-(V_2+V_r)G_0t_{2r}]^{-1}(V_2+V_r)G_0t_{2r}\}\Omega_1+\Omega_{2r}(V_1+v-v_{2r}), \qquad \text{(A2)}$$

where $t_{\mathrm{ex}}=v+t_{2r}G_0v$, and $t_{2r}=v_{2r}+v_{2r}G_0t_{2r}$, and all other quantities not explicitly defined, such as G_0, H_M, Ω, appear the same as before except the incident proton labeled 1 is exchanged with the proton from the residual nucleus labeled by r. Other than in the third (recoil) term, we see that the difference between Eqs. (6) and (A2) is the appearance of t_{ex} instead of t.

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