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Cluster-Model Vertex Functions for ^{16}O: Monopole Form Factors*

J. V. Noble
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
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We introduce a simple phenomenological model of the vertex functions (virtual decay amplitudes) describing the reactions ^{16}O(g.s.)$\rightarrow\alpha+^{12}$C, and ^{16}O(6.056 MeV)$\rightarrow\alpha+^{12}$C. These vertex functions (under certain assumptions) can be used to predict the elastic and inelastic electron scattering monopole form factors, as well as describing hadronic reactions such as α-cluster transfer and excitation. The monopole form factors are compared with recent data: The ratio of the inelastic form factor to experiment is about 4 (as compared with deformed and spherical shell-model predictions which give $<\frac{1}{2}$ for this ratio), owing to the excessive collectiveness of this description of the transition.

I. INTRODUCTION

The nuclear cluster model is useful for the insight it affords into the mechanisms of direct cluster transfer reactions,[1] and into certain structural features of light nuclei.[2] However, for practical purposes, it is as impossible to solve exactly the 4-boson problem as it is to solve the 16-fermion problem, so it is not remarkable that few authors have tried to describe ^{16}O as an α-cluster nucleus. (The early work of Dennison,[3] and the more recent treatments by Perring and Skyrme,[4] and by Brink[5] comprise essentially all of the literature on the subject.) Nevertheless, in view of the success of the 3-α cluster model in accounting for some of the gross properties of ^{12}C,[6] the time may be auspicious for reopening the question of the α-particle structure of ^{16}O. Ultimately, we may hope to

take into account such (obviously important) dynamical features as the small binding energy of ^{16}O with respect to dissociation into α particles, as well as the proximity to the ^{16}O ground state of the $\alpha + {}^{12}$C continuum states – features which are ignored in the conventional shell model. The aim of this note is much more modest: We propose "reasonable" vertex functions which describe the virtual decays ^{16}O(g.s.) $\to {}^{12}$C(g.s.) $+ \alpha$ and ^{16}O(6.056 MeV) $\to {}^{12}$C(g.s.) $+ \alpha$, and which satisfy certain general criteria associated with the cluster model. These (model) vertex functions are ultimately intended for phenomenological treatments of various hadronic reactions involving ^{16}O; here we shall assume that the intermediate states which arise in calculating the elastic and inelastic electron scattering monopole form factors are saturated by the asymptotic ($\alpha + {}^{12}$C) parts of the corresponding wave functions, and we shall compare the resulting form factors with recent data.

II. CLUSTER-MODEL FORM FACTORS

Suppose we were given the α-cluster wave functions of the (0^+) ground and first excited states of ^{16}O: Of necessity they would be totally symmetric scalar functions, Ψ_0 and Ψ_1, of the cluster position coordinates $\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4$. Let us also suppose that we probe this nucleus in a manner which measures only the spatial distribution of its constituent α particles; then we should require the function

$$M_{ba}(q) = \tfrac{1}{4}\int d\vec{r}_1 \cdots \int d\vec{r}_4 \sum_{\mu=1}^{4} e^{i\vec{q}\cdot\vec{r}_\mu} \times \delta\left(\tfrac{1}{4}(\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4)\right) \Psi_b{}^*\Psi_a(\vec{r}_1,\ldots,\vec{r}_4), \quad (b,a) = (0,0),(1,0). \tag{1}$$

The amplitude defined in (1) above is related to the elastic and inelastic electron scattering form factors by

$$F_{ba}(q) = F_\alpha(q) M_{ba}(q), \tag{2}$$

where $F_\alpha(q)$ is the ^{4}He elastic scattering form factor. [Moreover, $M_{ba}(q)$ is closely related to the amplitudes for foward scattering of strongly interacting particles, such as α particles.] Equation (1) can be simplified by changing to Jacobi variables

$$\vec{r} = \vec{r}_1 - \tfrac{1}{3}(\vec{r}_2+\vec{r}_3+\vec{r}_4), \quad \vec{s} = \vec{r}_2 - \tfrac{1}{2}(\vec{r}_3+\vec{r}_4), \quad \vec{t} = \vec{r}_3 - \vec{r}_4. \tag{3}$$

This coordinate transformation, together with the Bose symmetry of the wave functions, yields

$$M_{ba}(q) = \int d\vec{r}\, e^{(3/4)i\vec{q}\cdot\vec{r}} \rho_{ba}(r), \tag{4}$$

where

$$\rho_{ba}(r) = \int d\vec{s} \int d\vec{t}\, \Psi_b{}^* \times \Psi_a(\tfrac{3}{4}\vec{r}, -\tfrac{1}{4}\vec{r}+\tfrac{2}{3}\vec{s}, -\tfrac{1}{4}\vec{r}-\tfrac{1}{3}\vec{s}+\tfrac{1}{2}\vec{t}, -\tfrac{1}{4}\vec{r}-\tfrac{1}{3}\vec{s}-\tfrac{1}{2}\vec{t}), \tag{5}$$

so that

$$M_{ba}(q=0) = \int d\vec{r}\, \rho_{ba}(r) = \rho_{ba}. \tag{6}$$

In general the wave functions Ψ_0, Ψ_1 will be rather complicated functions of $\vec{r}$, $\vec{s}$, $\vec{t}$, and $\rho_{ba}(r)$ will therefore be hard to evaluate. Let us represent $M_{ba}(q)$, its Fourier transform, by the triangle part of the graph in Fig. 1, where the sum over all intermediate states of the unscattered clusters is implied. If we were to make the drastic assumption that this (three-α) intermediate state is saturated by its lowest-mass term, the ^{12}C ground-state pole, we could represent $M_{ba}(q)$ in the form[7]

$$M_{ba}(|\vec{k}-\vec{k}'|) = \int d\vec{k}'' \frac{v_b{}^*(\vec{k}''+\frac{3}{4}\vec{k}')}{B_b + \frac{2}{3}\hbar^2(\vec{k}''+\frac{3}{4}\vec{k}')^2/m_\alpha} \times \frac{v_a(\vec{k}''+\frac{3}{4}\vec{k})}{B_a + \frac{2}{3}\hbar^2(\vec{k}''+\frac{3}{4}\vec{k})^2/m_\alpha}, \tag{7}$$

where v_a and $v_b{}^*$ are the vertex amplitudes describing the (virtual) processes

$$^{16}\mathrm{O}(a) \to \alpha + {}^{12}\mathrm{C(g.s.)} - B_a(\mathrm{MeV}),$$

and

$$\alpha + {}^{12}\mathrm{C(g.s.)} \to {}^{16}\mathrm{O}(b) + B_b(\mathrm{MeV}).$$

The well-known relation[8] between vertex amplitudes and single-particle wave functions leads to the expression

$$\rho_{ba}(r) = \psi_b{}^*(r)\psi_a(r), \tag{8}$$

where $\psi_a(r)$ and $\psi_b(r)$ are "single-particle wave

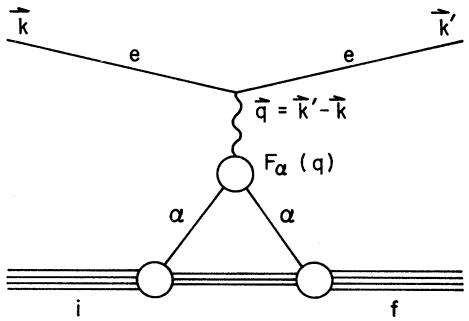

FIG. 1. Diagrammatic representation of the α-cluster model of electron scattering from ^{16}O, in the first Born approximation.

functions" in the α-^{12}C relative coordinate, r. Although this model for ρ_{b_a} is far removed from the wave functions Ψ_0 and Ψ_1, it is so much simpler than Eq. (5) that it merits consideration despite this flaw. Indeed, Eq. (8) clearly exhibits the correct asymptotic behavior in r since the longest-range components (in r) of the true wave functions, Ψ_0 and Ψ_1, correspond to the most weakly bound channel, $\alpha + {}^{12}\mathrm{C}(\mathrm{g.s.})$.

III. VERTEX FUNCTIONS

In order to apply this simple picture, we require "single-particle wave functions," $\psi_{0,1}(r)$. As we lack a theory of these functions, we shall have to conjecture them, guiding our guesses by our prejudices as to their behavior. We would generally expect that (ignoring Coulomb effects) a configuration-space wave function must be finite at the origin and should have the asymptotic form $e^{-\kappa r}/r$ at large spatial separations, where κ is related to the separation energy in the usual way:

$$\kappa = (2m_{\mathrm{red.}} E_{\mathrm{sep.}}/\hbar^2)^{1/2}. \tag{9}$$

Since we wish to avoid introducing the nucleon degrees of freedom explicitly, let us put in the effects of the exclusion principle by hand, as it were, to simulate the behavior of a properly antisymmetrized wave function. What sort of behavior should we aim at? Clearly, it must be a modification of the wave function at short distances, since the effects of antisymmetrization vanish when the clusters no longer overlap appreciably. Some time ago, Perring and Skyrme[4] in a remarkable paper showed that the isotropic harmonic-oscillator–model ground states for the $4n$ nuclei ^{8}Be, ^{12}C, and ^{16}O were exactly equivalent to properly antisymmetrized α-cluster wave functions. They were generally able to find two linearly independent functional forms for the cluster wave functions which became indentical upon antisymmetrization. When we isolate the dependence of these two wave functions on one α-particle coordinate, we obtain the functional forms $(1+ar^2+br^4)\times e^{-r^2}$ and $r^4 e^{-r^2}$. Clearly, when they are generalized to the case of localized clusters (that is, when the internal oscillator constants differ from that describing the relative motion), the two forms are no longer identical after antisymmetrization. To decide which is preferable, we may guide ourselves by two analogous previous cases: First, Kudeyarov *et al.*[2] found that the Coulomb quadrupole form factor of the transition $1^+(\mathrm{g.s.}) \rightarrow 3^+$ (3.56 MeV) in ^{6}Li could be fit with the function $r^2 e^{-r^2}$, but could not be fit with its shell-model equivalent, $(1-ar^2)e^{-r^2}$. Similarly, Scholz and Neogi[9] have found that the $^7\mathrm{Li} \rightarrow {}^3\mathrm{H} + {}^4\mathrm{He}$ vertex function, as measured by several direct reactions, could be fit with the function $r^3 Y_{1\mu}(\hat{r}) e^{-r^2}$ but not with the (shell-model equivalent) function $rY_{1\mu}(\hat{r}) \times (1-ar^2)e^{-r^2}$. If we generalize the ^{6}Li and ^{7}Li results to the present case, we should prefer a wave function proportional to r^4 at small distances, to describe α-particle motion relative to a nucleus with a filled $1s_{1/2}$ shell. For the moment, we leave the precise power behavior of the wave function at the origin as a parameter, and take for our ground-state wave function

$$\psi_0(r) = N_0(1-e^{-r/R})^{n+1} \frac{e^{-\kappa_0 r}}{r}, \tag{10}$$

and as the corresponding wave function of the first 0^+ excitation

$$\psi_1(r) = N_1(1-e^{-r/R})^{n+1}(1-e^{(r_0-r)/R}) \frac{e^{-\kappa_1 r}}{r}. \tag{11}$$

The position of the node, r_0, is fixed by the orthogonality condition

$$\int_0^\infty dr\, r^2\, \psi_0(r)\, \psi_1(r) = 0. \tag{12}$$

In (10) and (11), N_0 and N_1 are normalization constants. Assuming these functions somehow represent the ground and first excited states of ^{16}O, we determine their asymptotic decay constants κ_0 and κ_1 from the corresponding $^{16}\mathrm{O} \rightarrow \alpha + {}^{12}\mathrm{C}$ separation energies. The cutoff radius, R, and the exponent n remain as free parameters. The functional forms (10) and (11) interpolate smoothly between the conjectured short-range behavior and the correct asymptotic form; our hope is that they are reasonable at intermediate distances also. Furthermore, they are eigenstates of the Hamiltonian

$$H_{\mathrm{eff}} = -\frac{\hbar^2}{2m_r}\nabla^2 + \frac{\hbar^2}{2m_r R^2} n(n+1)[e^{r/R}-1]^{-2} - V_0[e^{r/R}-1]^{-1}, \tag{13}$$

with V_0 chosen appropriately. We note the appearance of the singular short-range repulsion, proportional to $[e^{r/R}-1]^{-2}$, which simulates the exclusion principle.

We can use the functions (10) and (11) to calculate the monopole form factors

$$M_{00}(q) = \int_0^\infty dr\, r^2 |\psi_0(r)|^2 j_0(\tfrac{3}{4}qr), \tag{14}$$

and

$$M_{10}(q) = \int_0^\infty dr\, r^2\, \psi_0(r) \psi_1{}^*(r)\, j_0(\tfrac{3}{4}qr), \tag{15}$$

which yield the following differential cross sec-

tions for elastic and inelastic electron scattering, in the first Born approximation (Fig. 1)[10]:

$$d\sigma_{\mathrm{el.}}/d\Omega = (d\sigma_R/d\Omega)\,|F_\alpha(q)M_{00}(q)|^2, \tag{16}$$

$$d\sigma_{\mathrm{in.}}/d\Omega = (d\sigma_R/d\Omega)\,|F_\alpha(q)M_{10}(q)|^2, \tag{17}$$

where $F_\alpha(q)$ is the empirical α-particle elastic form factor,[11] and $(d\sigma_R/d\Omega)$ is the elastic differential cross section for electrons scattering from a point charge with $Z=8$.

IV. COMPARISON WITH EXPERIMENT

Rather than trying to vary the parameters R and n to obtain best fits to the data, we shall test the model form factors, $M_{00}(q)$ and $M_{10}(q)$ by fixing R for each n, and then varying n to see what happens. How shall we fix R? We can use it to fit one of the three experimental numbers: (1) the position of the 0^+ excited state relative to the ground state; (2) the mean-square matter radius, $\langle r_\alpha^2\rangle$, of the ground state; or (3) the pair-emission monopole matrix element. The excitation energy of the 0^+ state is given in MeV by[12]

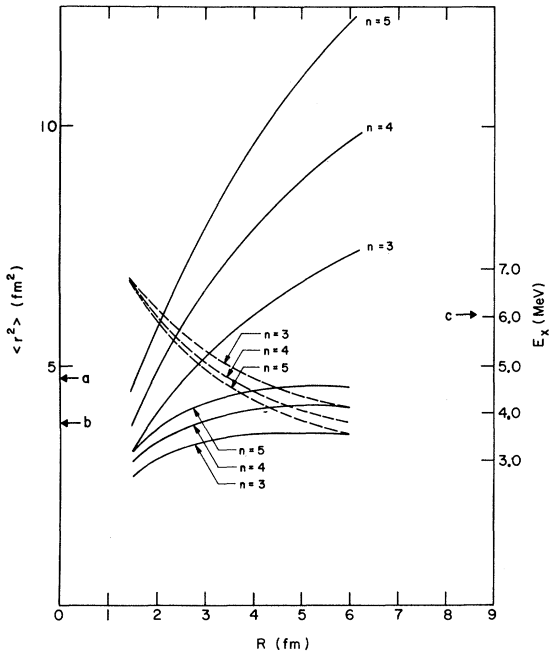

FIG. 2. Functional dependence of the excitation energy of the 0^+ state (dashed curves, right-hand vertical scale), mean-square matter radius (upper three solid curves), and monopole matrix element (lower three solid curves) on the interpolating distance, R. The left-hand vertical scale pertains to both sets of mean-square radii. The parameter n is explained in the text. The arrows labelled a, b, and c indicate, respectively, the experimental values of the mean-square matter radius, 4.74 fm^2, the monopole matrix element, 3.8 fm^2, and the excitation energy, 6.056 MeV.

$$E_x(n,R) = 7.15 - \left[\frac{(n+1)(2\kappa_0 R-1)}{n+2} - 1\right]^2 \frac{\hbar^2}{6m_\alpha R^2}, \tag{18}$$

(that is, by the mass less the binding energy), the mean-square matter radius[13] by

$$\langle r_\alpha^2\rangle = \tfrac{9}{16}\int_0^\infty dr\, r^4 |\psi_0(r)|^2, \tag{19}$$

and the pair-emission monopole matrix element by[14]

$$\langle 0^+, 6.056|\, \tfrac{1}{8}\sum_p r_p^2\,|\,\mathrm{g.s.}\rangle = \tfrac{16}{25}\int_0^\infty dr\, r^4 \psi_0(r)\psi_1(r) = 3.8\ \mathrm{fm}^2. \tag{20}$$

These quantities are plotted against R, for $n=3$, 4, and 5 in Fig. 2; their experimental values are indicated on the figure by arrows. The case $n=3$ can be ignored, since for no value of R is the observed monopole matrix element of 3.8 fm^2 reproduced. When $n=4$, the values of R which fit the three data are, respectively, 2.05, 1.95, and 2.8 fm; while at $n=5$ they are 1.95, 1.6, and 2.1 fm. The elastic form factor [Eq. (5) multiplied by the empirical ^{4}He elastic form factor] is plotted as a function of momentum transfer in Fig. 3 for $n=4$, $R=2.8$ fm, and $n=5$, $R=2.1$ fm.[15] The first and second diffraction minima occur at the correct values of q, but we observe that the form factor falls off too rapidly at large momentum transfers,

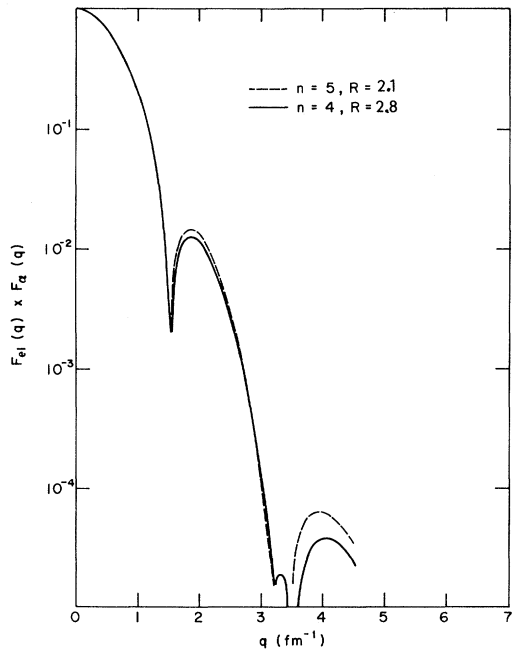

FIG. 3. Elastic form factor (theory) as a function of q, the momentum transfer to the electron, for the parameter sets $n=4$, $R=2.8$ fm (solid curve), and $n=5$, $R=2.1$ fm (dashed curve).

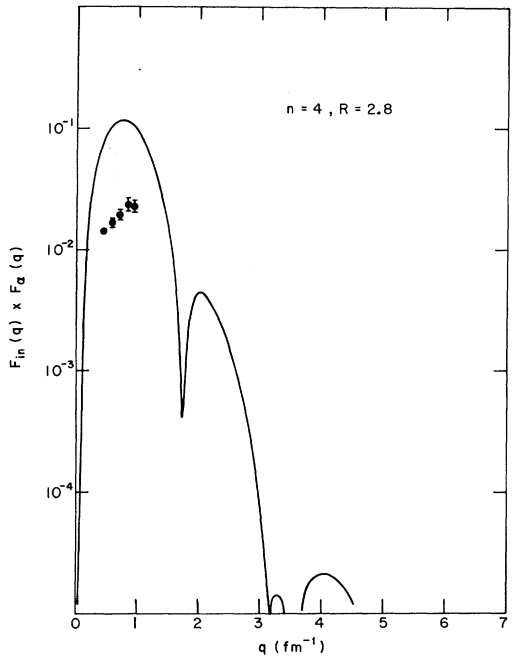

FIG. 4. Inelastic form factor, for $n=4$, $R=2.8$ fm, together with several data points from Ref. 13.

for both sets of parameters. At the first maximum, $q \simeq 1.9$ fm^{-1}, the experimental form factor[16] is about 2×10^{-2}, whereas the cluster form factors with $n=4$ and 5 give 1.25×10^{-2} and 1.5×10^{-2}, respectively. The fit becomes progressively worse at higher q – for example, at $q \simeq 4$ fm^{-1} the ratio of experiment to theory is between 5 and 10.[17]

Figure 4 displays the inelastic form factor for the parameters $n=4$, $R=2.8$ fm. The form factor with $n=5$, $R=2.1$ fm is only slightly different, so it was not plotted. Some of the experimental values recently measured by Bergstrom *et al.*[18] are included on the figure. The theoretical form factor is too large by a factor of 4 or 5 and has the correct general shape. We would surmise (by comparison with the elastic fit) that the position of the first diffraction minimum is correctly predicted (although, of course, one cannot ascertain this from the data). At least it agrees with other calculations using finite-well wave functions, as does the predicted position of the first maximum.[19] The size of the inelastic form factor at maximum is insensitive to the value chosen for R. Thus we could fit $\langle r_\alpha^2\rangle$ ($R=1.95$) or E_x($R=2.05$) without substantially modifying this prediction.

V. CONCLUSIONS

What can we deduce from the behavior of these form factors? Since the elastic form factor correctly predicts the positions of the diffraction minima, its length scale must be correct. However, the corresponding matter density $\rho_{00}(r) \equiv |\psi_0(r)|^2$ is clearly too smooth and/or insufficiently peaked at the nuclear surface, since its higher Fourier components decrease too rapidly. For, suppose we had $\rho_{00}(r) = \langle R\rangle^{-2}\delta(r-\langle R\rangle)$, which yields the corresponding form factor, $j_0(\frac{3}{4}q\langle R\rangle) \times F_\alpha(q)$: This expression, with $\langle R\rangle$ chosen to reproduce the minima, overestimates the experimental form factor by a factor of ~2 at $q=1.9$ fm^{-1}. The true matter density may be considered to lie somewhere between a surface δ function ($n=\infty$), and $\rho_{00}(r)$ (for $n=4$ or 5).

It is encouraging that the inelastic form factor comes out too large: For the $\alpha + {}^{12}$C intermediate states certainly do not span the 16-nucleon space, and we should therefore expect a certain amount of renormalization to take place. Several authors[19,20] have calculated the inelastic monopole form factor using shell-model wave functions, and have obtained results which underestimate the amplitude by factors of 2 or greater. That is, while the form factors calculated in this paper describes the transition as being strongly collective, the shell-model wave functions are not collective enough. Whether this result implies the necessity of distinct cluster components (with the asymptotic $\alpha + {}^{12}$C form) in the various ^{16}O states is not entirely clear; it *is* clear, however, that the conventional shell model does violence to the physics when it ignores the lowest-lying continuum states in the ^{16}O system.

While one would hesitate to make extravagant claims for the model form factors presented herein since their derivation lacked the foundation of basic theory, it is nevertheless true that they work suprisingly well. With suitable caution as to their normalization, we should be able to apply them and the vertex functions $v_0(q)$, $v_1(q)$ [corresponding to the "single-particle wave functions" (10) and (11)] to various hadronic reactions which emphasize the cluster properties of ^{16}O.

Finally, we should not ignore the potential usefulness of wave functions such as (10) and (11) based on eigenstates of the Eckart[21] potential $n(n+1)/(e^r-1)^2 - \lambda/(e^r-1)$, both as models of cluster vertex functions in transfer reactions, and as variational trial functions in microscopic cluster calculations. Singular short-range potentials of the Eckart type may also prove useful in analyzing composite-particle scattering, where singular short-range potentials of other forms have already proved to be necessary.[22]

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